Exploring the convergence radius of HBχPT

Karl Sallmen

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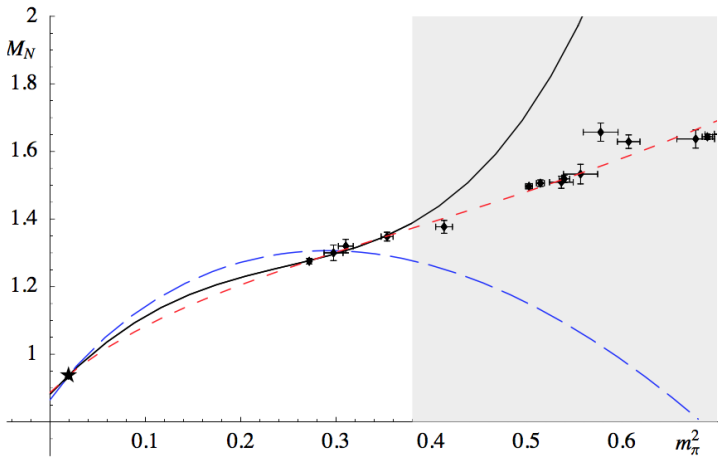
Introduction

Due to the non-perturbative nature of QCD at low-energies and the frequent use of chiral extrapolations of lattice QCD data, there exists looming questions about the convergence of chiral expansions. In particular for the range of quark-masses being used on the lattice.



motivation

Show that including higher order terms in the correction of m_N from chiral EFT ruins the agreement with data for values of the pion mass larger than ($m_\pi = 600 MeV$)

This is also the case for g_A!

With data for m_N and g_A in a range of "smaller" pion masses, perhaps we could attempt to address the question of convergence.

At higher orders in HBχPT, starting at N2LO ($\mathcal{O}\left(m_\pi^3\right)$), g_A couples to m_n

$$\delta m_n = -\frac{3g_A^2}{32\pi F_\pi} m_\pi^3$$

To capture the effect of the pion mass dependence of g_A, it is advantageous to perform a combined analysis (simultaneous fitting) of both m_N andg_A

HBχPT "quickly"

Incorporating heavy fermionic fields into χPT, despite being chirally consistent, unfortunately has the undesired effect of introducing a mass that is of the same order as the chiral symmetry breaking scale $m_{hf}/\Lambda_\chi \sim 1$, and thus there is no longer a consistent derivative expansion for processes involving these heavy fermionic fields.

Additionally Weinberg's power counting fails since higher order loop graphs can produce amplitudes which are no longer supressed by Λ_χ.

A solution to this problem was first provided by Elizabeth Jenkins and Aneesh V. Manohar.

HBχPT "quickly"

Nearly on-shell Baryon with velocity v_μ has a momentum given by

$p_\mu = m_B v_\mu + k_\mu$

Where k_μ is a small off-shell contribution

The effective theory is then written in terms Baryon fields B_v

$$B_v = e^{i m_B \not{v} v_\mu x^\mu} B(x)$$

Derivatives act on B_v by producing k, the higher derivative terms are thus suppressed by $1/\Lambda_\chi$ In addition the heavy Baryon Lagrangian has a $1/m_B$ expansion. For the lightest Baryon multiplets this can be combined with $1/\Lambda_\chi$ into a single expansion in $1/\Lambda_\chi$. Our theory has now a consistent power counting expansion.

Gathering expressions for m_N, g_A

Up to order m_π^5

$$m_N = m_0 + \delta m_N^{2+4} + \delta m_N^{3+5}$$

$$\delta m_N^{2+4} = -4c_1 m_\pi^2 + m_\pi^4 \left[-e'(L_\chi) + \frac{3}{128\pi^2 f_\pi^2} \left(c_2 - \frac{2g_A^2}{M} \right) \right] - \frac{m_\pi^4}{32\pi^2 f_\pi^2} \ln \frac{m_\pi}{L_\chi} \left(\frac{3g_A^2}{M} - 32c_1 + 3c_2 + 12c_3 \right)$$

and

$$\delta m_N^{3+5} = -\frac{3g_A m_\pi^3}{32\pi f_\pi^2} + \frac{3g_A m_\pi^5}{32\pi f_\pi^2} \left[\frac{2l_4^r(L_\chi)}{f^2} - \frac{4\left(2d_{16}^r(L_\chi) - d_{18}\right)}{g} + 16d_{28}^r(L_\chi) + \frac{g^2}{8\pi^2 f^2} + \frac{1}{8M^2} \right] + \frac{3g_A^4 m_\pi^5}{64\pi^3 f_\pi^4} \ln \frac{m_\pi}{L_\chi}$$

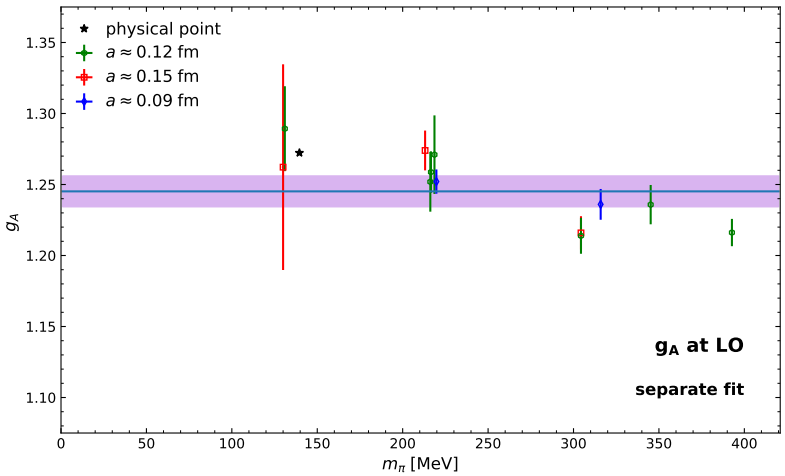
Gathering expressions for m_N, g_A

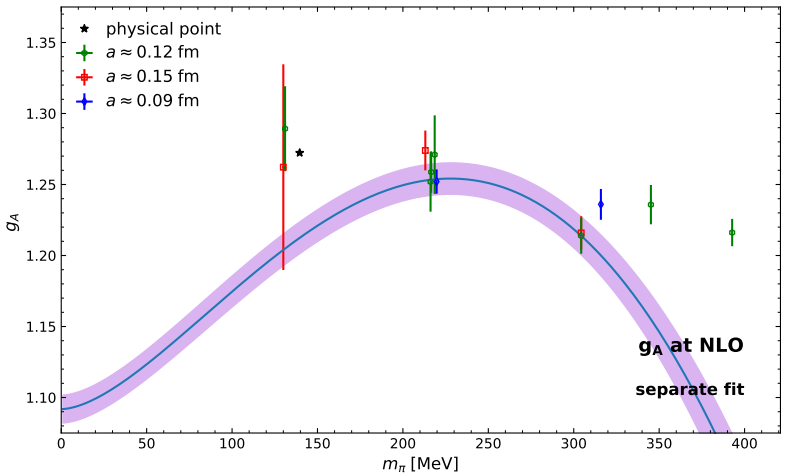
$$g_A = g_0 \left[1 + \left(\frac{\alpha_2}{(4\pi F_\pi)^2} \ln \frac{m_\pi}{L_\chi} + \beta_2 \right) m_\pi^2 + \alpha_3 m_\pi^3 \right.$$
$$\left. + \left(\frac{\alpha_4}{(4\pi F_\pi)^4} \ln^2 \frac{m_\pi}{L_\chi} + \frac{\gamma_4}{(4\pi F_\pi)^2} \ln \frac{m_\pi}{L_\chi} + \beta_4 \right) m_\pi^4 + \alpha_5 m_\pi^5 \right]$$

m_N	g_A	$\mathcal{O}(m_\pi)$
c_1	$d_{16}^r,\ d_{28}^r$	m_π^2
	$c_3,\ c_4$	m_π^3
$e',\ c_2,\ c_1,\ c_3$	$l_4^r,\ c_4,\ c_3,\ d_{16}^r$	m_π^4
$l_4^r,\ d_{16}^r,\ d_{18},\ d_{28}^r$	$c_3,\ c_4,\ l_4^r$	m_π^5

