Electric Dipole Moment Results from Lattice QCD

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QCD Lagrangian with $\not{C}\not{P}$

- Standard Model QCD Lagrangian has the form:

$$\mathcal{L}_{QCD} = \frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} + \sum_q \overline{\psi}_q \left(\gamma^\mu D_\mu - m_q\right) \psi_q.$$

- Induce CP violations in $\mathcal{L}_{QCD}$ by adding θ term:

$$\mathcal{L}_{\not{C}\not{P}} = \mathcal{L}_{QCD} - i\theta q(x)$$

- Where $q(x)$ is defined as:

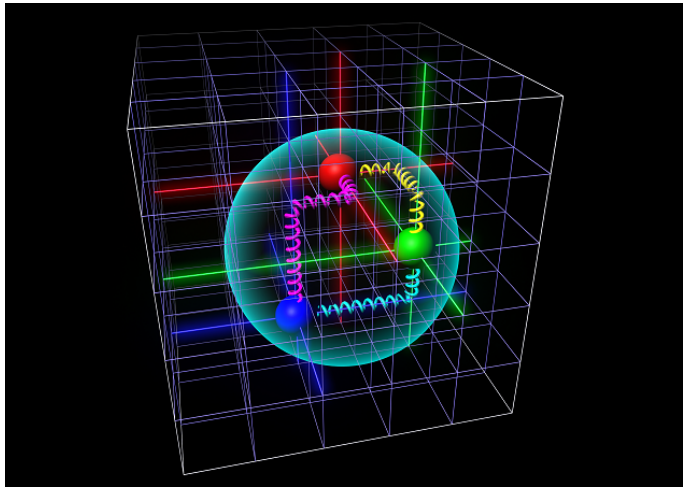
$$-i\theta q(x) \equiv -i\theta \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[G^{\mu\nu}(x) G^{\rho\sigma}(x)\right]$$

- For the fermion action and the small θ expansion, we use:

$$Q_t = \int dt Q(t) = \int d^4x q(x)$$

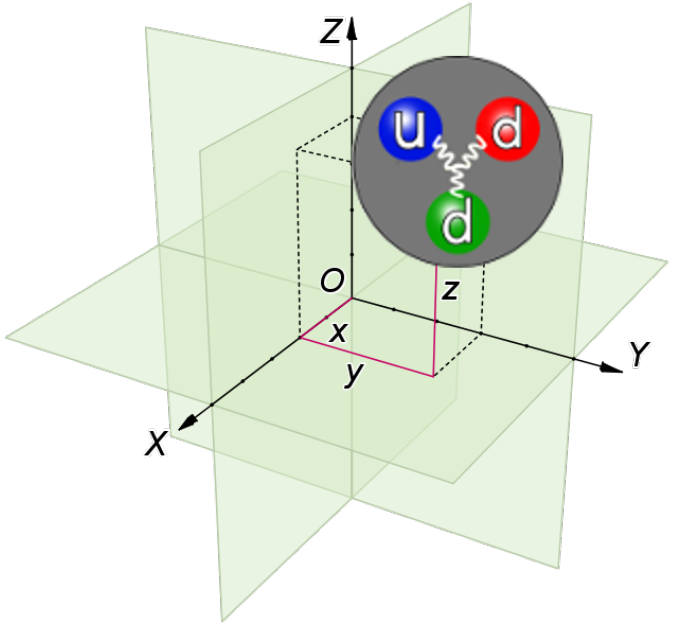
Key Systematics and Difficulties in Lattice QCD

Continuum limit	$a \to 0$
Infinite volume limit	$L_{x,y,z,t} \to \infty$
Chiral limit	$(m_\pi)_{Lat} \to (m_\pi)_{Phys}$
State isolation	$\sum_{E_{\vec{p}}} \to E_{\vec{p}}^{(n)}$ (for state n)
Signal to noise	$\Delta O(t) \xrightarrow{t\ large} \infty$



[AND You INC.]

Continuum Limit



[Wiki]

Calculation Parameters: m_π Ensembles.

- Publicly available PACS-CS gauge fields from [www.jldg.org].
- $N_f = 2 + 1$, Iwasaki gauge action, Clover fermion action $C_{sw} = 1.715$
- Vector current renormalisation of 0.7354, from work done in [Aoki,2010]
- Gauge-invariant Gaussian smearing at source and sink ($r_{rms} = 0.431$ fm)
- $a = 0.091$ fm , $32^3 \times 64$ volume, $L = 2.912$ fm
- Different m_π used to perform chiral extrapolation.

$m_\pi \approx$	411 MeV	570 MeV	701 MeV
G. Fields	444	400	322
Meas.	30,094	20,000	17,834

Calculation Parameters: a Ensembles

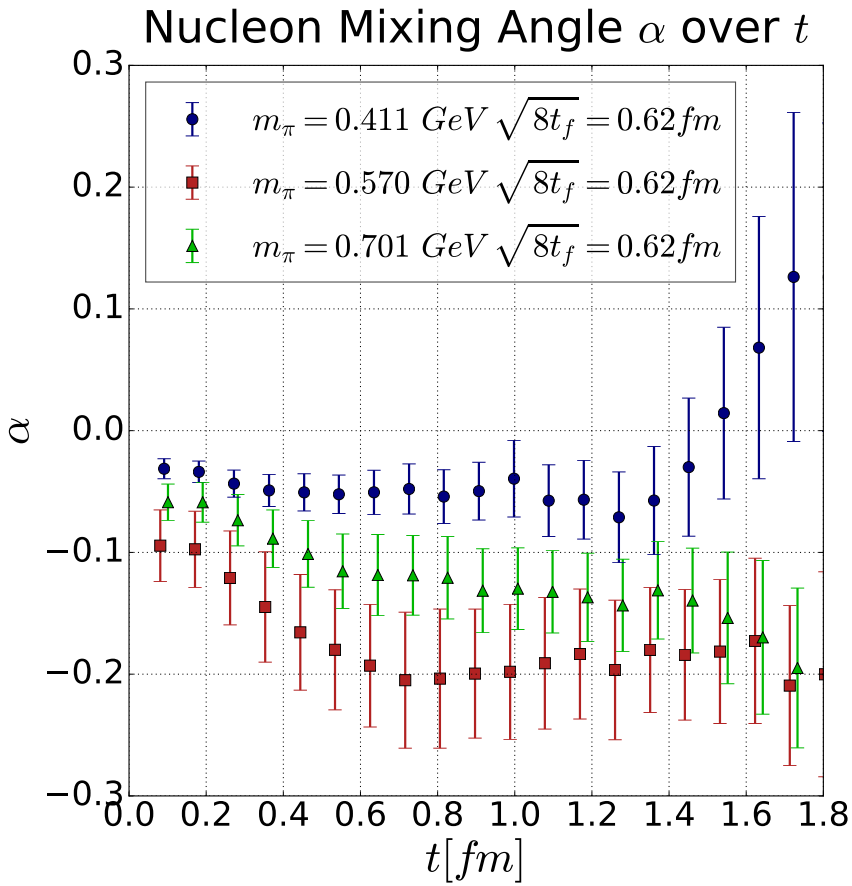
- Publicly available PACS-CS gauge fields from [www.jldg.org].
- $N_f = 2 + 1$, Iwasaki gauge action, Clover fermion action.
- Vector current renormalisation of 0.7354, from work done in [Aoki,2010]
- Gauge-invariant Gaussian smearing at source and sink.
- Different lattice spacing at $\approx$ equal box size to test discritization effects.

