Nucleon Physics with

All HISQ Fermions

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Collaborators

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and

Fermilab Lattice & MILC Collaborations
Outline

- Motivation
- Staggered baryon group theory
- Two-point fitting strategy
- Sample two-point fit
- Nucleon mass continuum extrapolation
- Preliminary data
Motivation
Motivation: why nucleon physics?

The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments

[Credit: DUNE Collaboration]
Motivation: why nucleon physics?

- The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments
- Nucleon axial form factor uncertainty is often underestimated by model

[Meyer, Betancourt, Gran, Hill arxiv:1603.03048]
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- The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments

- Nucleon axial form factor uncertainty is often underestimated by model
  [Meyer, Betancourt, Gran, Hill arxiv:1603.03048]

- Lattice QCD can provide ab-initio calculation
**Motivation: why all staggered?**

- Baryon correlators suffer from unfavorable S/N
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- 2+1+1 Highly Improved Staggered Fermions (HISQ) ensembles at physical pion mass
  [MILC collaboration arxiv: 1212.4768]
Motivation: why all staggered?

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- Staggered propagator inversion is fast
- 2+1+1 Highly Improved Staggered Fermions (HISQ) ensembles at physical pion mass
  [MILC collaboration arxiv: 1212.4768]
- Need to demonstrate controls over remaining fermion doublers from staggered formalism
Staggered Baryon Group Theory
Baryon operators (at zero momenta) are constructed as irreducible representation (irrep) of geometric time slice (GTS) group [Golterman and Smit 1985]

\[ S_F = \bar{\chi}(\mathcal{D} + m)\chi \]
**Staggered Baryon Group Theory**

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- GTS includes rotations, spatial inversion, and taste transformations
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GTS includes rotations, spatial inversion, and taste transformations

Three irreps for baryon: 8, 8’, and 16
### Staggered Baryon Group Theory

<table>
<thead>
<tr>
<th>$SU(2)_I \times GT S$</th>
<th>States Mixed</th>
<th>Number of Operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{2} \times 8$</td>
<td>$3N + 2\Delta$</td>
<td>5</td>
</tr>
<tr>
<td>$\frac{3}{2} \times 8'$</td>
<td>$2\Delta$</td>
<td>2</td>
</tr>
<tr>
<td>$\frac{3}{2} \times 16$</td>
<td>$1N + 3\Delta$</td>
<td>4</td>
</tr>
</tbody>
</table>

- Isospin 3/2 nucleon-like state has the same properties as isospin 1/2 nucleon in the continuum limit

[Bailey hep-lat/0611023]
Two-point Fits
Two-point Fitting Strategy

- Fit away correlator contribution by three taste-splitted $\Delta$'s and other excitations to focus on nucleon group state

$$C_{2pt}^+(t) = C_{Nucleon}(t) + A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} + C_{radial}(t)$$
**Two-point Fitting Strategy**

- Fit away correlator contribution by three taste-splitter Δ’s and other excitations to focus on nucleon group state

\[
C^{+}_{2pt}(t) = C_{\text{Nucleon}}(t) + A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} + C_{\text{radial}}(t)
\]

- The differences between \(M_1\), \(M_2\), and \(M_3\) are less than 100 MeV (\(O(\alpha_s a^2)\)). Impossible to resolve them given our statistics which destabilizes our fits
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- The differences between $M_1$, $M_2$, and $M_3$ are less than 100 MeV ($O(\alpha_s a^2)$). Impossible to resolve them given our statistics which destabilizes our fits

- Solution: replace our fitting model for $\Delta$'s
Two-point Fitting Strategy

\[ C_\Delta(t) = A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} \]

\[ C'_\Delta(t) = A' e^{-M' t} + B' e^{-(M' + \Delta M') t} \]

Taylor expand both models at \( M' \) and we find that leading model error is small comparing to statistical errors.
Two-point Fitting Strategy

![Graph showing two-point fitting strategy with data points and error markers. The graph plots fractional error against time divided by a. The legend indicates different data sets: 0.12fm data, Leading error, and Next-to-leading error.]

