Heavy quark scaling of $B \to \pi \ell \nu$ form factors with Möbius domain wall fermions

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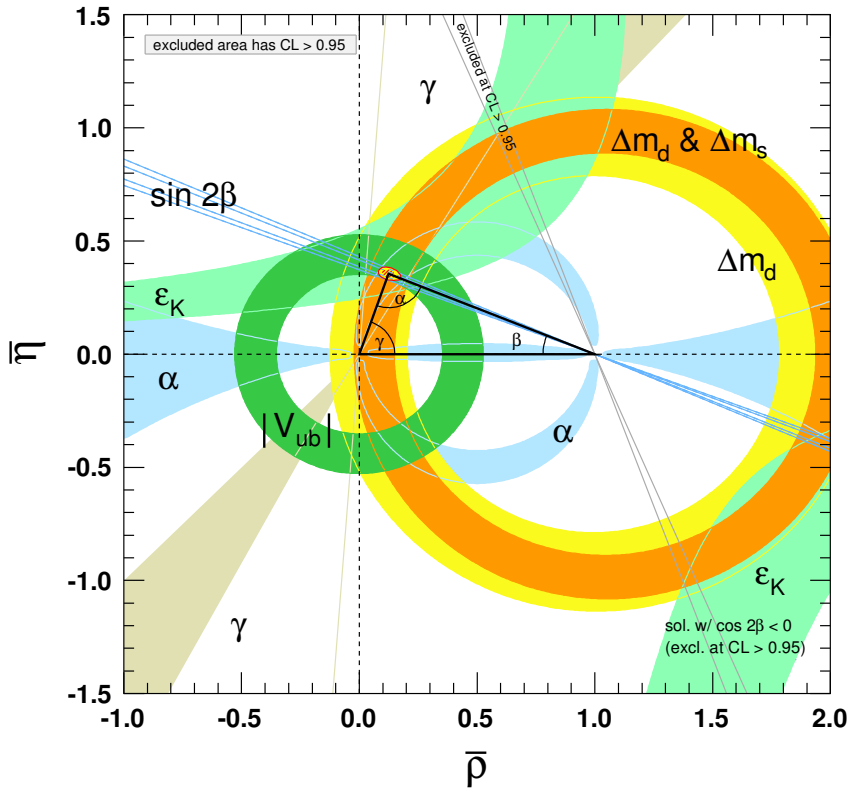
w/ Shoji Hashimoto, Takashi Kaneko
for the JLQCD Collaboration

July 26 2018

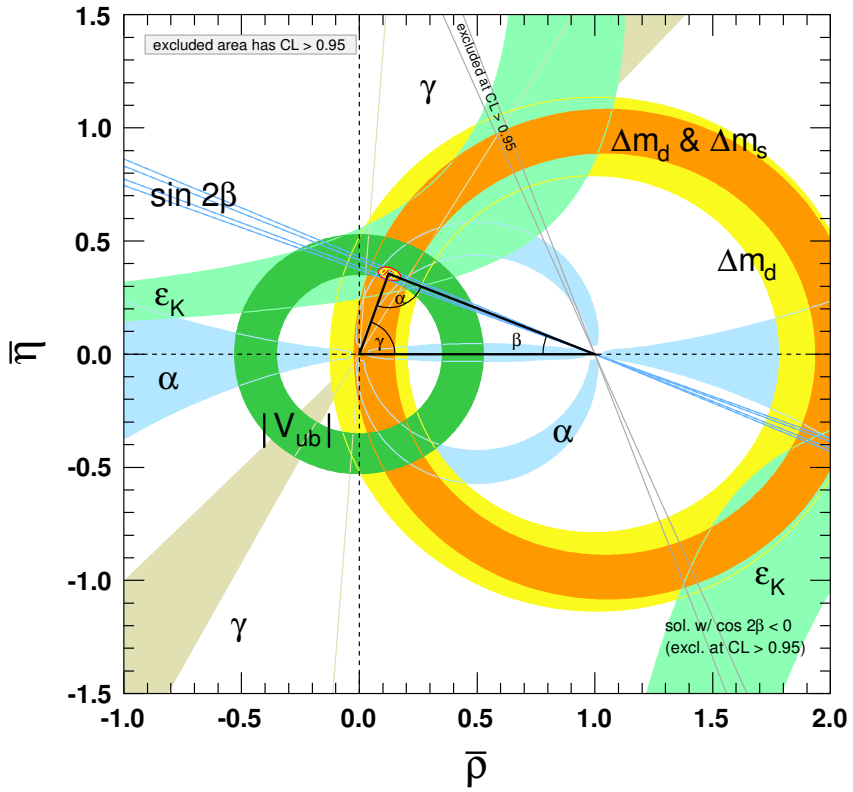
East Lansing, MI, USA

Motivation



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \bar{B}_s \rangle & \langle B_s | \bar{B}_s \rangle & \end{pmatrix}$$

Motivation



This talk

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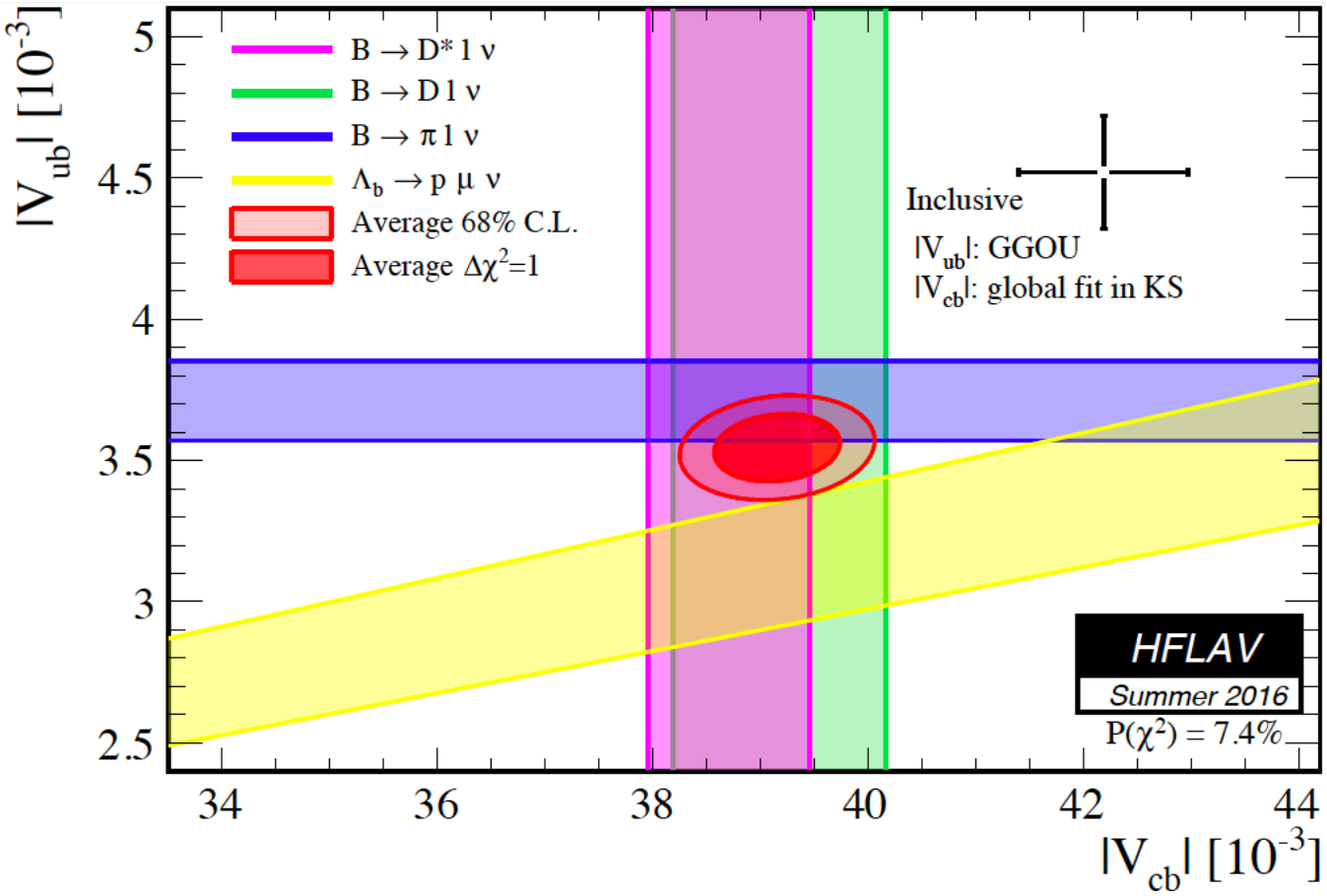
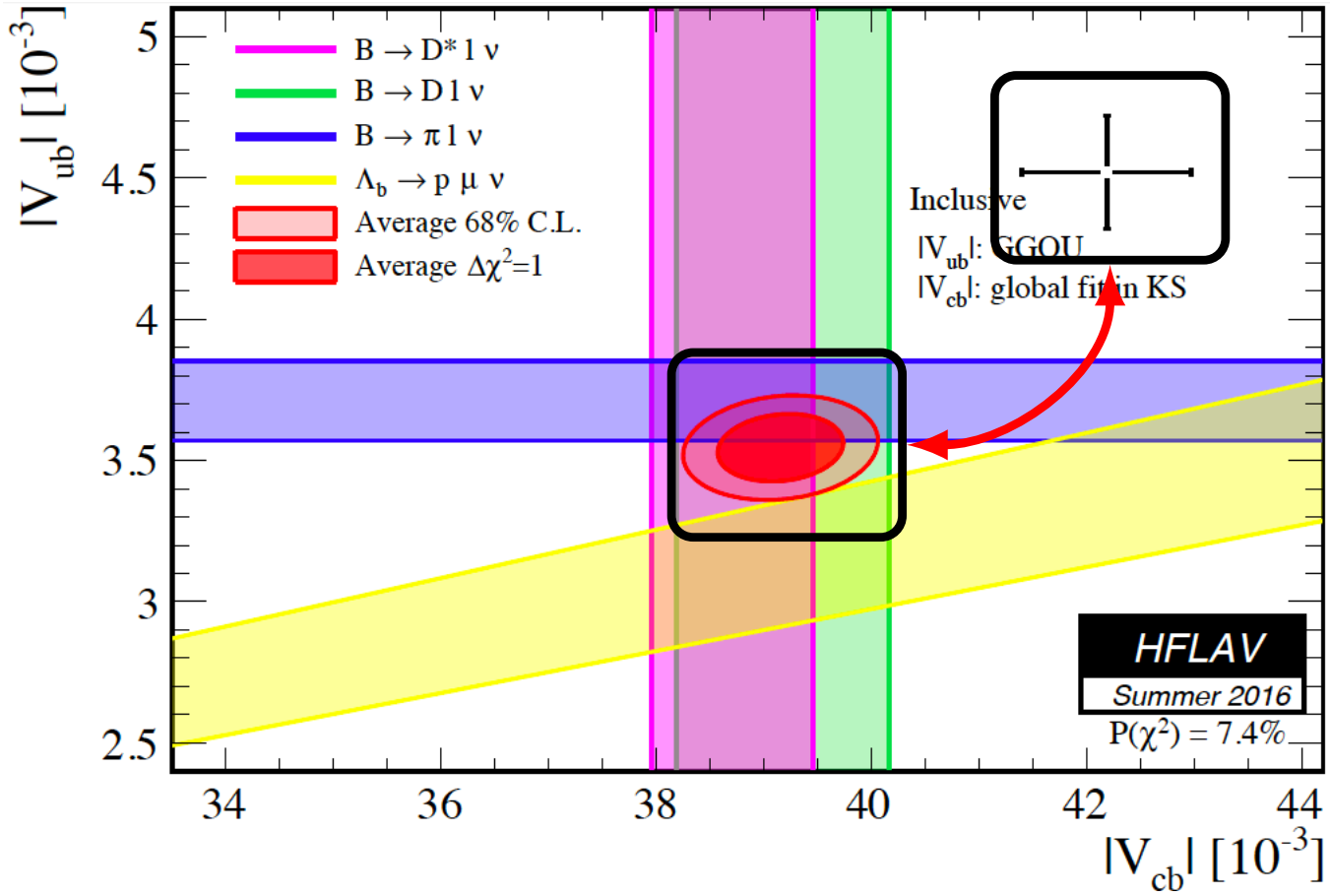
Tension


B → D* l ν
B → D l ν
B → π l ν
Λb → p μ ν
Average 68% C.L.
Average Δχ²=1
Inclusive
|Vub|: GGOU
|Vcb|: global fit in KS
HFLAV
Summer 2016
P(χ²) = 7.4%
|Vub| [10⁻³]
|Vcb| [10⁻³]
5
4.5
4
3.5
3
2.5
34
36
38
40
42
44

Tension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Background

CKM element V_{ub} relates to the differential decay rate:

$$\frac{\mathrm{d}\Gamma\left(B \to \pi \ell \nu\right)}{\mathrm{d}q^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{24\pi^3} \left|k_\pi\right|^3 \left|f_+(q^2)\right|^2$$

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$$\frac{\mathrm{d}\Gamma\left(B \to \pi \ell \nu\right)}{\mathrm{d}q^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{24\pi^3} \left|k_\pi\right|^3 \left|f_+(q^2)\right|^2$$

Experiment

CKM

Lattice

Semileptonic Decays

For pseudoscalar to pseudoscalar decays: :

$$\begin{aligned}\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle &= f_+(q^2)\left[(p_B+k_\pi)^\mu - \frac{m_B^2-m_\pi^2}{q^2}q^\mu\right] \\ &\quad + f_0(q^2)\frac{m_B^2-m_\pi^2}{q^2}q^\mu,\end{aligned}$$

- p_B and k_π: B and π four-momenta
- $q^\mu = p_B^\mu - k_\pi^\mu$: four-momentum transfer
- Constraint: $f_0(0) = f_+(0)$

Semileptonic Decays

In the context of HQET, a useful parametrisation is:

[Burdman et al. (1994)]

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{m_B}\left[f_1\left(v\cdot k_\pi\right)v^\mu + f_2\left(v\cdot k_\pi\right)\frac{k^\mu}{v\cdot k_\pi}\right]$$

- $v^\mu = \dfrac{p_B}{m_B}$: heavy quark velocity
- $E_\pi = v\cdot k_\pi = \dfrac{m_B^2 + m_\pi^2 - q^2}{2m_B}$

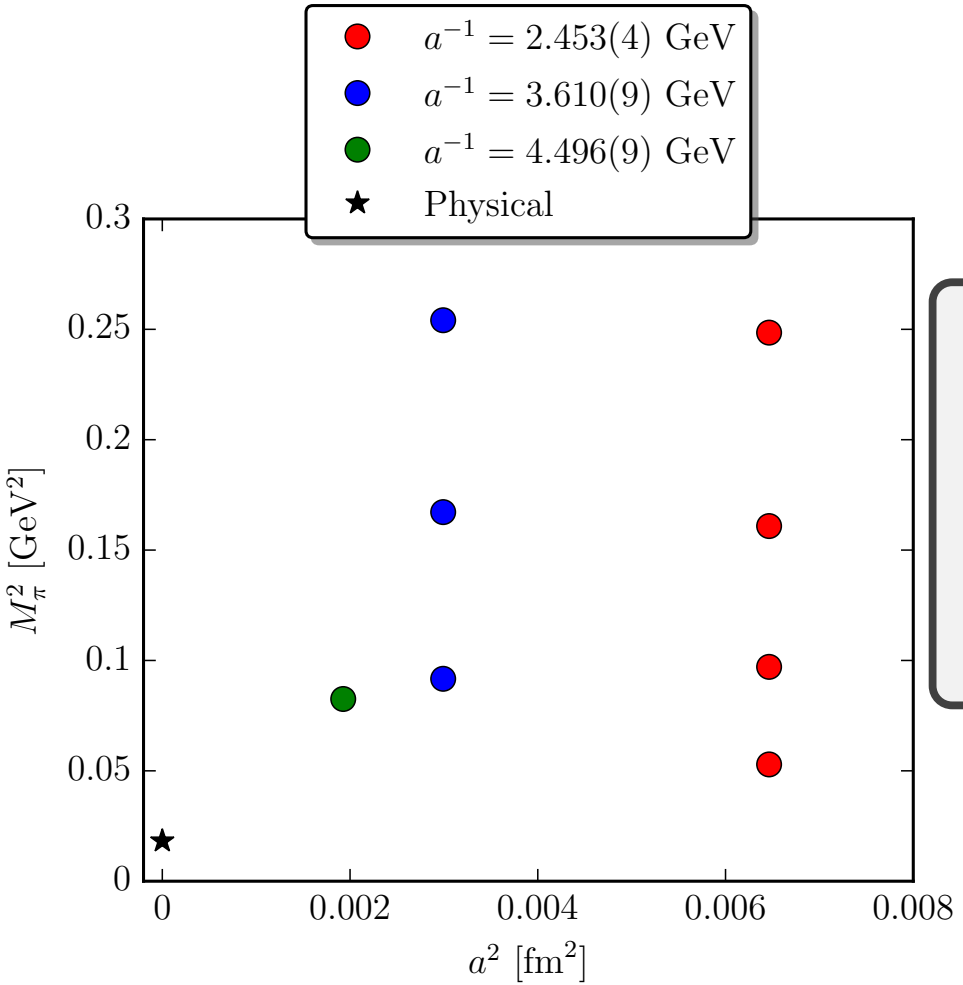
Semileptonic Decays

These two form factor definitions are related by:

$$
\begin{aligned}
f_+\left(q^2\right) =& \sqrt{m_B}\left[\frac{f_2\left(v\cdot k\right)}{v\cdot k_\pi}+\frac{f_1\left(v\cdot k\right)}{m_B}\right];\\
f_0\left(q^2\right) =& \frac{2}{\sqrt{m_B}}\frac{m_B^2}{m_B^2-m_\pi^2}\left[f_1\left(v\cdot k\right)+f_2\left(v\cdot k\right)\right.\\
&\left.-\frac{v\cdot k_\pi}{m_B}\left(f_1\left(v\cdot k\right)+\frac{m_\pi^2}{\left(v\cdot k_\pi\right)^2}f_2\left(v\cdot k\right)\right)\right]
\end{aligned}
$$

