Electric Dipole Moment Results from Lattice QCD

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July 22, 2018
QCD Lagrangian with $\mathcal{CR}$

- Standard Model QCD Lagrangian has the form:

$$\mathcal{L}_{QCD} = \frac{1}{4} G_{\mu\nu}^{(a)} G^{(a)\mu\nu} + \sum_q \bar{\psi}_q (\gamma^\mu D_\mu - m_q) \psi_q.$$ 

- Induce CP violations in $\mathcal{L}_{QCD}$ by adding $\theta$ term:

$$\mathcal{L}_{\mathcal{CR}} = \mathcal{L}_{QCD} - i\theta q(x)$$

- Where $q(x)$ is defined as:

$$-i\theta q(x) \equiv -i\theta \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [G^{\mu\nu}(x)G^{\rho\sigma}(x)]$$

- For the fermion action and the small $\theta$ expansion, we use:

$$Q_t = \int dt Q(t) = \int d^4x q(x)$$
# Key Systematics and Difficulties in Lattice QCD

<table>
<thead>
<tr>
<th>Continuum limit</th>
<th>( a \rightarrow 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite volume limit</td>
<td>( L_{x,y,z,t} \rightarrow \infty )</td>
</tr>
<tr>
<td>Chiral limit</td>
<td>( (m_\pi)<em>{\text{Lat}} \rightarrow (m</em>\pi)_{\text{Phys}} )</td>
</tr>
<tr>
<td>State isolation</td>
<td>( \sum E_{\vec{p}} \rightarrow E_{\vec{p}}^{(n)} ) (for state ( n ))</td>
</tr>
<tr>
<td>Signal to noise</td>
<td>( \Delta O(t) \xrightarrow{t \text{ large}} \infty )</td>
</tr>
</tbody>
</table>

### Continuum Limit

![Continuum Limit](image)

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[AND You INC.]

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[Wiki]
Calculation Parameters: $m_\pi$ Ensembles.

- Publicly available PACS-CS gauge fields from [www.jldg.org].
- $N_f = 2 + 1$, Iwasaki gauge action, Clover fermion action $C_{sw} = 1.715$
- Vector current renormalisation of 0.7354, from work done in [Aoki,2010]
- Gauge-invariant Gaussian smearing at source and sink ($r_{rms} = 0.431$ fm)
- $a = 0.091$ fm, $32^3 \times 64$ volume, $L = 2.912$ fm
- Different $m_\pi$ used to perform chiral extrapolation.

<table>
<thead>
<tr>
<th>$m_\pi \approx$</th>
<th>411 MeV</th>
<th>570 MeV</th>
<th>701 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>G. Fields</td>
<td>444</td>
<td>400</td>
<td>322</td>
</tr>
<tr>
<td>Meas.</td>
<td>30,094</td>
<td>20,000</td>
<td>17,834</td>
</tr>
</tbody>
</table>
Calculation Parameters: $\alpha$ Ensembles

- Publicly available PACS-CS gauge fields from [www.jldg.org].
- $N_f = 2 + 1$, Iwasaki gauge action, Clover fermion action.
- Vector current renormalisation of 0.7354, from work done in [Aoki,2010]
- Gauge-invariant Gaussian smearing at source and sink.
- Different lattice spacing at $\approx$ equal box size to test discretization effects.

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$16^3 \times 32$</th>
<th>$20^3 \times 40$</th>
<th>$28^3 \times 56$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \approx$</td>
<td>0.1215 fm</td>
<td>0.0980 fm</td>
<td>0.0685 fm</td>
</tr>
<tr>
<td>$\alpha L \approx$</td>
<td>1.944 fm</td>
<td>1.960 fm</td>
<td>1.918 fm</td>
</tr>
<tr>
<td>G. Fields</td>
<td>800</td>
<td>789</td>
<td>650</td>
</tr>
<tr>
<td>Meas.</td>
<td>15,220</td>
<td>15,407</td>
<td>12,867</td>
</tr>
</tbody>
</table>
Mixing Angle Induced by $\theta$ Term.

Gradient Flow is used on $Q_t$ for renormalization.

$$G_2^Q(\gamma_5 \Gamma_4; \vec{0}, t, t_f) \xrightarrow{t \gg 0} \alpha_N^{(1)}$$

Nucleon Mixing Angle $\alpha$ over $t$

- $m_\pi = 0.411 \text{ GeV} \sqrt{8t_f} = 0.62 \text{ fm}$
- $m_\pi = 0.570 \text{ GeV} \sqrt{8t_f} = 0.62 \text{ fm}$
- $m_\pi = 0.701 \text{ GeV} \sqrt{8t_f} = 0.62 \text{ fm}$

Nucleon Mixing Angle $\alpha$ over $t$

- $a = 0.1215 \text{ fm} \ 16^3 \times 32 \sqrt{8t_f} = 0.86 \text{ fm}$
- $a = 0.0980 \text{ fm} \ 20^3 \times 40 \sqrt{8t_f} = 0.68 \text{ fm}$
- $a = 0.0685 \text{ fm} \ 28^3 \times 56 \sqrt{8t_f} = 0.47 \text{ fm}$
Mixing Angle Induced by $\theta$ Term.