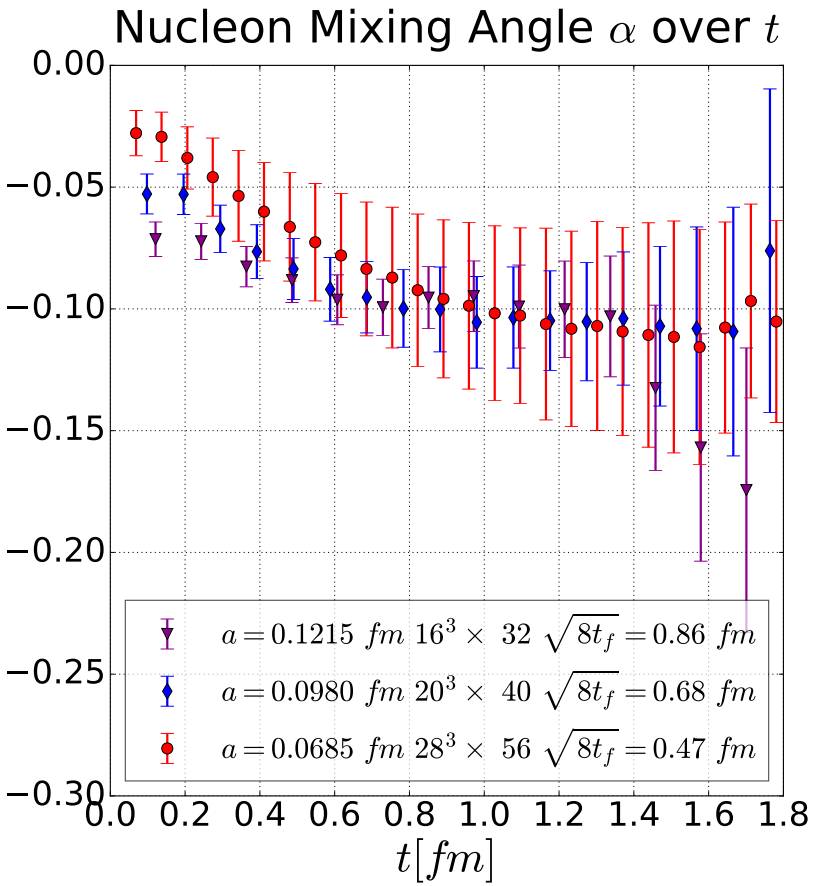
$L^3 \times T$	$16^3 \times 32$	$20^3 \times 40$	$28^3 \times 56$
$a \approx$	0.1215 fm	0.0980 fm	0.0685 fm
$aL \approx$	1.944 fm	1.960 fm	1.918 fm
G. Fields	800	789	650
Meas.	15,220	15,407	12,867

Mixing Angle Induced by θ Term.

Gradient Flow is used on Q_t for renormalization.

$$\frac{G_2^{Q_t}(\gamma_5\Gamma_4;\vec{0},t,t_f)}{G_2(\Gamma_4;\vec{0},t)} \xrightarrow{t\gg 0} \alpha_N^{(1)}$$





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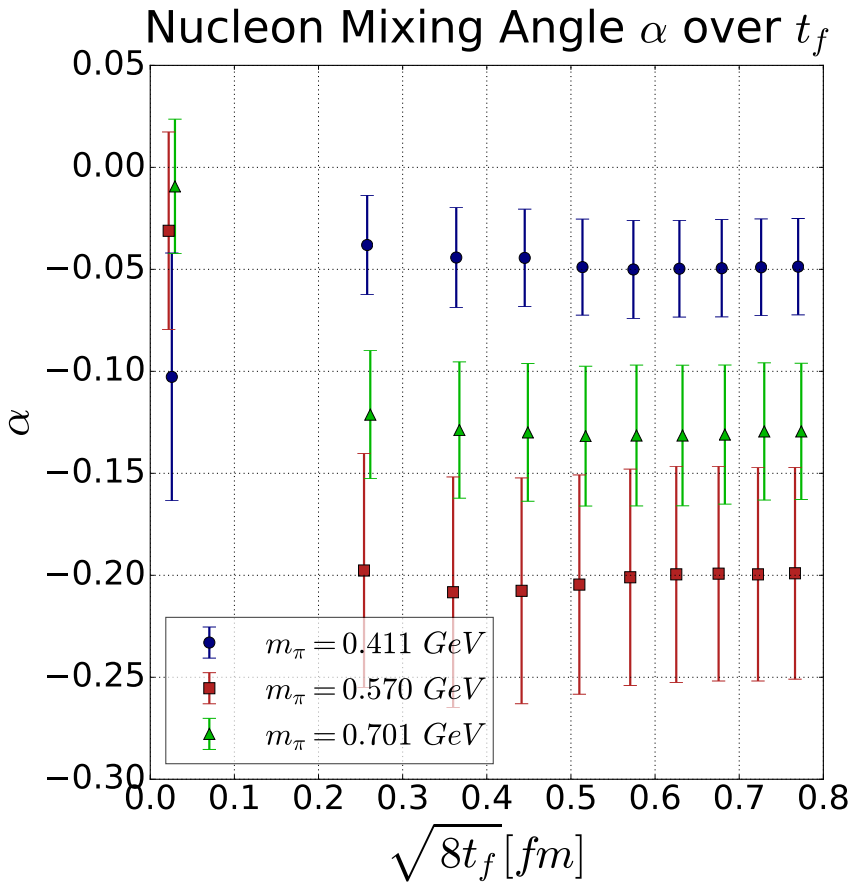
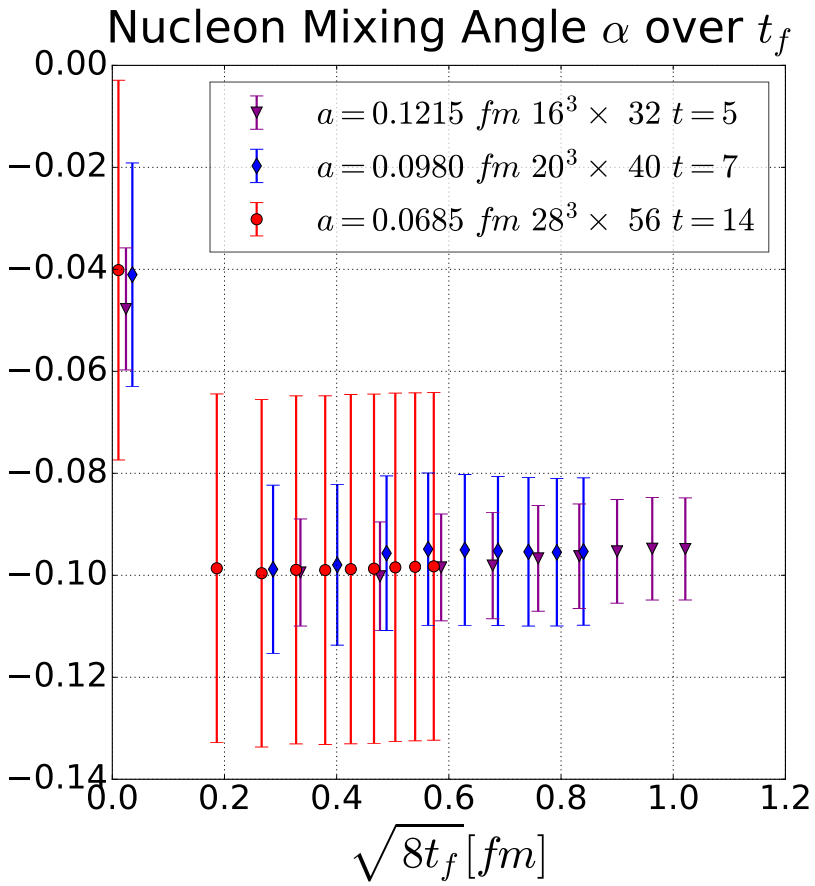
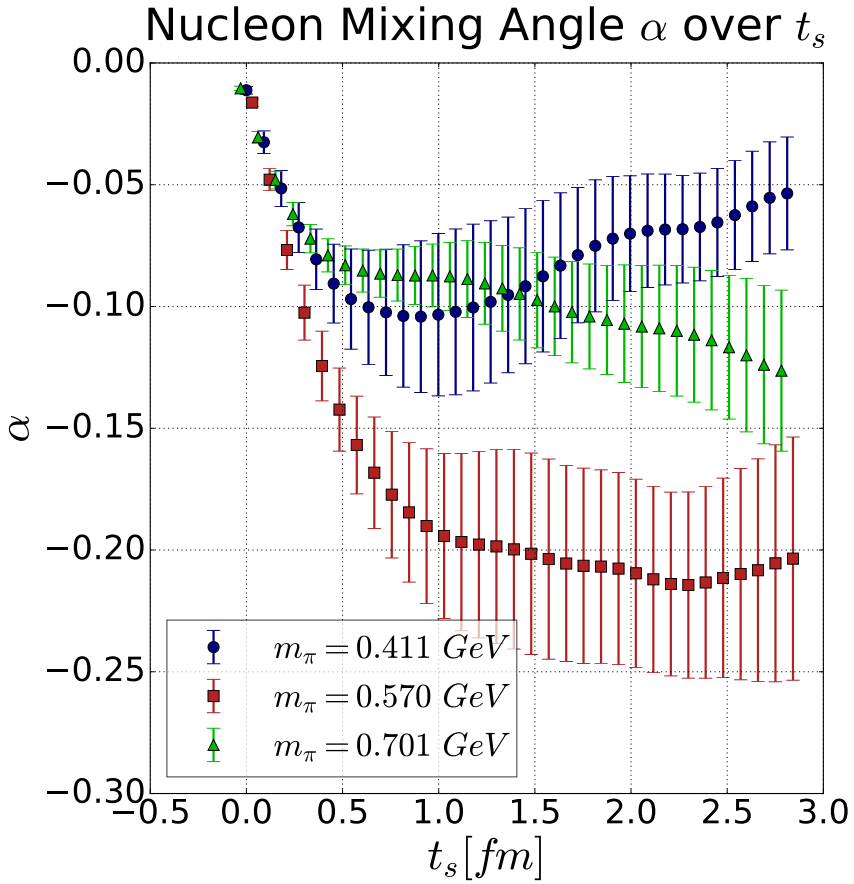
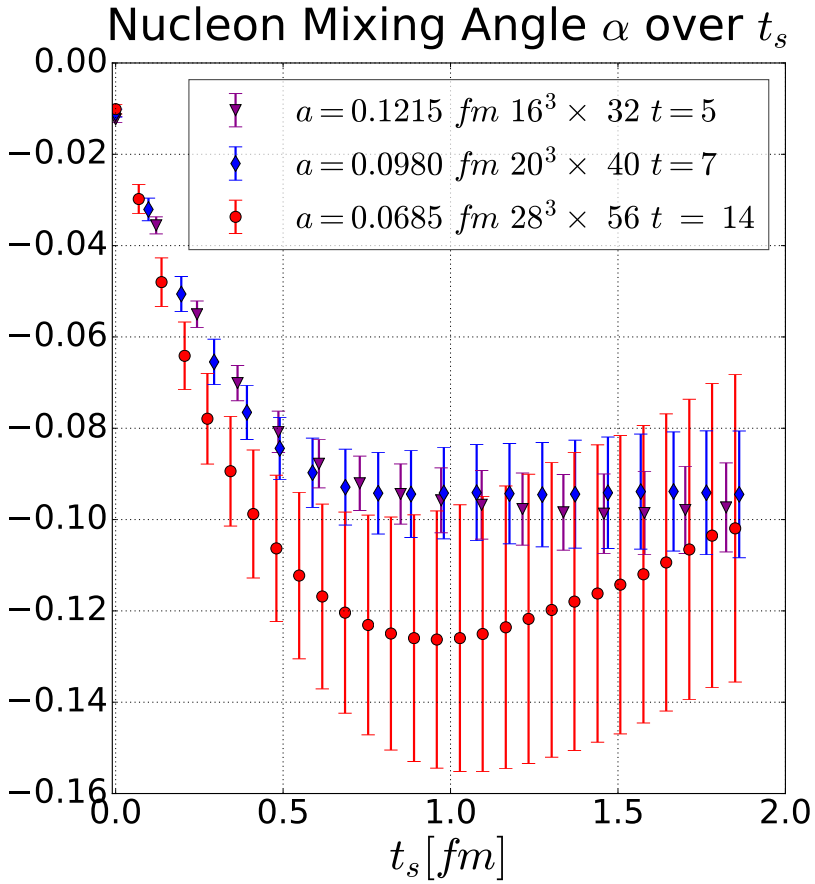




