Motivation

\[ \langle \pi^+(p_f) | V_\mu(x) | \pi^+(p_i) \rangle = (p_f + p_i)_\mu f_{\pi\pi}(q^2) \]

<table>
<thead>
<tr>
<th>Mean Square Radius</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.42(10)</td>
<td>ESCHRICH</td>
</tr>
<tr>
<td>0.439(8)</td>
<td>AMENDOLIA</td>
</tr>
<tr>
<td>0.440(30)</td>
<td>DALLY</td>
</tr>
<tr>
<td>0.452(11)</td>
<td>PDG</td>
</tr>
</tbody>
</table>
Pion Form Factor

\[ C_{3pt}(\tau, t_f, p_i, p_f) = \sum_{x_f, z} e^{-ip_f \cdot (x_f - z)} e^{i q \cdot z} \langle T[\chi_{\pi^+}(x_f, t_f)V_\mu(z, \tau)\chi_{\pi^+}^\dagger(0, 0)] \rangle \]

\[ \approx \frac{m^2 Z_{p_i} Z_{p_f}}{E(p_i)E(p_f)} e^{-(E(p_i)\tau - E(p_f)(t_f - \tau))} \langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle \]

\[ + C_1 e^{-(E(p_i)\tau - E(p_f)(t_f - \tau))} + C_2 e^{-(E(p_i)\tau - E^1(p_f)(t_f - \tau))} \]

\[ q = p_f - p_i \]

\[ C_{2pt}(t, p) = \sum_x e^{-ip \cdot x} \langle T[\chi_{\pi^+}(x, t)\chi_{\pi^+}^\dagger(0, 0)] \rangle \]

\[ \approx \frac{m |Z_p|^2}{E(p)} (e^{-E(p)t} + e^{-E(p)(T-t)}) \]

\[ + \frac{m |Z_p^1|^2}{E^1(p)} (e^{-E^1(p)t} + e^{-E^1(p)(T-t)}) \]

\[ \chi_{\pi^+}(x_f, t_f) = \bar{u}(x, t)\gamma_5 d(x, t) \]

\[ \chi_{\pi^+}^\dagger(x_f, t_f) = -\bar{d}(x, t)\gamma_5 u(x, t) \]

\[ V_\mu(x) = \bar{u}(x)\gamma_\mu u(x) \]
Simulation Details

- **Lattices**
  - 24I--Domain Wall Lattice, $24^3 \times 64$, $a = 0.11$ fm, Pion 337 MeV
  - 32ID--Domain Wall Lattice, $32^3 \times 64$, $a = 0.143$ fm, Pion 171 MeV
  - Overlap Fermion with several valence quark masses

- **Sources and Sinks**
  - Grid Source at time 0 with momentum $p_i = q$
  - Stochastic Sinks with $t_f$ with momentum $p_f = 0$
  - Local vector current at time $\tau$

- **Fitting Strategy**
  - Joint fit of $C_{2pt}$ and $C_{3pt}$ with excited state
Zero Momentum Transfer

By adding excited state to two point and three point, joint fit has been done with $\chi^2$ to be around 0.6.

$$\frac{1}{Z_V} = \langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle$$
$Z_V$ and $Z_A$ Compare

\[ Z_A = \frac{2m_q \langle 0|P|\pi \rangle}{m_\pi \langle 0|A_4|\pi \rangle} \]

\[ Z_V = \frac{1}{\langle \pi^+(0)|V^\mu(0)|\pi^+(0) \rangle} \]

Jian Liang, Yi-Bo Yang, Terrence Draper, Ming Gong, Keh-Fei Liu, arXiv:1806.08366
$32\text{ID } f_{\pi\pi}(Q^2 = 0.05 \text{ GeV}^2)$

$$f_{\pi\pi}(Q^2) = \frac{1}{E_i + E_f} \frac{\langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle}{\langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle}$$

With small momentum transfer, we get stable results at small pion mass.
Z-expansion Fitting

\[ f_{\pi\pi}(Q^2) = 1 + \sum_{k=1}^{k_{\text{max}}} a_k z^k \]

\[ z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t_0} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t_0} + \sqrt{t_{\text{cut}} - t_0}} \]

\[ t = -Q^2, \ t_{\text{cut}} = 4\pi^2 \]
Pion Radius from 24I and 32ID

Strong valence pion mass dependence observed here

\[ \langle r^2 \rangle = 6 \frac{d}{dQ^2} f_{\pi\pi}(Q^2) \]

Need more lattices to explain the difference between 24I and 32ID
JLQCD, Phys. Rev. D 93, 034504 (2016); arXiv:1510.06470
Summary

- We have a preliminary result of pion form factor with sea pion mass 171 MeV and 337 MeV
- 32ID result agrees with experiment after extracted to physical pion mass point
- Production with different lattice spacing and sea pion masses are needed to control and understand varies systematics
- Momentum smearing source could be used to approach large momentum in further production