Gradient Flow is used on $Q_t$ for renormalization.

\[
\frac{G^Q_t (\gamma_5 \Gamma_4; \vec{0}, t, t_f)}{G_2 (\Gamma_4; \vec{0}, t)} \xrightarrow{t \gg 0} \alpha^{(1)}_N
\]
Improving the Mixing Angle

- Look at convergence as $Q_t$ move away from $NN$ in time.
- Involves symmetrically summing $Q_t$ about creation operator $N$.

![Nucleon Mixing Angle $\alpha$ over $t_s$](image)

- $m_\pi = 0.411$ GeV
- $m_\pi = 0.570$ GeV
- $m_\pi = 0.701$ GeV
- $a = 0.1215$ fm $16^3 \times 32$ $t = 5$
- $a = 0.0980$ fm $20^3 \times 40$ $t = 7$
- $a = 0.0685$ fm $28^3 \times 56$ $t = 14$
The EDM of P/N is related to the CP-odd Form Factor:

\[
\frac{F_{3}^{P/N}(Q^2)}{2M_N} \xrightarrow{small \; Q^2} d_{P/N} + S_{P/N}Q^2 + \mathcal{O}(Q^4)
\]

\(F_{3}(Q^2)\) is contained in the combination of \(G_3\) and \(Q\).

\[
G_{3}^{Q_t}(\Gamma; \vec{p}', t; \vec{q}, \tau; \mathcal{J}_{\mu}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}' \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \text{Tr} \{\Gamma \langle \chi(\vec{x}, t) \mathcal{J}_{\mu}(\vec{y}, \tau) \chi(0) Q_{t}(t_f) \rangle\}
\]

But \(G_{3}^{Q_t}(\Gamma; \vec{p}', t; \vec{q}, \tau; \mathcal{J}_{\mu}) = 0\) for all cases when \(Q^2 = 0\)

To fix this, we fit the resulting \(F_{3}\) at \(Q^2 > 0\) using the form above.
Results for Different $m_\pi$ Ensembles

Neutron $\frac{\bar{F}_3}{2m_N}$ vs $m_\pi$

- $\text{Par}_2 = 59.2(691) \times 10^{-4}$
- $\text{Par}_2 = 60.5(616) \times 10^{-4}$
- $\text{Par}_2 = 18.6(478) \times 10^{-4}$

Proton $\frac{\bar{F}_3}{2m_N}$ vs $m_\pi$

- $\text{Par}_2 = -94(114) \times 10^{-4}$
- $\text{Par}_2 = -32(108) \times 10^{-4}$
- $\text{Par}_2 = 50.0(8620.0) \times 10^{-6}$
Results for Different Lattice Spacing Ensembles

\[ \frac{F_3(Q^2)}{2m_N} \]

**Neutron** \( \frac{F_3}{2m_N} \) vs \( a \)

- \( Par^2 = 81.2(292) \times 10^{-4} \)
  - \( a = 0.1215 \text{ fm} \times 32 \)
- \( Par^2 = 91.8(293) \times 10^{-4} \)
  - \( a = 0.0980 \text{ fm} \times 40 \)
- \( Par^2 = 26.8(168) \times 10^{-4} \)
  - \( a = 0.0685 \text{ fm} \times 56 \)

**Proton** \( \frac{F_3}{2m_N} \) vs \( a \)

- \( Par^2 = -71.6(419) \times 10^{-4} \)
  - \( a = 0.1215 \text{ fm} \times 32 \)
- \( Par^2 = -47.1(384) \times 10^{-4} \)
  - \( a = 0.0980 \text{ fm} \times 40 \)
- \( Par^2 = -34.9(258) \times 10^{-4} \)
  - \( a = 0.0685 \text{ fm} \times 56 \)
Improving the Ratio Functions

- We can study the three-point correlator as $Q(\tau_Q, t_f)$ moves away from $N(t)J(\tau)\bar{N}(0)$.
- Similar to improving the nucleon mixing angle $\alpha$.
Proton $\frac{F_3(Q^2)}{2m_N}$ from Improve Ratio Functions

Proton $\frac{F_3}{2m_N}$ at $m_\pi = 411\text{MeV}$

- $\text{Par}_2 = -32(108) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -57.6(526) \times 10^{-4}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $m_\pi = 570\text{MeV}$

- $\text{Par}_2 = -12.6(397) \times 10^{-4}$
- Standard
- $\text{Par}_2 = 50.0(8620.0) \times 10^{-6}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $m_\pi = 701\text{MeV}$

- $\text{Par}_2 = -69.0(192) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -12.6(397) \times 10^{-4}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $a = 0.0685\text{fm}$