table of LEC's appearing at different orders of m_π

plots A


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
g_A at LO
separate fit
g_A
m_π [MeV]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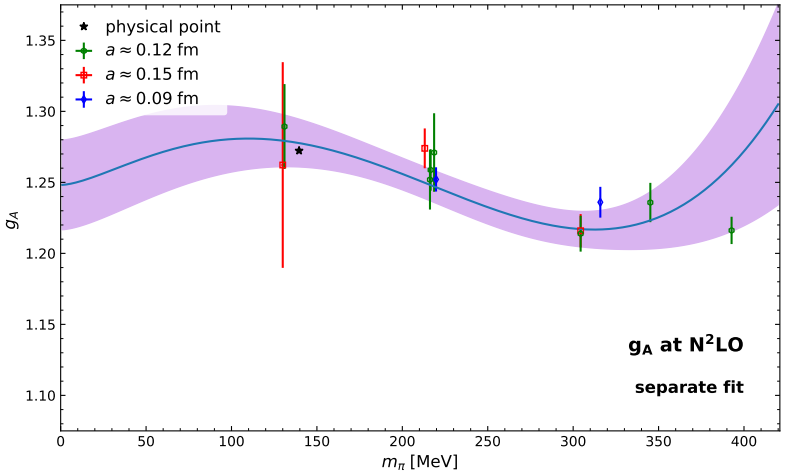
g_A at NLO
separate fit
g_A
m_π [MeV]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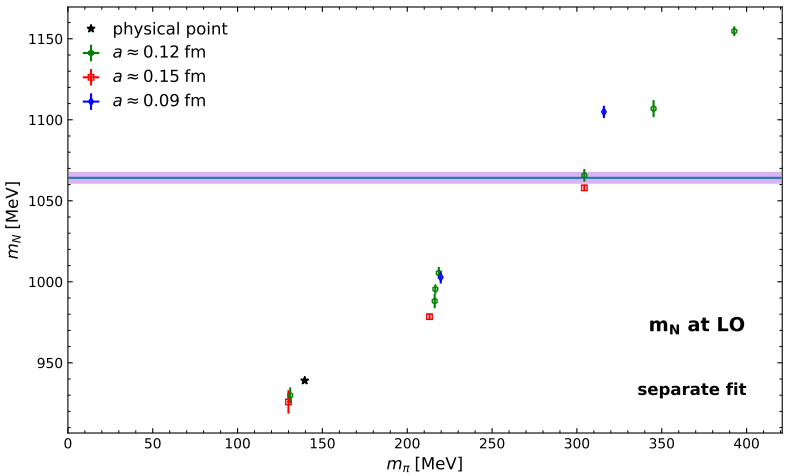
g_A at N^{2}LO
separate fit
g_A
m_π [MeV]

plots B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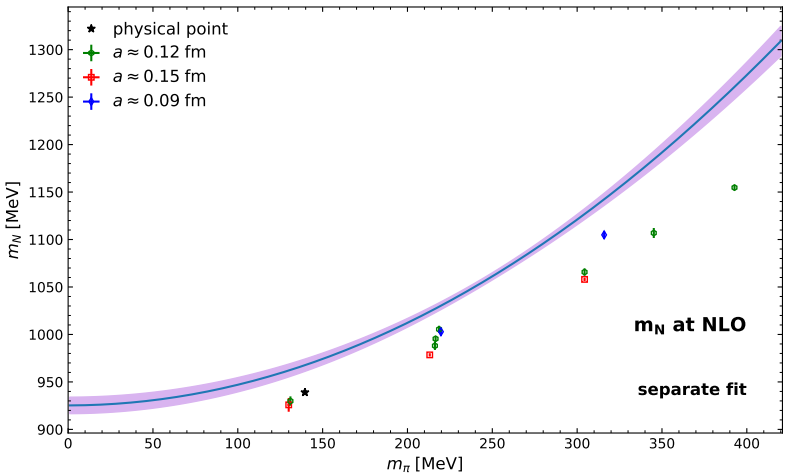
m_N [MeV]
m_π [MeV]
$\mathbf{m_N}$ **at LO**
separate fit


physical point
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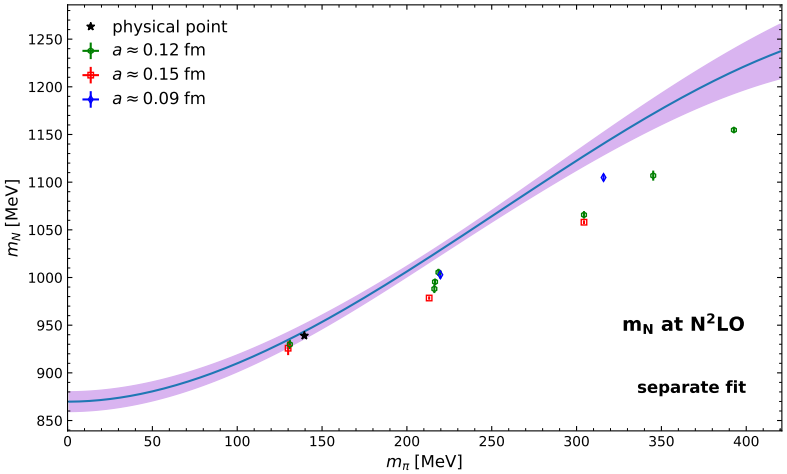
m_N [MeV]
m_π [MeV]
$\mathbf{m_N}$ **at NLO**
separate fit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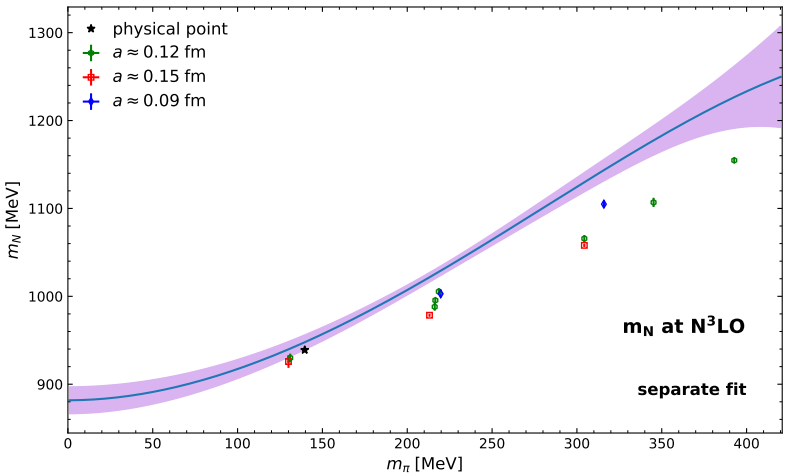
m_N [MeV]
m_π [MeV]
$\mathbf{m_N}$ **at N^{2}LO**
separate fit

