Pion distribution amplitude from Euclidean correlation functions: Exploring universality and higher-twist effects

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Definition of distribution amplitudes

\[ |\pi\rangle = |\bar{q}q\rangle + |\bar{q}gq\rangle + \ldots \]

- hard exclusive processes are sensitive to
  - Fock states with smallest number of partons
  - the distribution of the momentum within a Fock state at small transverse distances
- this information is contained in light-cone DAs; leading twist DA \( \phi_\pi \)

\[
\langle 0|\bar{u}(z)[z, -z]\not\!\gamma_5 u(-z)|\pi(p)\rangle = iF_\pi p \cdot z \int_0^1 du e^{i(2u-1)p \cdot z} \phi_\pi(u, \mu) \quad \bar{z}^2 = 0
\]

- quark and antiquark carry the momentum fraction \( u \) and \( \bar{u} = 1 - u \), respectively
- physical information: complementary to PDFs
- lattice technique: very similar to PDFs
**Problem:** on a Euclidean space-time one cannot realize nontrivial lightlike distances

- **traditional solution:** calculate Mellin moments of the DAs ($\hat{=}$ local derivative ops.)
  - higher moments $\rightarrow$ problems with renormalization (operator mixing)

- **new approach:** relate DAs to correlation functions at spacelike distance
  - requires large hadron momenta
  - relies heavily on pQCD

  - **Option 1:** use a nonlocal operator
    \[ \langle 0 | \bar{q}(z) \Gamma [z, 0] q(0) | \pi \rangle \quad z^2 < 0 \]
    Ji arXiv:1305.1539

  - **Option 2:** use two local operators
    \[ \langle 0 | \bar{q}(z) \Gamma_1 q(z) \bar{q}(0) \Gamma_2 q(0) | \pi \rangle \quad z^2 < 0 \]
    Braun, Müller arXiv:0709.1348
    Ma, Qiu arXiv:1709.03018

  - ...
DA \leftrightarrow \text{correlation function (schematically & oversimplified)}

\[
\begin{align*}
\text{DA} & \quad \overset{\text{LaMET}}{\longleftrightarrow} \quad \text{quasi-DA} & \quad \overset{\text{FT}}{\longleftrightarrow} \quad \text{lattice data} \\
\text{Hadron structure, Mon 14:00 (K. Cichy), 14:20 (A. Scapellato), 14:40 (Y. Zhao), 15:00 (Y. Yang), 15:20 (Y. Liu)} \\
\text{Hadron structure, Tue 14:40 (R. Zhang), 16:10 (J. Zhang), 16:30 (N. Karthik), 16:50 (C. Shugert)} \\
\text{or} \\
\text{DA} & \quad \overset{\text{FT}}{\longleftrightarrow} \quad \text{loffetime-DA} & \quad \overset{\text{pQCD}}{\longleftrightarrow} \quad \text{lattice data} \\
\text{Die structure, Mon 16:10 (S. Zafeiropoulos), 16:30 (J. Karpie)} \\
\text{or} \\
\text{DA} & \quad \overset{\text{pQCD (directly in position space)}}{\longrightarrow} \quad \text{lattice data}
\end{align*}
\]

\text{our ansatz: (also works when using the Wilson-line operator)}

- parametrize DA (\& higher twist effects) and fit directly to the lattice data
- basic idea very similar to “lattice cross section” talks on PDFs

\text{Hadron structure, Tue 14:00 (R. Sufian), 14:20 (B. Chakraborty)}

\text{(feel free to replace DA by PDF on this slide)}
Matrix elements ↔ DAs

\[ \mathbb{T}_{XY}(p \cdot z, z^2) = \langle 0| J_X^\dagger \left( \frac{z}{2} \right) J_Y \left( -\frac{z}{2} \right) | \pi^0(p) \rangle \]

\[ J_S = \bar{q} u, \quad J_P = \bar{q} \gamma_5 u, \quad J_V^\mu = \bar{q} \gamma^\mu u \equiv J_{V\mu}, \quad J_A^\mu = \bar{q} \gamma^\mu \gamma_5 u \equiv J_{A\mu} \]

\[ \mathbb{T}_{SP} = T_{SP} \]

\[ \mathbb{T}_{VV}^{\mu\nu} = \frac{i \varepsilon^{\mu\nu\rho\sigma} p_\rho z_\sigma}{p \cdot z} T_{VV} \]

\[ \mathbb{T}_{VA}^{\mu\nu} = \frac{p_\mu z_\nu + z_\mu p_\nu - g^{\mu\nu} p \cdot z}{p \cdot z} T_{VA}^{(2)} + \frac{p_\mu z_\nu - z_\mu p_\nu}{p \cdot z} T_{VA}^{(3)} + \frac{2 z_\mu z_\nu - g^{\mu\nu} z^2}{z^2} T_{VA}^{(3)} \]

\[ + \frac{2 p_\mu p_\nu - g^{\mu\nu} p^2}{p^2} T_{VA}^{(4)} + g^{\mu\nu} T_{VA}^{(5)} \]