Lattice Calculation

Gauge Configurations



$N_f = 2 + 1$

$m_\pi \approx 225, 300, 400, 500$ MeV

$\beta = 4.17, 4.35, 4.47$

$a^{-1} \approx 2.453, 3.610, 4.496$ GeV

$L^3 \times T = 32^3 \times 64, 48^3 \times 96, 64^3 \times 128$

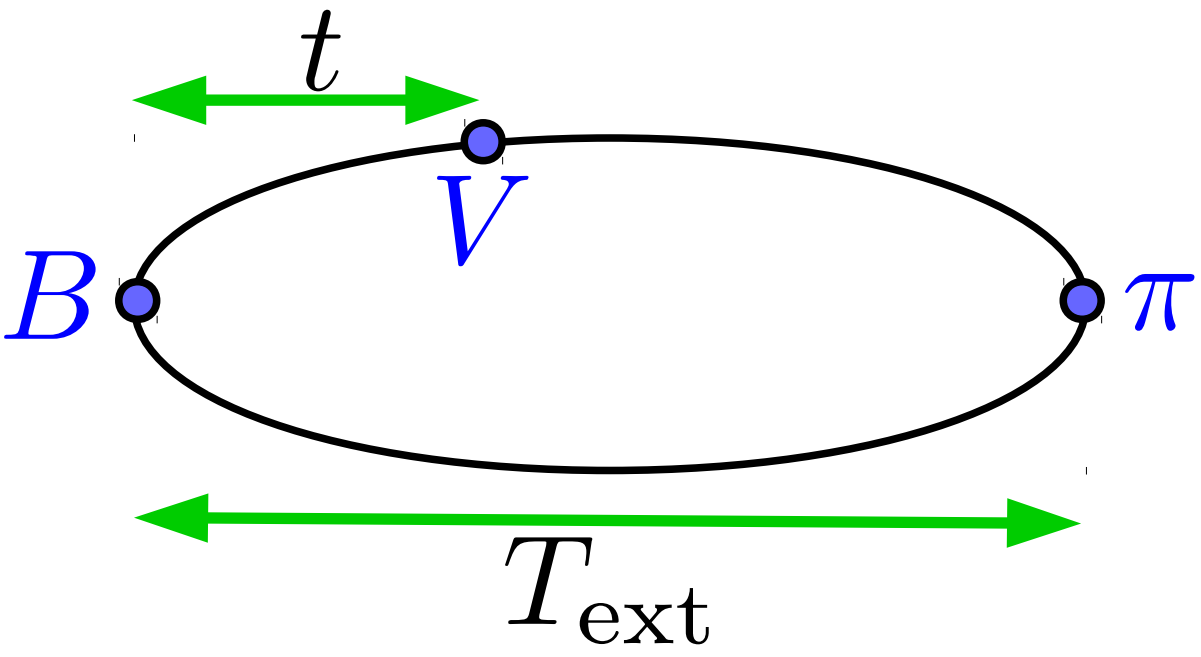
Heavy Quarks

β	a [fm]	am_c	$1.25^2 \times am_c$	$1.25^4 \times am_c$
4.17	0.0804	0.44037	0.68808	-
4.35	0.0547	0.27287	0.42636	0.66619
4.47	0.0439	0.210476	0.328869	0.5138574

- Möbius Domain Wall Fermions
- m_h up to $2.44m_c$: $m_c,\ 1.25^2 m_c,\ 1.25^4 m_c$
  - $m_{H_l} \approx 1.95$ GeV, 2.55 GeV, 3.40 GeV
- Smeared sources with $\mathbb{Z}_2$ noise; local/smeared sink
- Time sources: 1, 2 or 4

Correlators: 3-point

- B and π mesons are separated by time T_{ext}
- Operators V^μ are inserted at $0 \leq t \leq T_{\text{ext}}$



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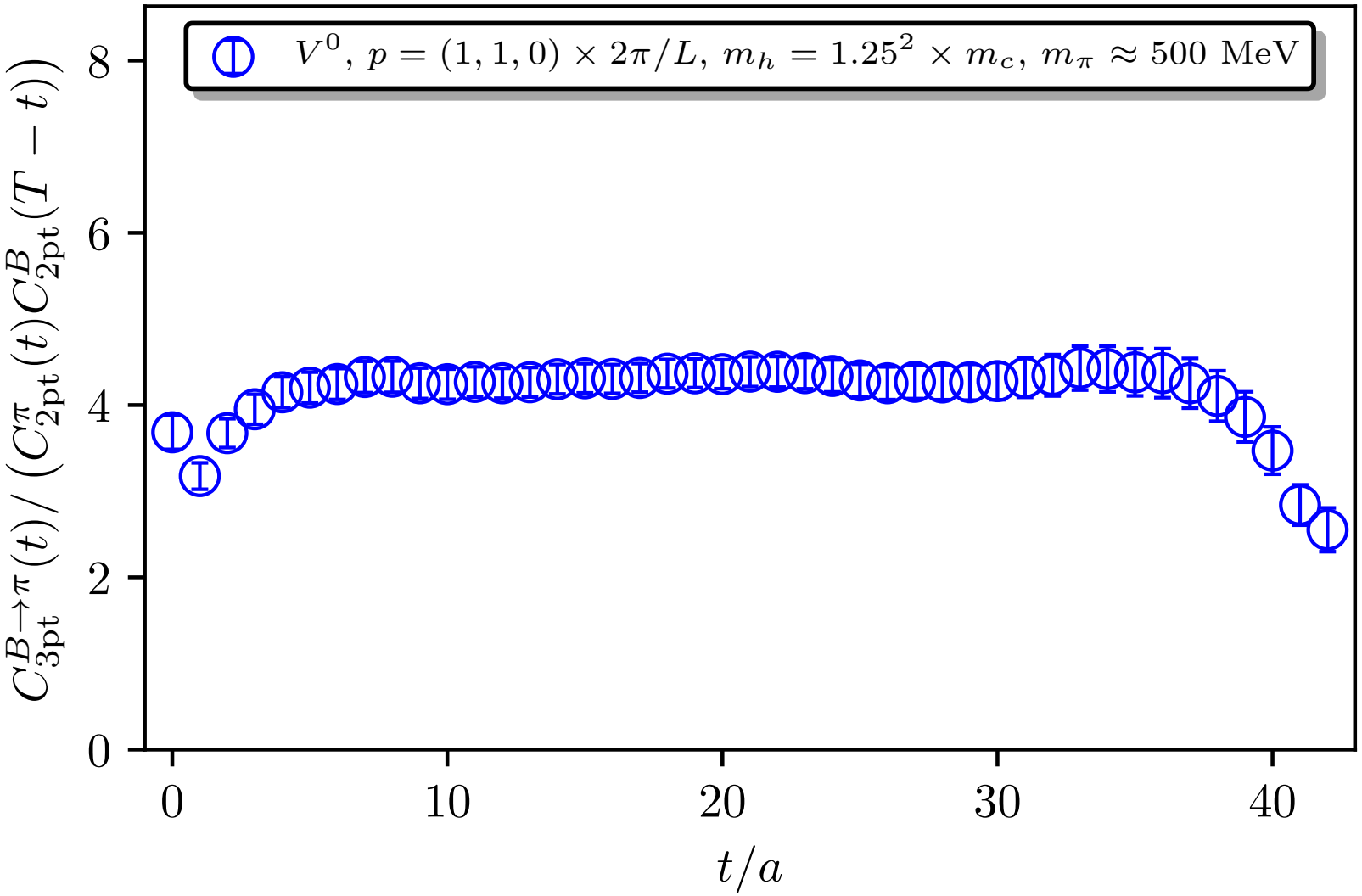
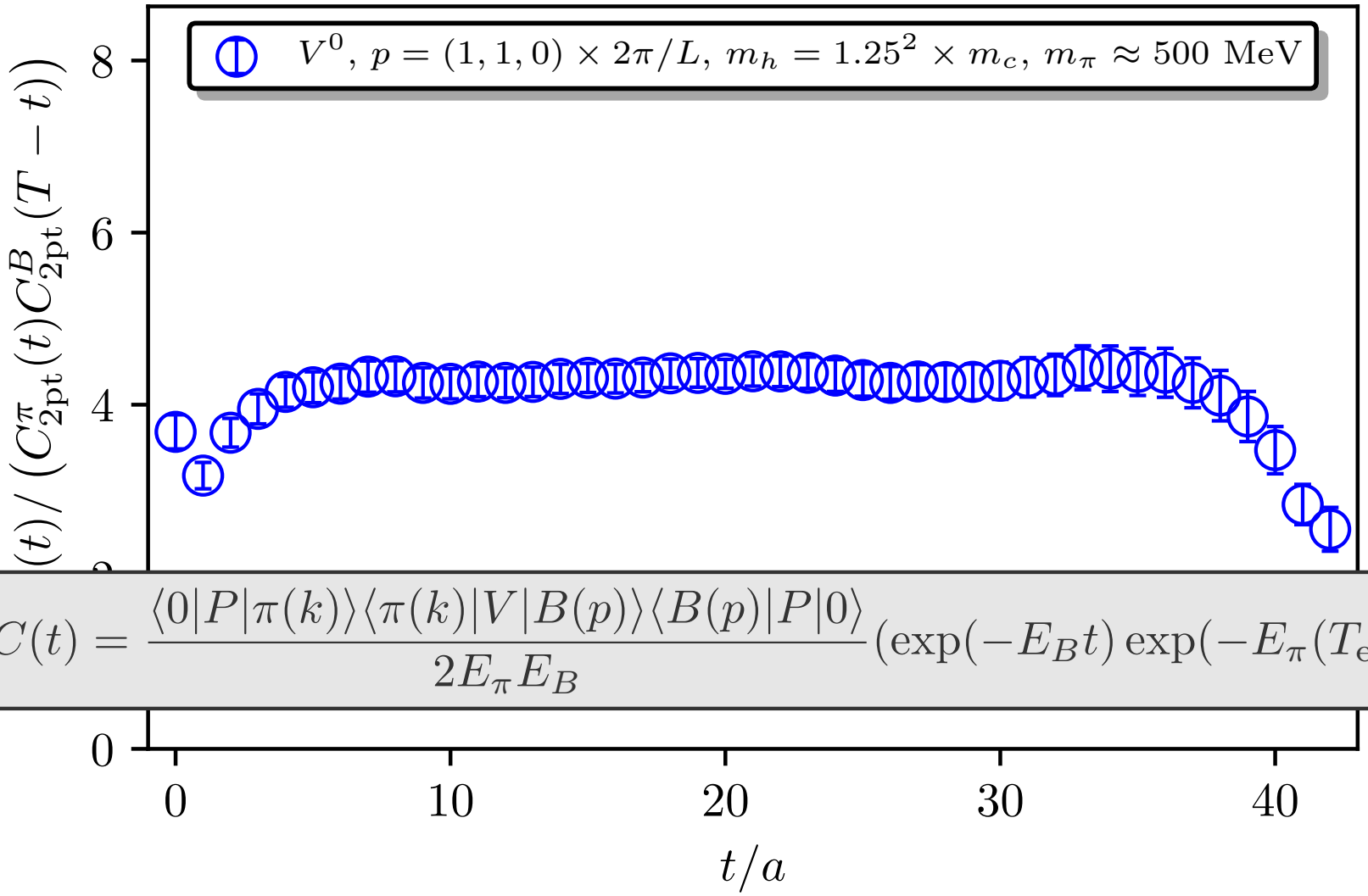


q

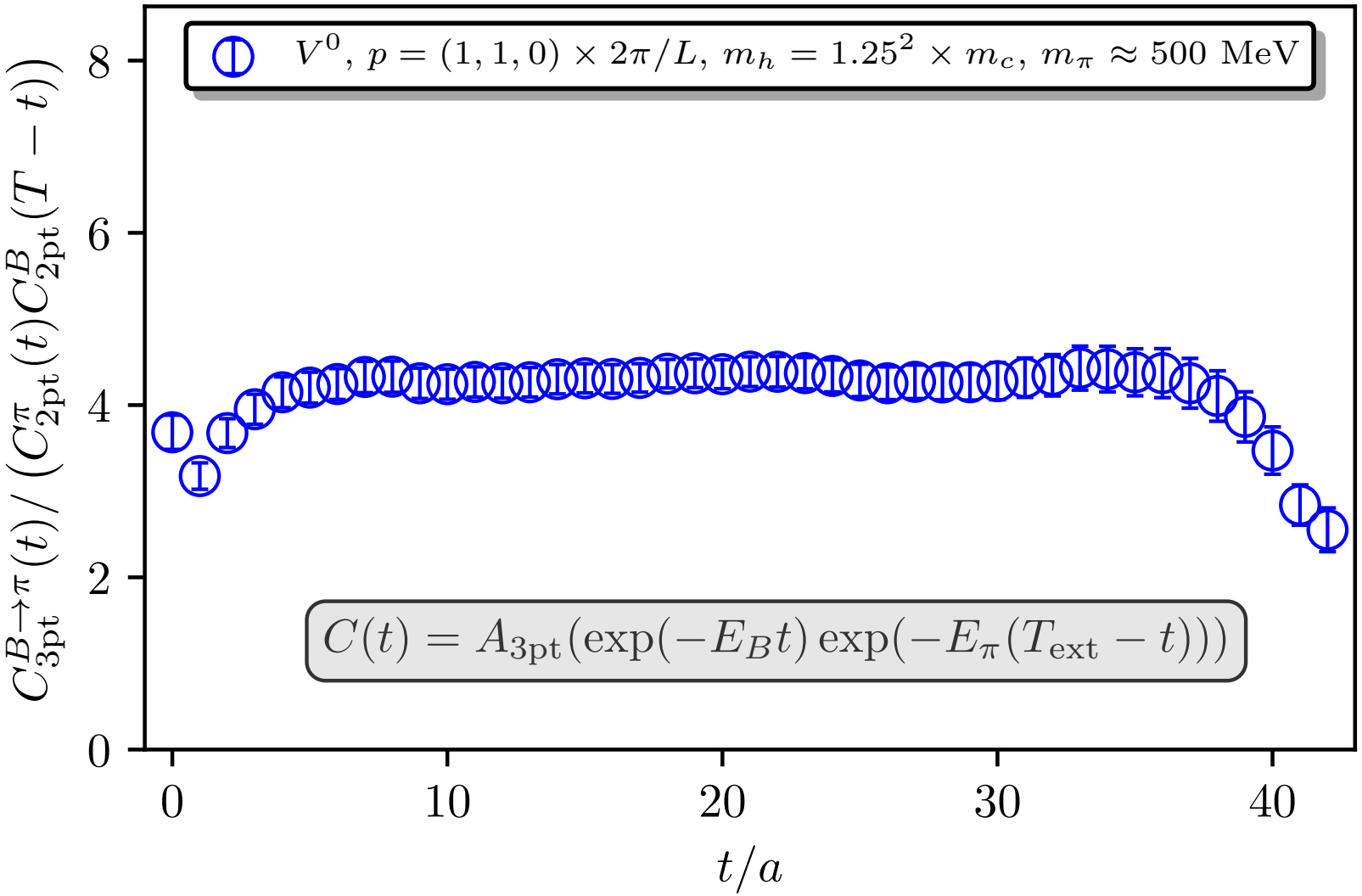
Correlators: 3-point



$$C(t) = \frac{\langle 0|P|\pi(k)\rangle\langle\pi(k)|V|B(p)\rangle\langle B(p)|P|0\rangle}{2E_\pi E_B}\left(\exp(-E_B t)\exp(-E_\pi(T_{\mathrm{ext}} - t))\right)$$