plot C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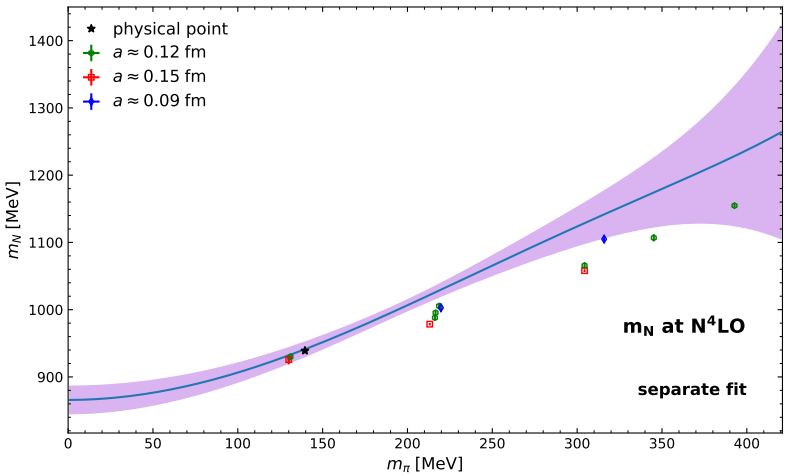
m_N at N^{3}LO
separate fit
m_N [MeV]
m_π [MeV]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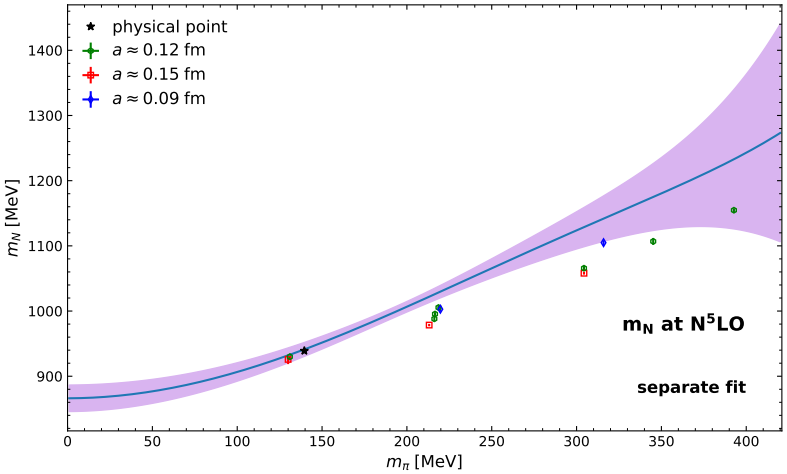
m_N at N^{4}LO
separate fit
m_N [MeV]
m_π [MeV]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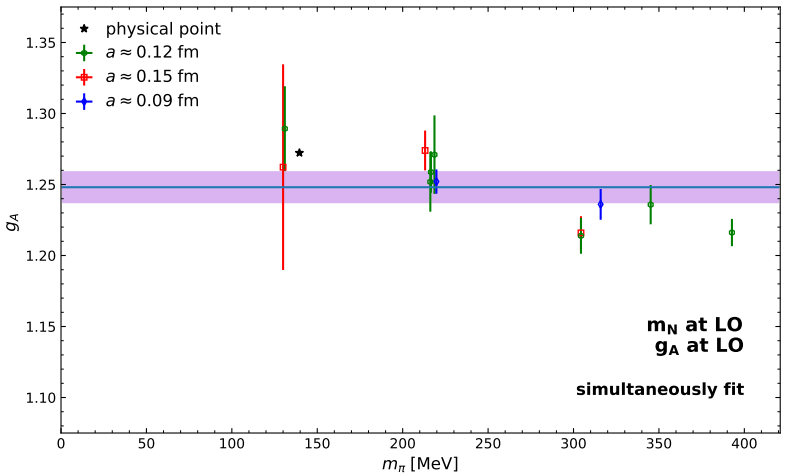
m_N at N^{5}LO
separate fit
m_N [MeV]
m_π [MeV]

plot D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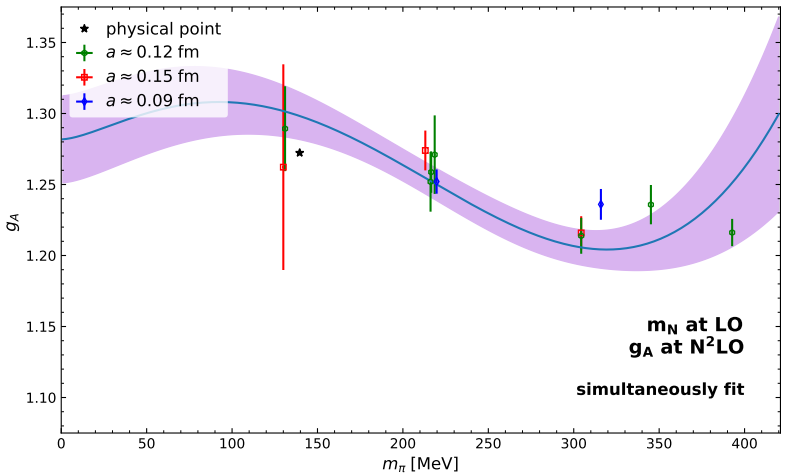
g_A
m_π [MeV]
m_N at LO
g_A at LO
simultaneously fit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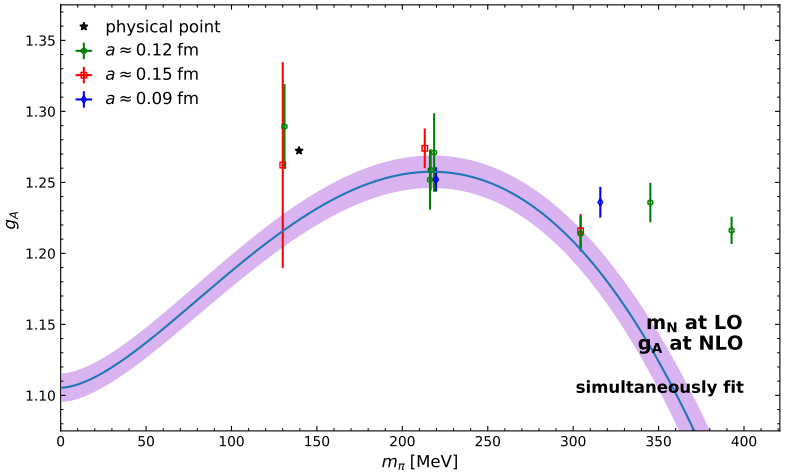
g_A
m_π [MeV]
m_N at LO
g_A at N^2LO
simultaneously fit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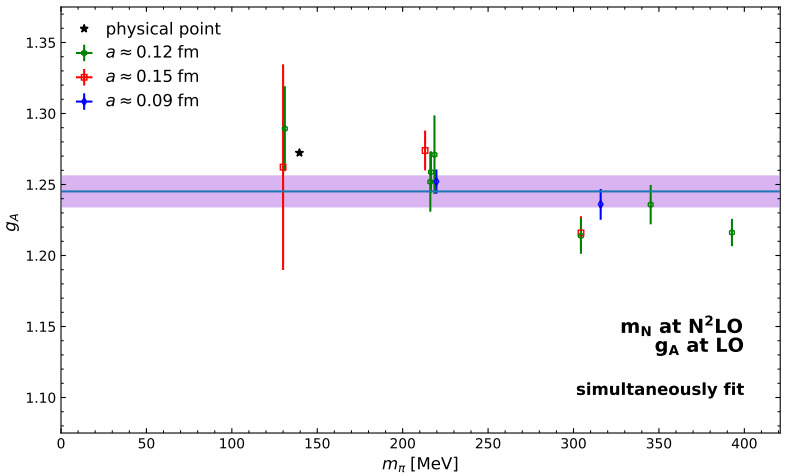
g_A
m_π [MeV]
m_N at LO
g_A at NLO
simultaneously fit

plot E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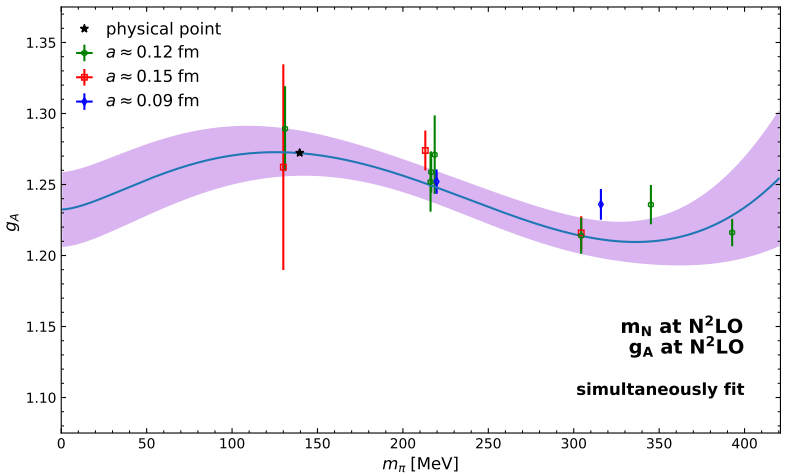
g_A
m_π [MeV]
m_N at N^2LO
g_A at LO
simultaneously fit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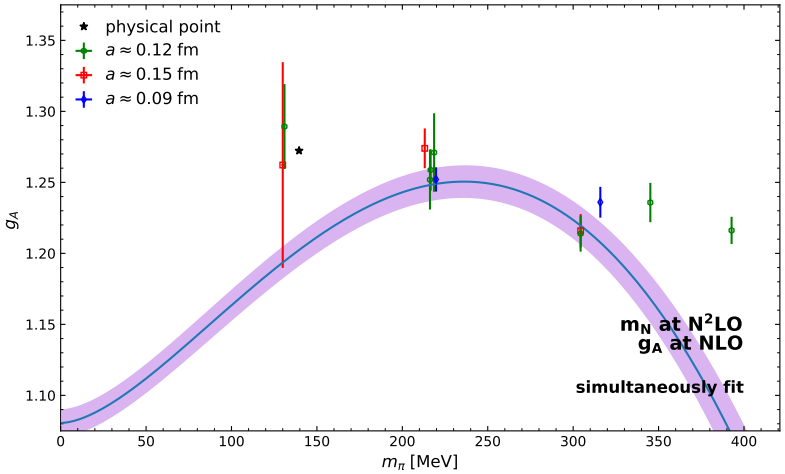
g_A
m_π [MeV]
m_N at N^2LO
g_A at N^2LO
simultaneously fit