- $\text{Par}_2 = -69.0(192) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -34.9(258) \times 10^{-4}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $a = 0.0980\text{fm}$

- $\text{Par}_2 = -34.9(258) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -71.6(419) \times 10^{-4}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $a = 0.1215\text{fm}$

- $\text{Par}_2 = -71.6(419) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -11(133) \times 10^{-5}$
- R Improved

Proton $\frac{F_3}{2m_N}$ at $a = 0.0685\text{fm}$

- $\text{Par}_2 = -34.9(258) \times 10^{-4}$
- Standard
- $\text{Par}_2 = -71.6(419) \times 10^{-4}$
- R Improved
Continuum Extrapolation Fit Function

- From $\chi$PT, we use the $m_\pi$ dependence of the EDM up to second order:

$$d_{N/P}(m_\pi) = c + d m_\pi^2 + e m_\pi^2 \log(m_\pi^2)$$

[E. Mereghetti, 2010] [K. Ottnad, 2010]

- Combining with a $O(a)$ improved fermion action, the total extrapolation over all ensembles is:

$$d_{N/P}(a, m_\pi) = d m_\pi^2 + e m_\pi^2 \log(m_\pi^2) + f a^2$$

- **NOTE:** $c$ has dropped, as $d_{N/P}$ vanishes in the chiral limit at $a = 0$. 
Continuum Extrapolation of $d_{N/P}$, $m_\pi$ Evaluation

- **Blue** is at $a = 0$
- **Red** is $a = 0.091$ fm, which is the lattice spacing of the $m_\pi$ ensembles
Continuum Extrapolation of $d_{N/P}$, Lattice Spacing Evaluation

Neutron $\frac{F_3(Q^2 = 0)}{2m_N}$, $f(a, m_\pi)$ Fit

- Blue is at $m_\pi = m_\pi^{phys}$
- Red is at $m_\pi = 411$ MeV

Proton $\frac{F_3(Q^2 = 0)}{2m_N}$, $f(a, m_\pi)$ Fit

- Green is at $m_\pi = 701$ MeV
- Purple is at chiral limit.
Comparisons

Results excluding the "α rotation" described in [M. Abramczyk, 2017] for comparison.

Besides purple [F. K. Guo, 2015], the second order term $m_\pi^2 \log(m_\pi^2)$ may be taking effect at $m_\pi \approx 411$ MeV??
Comparisons

Rotation uses $F_2$ and $\alpha$.

Unfair comparison as our "$\alpha$ rotation" uses the correlations in the data.

Dark blue band is our fit result,
Light blue band is fit to only $m_\pi$ ensembles with no vanishing chiral limit.

**Neutron EDM Chiral Plot ($\alpha$ Rotated)**

Rotation uses $F_2$ and $\alpha$.

Unfair comparison as our "$\alpha$ rotation" uses the correlations in the data.

Dark blue band is our fit result,
Light blue band is fit to only $m_\pi$ ensembles with no vanishing chiral limit.
Schiff Moment $S_{N/P}$ and its Continuum Extrapolation

- Schiff moment is linear term in $Q^2$ from:

$$\frac{F_{3}^{P/N}(Q^2)}{2M_N} \xrightarrow{\text{small } Q^2} d_{P/N} + S_{P/N}Q^2 + \mathcal{O}(Q^4) \quad *$$

- Difficult from $\chi$PT to understand $m_\pi$ dependence, so we use:

$$S_{N/P}(a, m_\pi) = cm_\pi + dm_\pi^2 + fa^2$$

[E. Mereghetti,2010]  [K. Ottnad,2010]

- Units have been presented in $[S_{N/P}] = efm^3$. 

Continuum Extrapolation of $S_{N/P}$, $m_\pi$ Evaluation

- **Blue** is at $a = 0$
- **Red** is $a = 0.091$ fm, which is the lattice spacing of the $m_\pi$ ensembles
Continuum Extrapolation of $S_{N/P}$, Lattice Spacing Evaluation

- **Blue** is at $m_\pi = m_\pi^{phys}$
- **Red** is at $m_\pi = 411$ MeV
Conclusion

- This computation utilized the gradient flow, and the small $\theta$ expansion to access the $d_{N/P}$ and $S_{N/P}$.
- We improved our results by understanding how the topological charge and the nucleon fields interact.
- Performing fits over both our $m_\pi$ and lattice spacing ensembles enables us to extrapolate to the continuum.
- Our final extracted value for the neutron and proton EDM are:
  \[ d_N = 0.0029(21) \ \text{e fm} , \quad d_P = 0.0007(27) \ \text{e fm} \]

- Our final extracted value for the neutron and proton Schiff moments are:
  \[ S_N = -0.00058(49) \ \text{e fm}^3 , \quad S_P = -0.0004(11) \ \text{e fm}^3 \]
Acknowledgements

- This work was supported in part by Michigan State University through computational resources, provided by the Institute for Cyber-Enabled Research.