q

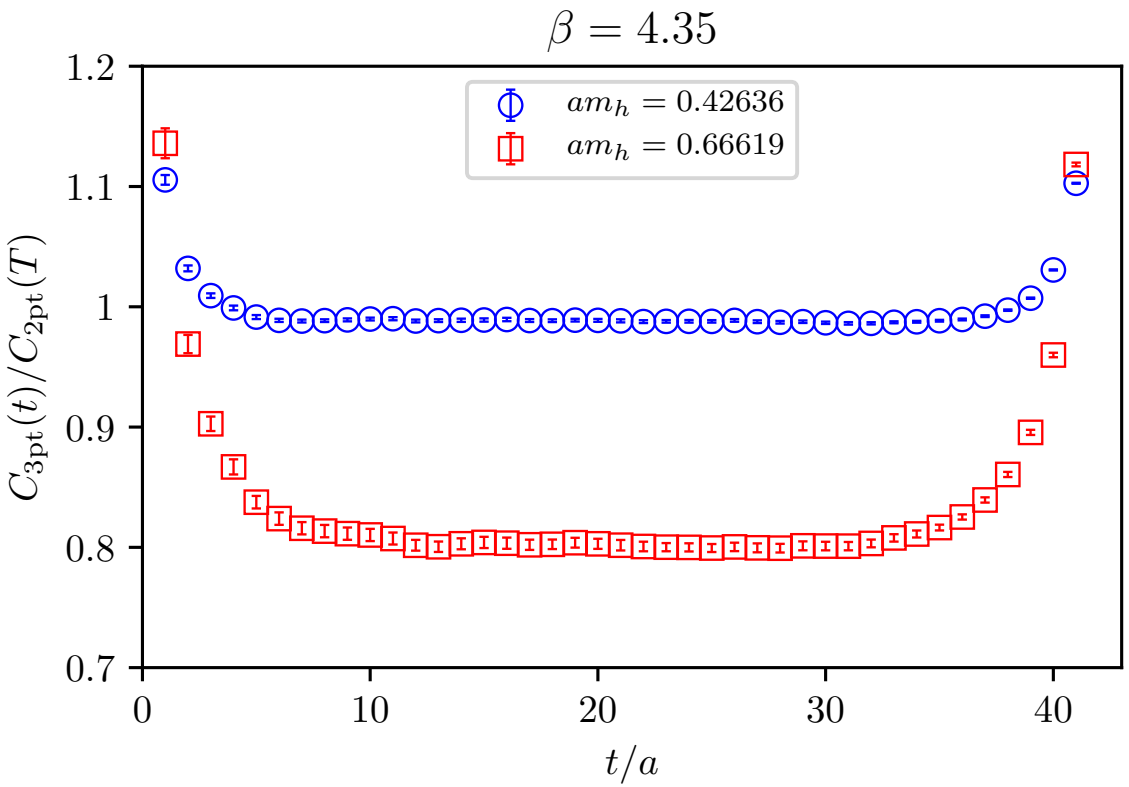
Correlators: 3-point



q

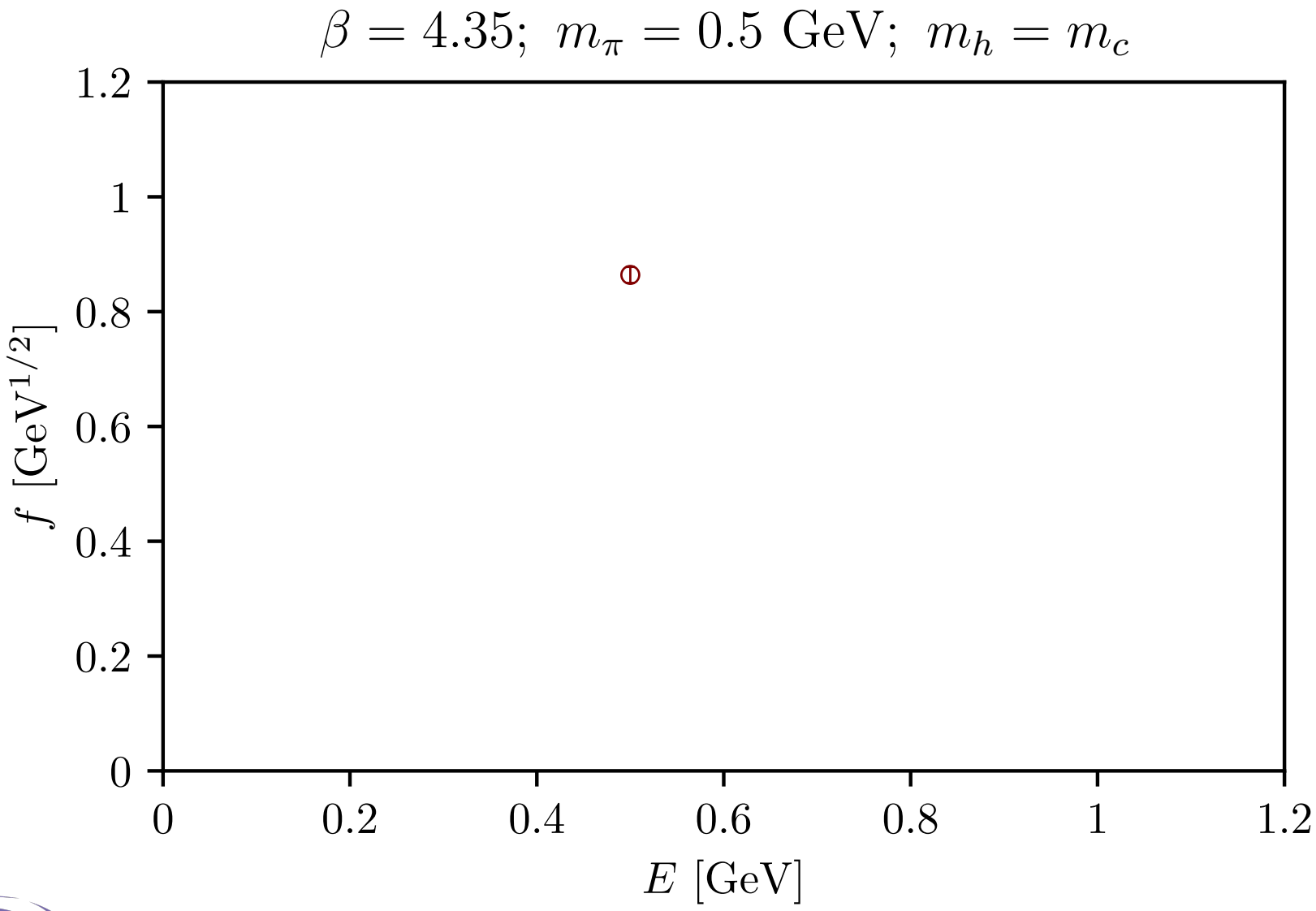
Renormalisation

- Renormalise with $Z_{V,bl} = \sqrt{Z_{V,bb} Z_{V,ll}}$.
- Require $C_{3\mathrm{pt}}(t)/C_{2\mathrm{pt}}(T_{\mathrm{ext}}) = Z_{V,bb}^{-1}$.



Results

E_π Dependence



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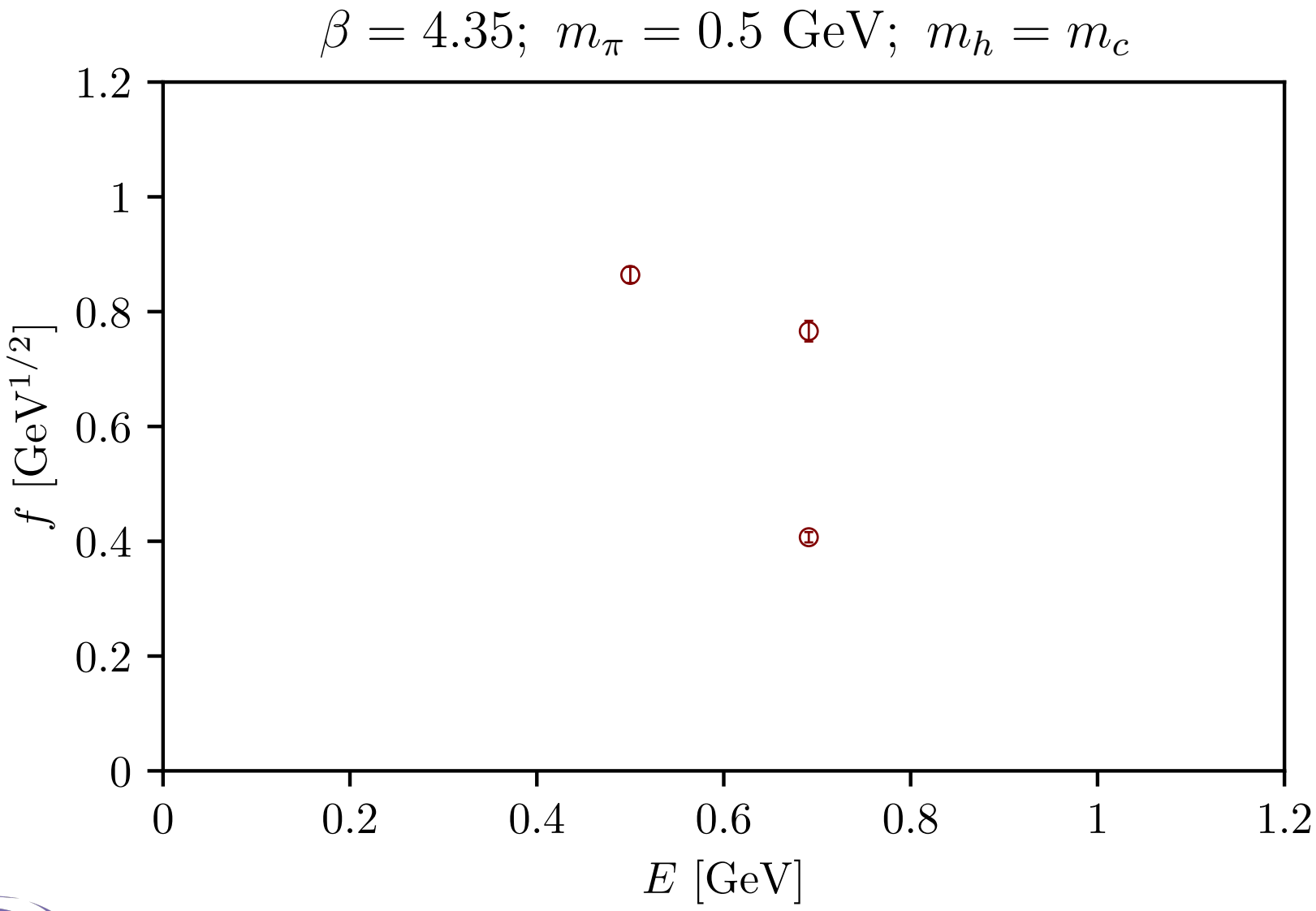

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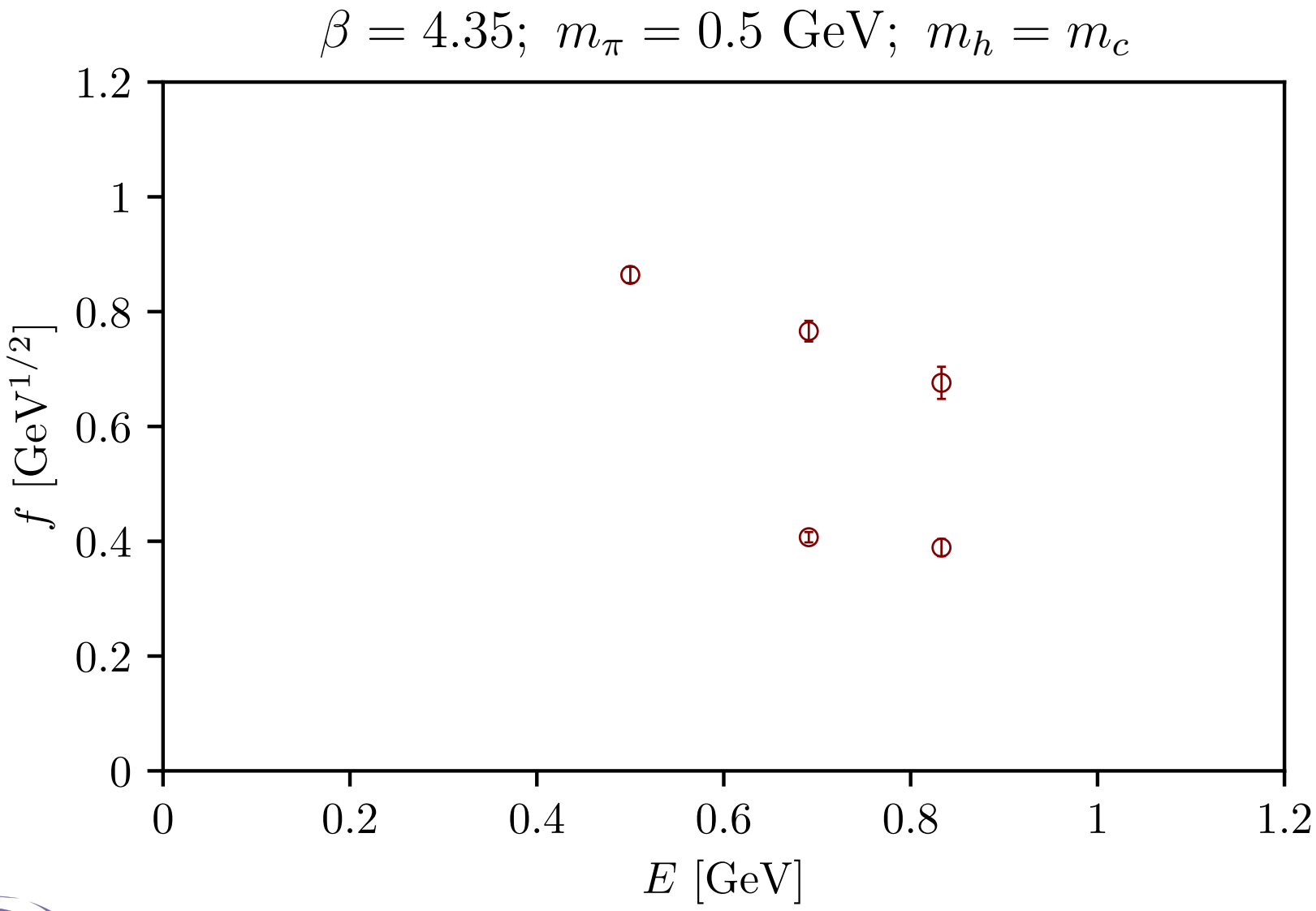

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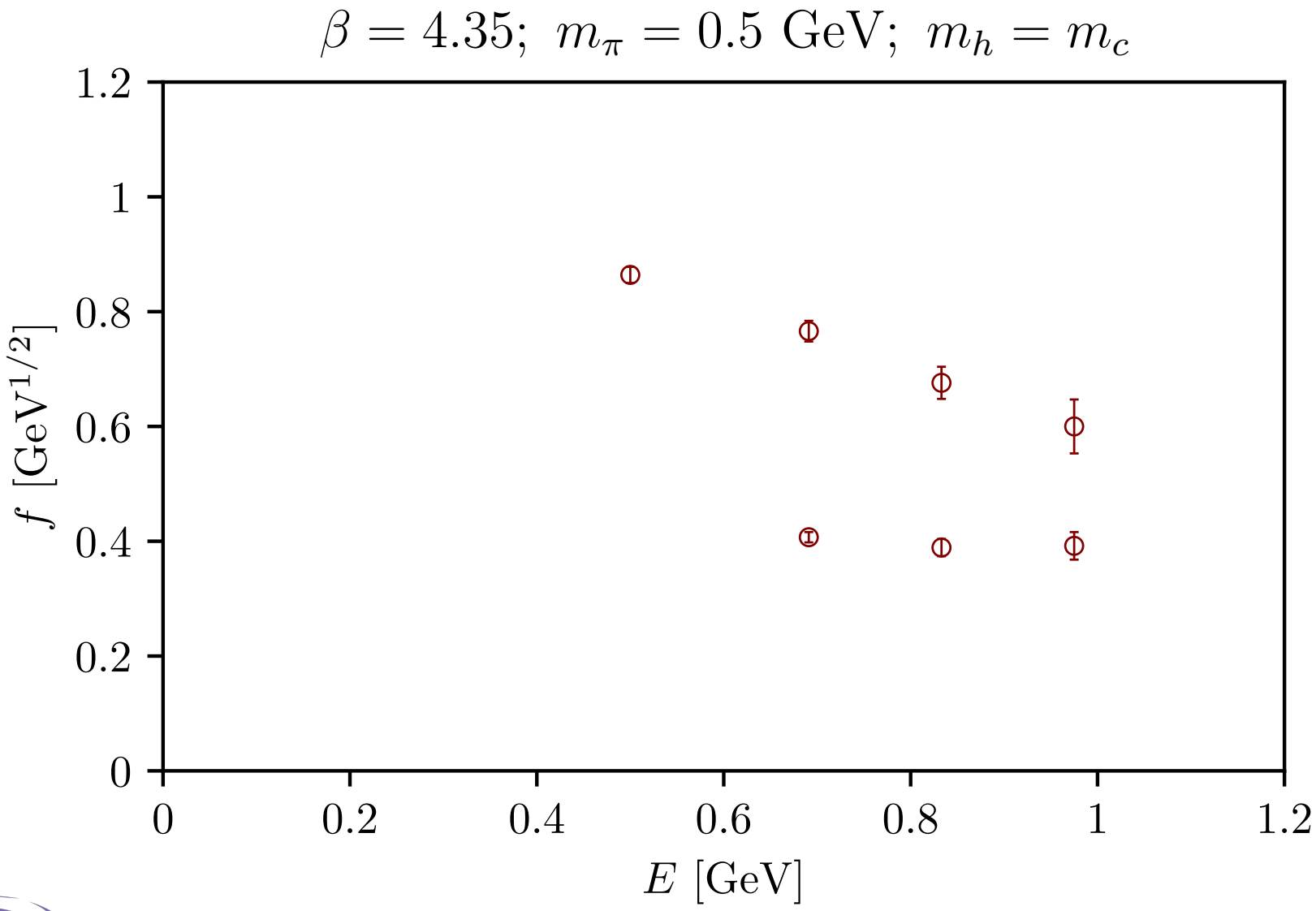
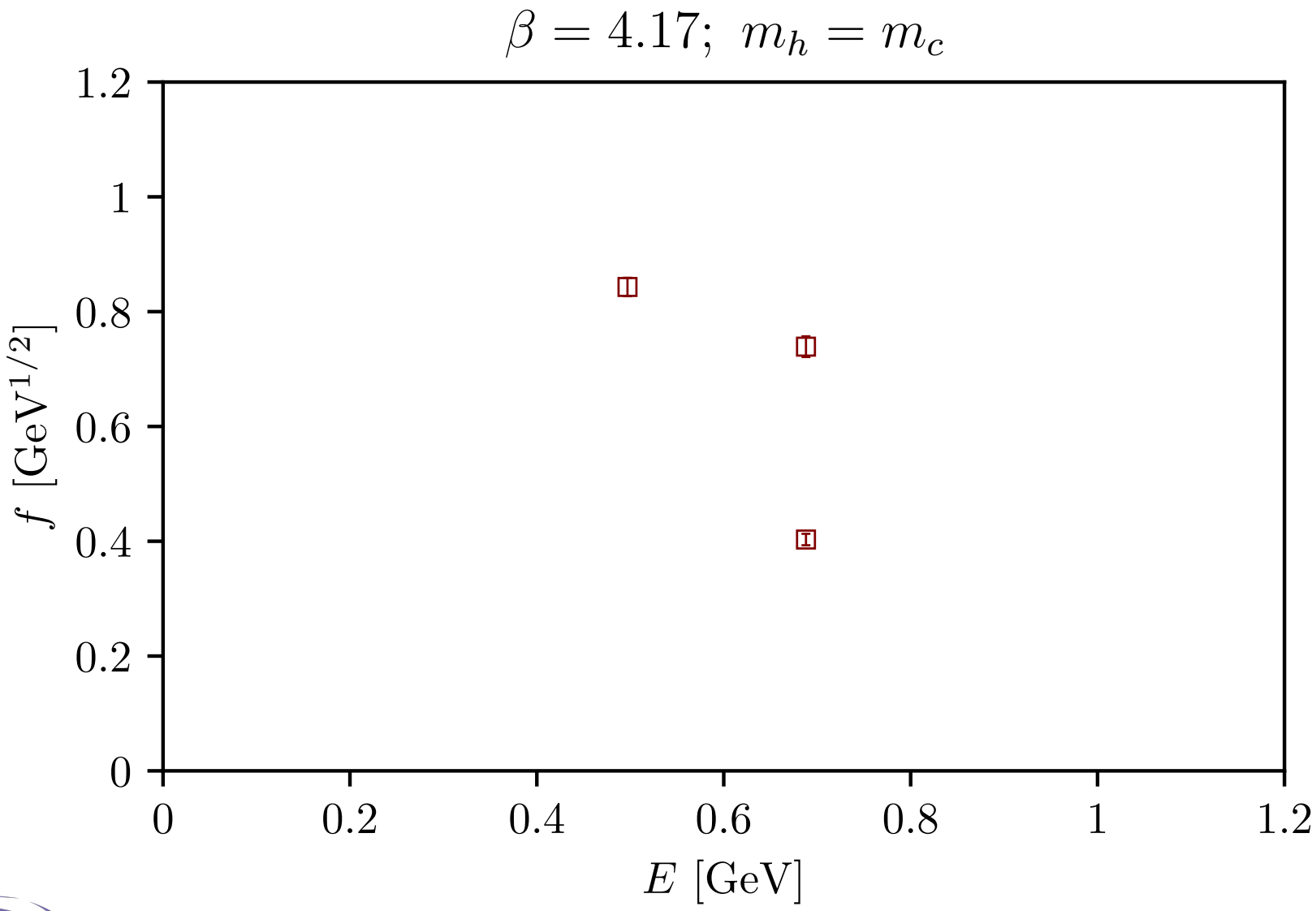