physical point
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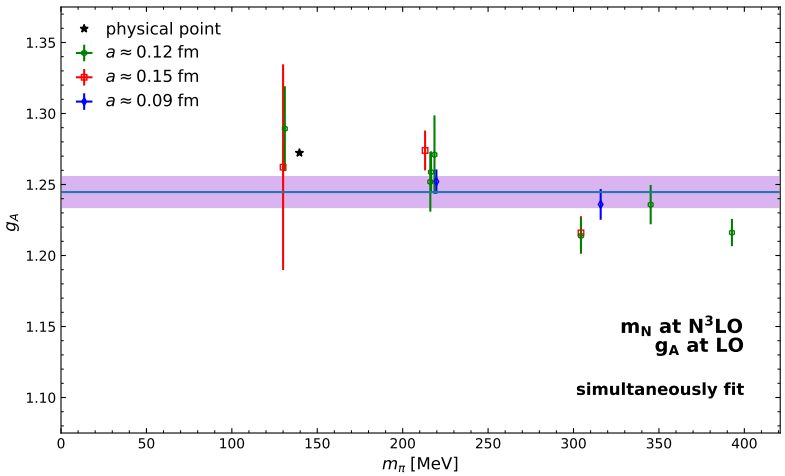
g_A
m_π [MeV]
m_N at N^2LO
g_A at NLO
simultaneously fit

plot F


physical point
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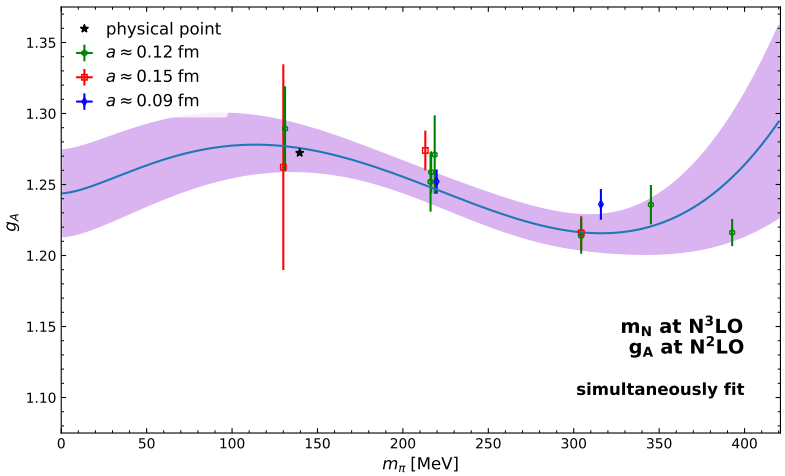
m_N at N^{3}LO
g_A at LO
simultaneously fit
g_A
m_π [MeV]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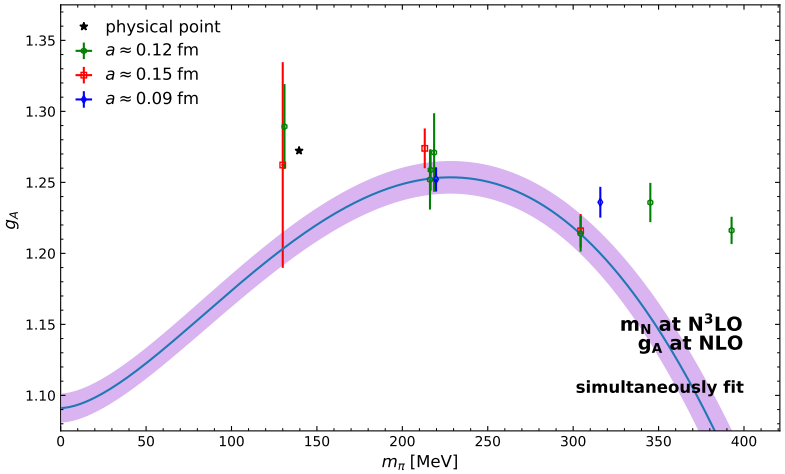
m_N at N^{3}LO
g_A at N^{2}LO
simultaneously fit
g_A
m_π [MeV]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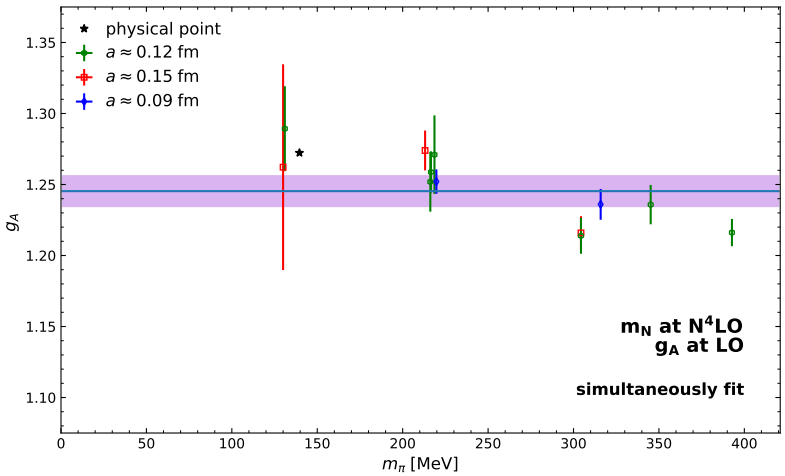
m_N at N^{3}LO
g_A at NLO
simultaneously fit
g_A
m_π [MeV]

plot G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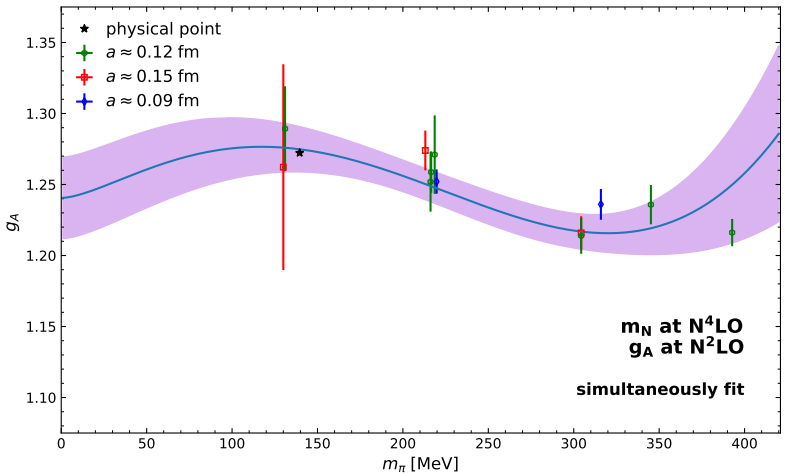
m_N at N^4LO
g_A at LO
simultaneously fit
g_A
m_π [MeV]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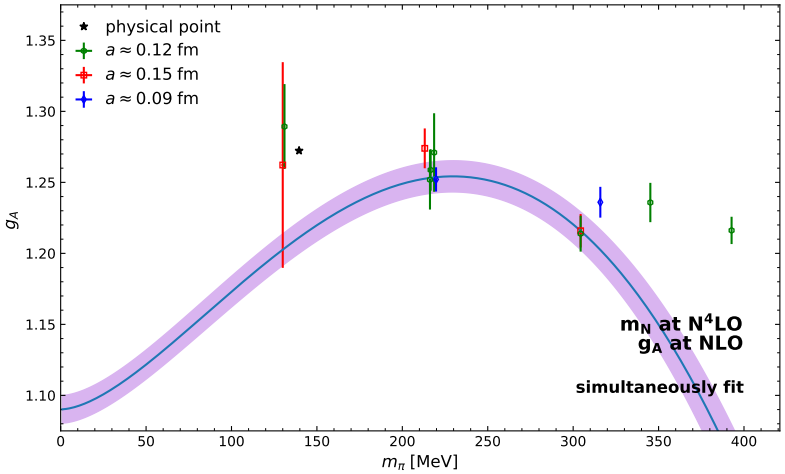
m_N at N^4LO
g_A at N^2LO
simultaneously fit
g_A
m_π [MeV]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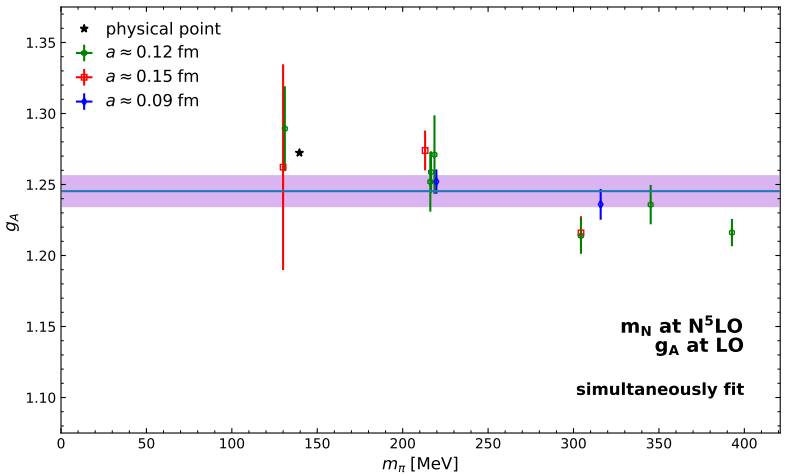
m_N at N^4LO
g_A at NLO
simultaneously fit
g_A
m_π [MeV]