Improving the Mixing Angle

- Look at convergence as Q_t move away from NN in time.
- Involves symmetrically summing Q_t about creation operator N.





Extracting the EDM from Lattice QCD.

- The EDM of P/N is related to the CP-odd Form Factor:

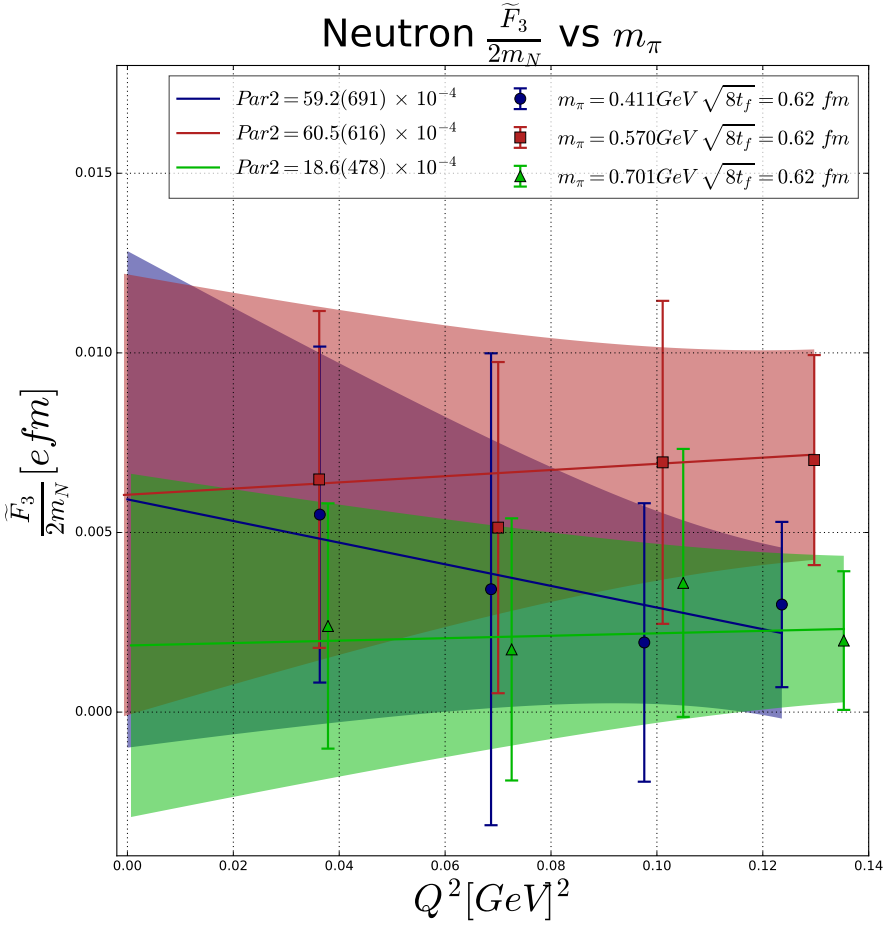
$$\frac{F_3^{P/N}(Q^2)}{2M_N} \xrightarrow{small\ Q^2} d_{P/N} + S_{P/N}Q^2 + \mathcal{O}(Q^4) \quad *$$

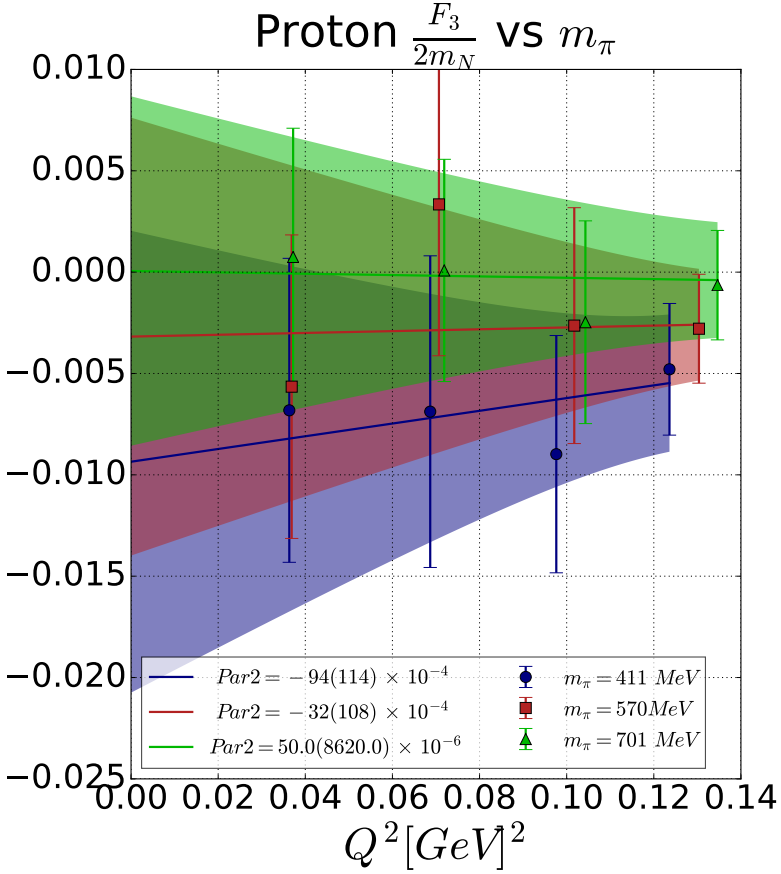
- $F_3(Q^2)$ is contained in the combination of G_3 and Q.

$$G_3^{Q_t}(\Gamma;\vec{p}',t;\vec{q},\tau;\mathcal{J}_\mu) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\Gamma\left\langle \chi(\vec{x},t)\mathcal{J}_\mu(\vec{y},\tau)\overline{\chi}(0)Q_t(t_f)\right\rangle\right\}$$