m_π Dependence



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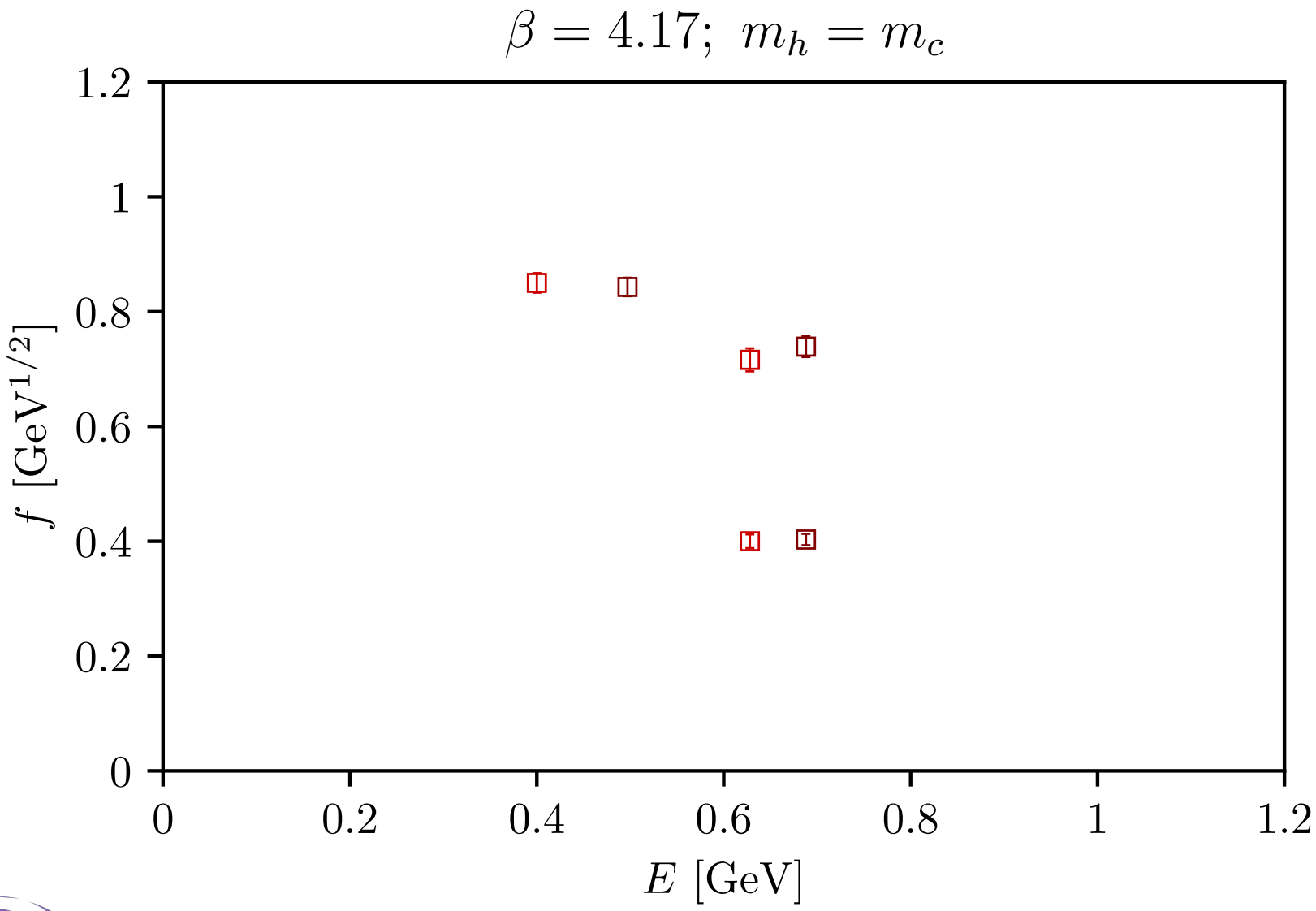

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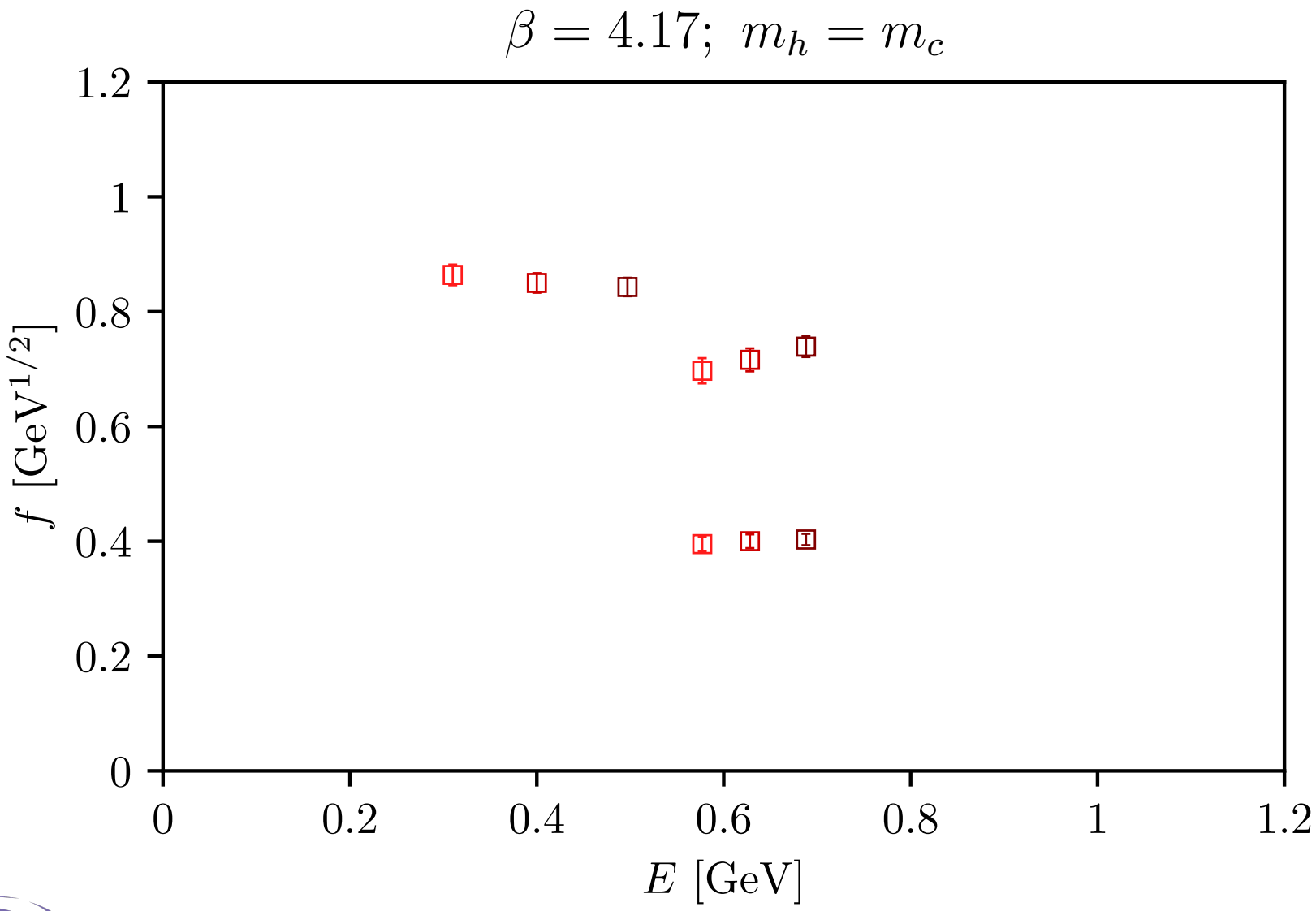

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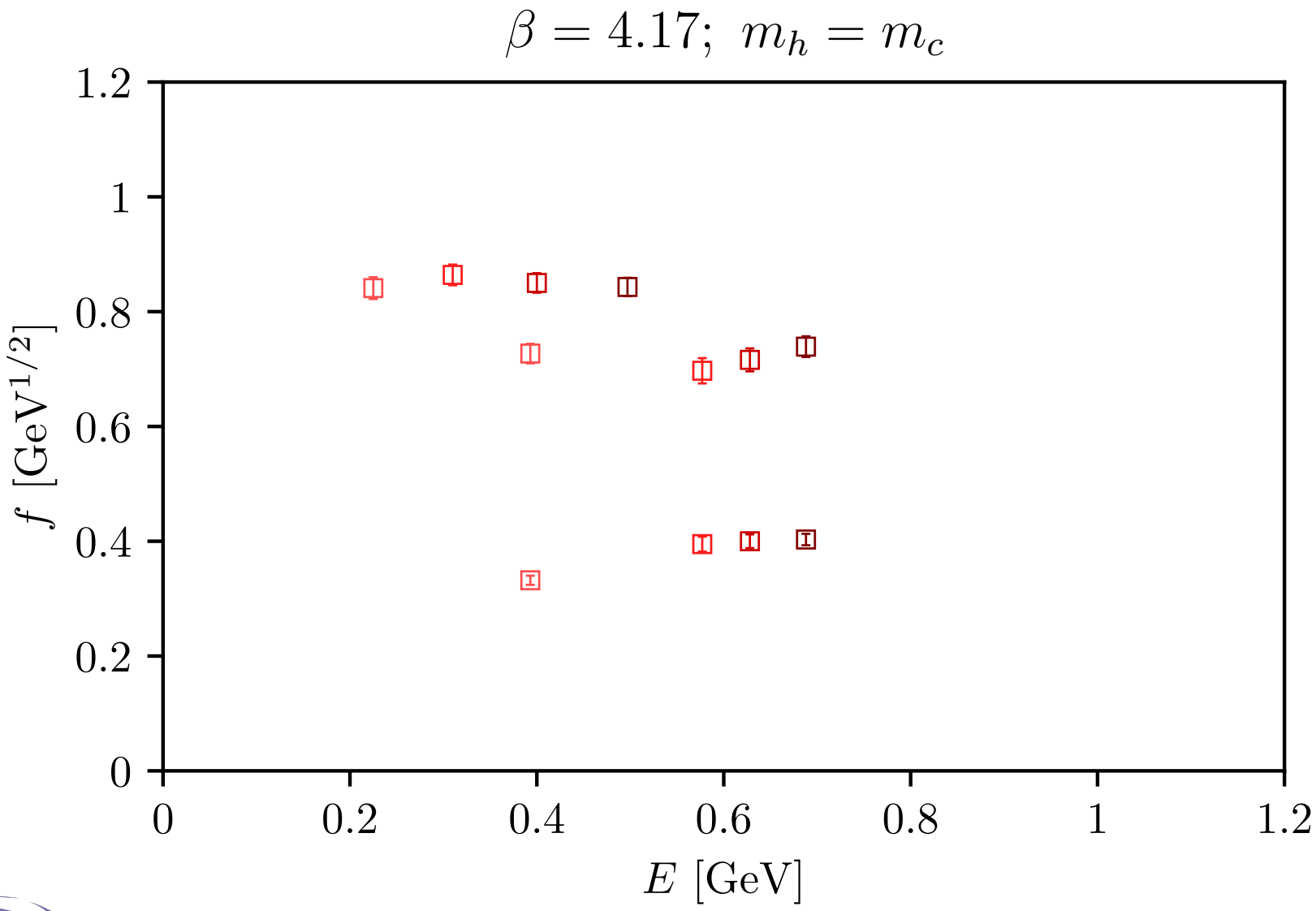
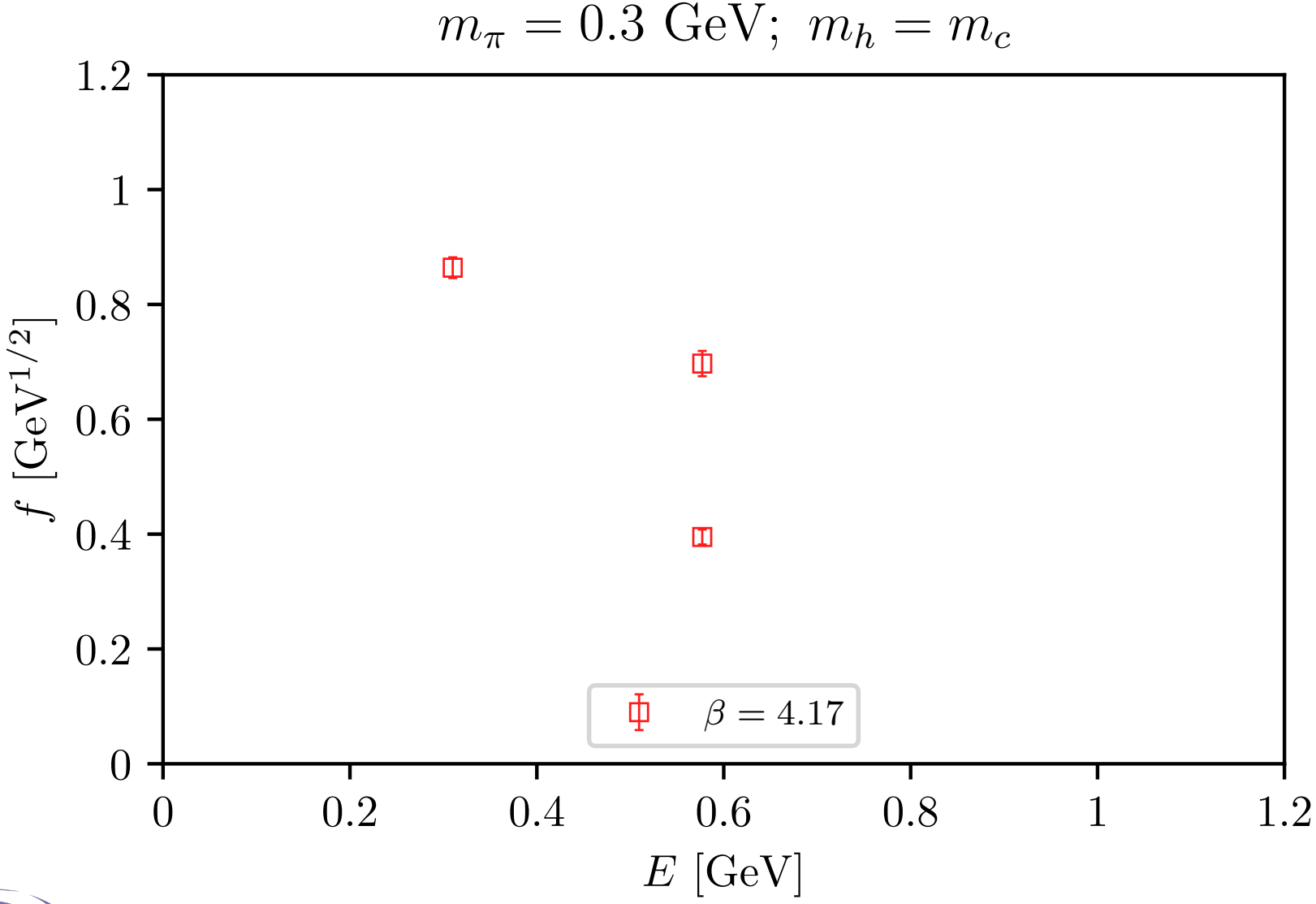


Lattice Spacing Dependence



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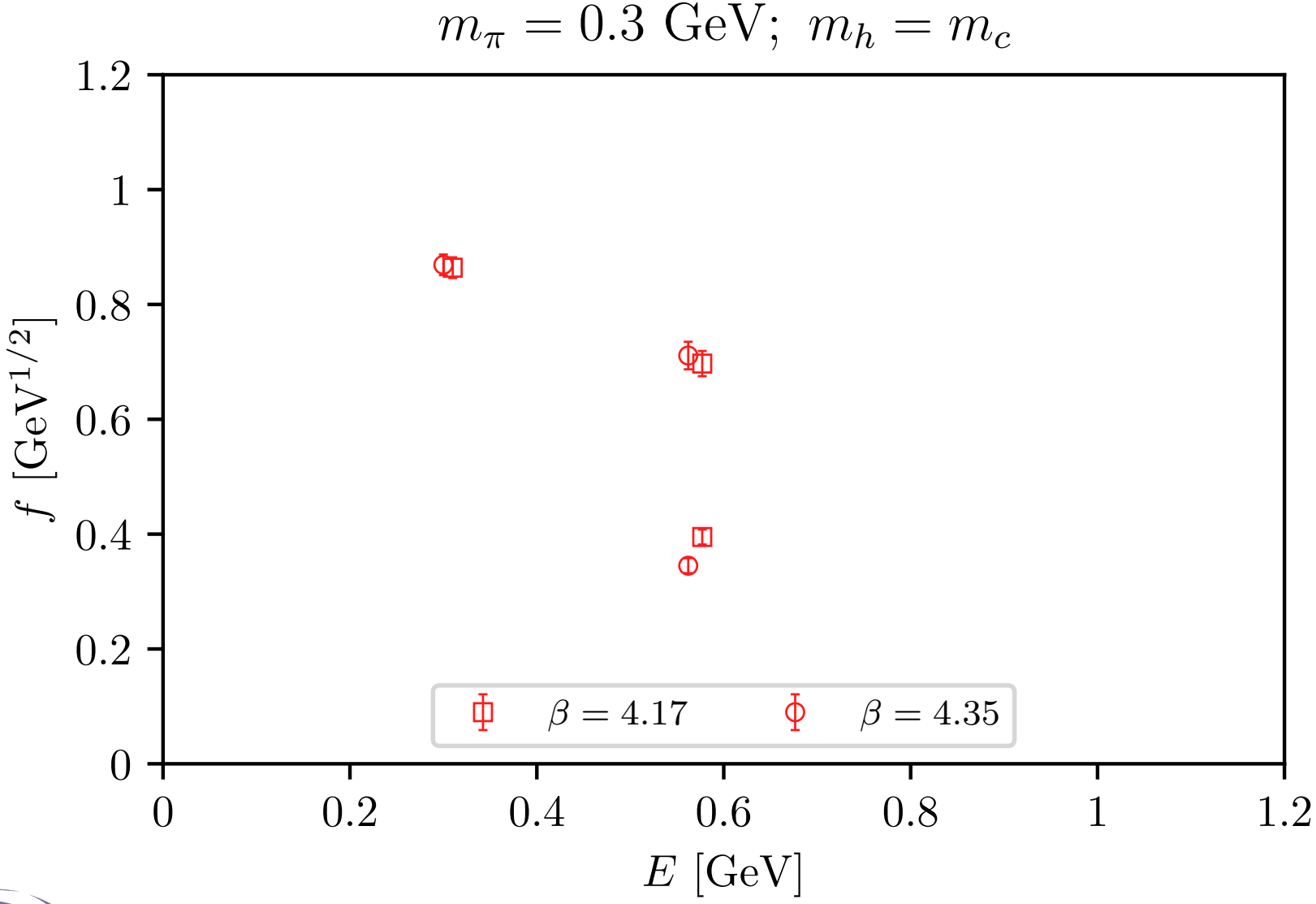