plot H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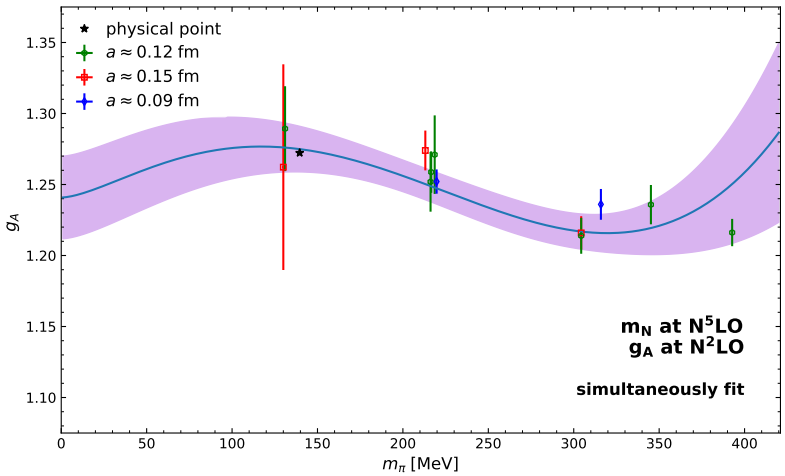
g_A
m_π [MeV]
m_N at N^5LO
g_A at LO
simultaneously fit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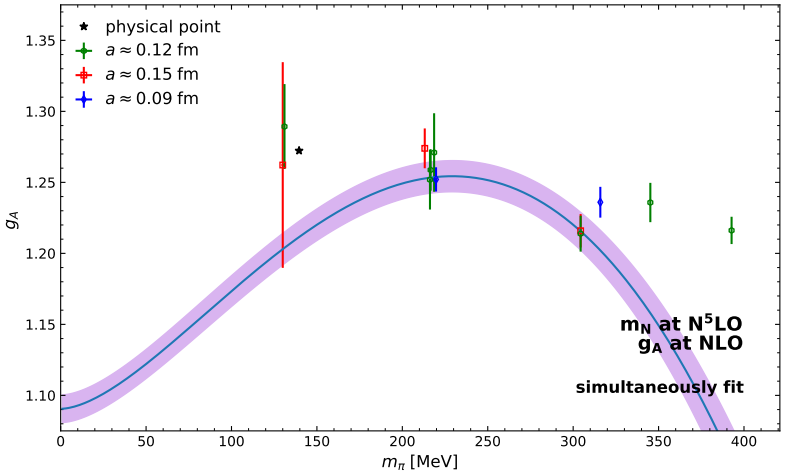
g_A
m_π [MeV]
m_N at N^5LO
g_A at N^2LO
simultaneously fit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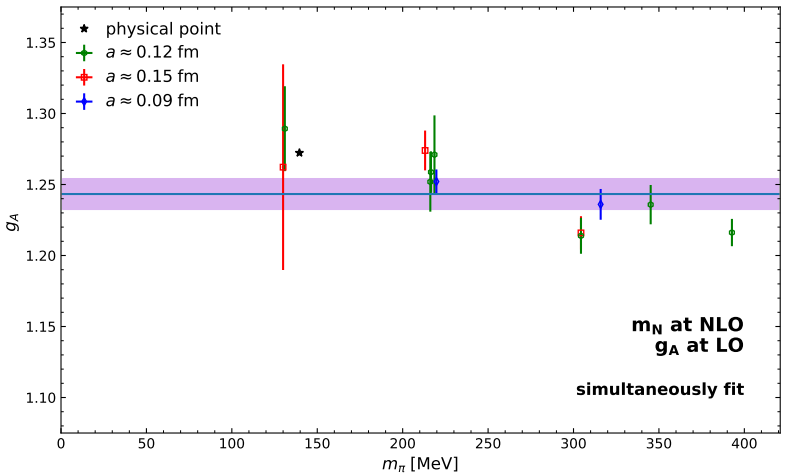
g_A
m_π [MeV]
m_N at N^5LO
g_A at NLO
simultaneously fit

plot I


physical point
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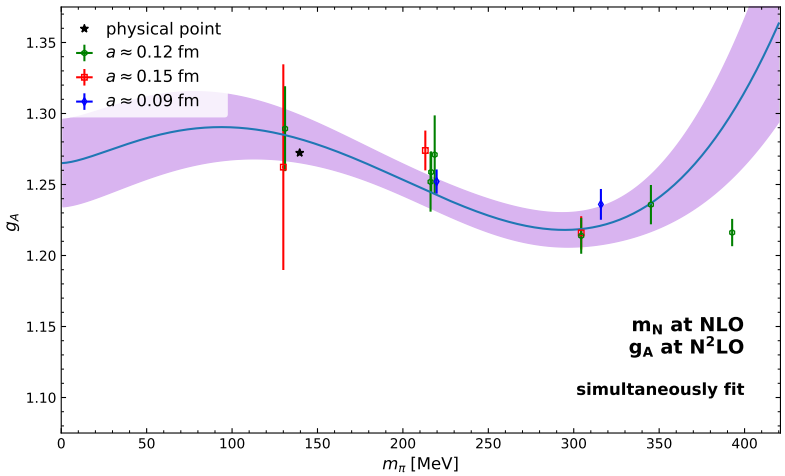
m_N at NLO
g_A at LO
simultaneously fit
g_A
m_π [MeV]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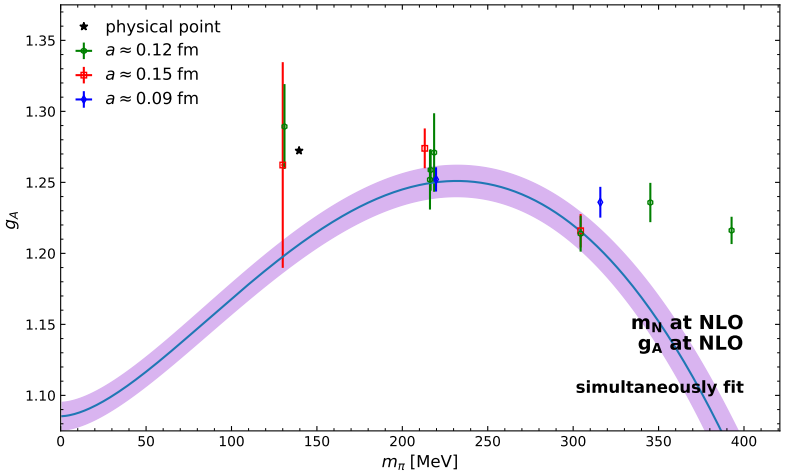
m_N at NLO
g_A at N^{2}LO
simultaneously fit
g_A
m_π [MeV]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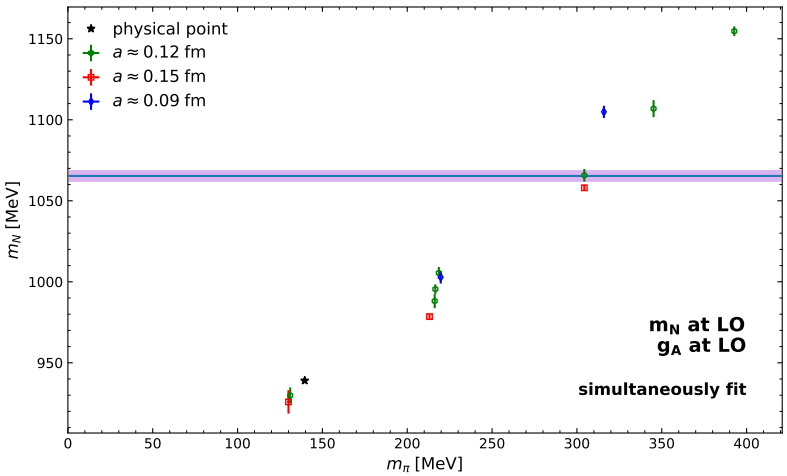
m_N at NLO
g_A at NLO
simultaneously fit
g_A
m_π [MeV]