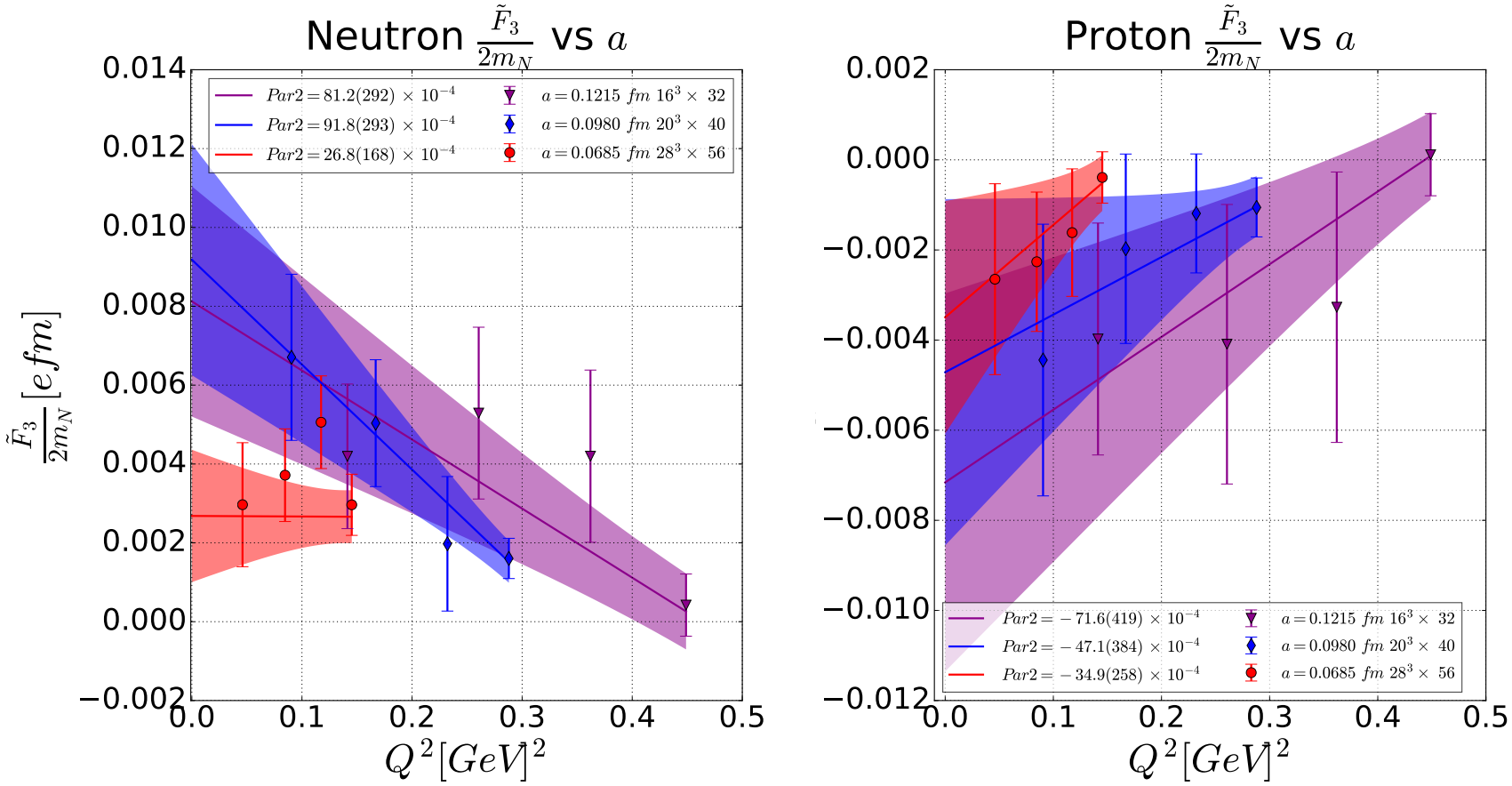
- But $G_3^{Q_t}(\Gamma;\vec{p}',t;\vec{q},\tau;\mathcal{J}_\mu) = 0$ for all cases when $Q^2 = 0$
- To fix this, we fit the resulting F_3 at $Q^2 > 0$ using the form above *.

$\frac{F_3(Q^2)}{2m_N}$ Results for Different m_π Ensembles



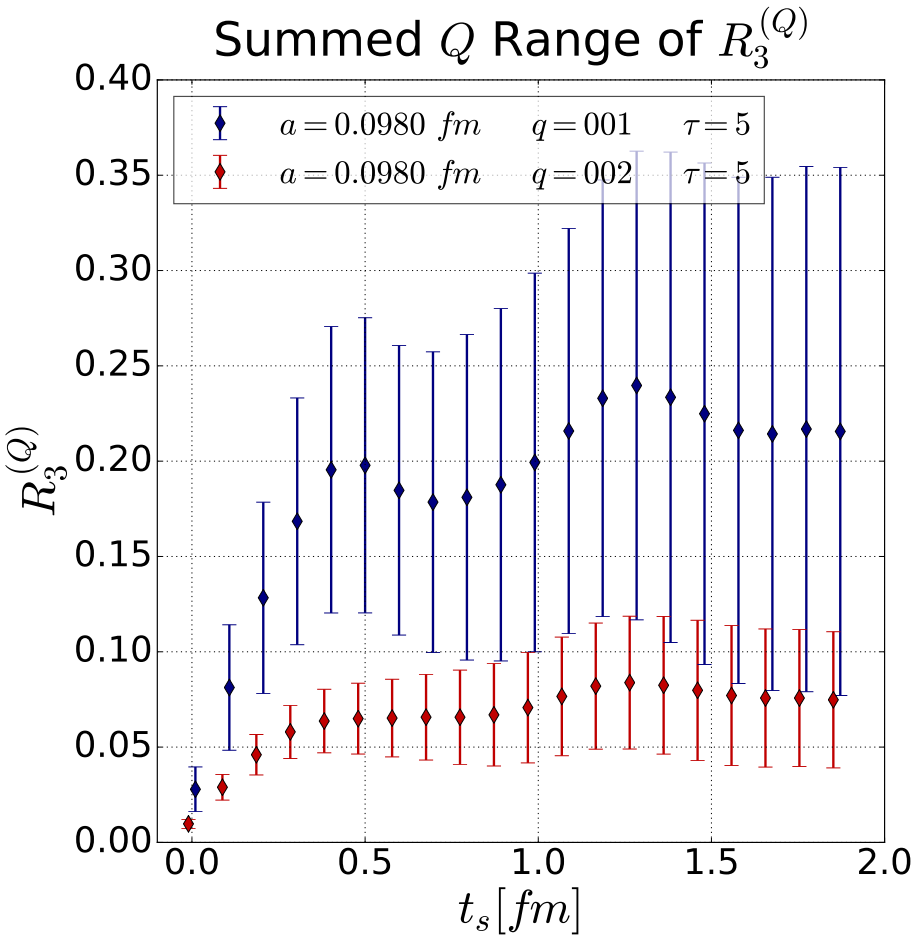


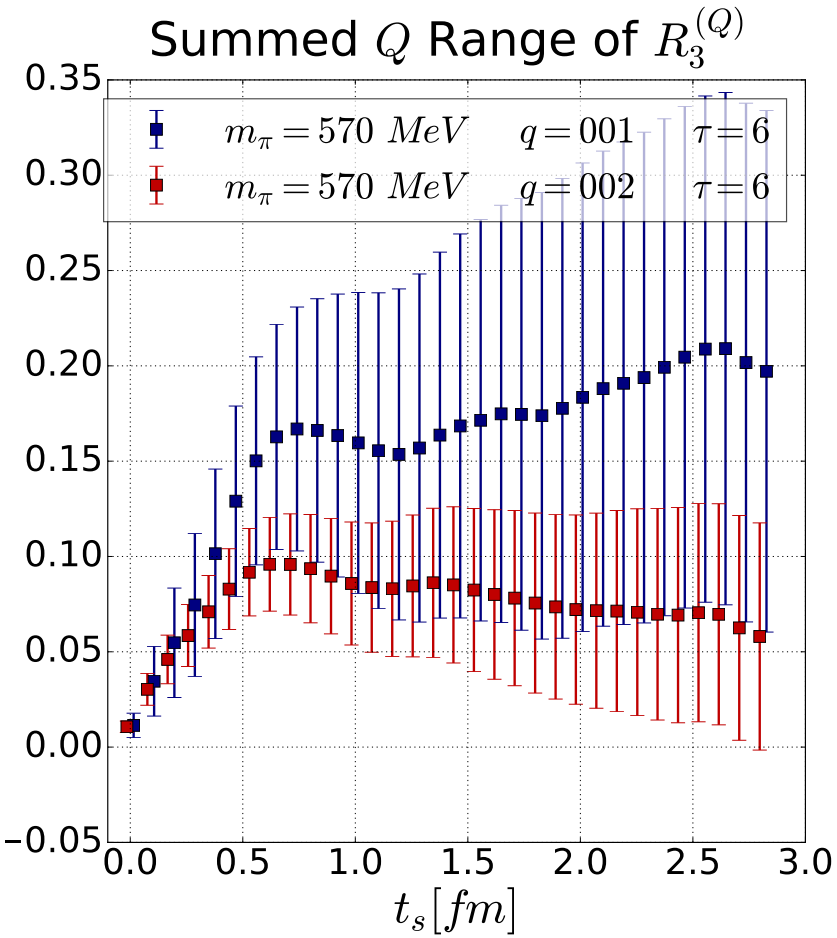
$\frac{F_3(Q^2)}{2m_N}$ Results for Different Lattice Spacing Ensembles