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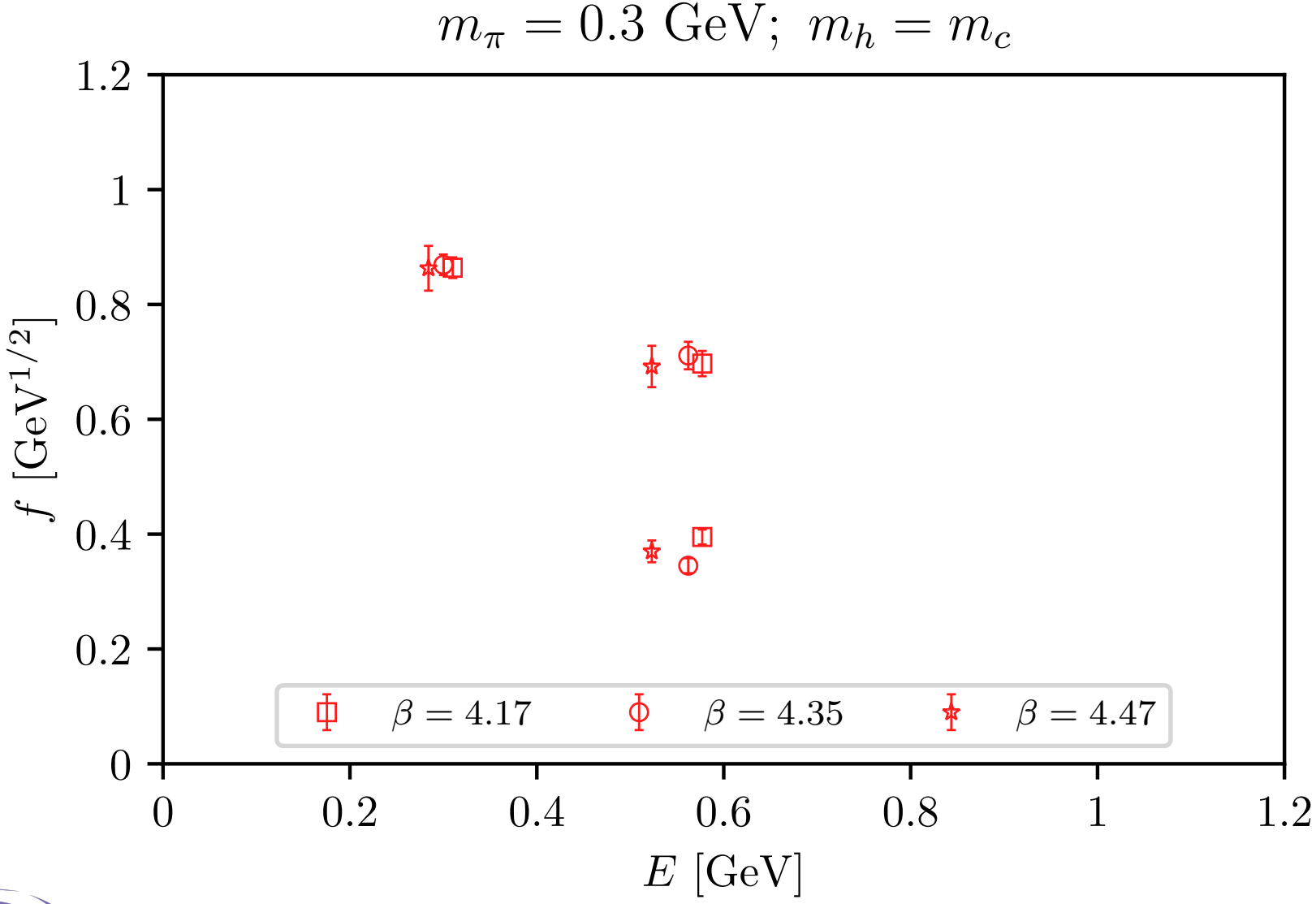
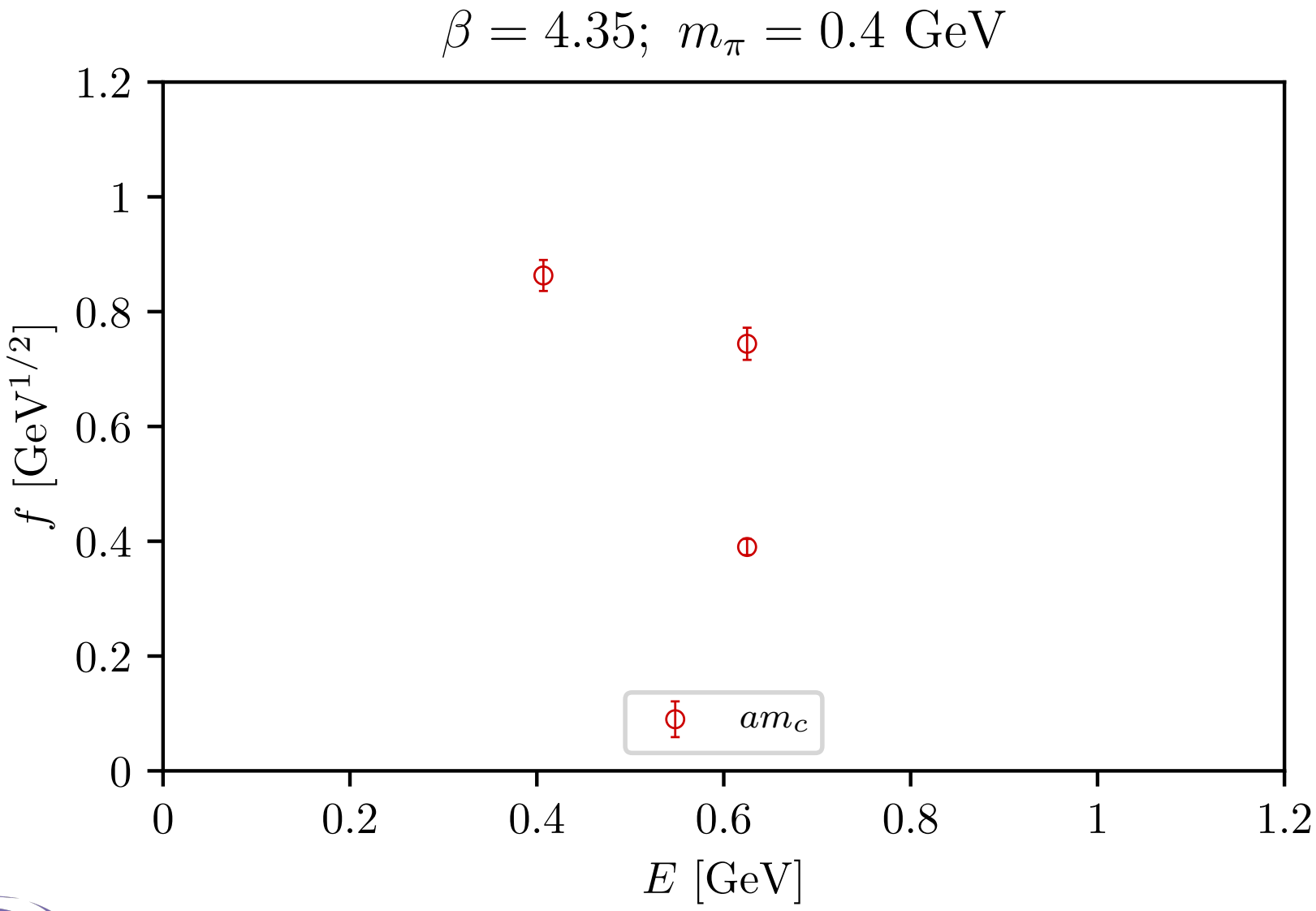


m_b Dependence



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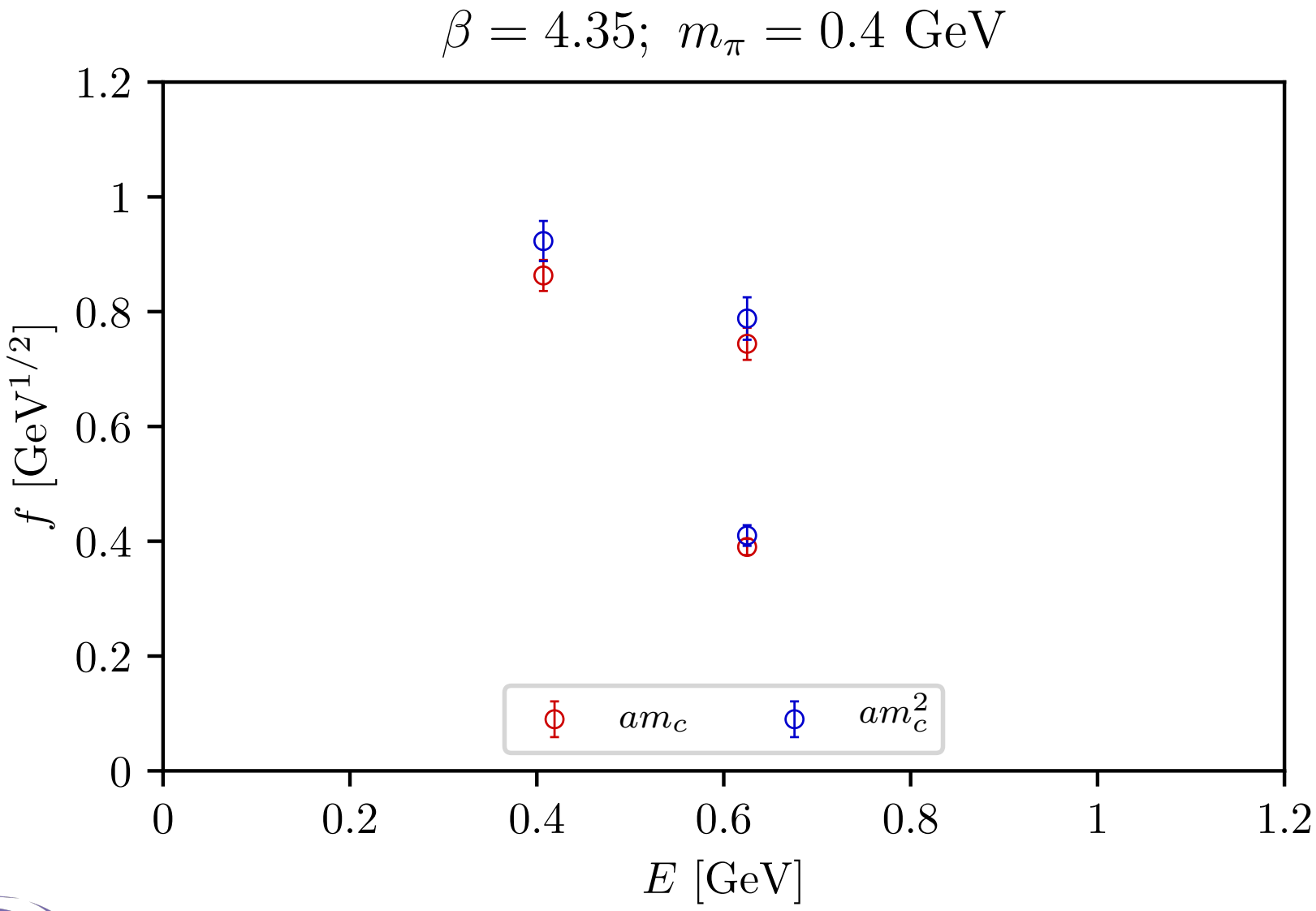

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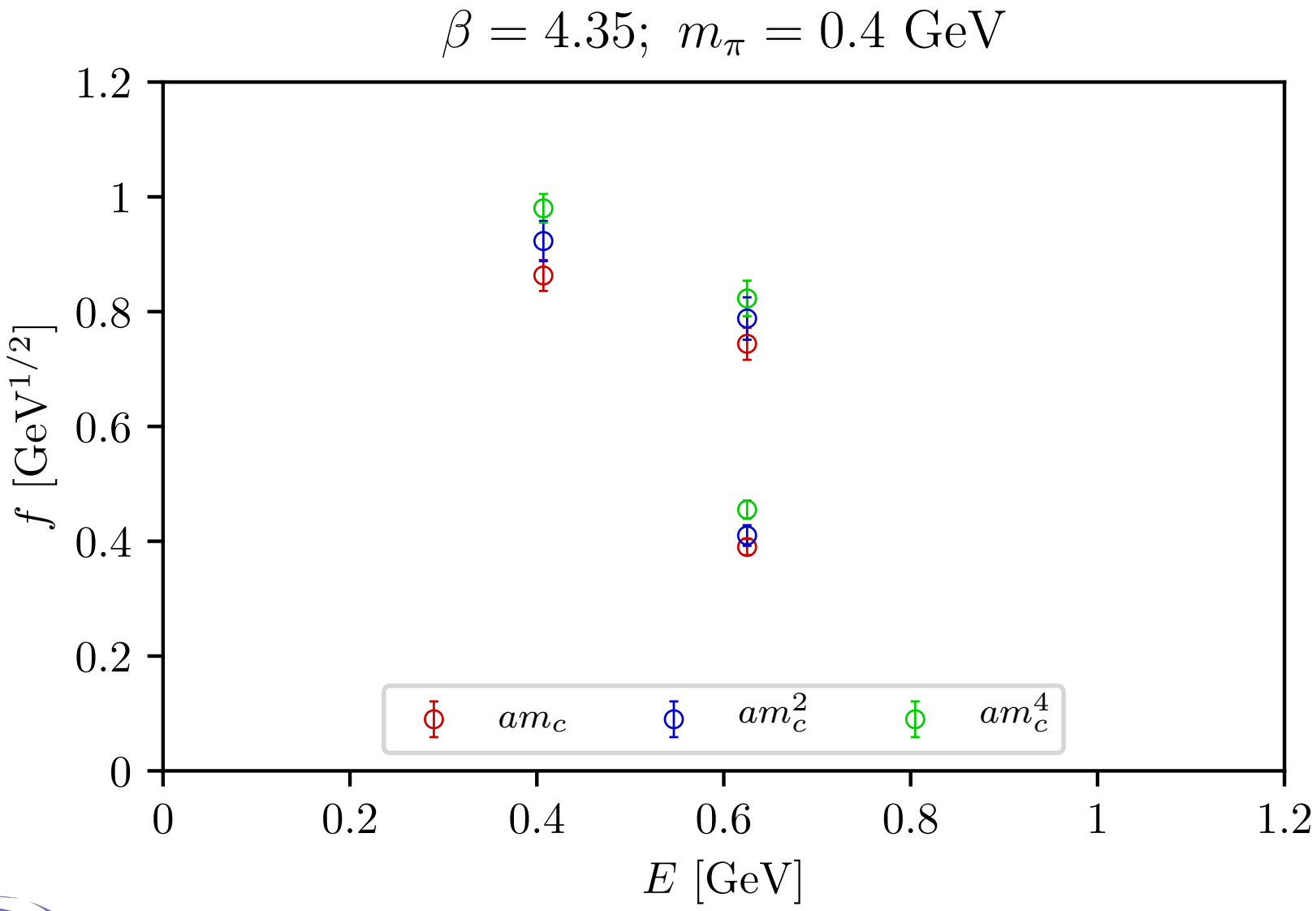
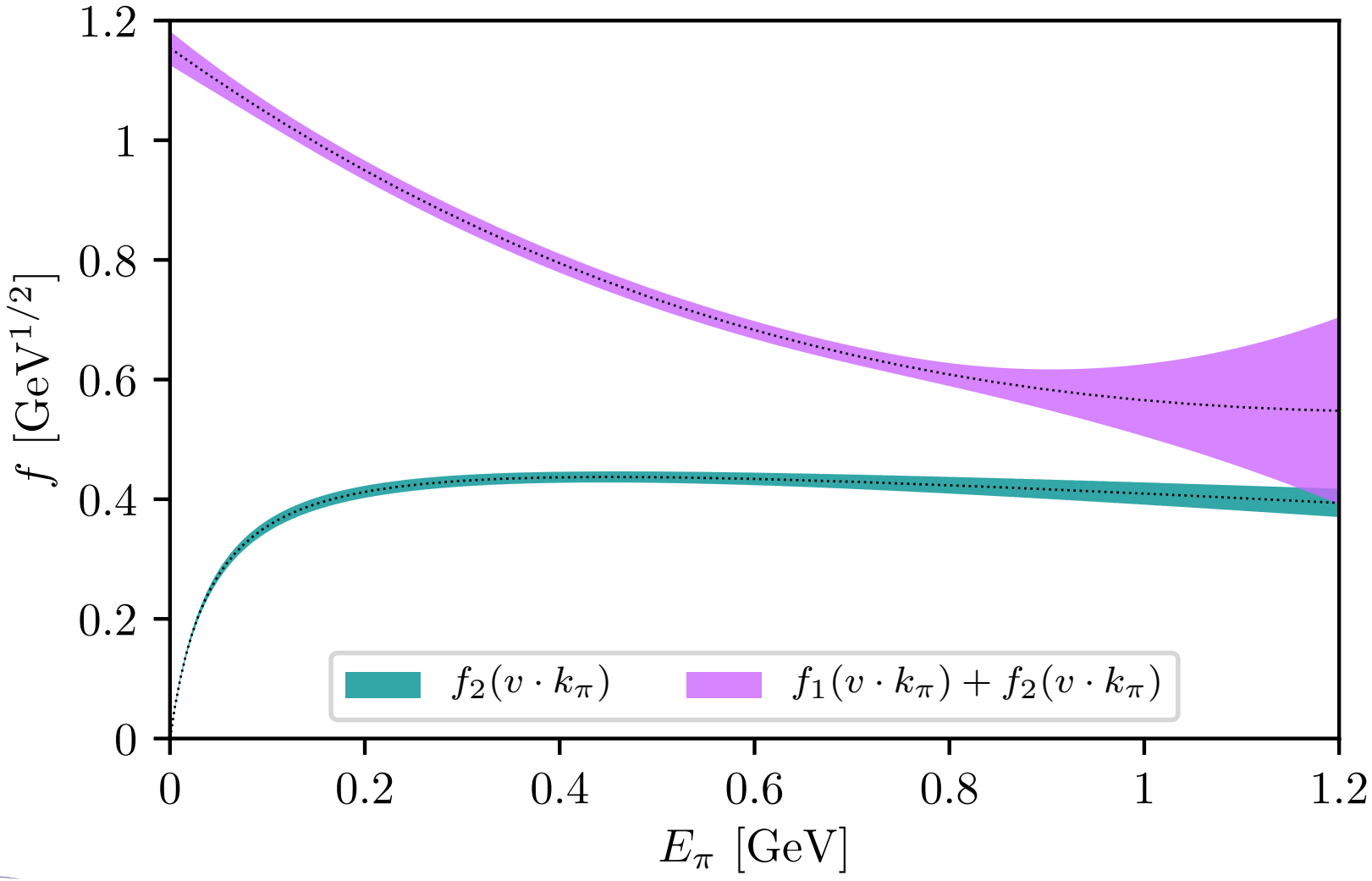
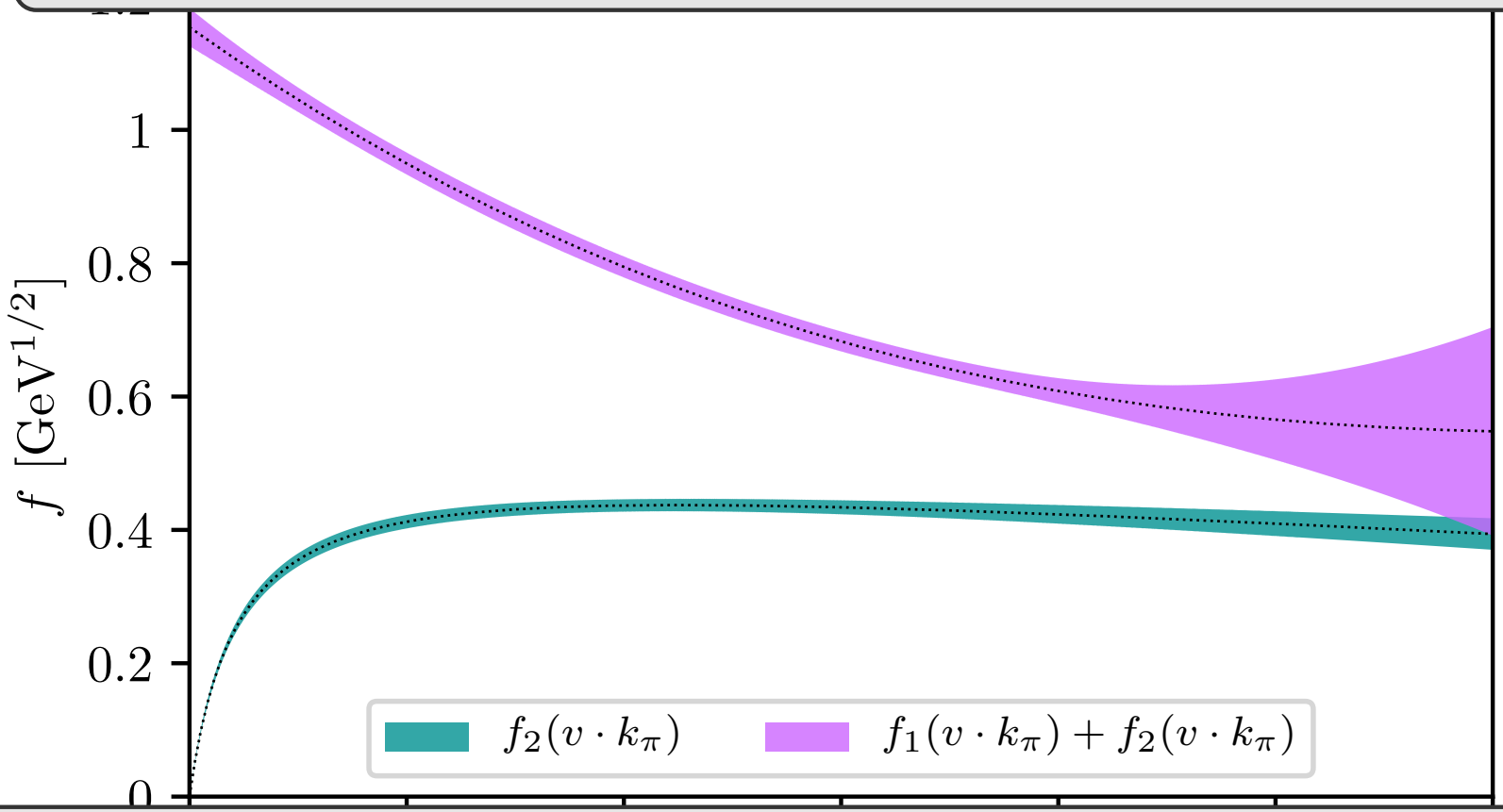