plot J


physical point
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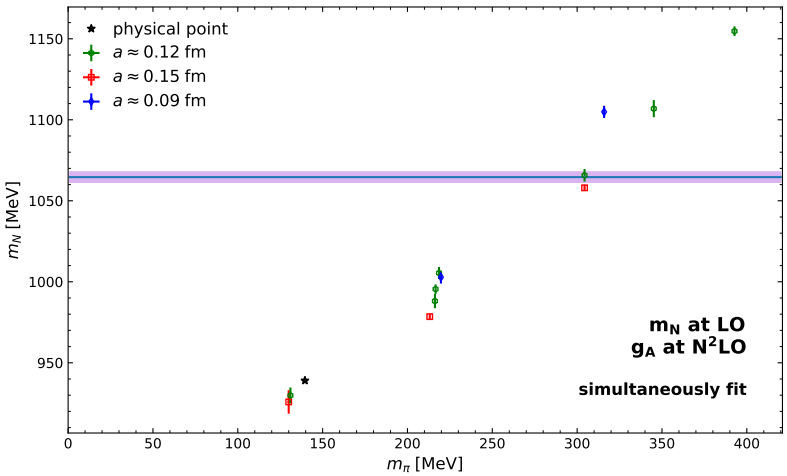
m_N [MeV]
m_π [MeV]
m_N at LO
g_A at LO
simultaneously fit


physical point
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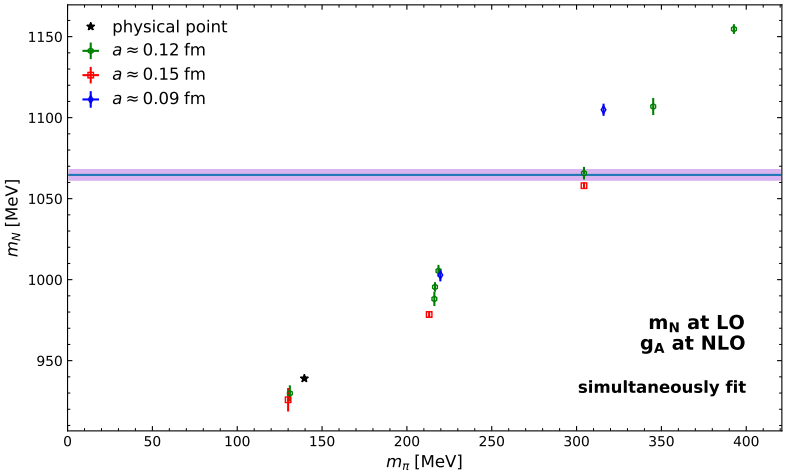
$a \approx 0.09$ fm
m_N [MeV]
m_π [MeV]
m_N at LO
g_A at N^2LO
simultaneously fit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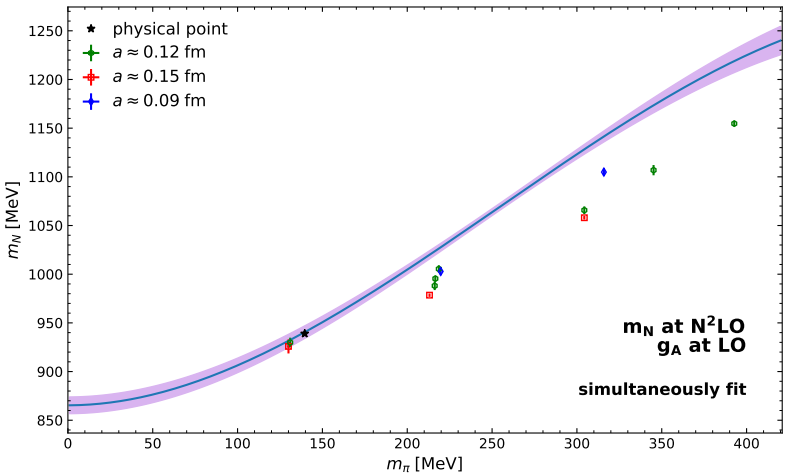
$a \approx 0.09$ fm
m_N [MeV]
m_π [MeV]
m_N at LO
g_A at NLO
simultaneously fit