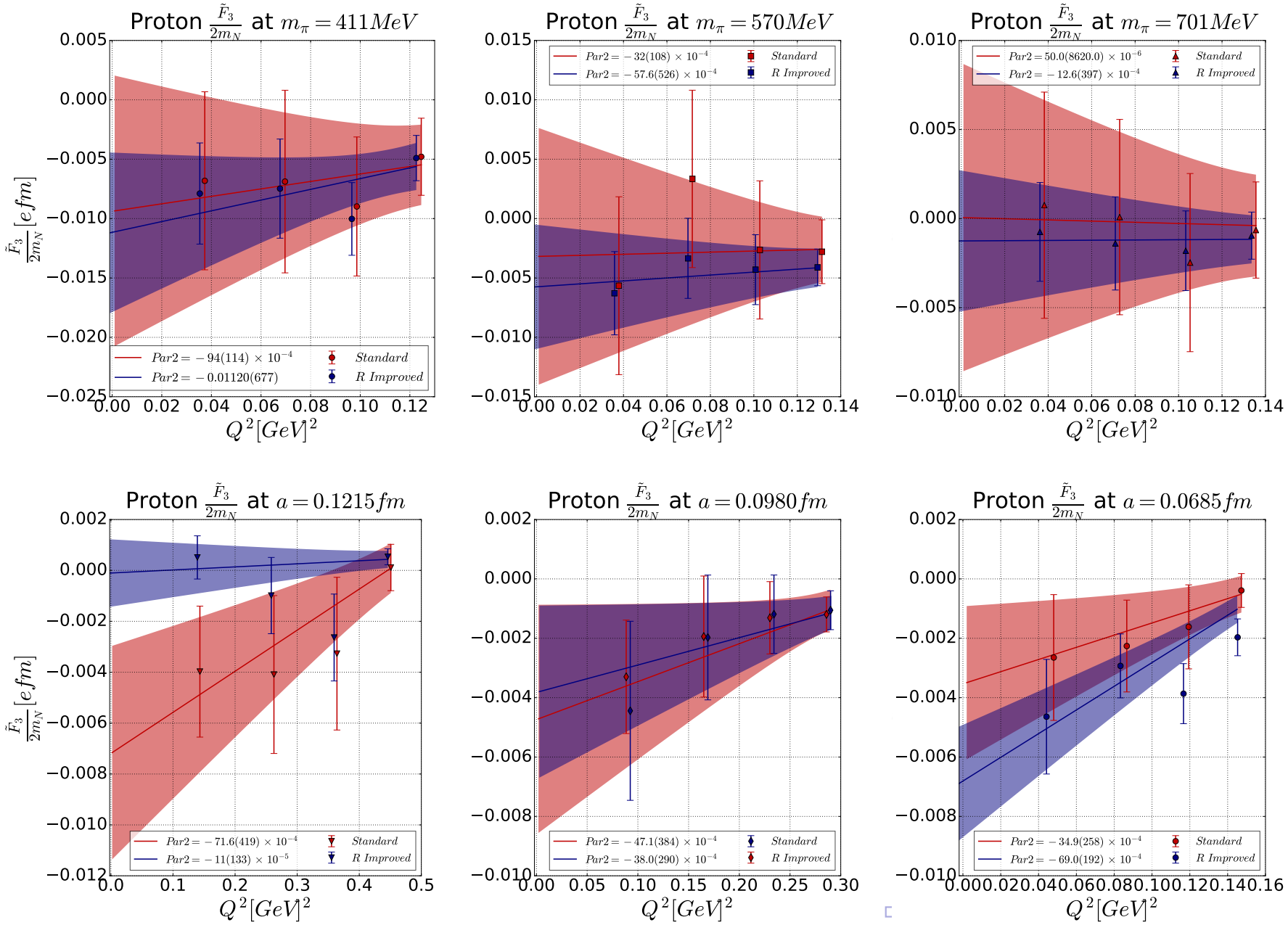
Improving the Ratio Functions

- We can study the three-point correlator as $Q(\tau_Q, t_f)$ moves away from $N(t)J(\tau)\bar{N}(0)$.
- Similar to improving the nucleon mixing angle α.





Proton $\frac{F_3(Q^2)}{2m_N}$ from Improve Ratio Functions



Continuum Extrapolation Fit Function

- From χPT, we use the m_π dependence of the EDM up to second order:

$$d_{N/P}(m_\pi) = c + d\, m_\pi^2 + e\, m_\pi^2 log(m_\pi^2)$$

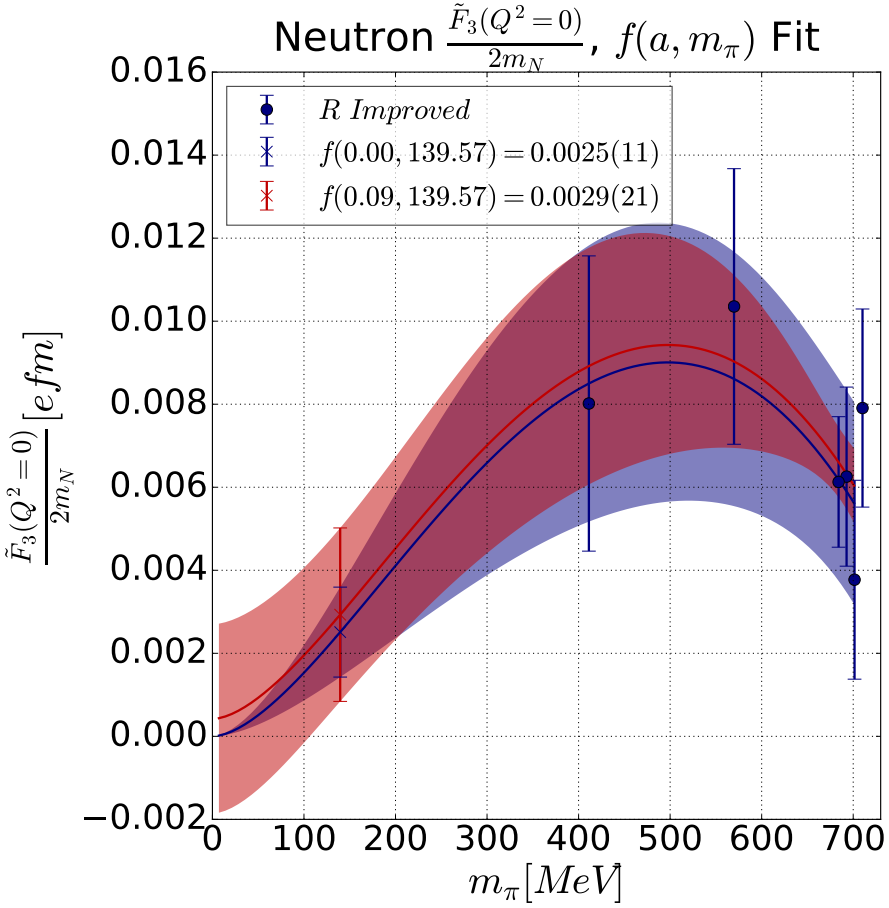
[E. Mereghetti,2010] [K. Ottnad,2010]

