Exploring the convergence radius of HB$\chi$PT

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Due to the non-perturbative nature of QCD at low-energies and the frequent use of chiral extrapolations of lattice QCD data, there exists looming questions about the convergence of chiral expansions. In particular for the range of quark-masses being used on the lattice.
Show that including higher order terms in the correction of \( m_N \) from chiral EFT ruins the agreement with data for values of the pion mass larger than \( m_\pi = 600 \text{MeV} \)

This is also the case for \( g_A \)!

With data for \( m_N \) and \( g_A \) in a range of "smaller" pion masses, perhaps we could attempt to address the question of convergence.

At higher orders in HB\( \chi \)PT, starting at N2LO ( \( \mathcal{O}(m_\pi^3) \)), \( g_A \) couples to \( m_n \)

\[
\delta m_n = -\frac{3g_A^2}{32\pi F_\pi} m_\pi^3
\]

To capture the effect of the pion mass dependence of \( g_A \), it is advantageous to perform a combined analysis (simultaneous fitting) of both \( m_N \) and \( g_A \)
Incorporating heavy fermionic fields into $\chi PT$, despite being chirally consistent, unfortunately has the undesired effect of introducing a mass that is of the same order as the chiral symmetry breaking scale $m_{hf}/\Lambda_\chi \sim 1$, and thus there is no longer a consistent derivative expansion for processes involving these heavy fermionic fields.

Additionally Weinberg’s power counting fails since higher order loop graphs can produce amplitudes which are no longer supressed by $\Lambda_\chi$.

A solution to this problem was first provided by Elizabeth Jenkins and Aneesh V. Manohar.
HB$\chi$PT "quickly"

Nearly on-shell Baryon with velocity $v_\mu$ has a momentum given by

$$p_\mu = m_B v_\mu + k_\mu$$

Where $k_\mu$ is a small off-shell contribution.

The effective theory is then written in terms Baryon fields $B_\nu$

$$B_\nu = e^{im_B p_\nu x^\mu} B(x)$$

Derivatives act on $B_\nu$ by producing $k$, the higher derivative terms are thus suppressed by $1/\Lambda_\chi$. In addition the heavy Baryon Lagrangian has a $1/m_B$ expansion. For the lightest Baryon multiplets this can be combined with $1/\Lambda_\chi$ into a single expansion in $1/\Lambda_\chi$. Our theory has now a consistent power counting expansion.
Gathering expressions for $m_N, g_A$

Up to order $m_\pi^5$

$$m_N = m_0 + \delta m_N^{2+4} + \delta m_N^{3+5}$$

$$\delta m_N^{2+4} = -4c_1 m_\pi^2 + m_\pi^4 \left[-e'(L_\chi) + \frac{3}{128\pi^2 f_\pi^2} \left(c_2 - \frac{2g_A^2}{M}\right)\right]$$

$$- \frac{m_\pi^4}{32\pi^2 f_\pi^2} \ln \frac{m_\pi}{L_\chi} \left(\frac{3g_A^2}{M} - 32c_1 + 3c_2 + 12c_3\right)$$

and

$$\delta m_N^{3+5} = -\frac{3g_A m_\pi^3}{32\pi f_\pi^2} + \frac{3g_A m_\pi^5}{32\pi f_\pi^2} \left[\frac{2l_4^r (L_\chi)}{f^2} - \frac{4(2d_{16}^r (L_\chi) - d_{18})}{g}\right]$$

$$+ 16d_{28}^r (L_\chi) + \frac{g^2}{8\pi^2 f^2} + \frac{1}{8M^2} \right] + \frac{3g_A^4 m_\pi^5}{64\pi^3 f_\pi^4} \ln \frac{m_\pi}{L_\chi}$$
Gathering expressions for $m_N, g_A$

\[
g_A = g_0 \left[ 1 + \left( \frac{\alpha_2}{(4\pi F_\pi)^2} \ln \frac{m_\pi}{L_\chi} + \beta_2 \right) m_\pi^2 + \alpha_3 m_\pi^3 \right. \\
+ \left. \left( \frac{\alpha_4}{(4\pi F_\pi)^4} \ln^2 \frac{m_\pi}{L_\chi} + \frac{\gamma_4}{(4\pi F_\pi)^2} \ln \frac{m_\pi}{L_\chi} + \beta_4 \right) m_\pi^4 + \alpha_5 m_\pi^5 \right]
\]