- similar for PS, AA, AV
- \( q \) is an auxiliary quark \( q \neq u, d \), but \( m_q = m_u = m_d \)
Matrix elements $\leftrightarrow$ DAs

\[
T_{XY}(p \cdot z, z^2) = F_\pi \frac{p \cdot z}{2\pi^2 z^4} \int_0^1 du \, e^{i(u-1/2)p \cdot z} \phi_\pi(u) + \mathcal{O}(\alpha_s) + \text{higher twist}
\]

\[
\equiv \Phi_{XY}(p \cdot z, z^2)
\]

- twist 4 effects estimated using asymptotic shape for chiral-odd twist three DAs
  → one parameter $\delta_2^\pi = 0.17 \text{ GeV}^2$ (at $\mu = 2 \text{ GeV}$, QCD sum rule estimate)
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Obtaining the matrix elements from Lattice

\[ C_{XY}^{3pt}(p, z) \]

\[ \sum_x e^{ip \cdot x} \]

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\[ \frac{T_{XY}(p \cdot z, z^2)}{F_\pi} = \frac{Z_X(\mu)Z_Y(\mu)}{Z_A} \frac{C_{XY}^{3pt}(p, z)e^{i\frac{p \cdot z}{2}}}{C^{2pt}(p)} E(p) + \text{excited states} \]

- the \( Z_X \) is the renormalization factor for the respective current (nonperturbatively calculated in RI’-MOM → conversion to \( \overline{\text{MS}} \) in 3-loop PT)
- we set both, the renormalization and the factorization scale to \( \mu = 2/|z| \)
- phase factor shifts the currents to the symmetric position
Obtaining the matrix elements from Lattice

\[ C_{XY}^{3\text{pt}}(p, z) \]

\[ \sum_x e^{ip \cdot x} \]

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\[ \Gamma_X \]

\[ \Gamma_Y \]

\[ z \]

\[ \sum_x \]

\[ e^{ip \cdot x} \]

\[ x \]

\[ x \]

\[ sm \]

\[ \gamma_4 \gamma_5 \]

\[ 0 \]

\[ C_{2\text{pt}}(p) \]

\[ \Gamma_Y \]

\[ \Gamma_X \]

\[ z \]

\[ \sum_x \]

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- **smearing**: momentum smearing
  → improved overlap with hadrons at large momentum
- **new**: we use stochastic estimation
  → get a volume average at the cost of some stochastic noise
  → much smaller statistical error
Discretization effects of the free Wilson propagator

propagator comparison:

\textbf{free Wilson vs. free continuum}

- large effects in \textbf{chiral even} (blue, $\propto \not \! \not \! \partial$) and \textbf{chiral odd} (red, $\propto \mathbb{1}$) part
- in continuum: chiral odd part strongly suppressed
- \textbf{problem on lattice}: large artefacts from terms removing the doublers

solution:

1. use observables, where the \textbf{chiral odd} part does not contribute at tree-level

   \[ \frac{1}{2} (T_{SP} + T_{PS}) , \quad \frac{1}{2} (T_{VA} + T_{AV}) , \quad \frac{1}{2} (T_{VV} + T_{AA}) \]

2. introduce correction factor for \textbf{chiral even} part

3. most important: ignore distances where the correction $> 10\%$ or $|z| < 3a$
Discretization effects of the free Wilson propagator

propagator comparison:

free Wilson vs. free continuum

- large effects in chiral even (blue, $\propto \not{\!}\!\!\!\!\!z$) and chiral odd (red, $\propto 1$) part
- in continuum: chiral odd part strongly suppressed
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note:

1. upper limit of range determined by $\mu = 2/|z| \geq 1$ GeV
   $\Rightarrow a \rightarrow a/2$ shifts the upper limit by a factor 4 to the right
2. discretization effects are strongest along the axes (crosses)
   $\rightarrow$ similar for Wilson-line operators?
Simulation details:

- mass-degenerate $N_f = 2$ nonperturbatively improved Wilson (clover) fermions and Wilson gluon action
- $L^3 \times T = 32^3 \times 64$
- coupling parameter $\beta = 5.29 \approx$ lattice spacing $a \approx 0.071 \text{ fm} = (2.76 \text{GeV})^{-1}$
- mass parameter $\kappa = 0.13632 \approx$ pion mass $m_\pi = 0.10675(59)/a \approx 295 \text{ MeV}$
- 12 momenta in different directions with $0.54 \text{ GeV} \leq |p| \leq 2.03 \text{ GeV}$

DA parametrizations: at the scale $\mu = 2 \text{ GeV}$

- Expansion in orthogonal (Gegenbauer) polynomials (truncated at $n = 2$ or $n = 4$)

$$\phi_\pi(u, \mu) = 6u(1-u) \sum_{n=0,2,...}^{\infty} a_\pi^n(\mu) C_n^{3/2}(2u - 1), \quad a_\pi^0 = 1 \text{ (normalization)}$$

- alternatively we try

$$\phi_\pi(u, \mu) \propto [u(1-u)]^\alpha, \text{ normalized to one}$$
Combined fit to all channels (Legacy Plot)

- two parameters: $\alpha$, $\delta_2^\pi$
- two parameters: $a_2^\pi$, $\delta_2^\pi$
- three parameters: $a_2^\pi$, $a_4^\pi$, $\delta_2^\pi$ $\leftarrow$ yields unreasonable values for $a_4^\pi$
Combined fit to all channels

- splitting between $SP+PS$ and $VV+AA$ data is consistent with the pQCD expectation
- "jumping" of the points shows large discretization effects
- probably 2 loop perturbative effects are crucial

Fixed distance: $|z| = 0.39 \text{ fm} \quad \hat{=} \quad 2/|z| = 1.03 \text{ GeV}
Combined fit to all channels

- splitting between SP+PS and VV+AA data is consistent with the pQCD expectation
- “jumping” of the points shows large discretization effects
- probably 2 loop perturbative effects are crucial
Motivation Calculation Results Summary

Combined fit to all channels

Fixed distance: $|z| = 0.36 \text{ fm} \quad \hat{=} \quad 2/|z| = 1.08 \text{ GeV}$

- splitting between SP+PS and VV+AA data is consistent with the pQCD expectation
- “jumping” of the points shows large discretization effects
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Combined fit to all channels

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Result for DAs

- errorbands show only the statistical error
- parameters: \( \alpha = 0.13(5) \), \( \delta_2 = 0.223(4) \text{ GeV}^2 \) \( a_2^{\pi} = 0.30(3) \), \( \delta_2^{\pi} = 0.223(4) \text{ GeV}^2 \)
- both agree perfectly well with our data: Why?
- only relevant information from DA for our data points is \( a_2^{\pi} \) and \( a_2^{\pi} = 0.31(3) \)
- **Disclaimer**: current systematic uncertainty for \( a_2^{\pi} \), \( \delta_2^{\pi} \) is at least \( \approx 50\% \)
  (fit range variation, estimate for two-loop correction)
What's the problem with $a_{\pi}^4$?

\[
\phi_\pi(u, \mu) = 6u(1 - u) \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu) C_n^{3/2}(2u - 1)
\]

\[
\Rightarrow \Phi^{XY} = \sum_{n=0,2,\ldots}^{\infty} a_n^\pi(\mu) F_n(p \cdot z/2) + O(\alpha_s) + \text{higher twist}
\]

Expansion in conformal partial waves $F_n$

- one needs $|p \cdot z| \gtrsim 5$ to constrain $a_{\pi}^4$ to reasonable values
- to discriminate between DAs on last slide: $|p \cdot z| \gtrsim 8$?
Summary

we have analysed Euclidean correlation functions with two local currents

global fit to multiple channels yields qualitatively reasonable results (universality)

first determination of HT normalization $\delta_2^\pi$ from lattice QCD
(in the ballpark of QCD sum rule estimates)

statistical accuracy very good for $a_2^\pi$ and $\delta_2^\pi$

BUT:

systematic uncertainty for $a_2^\pi$ and $\delta_2^\pi$ is very large
(discretization effects, two-loop perturbative correction not taken into account)

with current data no determination of $a_4^\pi$ possible

Next steps:

goto smaller lattice spacings ($a \approx 0.04$ fm would be nice)

perturbative two-loop calculation for coefficient functions

to be sensitive to $a_4^\pi$: goto larger momenta ($|p| > 3$ GeV would be nice)
Plateau Fits

- no sign of excited states (so far)
- for large momentum they might be hidden under bad statistics