plot K


physical point
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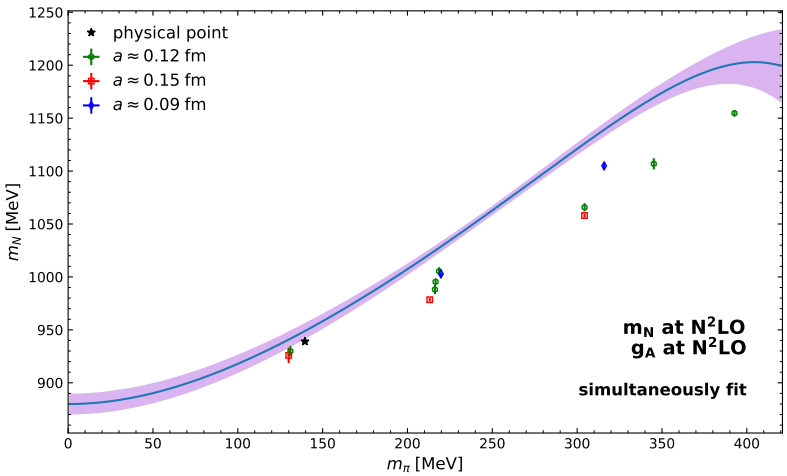
m_N at N^2LO
g_A at LO
simultaneously fit
m_N [MeV]
m_π [MeV]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m_N at N^2LO
g_A at N^2LO
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m_N [MeV]
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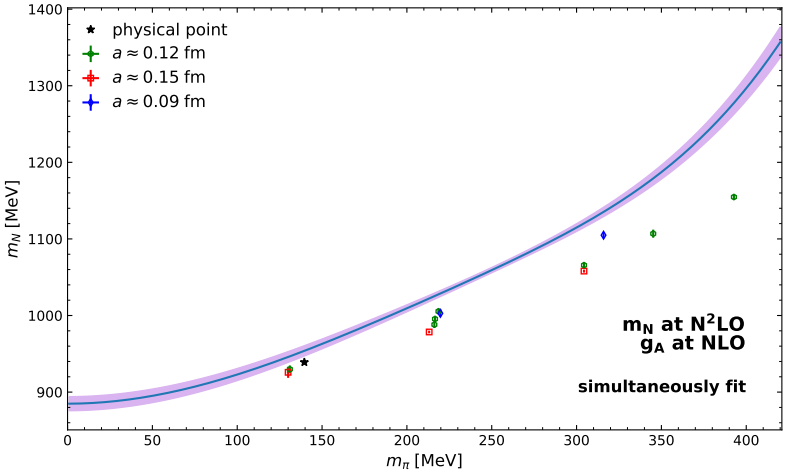
physical point
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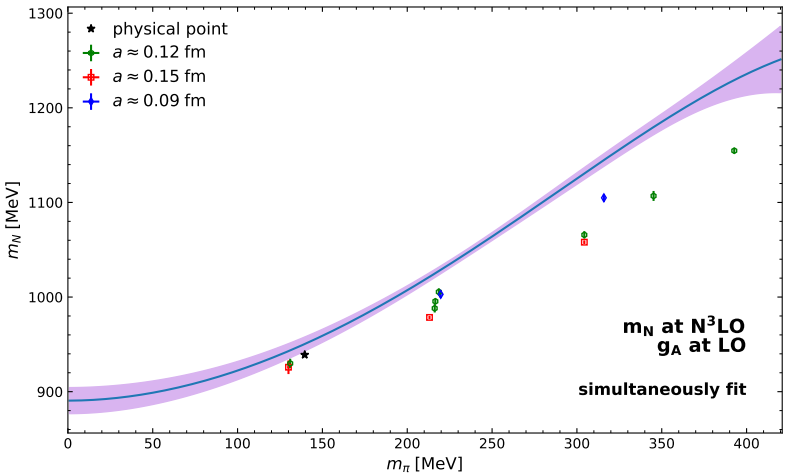
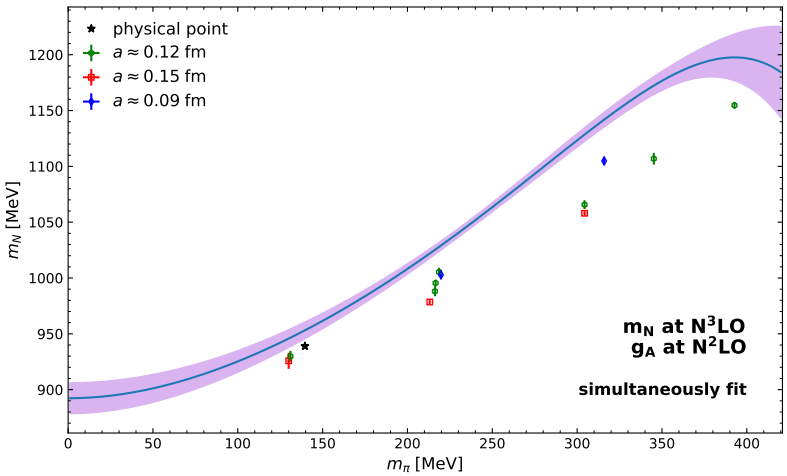
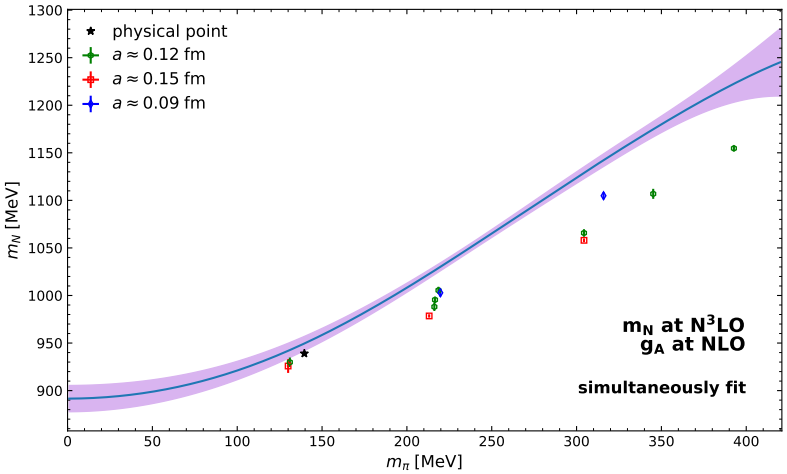
$a \approx 0.09$ fm
m_N at N^2LO
g_A at NLO
simultaneously fit
m_N [MeV]
m_π [MeV]

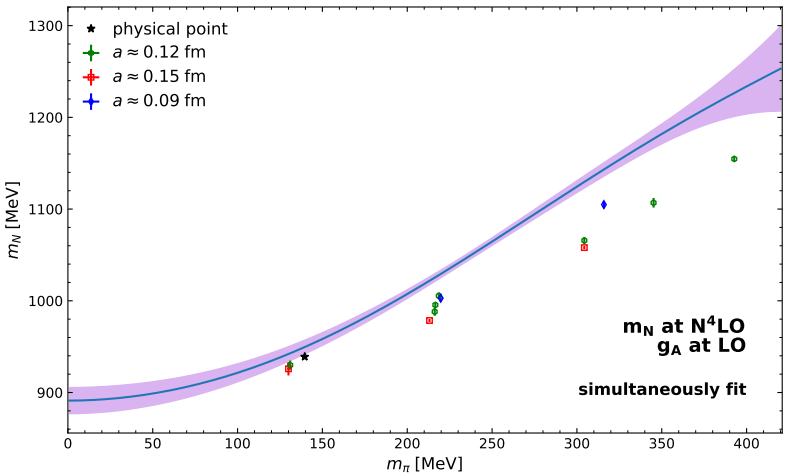
plot L

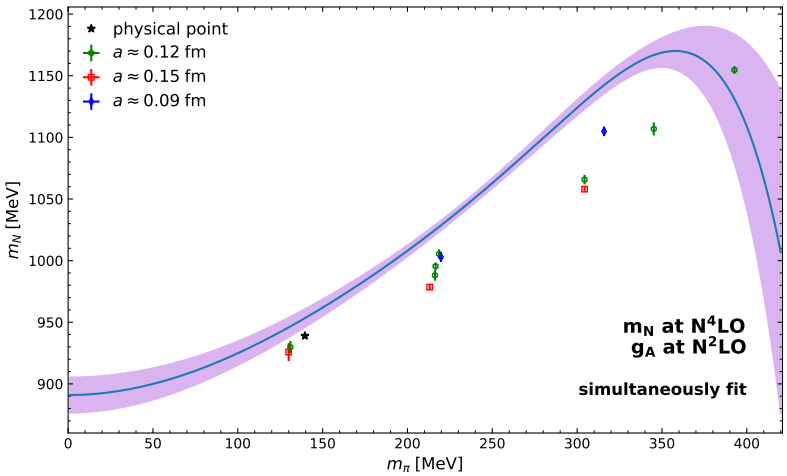

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^3LO
g_A at LO
simultaneously fit
m_N [MeV]
m_π [MeV]

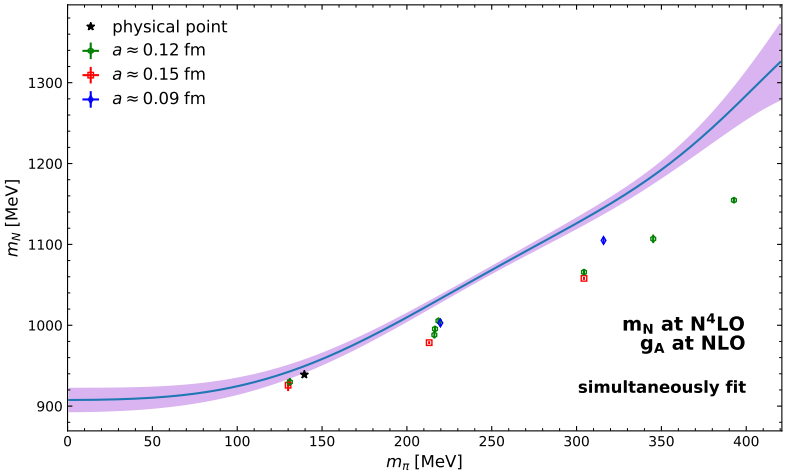

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^3LO
g_A at N^2LO
simultaneously fit
m_N [MeV]
m_π [MeV]


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^3LO
g_A at NLO
simultaneously fit
m_N [MeV]
m_π [MeV]