<table>
<thead>
<tr>
<th>$m_N$</th>
<th>$g_A$</th>
<th>$O(m_\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>$d_{16}^r, d_{28}^r$</td>
<td>$m_\pi^2$</td>
</tr>
<tr>
<td></td>
<td>$c_3, c_4$</td>
<td>$m_\pi^3$</td>
</tr>
<tr>
<td>$e^{'}, c_2, c_1, c_3$</td>
<td>$l_4^r, c_4, c_3, d_{16}^r$</td>
<td>$m_\pi^4$</td>
</tr>
<tr>
<td>$l_4^r, d_{16}^r, d_{18}, d_{28}^r$</td>
<td>$c_3, c_4, l_4^r$</td>
<td>$m_\pi^5$</td>
</tr>
</tbody>
</table>

Table of LEC’s appearing at different orders of $m_\pi$
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plots A

$g_A$ at LO
separate fit

$g_A$ at NLO
separate fit

$g_A$ at $N^2$LO
separate fit
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plots B
plot C

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plot D
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plot F

$m_n$ at N⁴LO
$g_A$ at LO
simultaneously fit

$m_n$ at N³LO
$g_A$ at LO
physical point
$a = 0.12$ fm
$a = 0.15$ fm
$a = 0.09$ fm

$m_n$ at N³LO
$g_A$ at N³LO
simultaneously fit

$g_A$ at NLO
simultaneously fit
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plot G

- Physical point
- $a = 0.12$ fm
- $a = 0.15$ fm
- $a = 0.09$ fm

$m_N$ at N$^4$LO
$g_A$ at LO

simultaneously fit

$m_N$ at N$^4$LO
$g_A$ at N$^4$LO

simultaneously fit

$m_N$ at N$^4$LO
$g_A$ at NLO

simultaneously fit
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plot H

- $m_N$ at N$^5$LO
- $g_A$ at LO
simultaneously fit

- physical point
- $a = 0.12$ fm
- $a = 0.15$ fm
- $a = 0.09$ fm

- $m_N$ at N$^5$LO
- $g_A$ at N$^3$LO
simultaneously fit

- physical point
- $a = 0.12$ fm
- $a = 0.15$ fm
- $a = 0.09$ fm

- $m_N$ at N$^5$LO
- $g_A$ at NLO
simultaneously fit

- physical point
- $a = 0.12$ fm
- $a = 0.15$ fm
- $a = 0.09$ fm
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plot J

 simultaneuously fit

 simultaneuously fit

 simultaneuously fit

 simultaneuously fit
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simultaneously fit \(m_N\) at N^2LO
\(g_A\) at LO
physical point
\(a = 0.12 \text{ fm}\)
\(a = 0.15 \text{ fm}\)
\(a = 0.09 \text{ fm}\)

\(m_N\) at N^2LO
\(g_A\) at LO
simultaneously fit

\(m_N\) at N^2LO
\(g_A\) at N^2LO
simultaneously fit

\(m_N\) at N^2LO
\(g_A\) at NLO
simultaneously fit
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plot L

 simultaneosuly fit

$m_N$ at $N^3$LO
$g_A$ at $N^2$LO

physical point
$a = 0.12 \text{ fm}$
$a = 0.15 \text{ fm}$
$a = 0.09 \text{ fm}$

$m_N$ at $N^2$LO
$g_A$ at LO

physical point
$a = 0.12 \text{ fm}$
$a = 0.15 \text{ fm}$
$a = 0.09 \text{ fm}$

$m_N$ at $N^3$LO
$g_A$ at NLO

physical point
$a = 0.12 \text{ fm}$
$a = 0.15 \text{ fm}$
$a = 0.09 \text{ fm}$
plot M

Simultaneously fit $m_N$ at N$^4$LO
$g_A$ at LO

physical point $a = 0.12$ fm
$a = 0.15$ fm
$a = 0.09$ fm
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plot N

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plotN.png}
\caption{Simultaneously fit $m_N$ at N$^2$LO and $g_A$ at LO/2LO physical point $a = 0.12$ fm, $a = 0.15$ fm, $a = 0.09$ fm.}
\end{figure}
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plot O

- Physical point
- $a = 0.12$ fm
- $a = 0.15$ fm
- $a = 0.09$ fm

$m_N$ at NLO
$g_A$ at LO

$m_N$ at NLO
$g_A$ at N$^2$LO

$m_N$ at NLO
$g_A$ at NLO

simultaneously fit