$f_1(v \cdot k_\pi)$ and $f_1(v \cdot k_\pi)$



$f_1(v \cdot k_\pi)$ and $f_1(v \cdot k_\pi)$

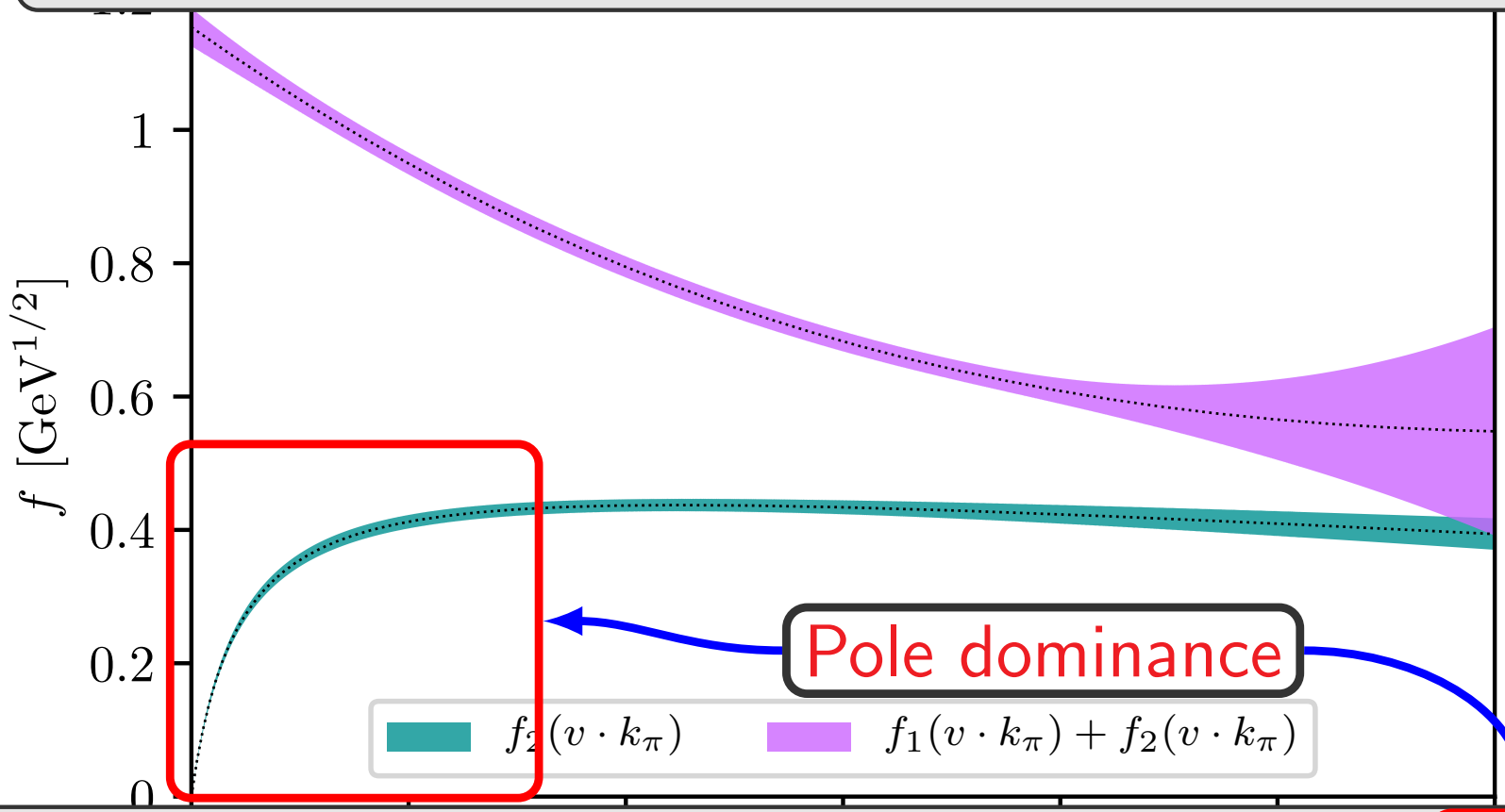
$$C_0 \left(1 + \sum_{n=1}^{3} C_{E_\pi^n} E_\pi^n\right) \left(1 + C_{m_\pi} m_\pi^2\right) \left(1 + C_{a^2} a^2\right) \left(1 + \frac{C_{m_b}}{m_b}\right) \left(1 + \chi_{\log}\right)$$



$$\left[C_0 \left(1 + C_a a^2\right) \left(1 + \frac{C_{m_b}}{m_b}\right) \left(1 + C_\pi m_\pi^2\right) \left(1 + C_{E_\pi} E_\pi\right) \left(1 + \chi_{\log}\right)\right] \frac{E_\pi}{E_\pi + \Delta_B}$$

Pole Dominance

$$C_0 \left(1 + \sum_{n=1}^{3} C_{E_\pi^n} E_\pi^n\right) \left(1 + C_{m_\pi} m_\pi^2\right) \left(1 + C_{a^2} a^2\right) \left(1 + \frac{C_{m_b}}{m_b}\right) \left(1 + \chi_{\log}\right)$$



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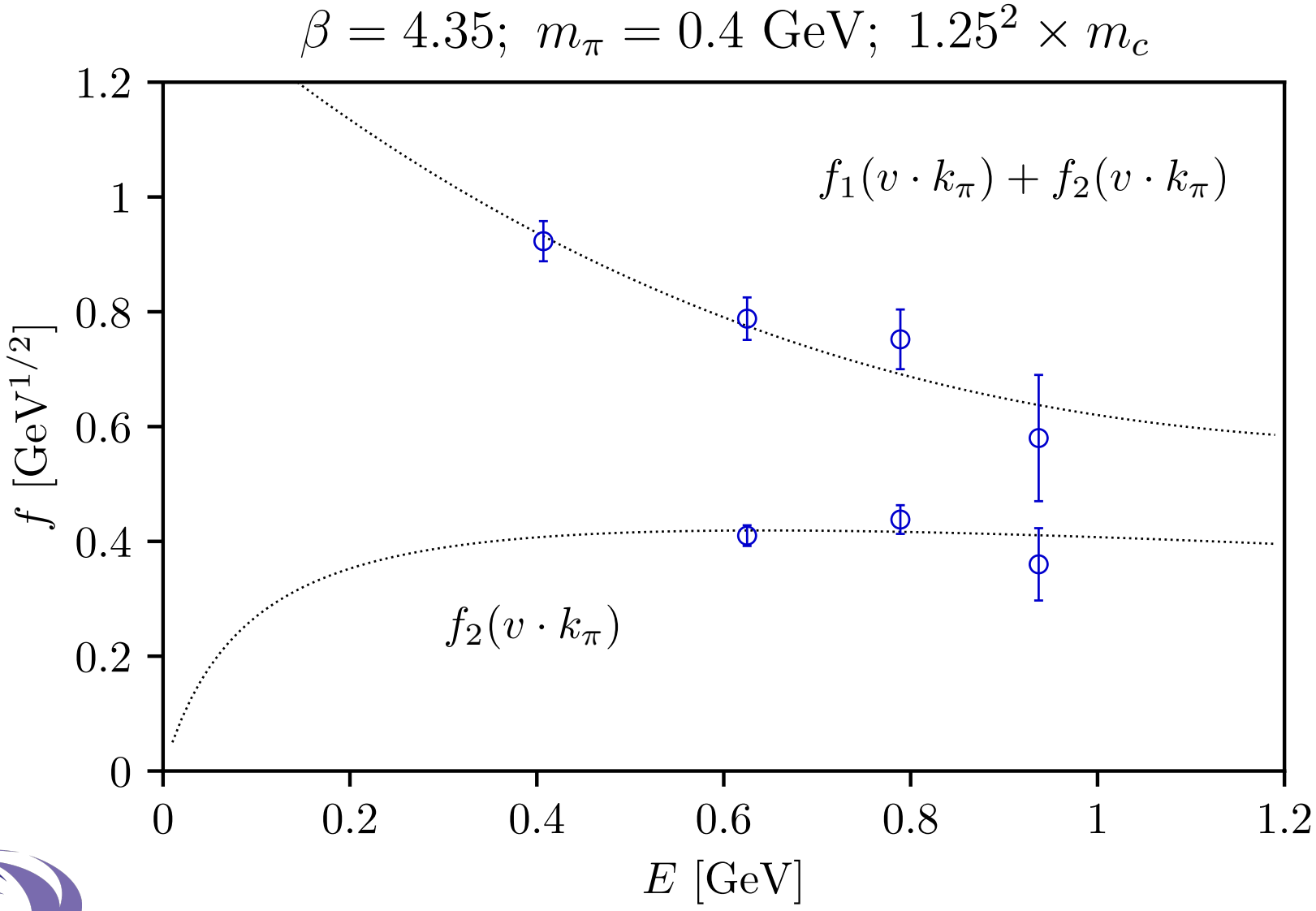
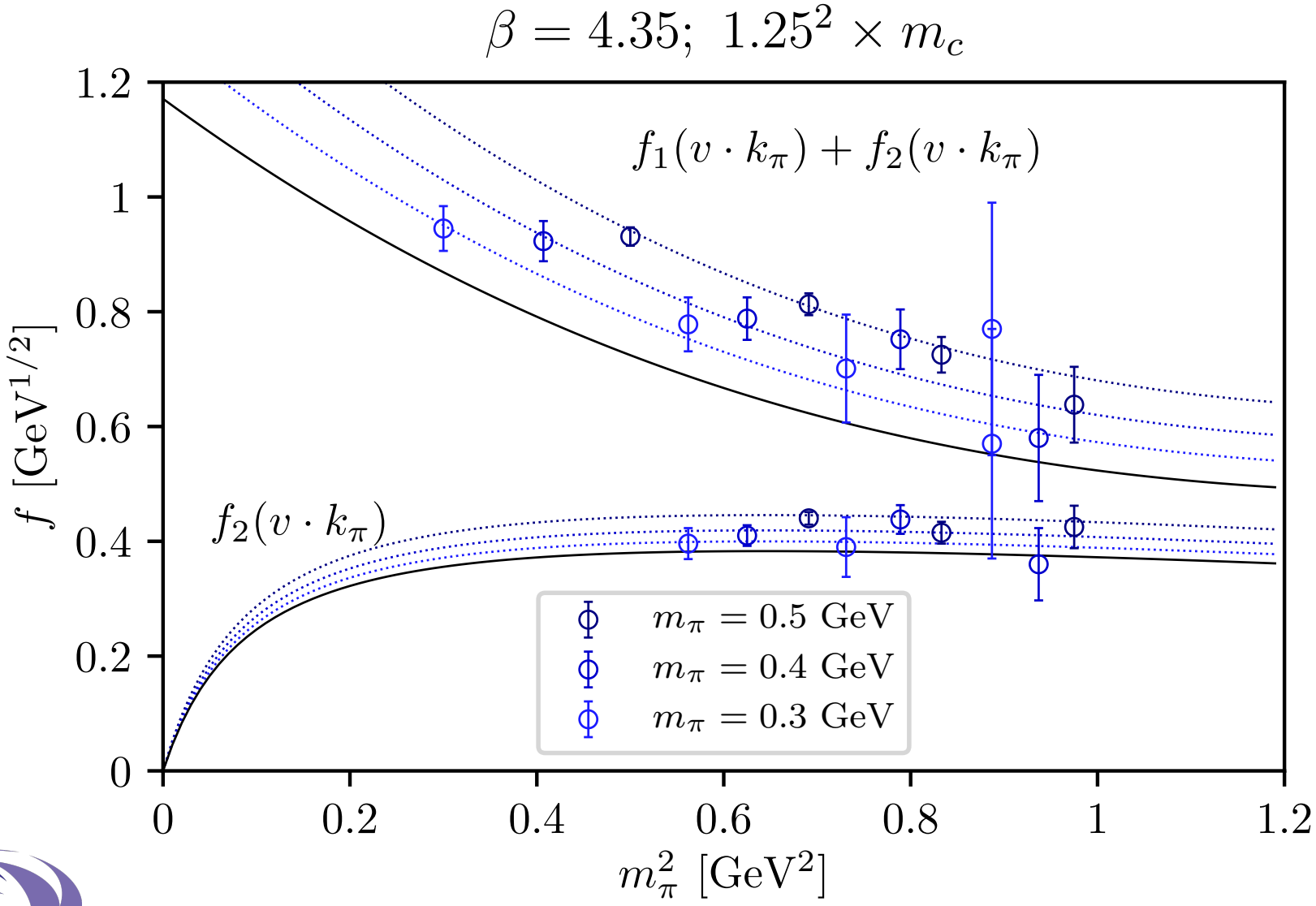
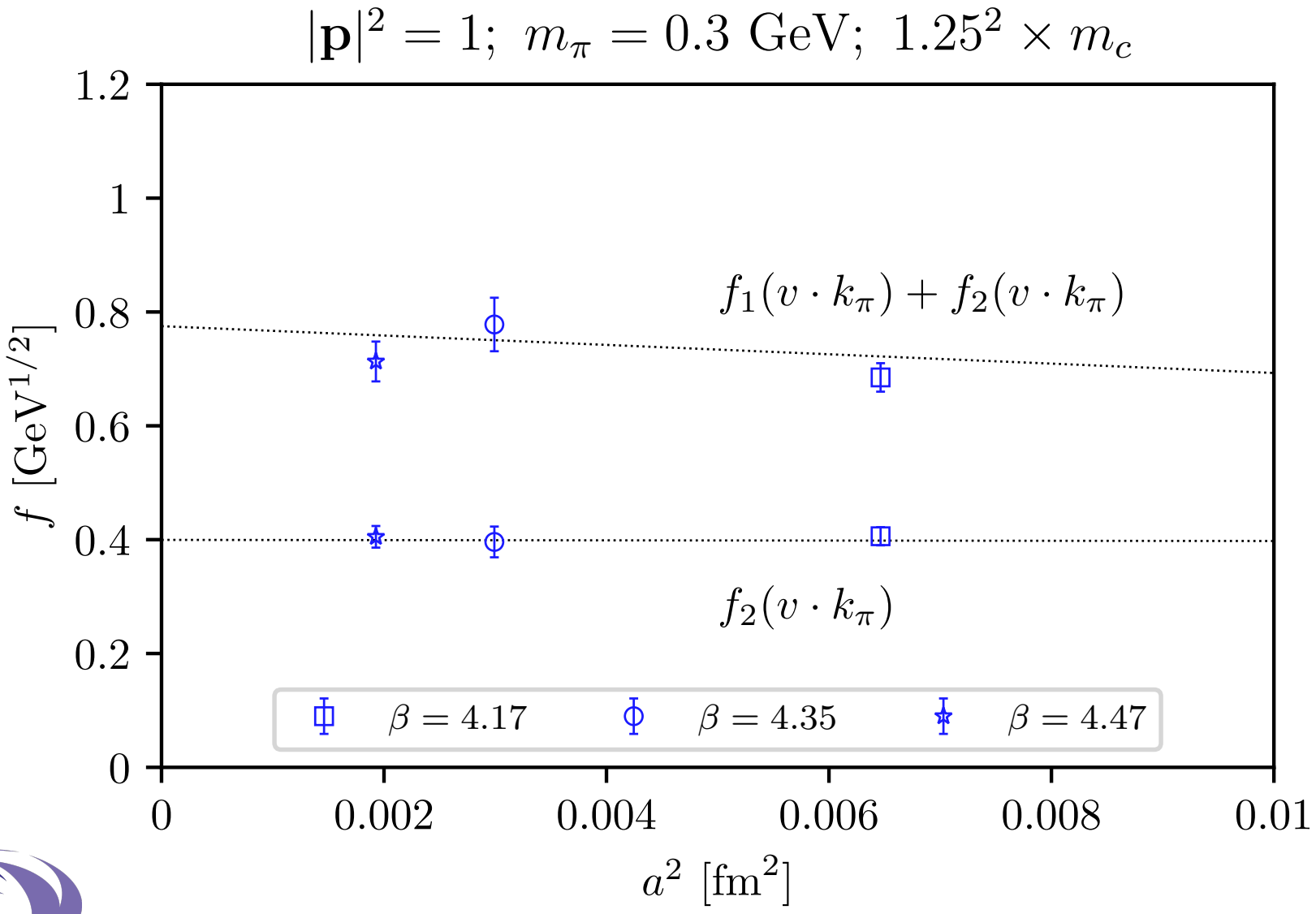
E_π Dependence