- 0.12fm data
- Leading error
- Next-to-leading error
Two-point Fitting Strategy

- Four fitting parameters: $A'$, $B'$, $M'$, $\Delta M'$
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- This method works for $i > 3$ taste-splitted states
Two-point Fitting Strategy

- Four fitting parameters: $A'$, $B'$, $M'$, $\Delta M'$
- The leading error of $O((\Delta M' t)^3)$ is likely overestimated
- This method works for $i > 3$ taste-splitted states
- We will treat the lowest lying three $\Delta$ states in 16 irrep using this two states model (radial excitations are treated as one states)
## Two-point Data

<table>
<thead>
<tr>
<th>$a \simeq$ (fm)</th>
<th>$(L/a)^3 \times (T/a)$</th>
<th>$a m_l$</th>
<th>$a m_s$</th>
<th>$a m_c$</th>
<th>$N_{\text{conf}}$</th>
<th>$N_{\text{tsrc}}$</th>
<th>$N_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>$32^3 \times 48$</td>
<td>0.002426</td>
<td>0.0673</td>
<td>0.8447</td>
<td>3224</td>
<td>2</td>
<td>6448</td>
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<tr>
<td>0.12</td>
<td>$48^3 \times 64$</td>
<td>0.001907</td>
<td>0.05252</td>
<td>0.6382</td>
<td>952</td>
<td>2</td>
<td>1904</td>
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<tr>
<td>0.09</td>
<td>$64^3 \times 96$</td>
<td>0.0012</td>
<td>0.0363</td>
<td>0.432</td>
<td>940</td>
<td>1</td>
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- **2+1+1 MILC HISQ ensembles** with valence HISQ quarks
Two-point Data

<table>
<thead>
<tr>
<th>$a \approx (\text{fm})$</th>
<th>$(L/a)^3 \times (T/a)$</th>
<th>$am_l$</th>
<th>$am_s$</th>
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- 2+1+1 MILC HISQ ensembles with valence HISQ quarks [MILC collaboration arxiv: 1212.4768]

- Three lattice spacings at physical light-quark masses
### Two-point Data

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- **2+1+1 MILC HISQ ensembles with valence HISQ quarks**
  - [MILC collaboration arxiv: 1212.4768](arxiv:1212.4768)

- Three lattice spacings at physical light-quark masses

- All ensembles are **Coulomb gauge fixed**
16 Irrep Correlators Fit

\[ C_{ij}(t) = C^+_{ij}(t) + C^-_{ij}(t) \]

\[ C^+_{ij}(t) = \sum_{n=0}^{N_E} \langle n^+ | (\mathcal{O}_i^{\text{src}})^\dagger | \Omega \rangle \langle \Omega | \mathcal{O}_j^{\text{snk}} | n^+ \rangle \left( e^{-M_{n+t}} - (-1)^t e^{-M_{n+(T-t)}} \right) \]

\[ C^-_{ij}(t) = \sum_{n=0}^{N_O} \langle n^- | (\mathcal{O}_i^{\text{src}})^\dagger | \Omega \rangle \langle \Omega | \mathcal{O}_j^{\text{snk}} | n^- \rangle \left( e^{-M_{n-(T-t)}} - (-1)^t e^{-M_{n-t}} \right) \]

- Simultaneous fit to 9 correlators using Bayesian methodology
- 16 irrep: 1N, 3\Delta \rightarrow 1N, 2\Delta' for the even parity channel
16 Irrep Correlators Fit

\[ C_{22}(t) \]

![Graph showing \[ C_{22}(t) \] with data points and fit to nucleon]
Nucleon Continuum Extrapolation
Nucleon Mass Continuum Extrapolation

\[ M(a) = M_{phy} \left( 1 + b\alpha_s(\Lambda_{QCD}a)^2 + c(\Lambda_{QCD}a)^4 \right) \]