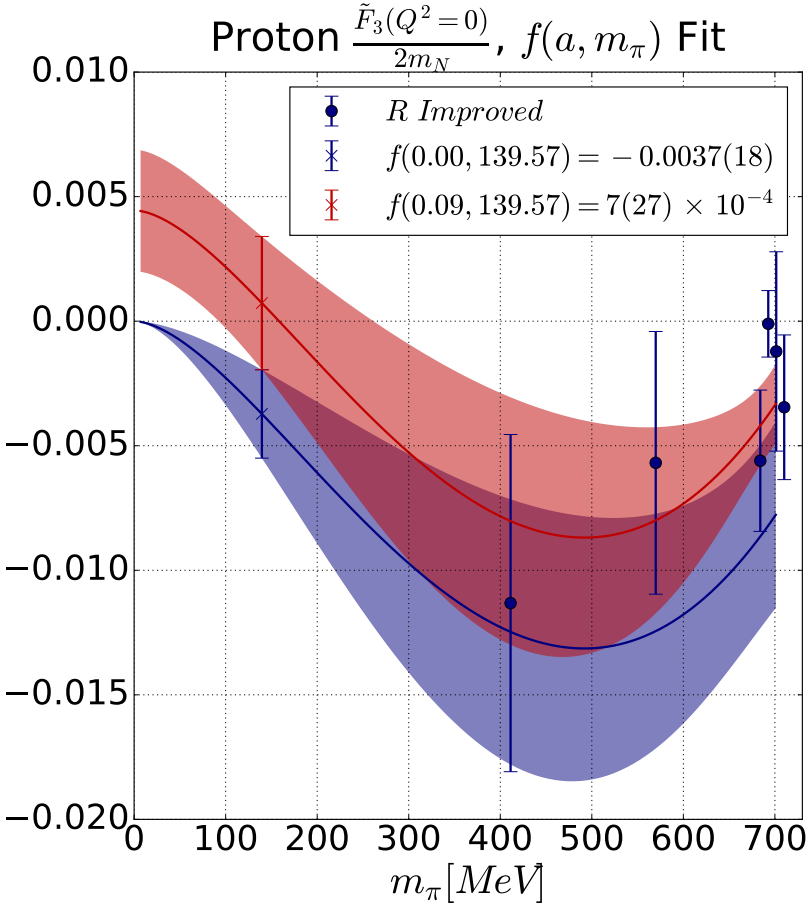
- Combining with a $\mathcal{O}(a)$ improved fermion action, the total extrapolation over all ensembles is:

$$d_{N/P}(a, m_\pi) = d\, m_\pi^2 + e\, m_\pi^2 log(m_\pi^2) + f\, a^2$$

- NOTE: c has dropped, as $d_{N/P}$ vanishes in the chiral limit at $a = 0$.

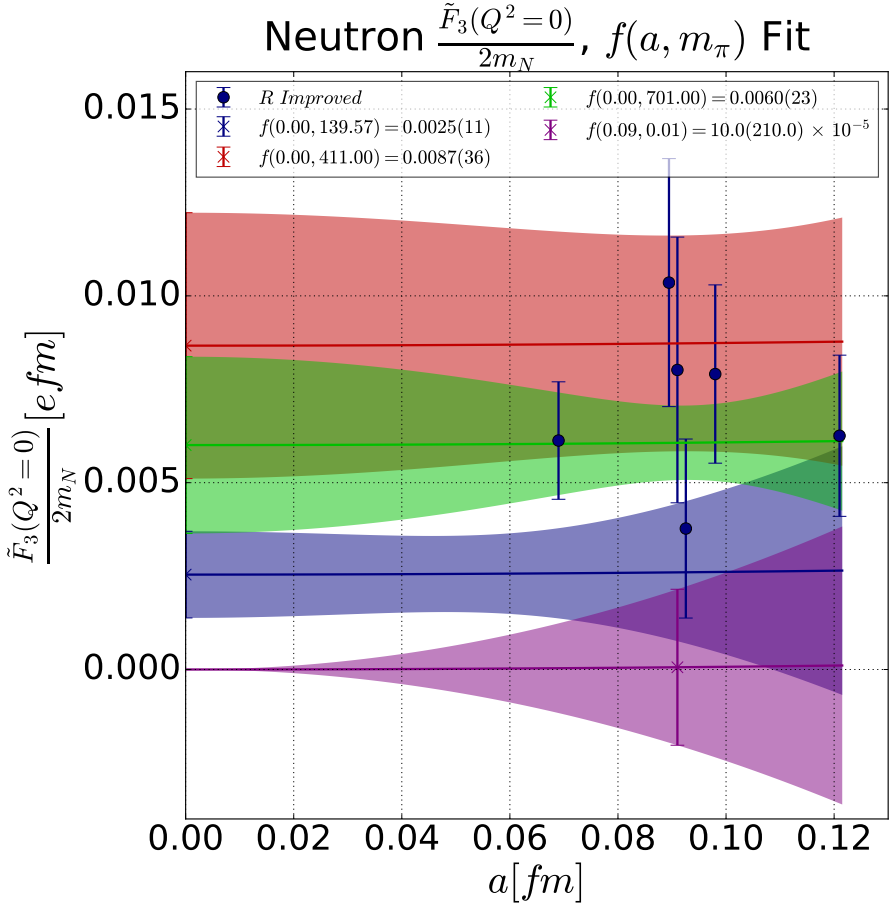
Continuum Extrapolation of $d_{N/P}$, m_π Evaluation

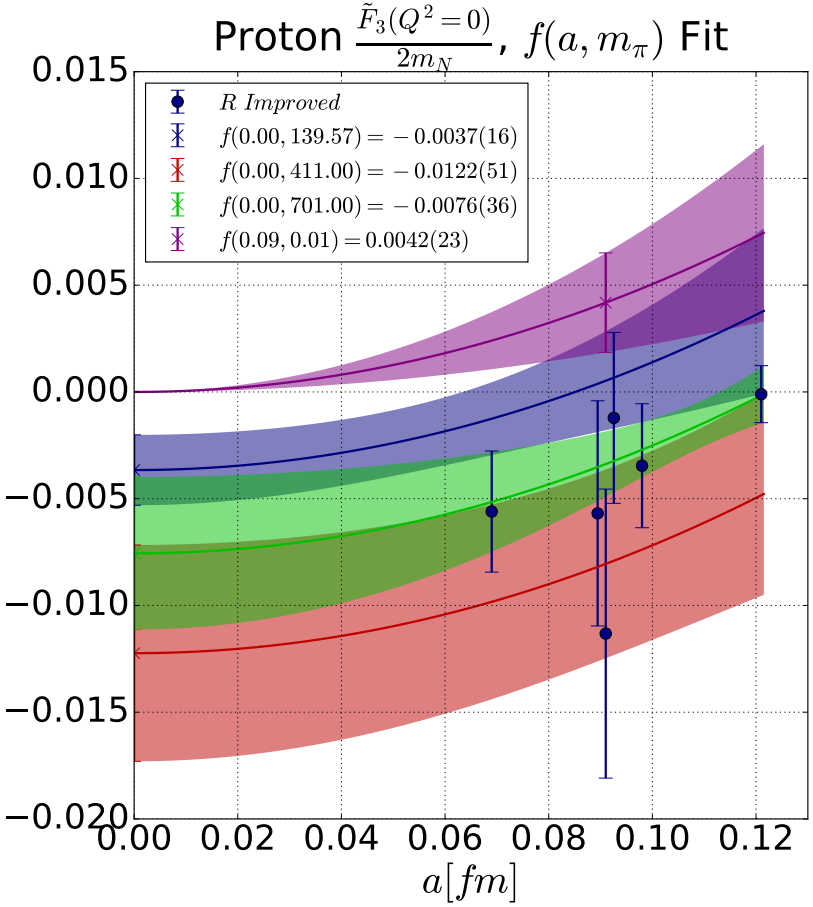




- Blue is at $a = 0$
- Red is $a = 0.091$ fm, which is the lattice spacing of the m_π ensembles