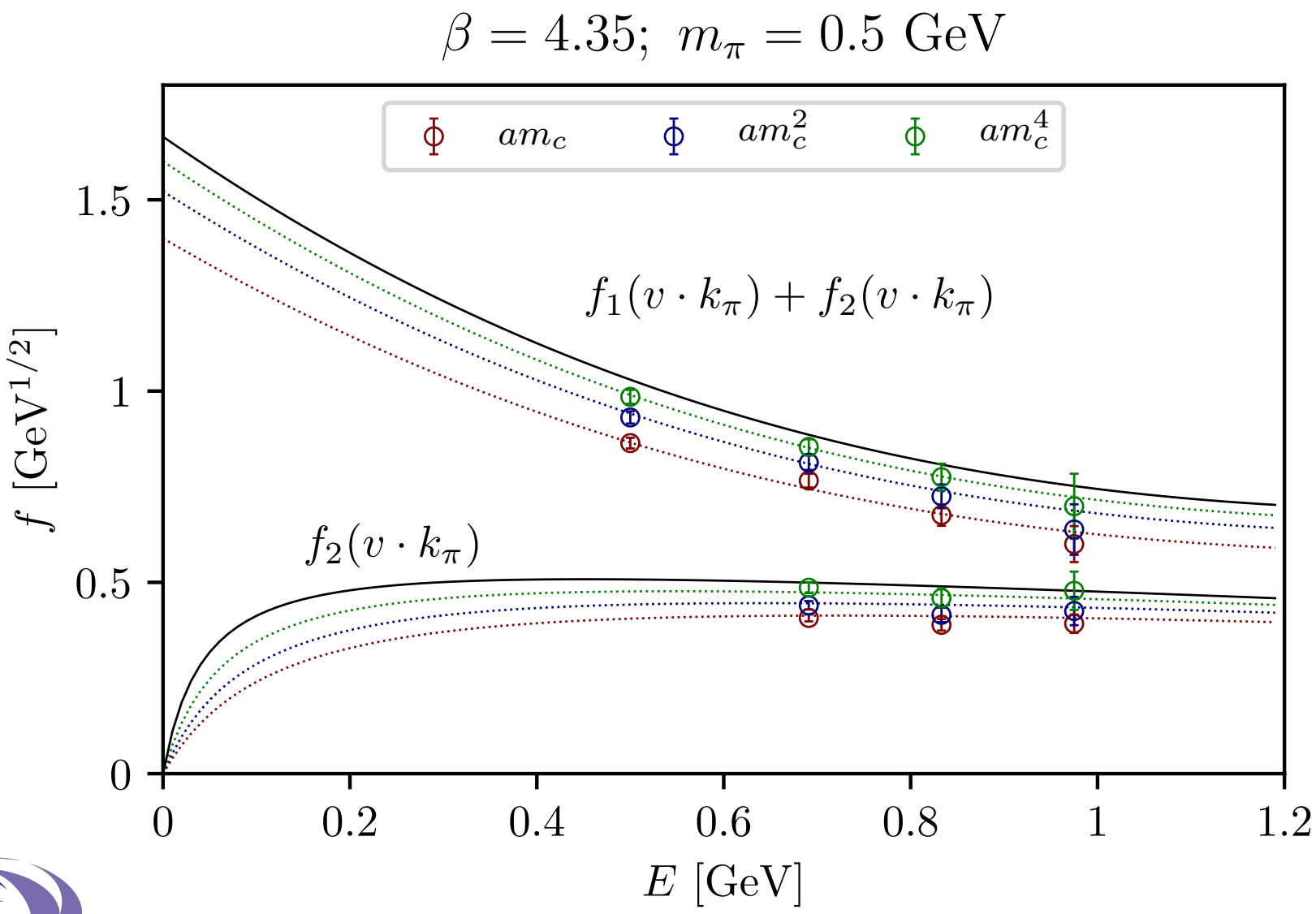
m_π dependence



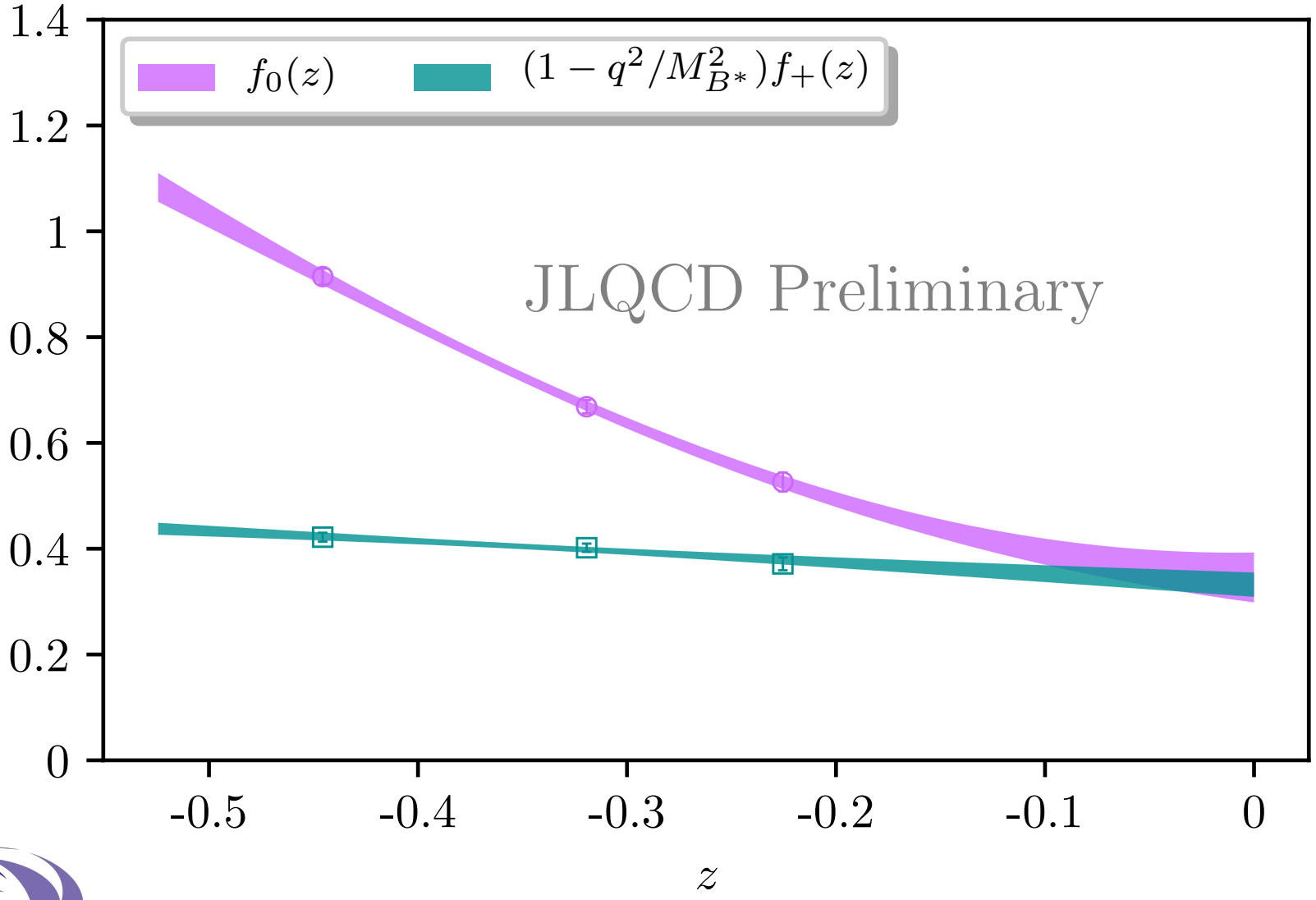
a^2 dependence



m_h dependence



z-parameter expansion



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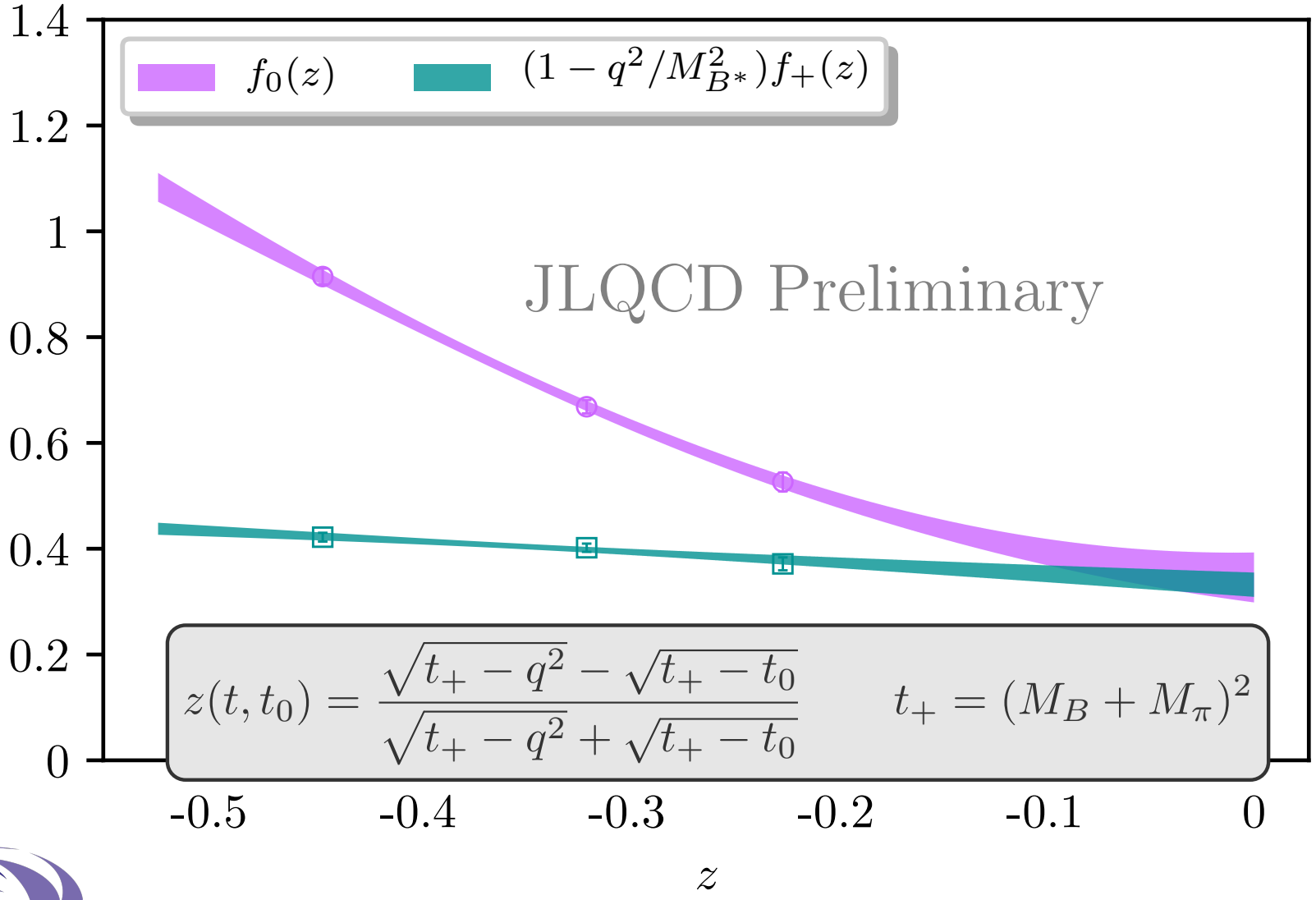
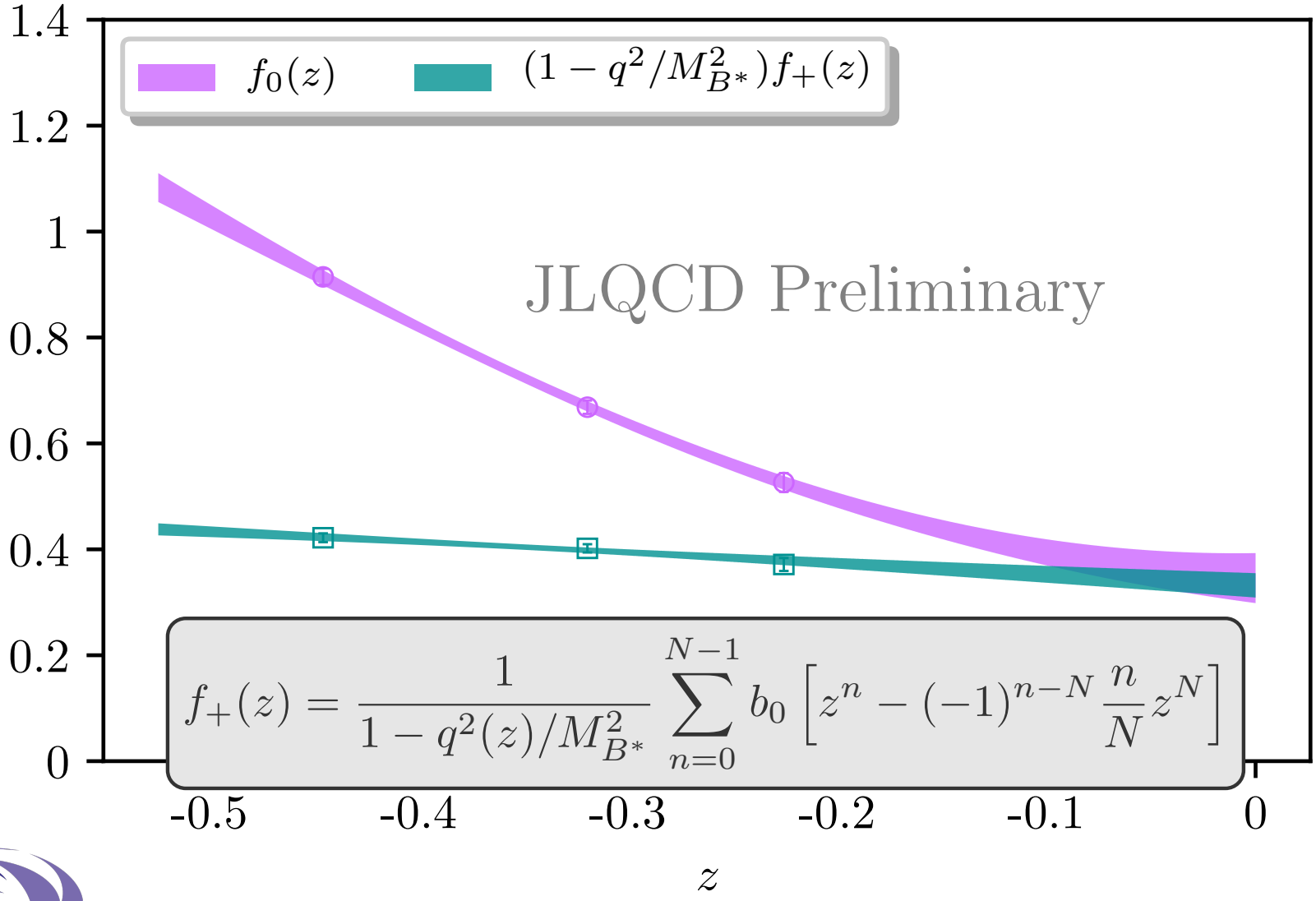