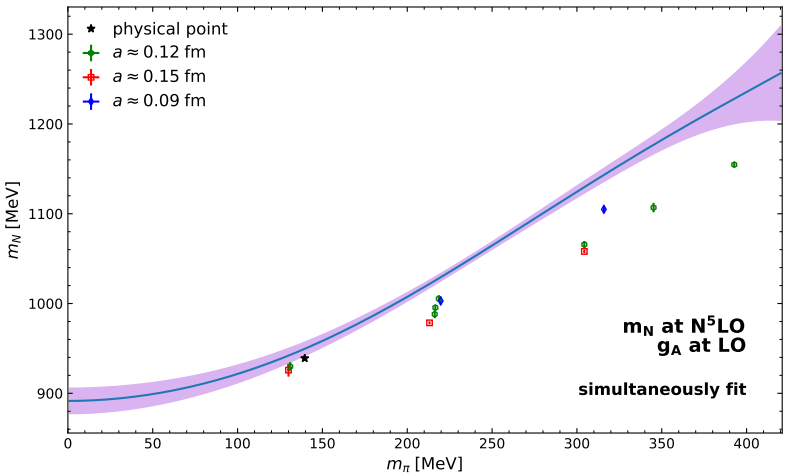
plot M

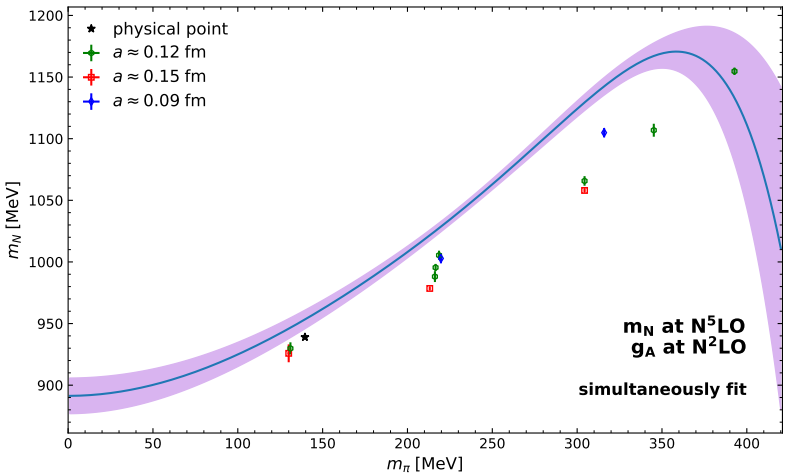

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^4LO
g_A at LO
simultaneously fit
m_N [MeV]
m_π [MeV]

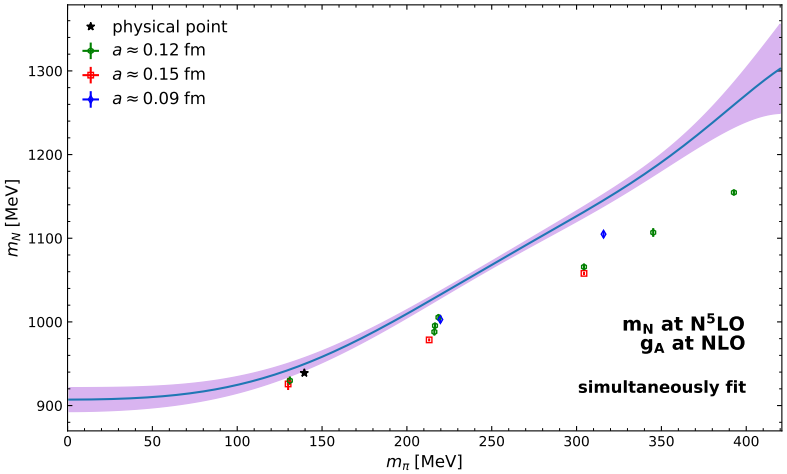

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^4LO
g_A at N^2LO
simultaneously fit
m_N [MeV]
m_π [MeV]


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^4LO
g_A at NLO
simultaneously fit
m_N [MeV]
m_π [MeV]

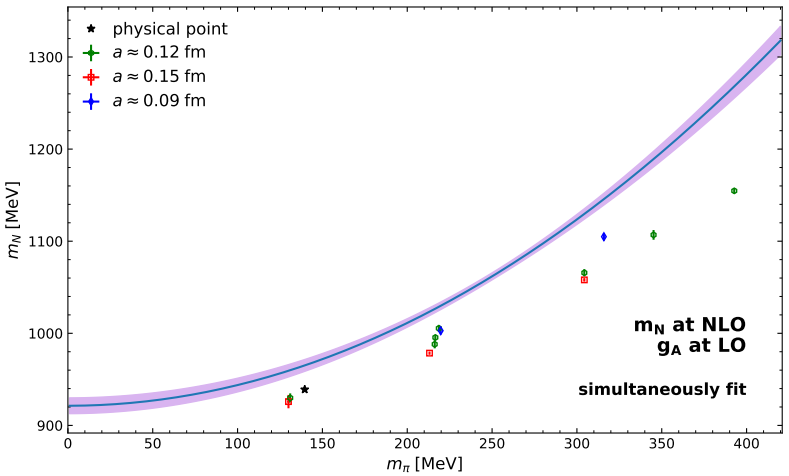
plot N

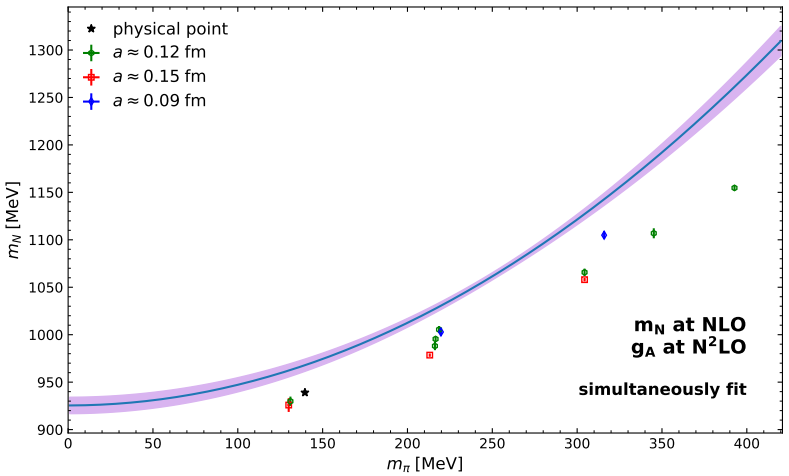

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^5LO
g_A at LO
simultaneously fit
m_N [MeV]
m_π [MeV]

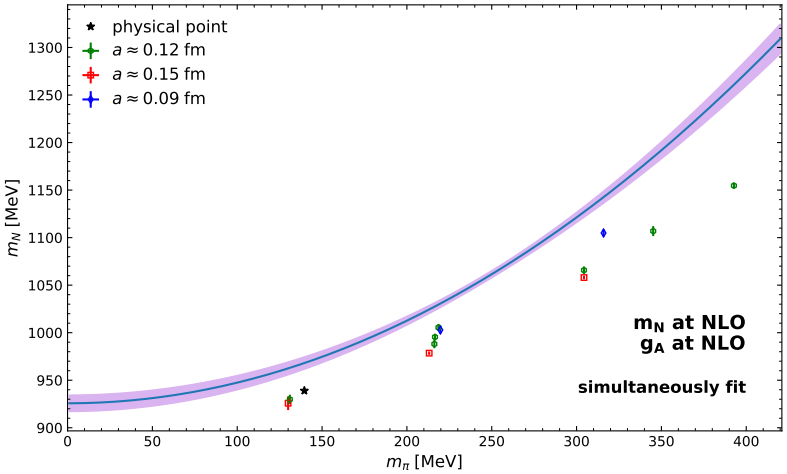

physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^5LO
g_A at N^2LO
simultaneously fit
m_N [MeV]
m_π [MeV]


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N at N^5LO
g_A at NLO
simultaneously fit
m_N [MeV]
m_π [MeV]

plot O


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N [MeV]
m_π [MeV]
m_N at NLO
g_A at LO
simultaneously fit


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N [MeV]
m_π [MeV]
m_N at NLO
g_A at N^2LO
simultaneously fit


physical point
$a \approx 0.12$ fm
$a \approx 0.15$ fm
$a \approx 0.09$ fm
m_N [MeV]
m_π [MeV]
m_N at NLO
g_A at NLO
simultaneously fit