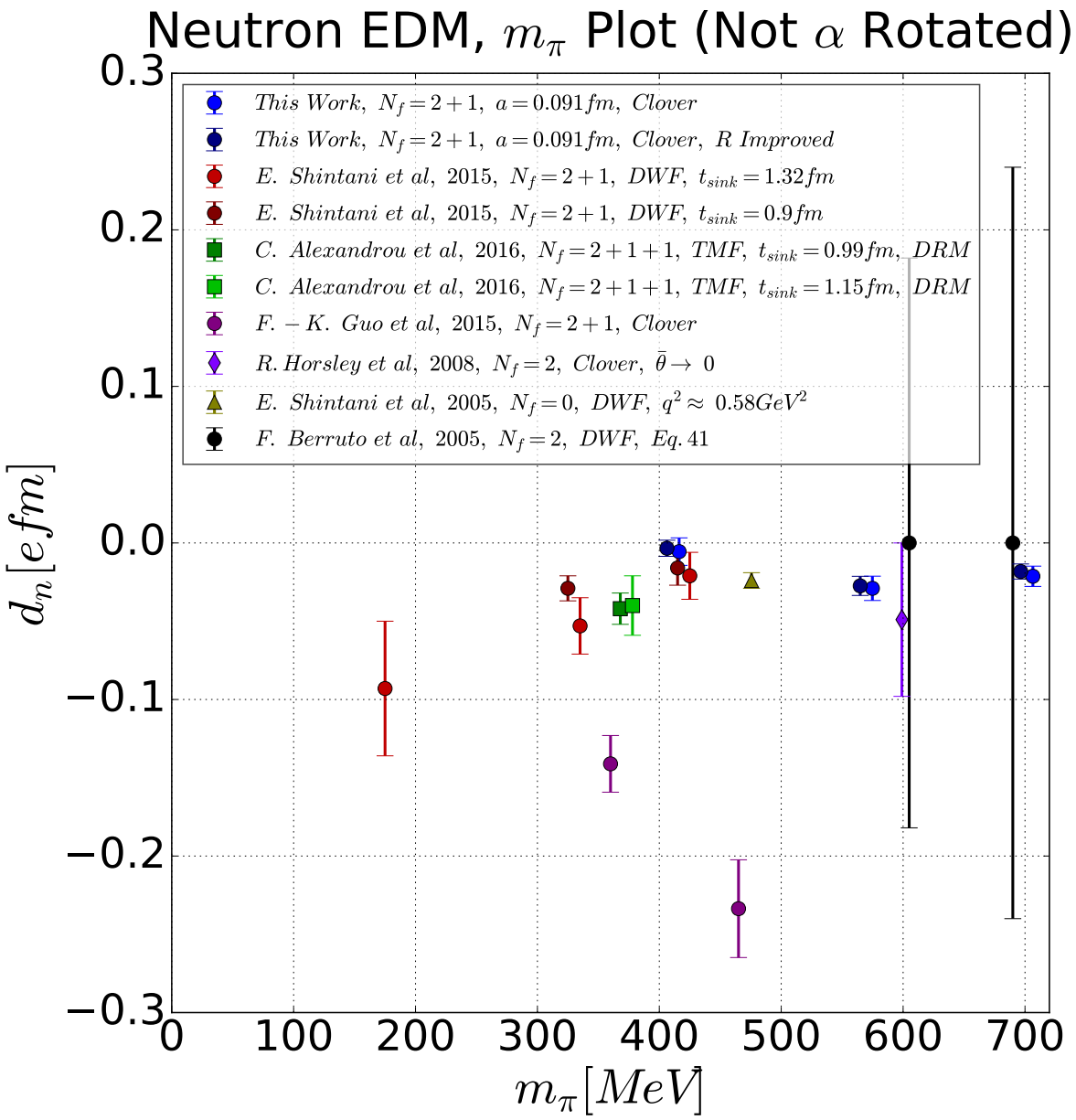
Continuum Extrapolation of $d_{N/P}$, Lattice Spacing Evaluation





- Blue is at $m_\pi = m_\pi^{phys}$
- Red is at $m_\pi = 411$ MeV
- Green is at $m_\pi = 701$ MeV
- Purple is at chiral limit.

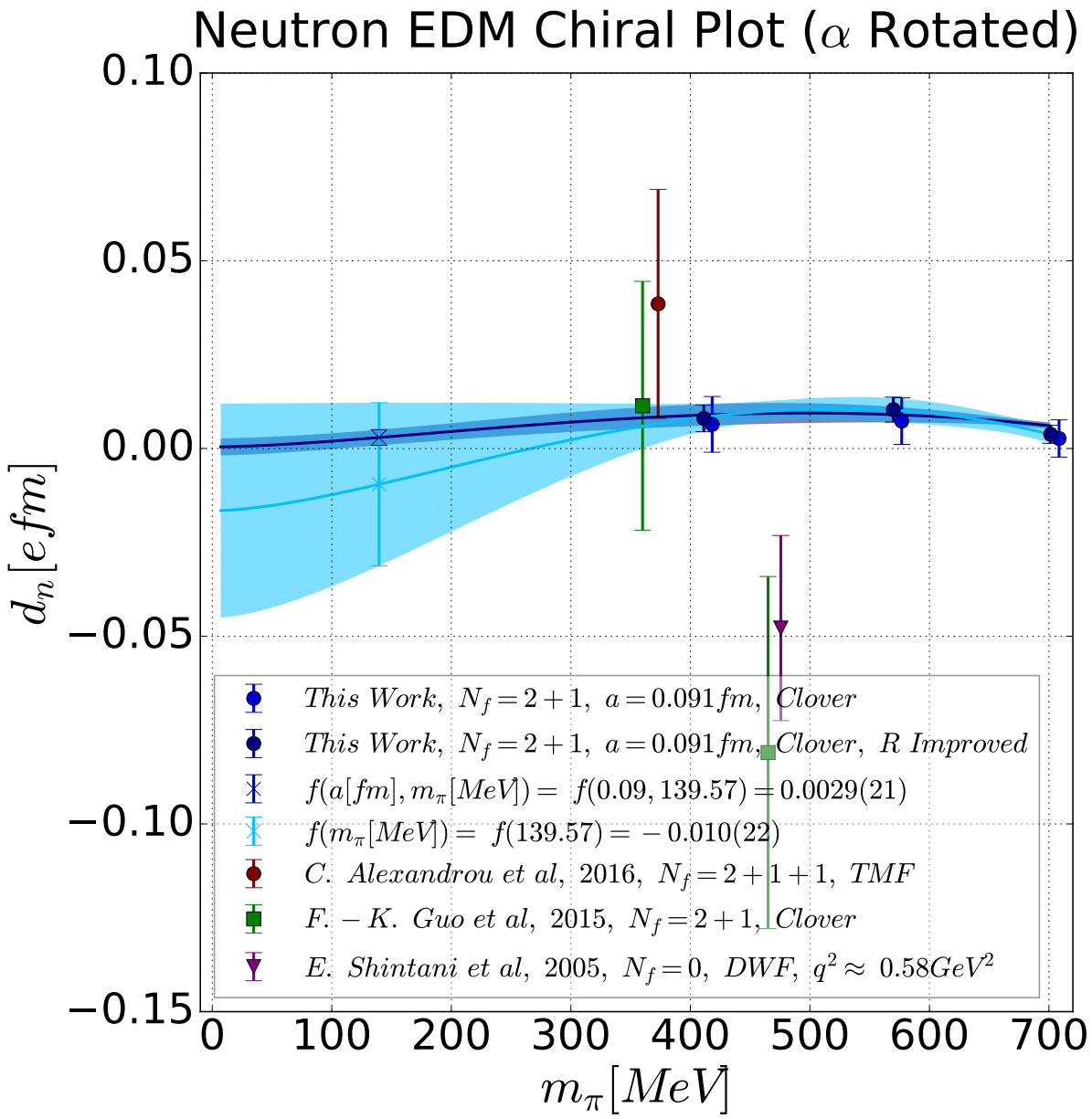
Comparisons



Results excluding the "α rotation" described in [M. Abramczyk,2017] for comparison.

Besides purple [F. K. Guo,2015], the second order term $m_\pi^2 log(m_\pi^2)$ may be taking effect at $m_\pi \approx 411$ MeV???

Comparisons



Rotation uses F_2 and α.

Unfair comparison as our "α rotation" uses the correlations in the data.

Dark blue band is our fit result,
Light blue band is fit to only m_π ensembles with no vanishing chiral limit.