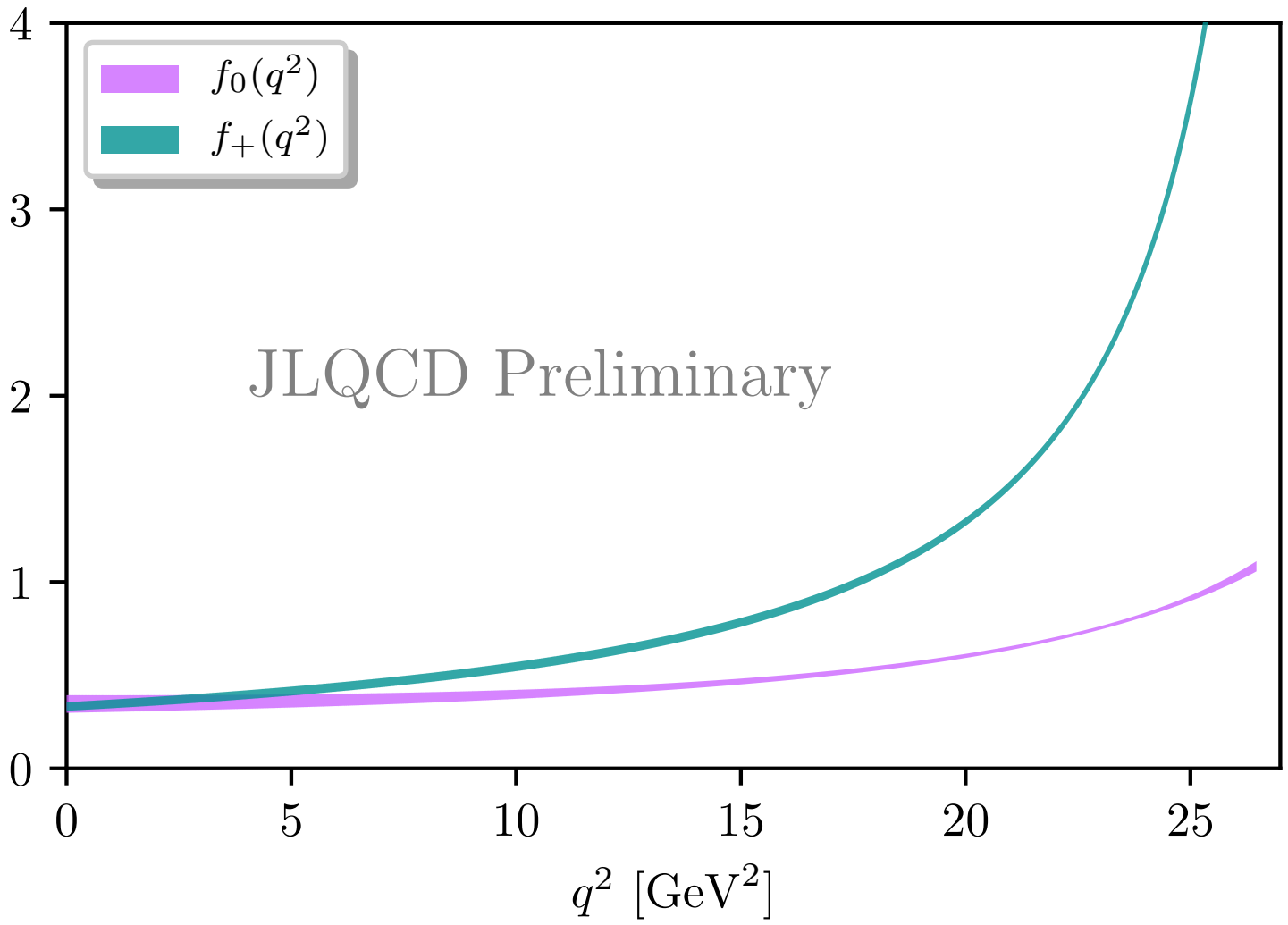
$$z(t, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \qquad t_+ = (M_B + M_\pi)^2$$

z-parameter expansion

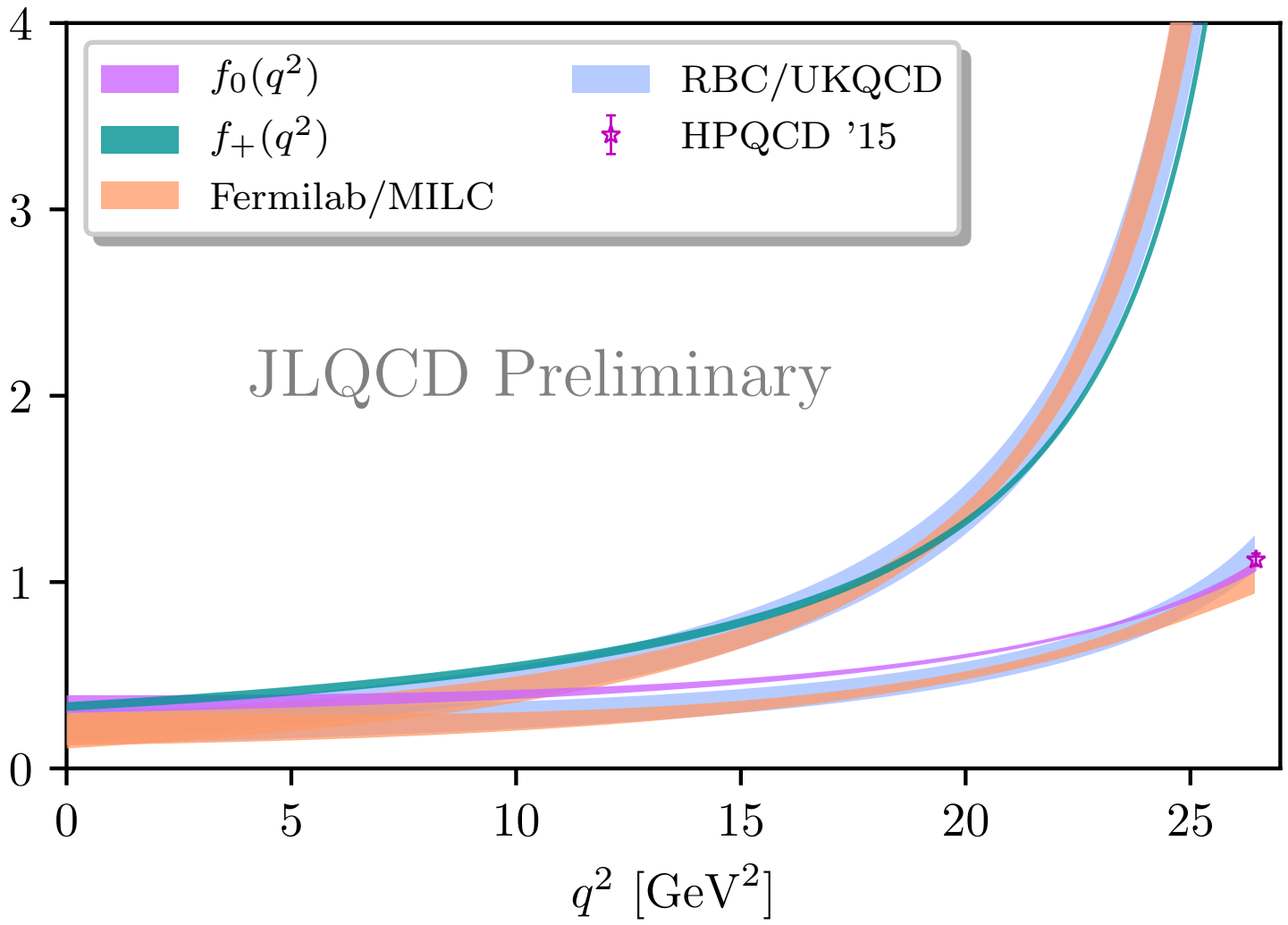


$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N-1} b_0 \left[z^n - (-1)^{n-N} \frac{n}{N} z^N \right]$$

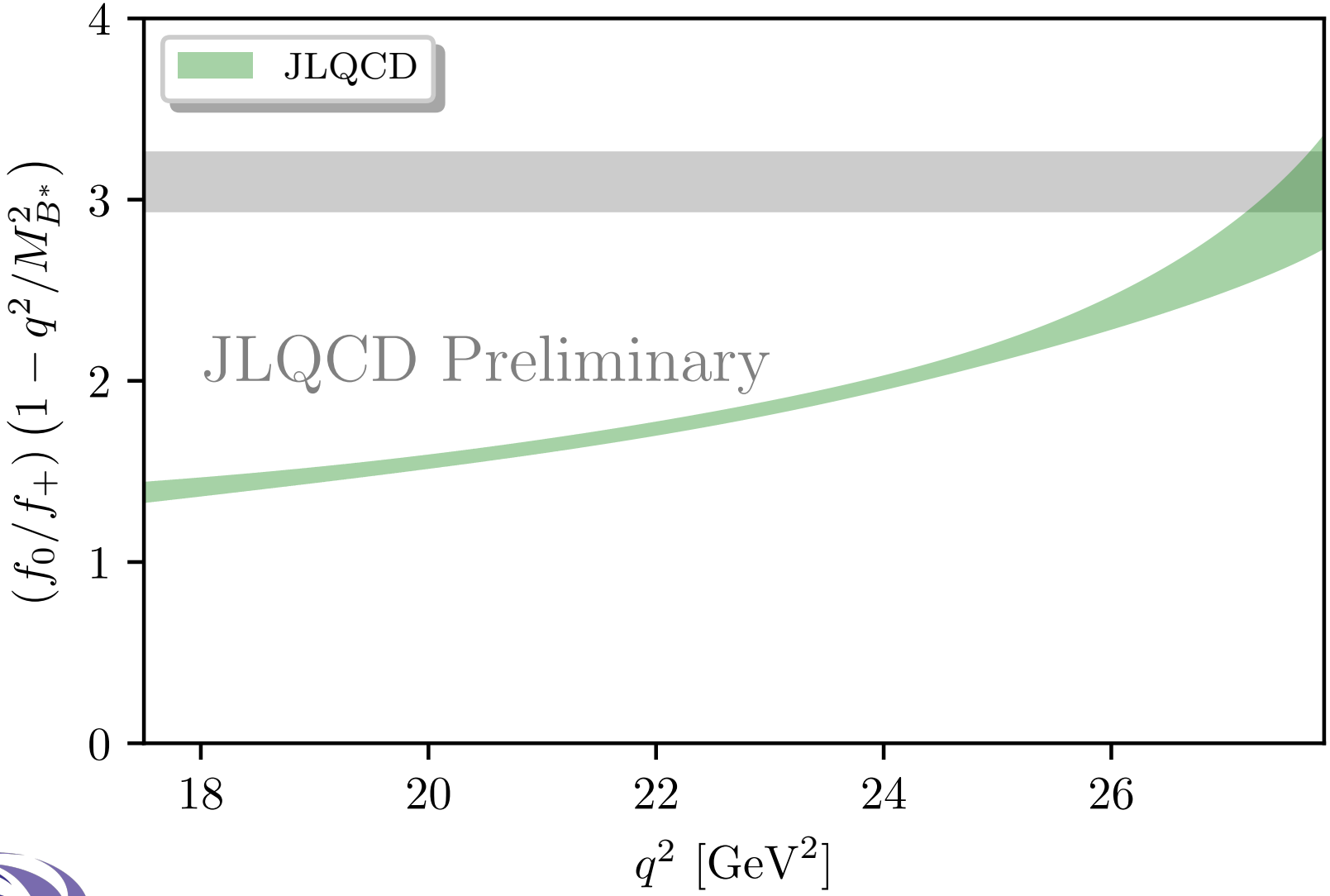
Fitted Form Factors



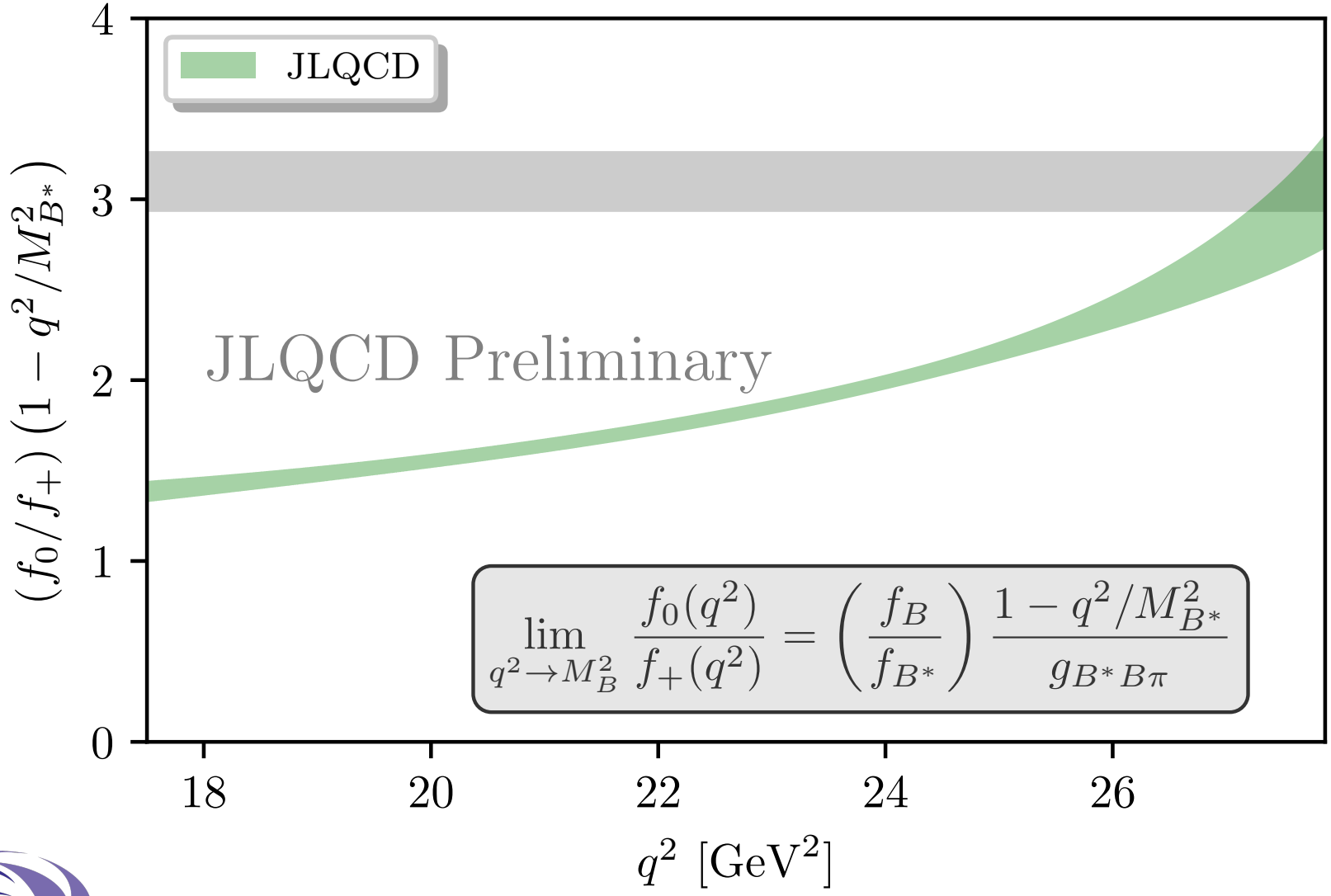
Fitted Form Factors



Soft pion relation



Soft pion relation



Summary

- We can use MDWF for semileptonic B decays
- Now we will push towards $|V_{ub}|$
  - Fit z-parameter expansions with experiment $\rightarrow |V_{ub}|$
- Effectively an extension of JLQCD $D \rightarrow \pi \ell \nu$ calculation
- Challenges
  - HQET behaviour to reach m_b
    - $1/m_b$ term explains data
    - (Possible: Push to higher m_h on finest lattice: $am_h \lesssim 0.8$)
  - Chiral extrapolation: chiral logs
  - Limited to high q^2

Thank You

Backup

More form factor relationships

Matrix Elements

Working in the B meson rest frame:

$$
\begin{aligned}
f_1\left(v\cdot k\right)+f_2\left(v\cdot k\right) &= \frac{\langle\pi(k_\pi)|V^0|B(p_B)\rangle}{2\sqrt{m_B}};\\
f_2\left(v\cdot k\right) &= \frac{\langle\pi(k_\pi)|V^i|B(p_B)\rangle}{2\sqrt{m_B}}\frac{v\cdot k_\pi}{k_\pi^i}
\end{aligned}
$$

Pole dominance model:

$$
\lim_{v\cdot k_\pi\to 0} f_2\left(v\cdot k\right) = g\frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\frac{v\cdot k_\pi}{v\cdot k_\pi+\Delta_B},
$$

with,

$$
g:\ B^*B\pi \text{ coupling;}\qquad \Delta_B:\ m_{B^*}-m_B.
$$

Semileptonic Decays

These two form factor definitions are related by:

$$
\begin{aligned}
f_+\left(q^2\right) =& \sqrt{m_B}\left[\frac{f_2\left(v\cdot k\right)}{v\cdot k_\pi}+\frac{f_1\left(v\cdot k\right)}{m_B}\right];\\
f_0\left(q^2\right) =& \frac{2}{\sqrt{m_B}}\frac{m_B^2}{m_B^2-m_\pi^2}\left[f_1\left(v\cdot k\right)+f_2\left(v\cdot k\right)\right.\\
&\left.-\frac{v\cdot k_\pi}{m_B}\left(f_1\left(v\cdot k\right)+\frac{m_\pi^2}{\left(v\cdot k_\pi\right)^2}f_2\left(v\cdot k\right)\right)\right]
\end{aligned}
$$

Semileptonic Decays

These two form factor definitions are related by:

In the heavy quark limit, then:

$$f_+\left(q^2\right) \sim \sqrt{m_B} f_2\left(v \cdot k\right);$$
$$f_0\left(q^2\right) \sim \frac{f_1\left(v \cdot k\right) + f_2\left(v \cdot k\right)}{\sqrt{m_B}}$$