Schiff Moment $S_{N/P}$ and its Continuum Extrapolation

- Schiff moment is linear term in Q^2 from:

$$\frac{F_3^{P/N}(Q^2)}{2M_N} \xrightarrow{small\ Q^2} d_{P/N} + S_{P/N}Q^2 + \mathcal{O}(Q^4) \ *$$

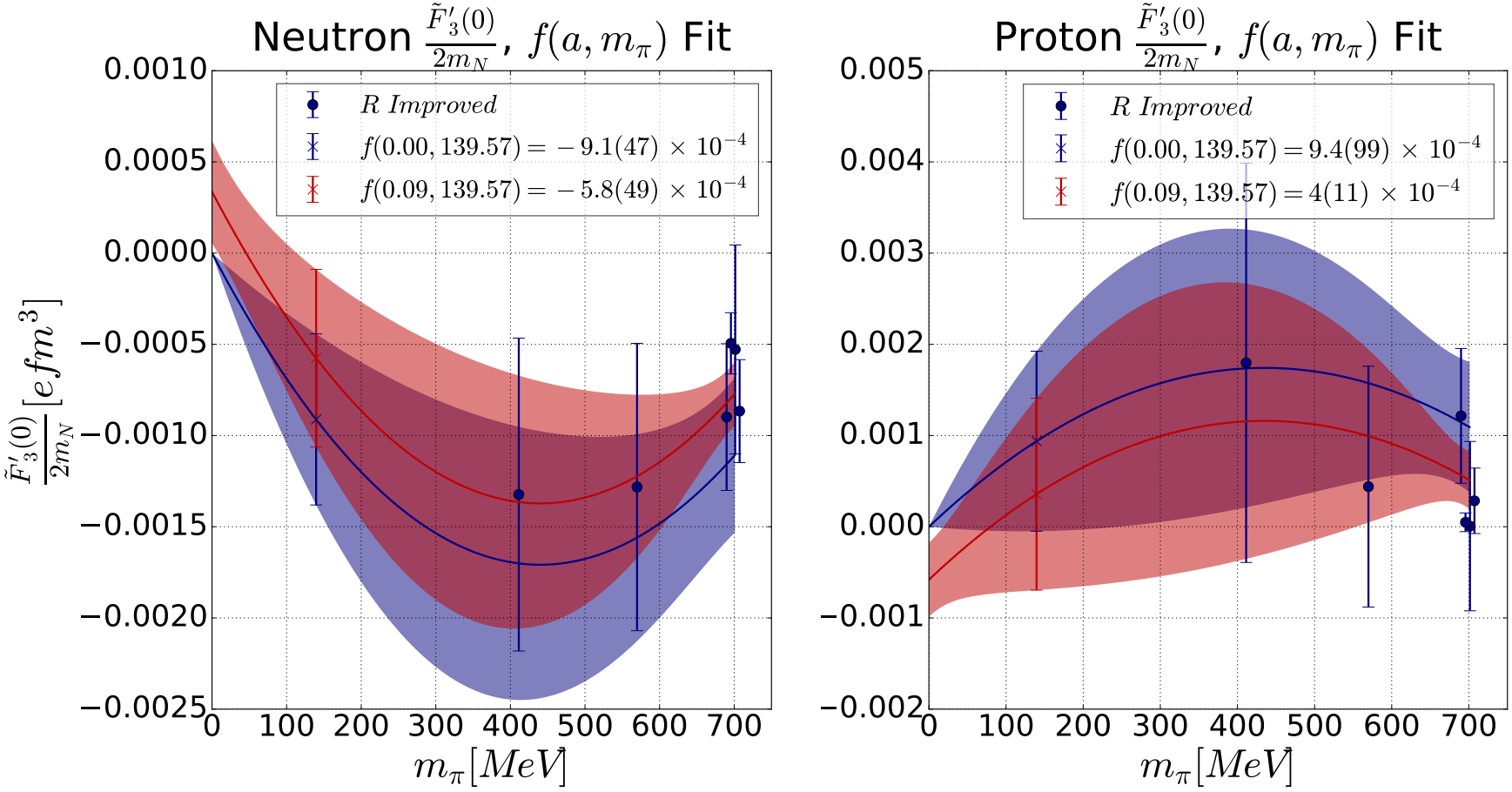
- Difficult from χPT to understand m_π dependence, so we use:

$$S_{N/P}(a, m_\pi) = c\, m_\pi + d\, m_\pi^2 + f\, a^2$$

[E. Mereghetti,2010] [K. Ottnad,2010]

- Units have been presented in $[S_{N/P}] = efm^3$.

Continuum Extrapolation of $S_{N/P}$, m_π Evaluation



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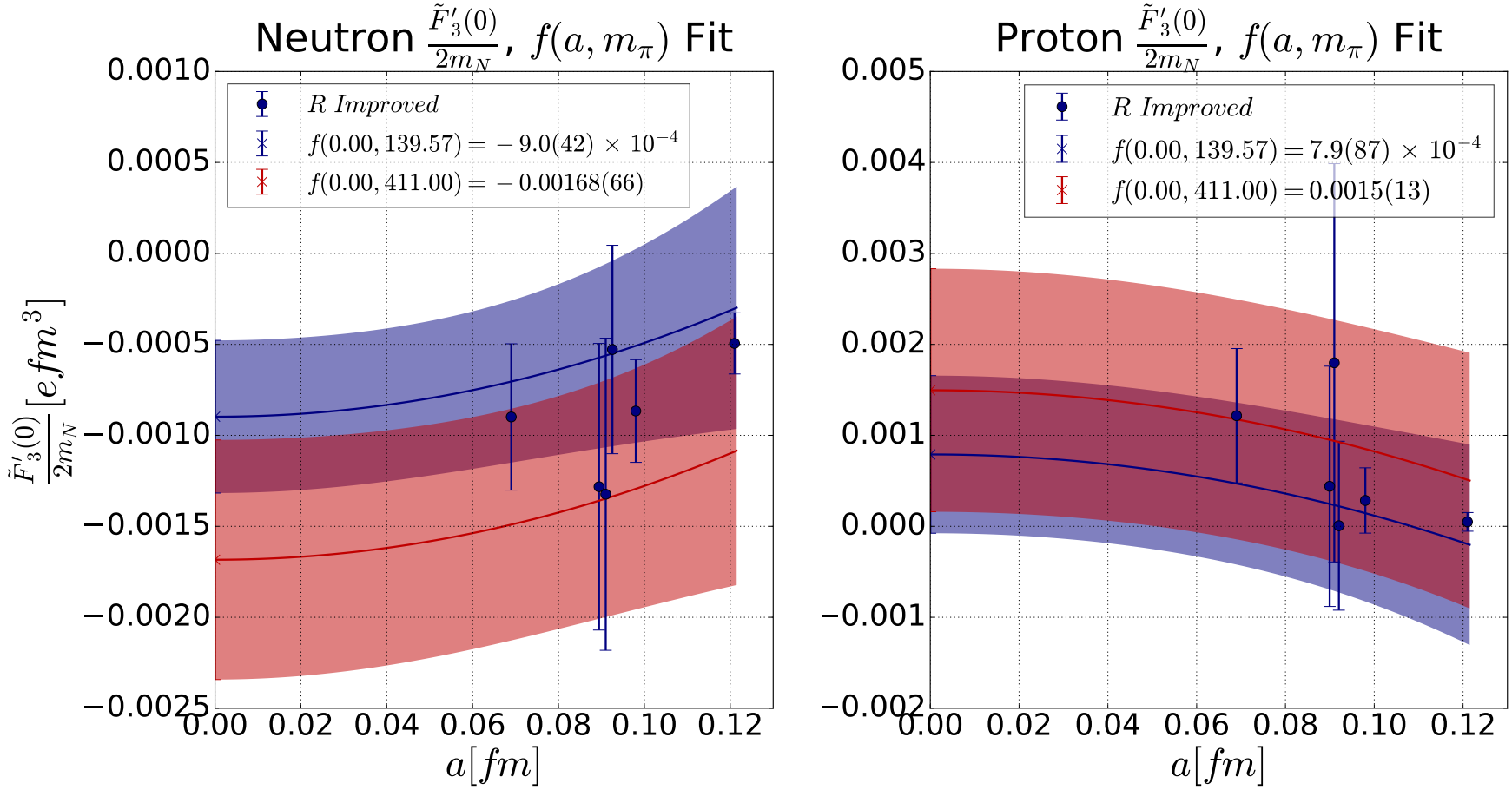
Continuum Extrapolation of $S_{N/P}$, Lattice Spacing Evaluation



- Blue is at $m_\pi = m_\pi^{phys}$
- Red is at $m_\pi = 411$ MeV

Conclusion

- This computation utilized the gradient flow, and the small θ expansion to access the $d_{N/P}$ and $S_{N/P}$.
- We improved our results by understanding how the topological charge and the nucleon fields interact.
- Performing fits over both our m_π and lattice spacing ensembles enables us to extrapolate to the continuum.
- Our final extracted value for the neutron and proton EDM are:

$$d_N = 0.0029(21)\,\theta\,efm \qquad , \qquad d_P = 0.0007(27)\,\theta\,efm$$

- Our final extracted value for the neutron and proton Schiff moments are:

$$S_N = -0.00058(49)\,\theta\,efm^3 \qquad , \qquad S_P = -0.0004(11)\,\theta\,efm^3$$

Acknowledgements

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- Jülich Supercomputing Centre. (2015). JUQUEEN: IBM Blue Gene/Q Supercomputer System at the Jlich Supercomputing Centre. Journal of large-scale research facilities, 1, A1. http://dx.doi.org/10.17815/jlsrf-1-18