- Three lattice spacing: 0.15, 0.12, and 0.09fm
\[ M(a) = M_{\text{phy}} \left( 1 + b\alpha_s(\Lambda_{\text{QCD}}a)^2 + c(\Lambda_{\text{QCD}}a)^4 \right) \]

- Three lattice spacing: 0.15, 0.12, and 0.09fm
- Quadratic Bayesian fit to 3 parameters: \( M_{\text{phy}}, b, \text{ and } c \)
\[ M_{\text{phy}} = 938 \pm 22 \text{ MeV} \]
Preliminary Data
Non-zero Momentum Two-point

(a=0.15fm, one lattice momentum = 230 MeV)
Zero Momentum Three-point

\( (a=0.15\text{fm}, \text{Az-Az current}) \)

[Credit: Aaron Meyer]
Conclusion

- New proven two-point fitting strategy for taste-splitted states
- Continuum extrapolated to physical nucleon mass
- Many more analyses are underway (axial charge, non-zero momenta two-point and three-point)
Backup Slides
**Two-point Fitting Model**

\[
\frac{\delta C_{\Delta}(t)}{C_{\Delta}(t)} \equiv \frac{C_{\Delta}(t) - C'_{\Delta}(t)}{C_{\Delta}(t)} = \frac{1}{6} \left( \sum_{i=1}^{3} \sum_{j=1}^{3} A_i A_j(\Delta M'_i)^2(\Delta M'_j)^2 - \Delta M'_i(\Delta M'_j)^3 \right) t^3 + O((\Delta M'_i)^4 t^4)
\]

- \(\Delta M'_i\) is the difference between \(M'\) and \(M_i\)
- This will match the new model to our old model exactly up to second order in the Taylor expansion
- The leading error comes in at the third order
More On Staggered Group Theory

**Wilson Mesons**

\[
\begin{array}{l}
J \text{ irrep} \\
0 \quad A_1(1) \\
1 \quad T_1(3) \\
2 \quad T_2(3) \oplus E(2) \\
3 \quad T_1(3) \oplus T_2(3) \oplus A_2(1) \\
4 \quad A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)
\end{array}
\]

TABLE III: Continuum spins subduced into lattice irreps \(\Lambda(\text{dim})\).

[arXiv:1004.4930]

**Staggered Baryons**

\[
\begin{align*}
\left(\frac{1}{2}, 572_M\right) & \rightarrow 3(10_S, 8) \oplus (10_S, 16) \oplus 5(8_M, 8) \\
& \oplus 3(8_M, 16) \oplus 3(1_A, 8) \oplus (1_A, 16) \\
\left(\frac{3}{2}, 364_S\right) & \rightarrow 2(10_S, 8) \oplus 2(10_S, 8') \oplus 3(10_S, 16) \\
& \oplus (8_M, 8) \oplus (8_M, 8') \oplus 4(8_M, 16) \\
& \oplus (1_A, 16).
\end{align*}
\]

(10s,16) mixes one nucleon and three \(\Delta\)'s

[hep-lat/0611023]

\(A_1\) mixes \(J=0\) and \(J=4\) states
Source and Sink Operators Construction

- **Sink operators:** Exact projection onto lattice irrep with appropriate gauge links

\[
\mathcal{O}^{\text{sink}}(t) = N \sum_{x_i \in \text{even} \ A, B, C} \sum \sum C^{(r)}_{ABC} \chi_A(x, t) U(x + A, x + B; t) \chi_B(x, t) U(x + A, x + C; t) \chi_C(x, t)
\]

- **Source operators:** Approximate projection onto lattice irrep without gauge links \(\rightarrow\) need gauge fixing

\[
\mathcal{O}^{\text{src}}(t) = N \sum_{x_i \in \text{even} \ z_j \in \text{even} \ y_i \in \text{even} \ A, B, C} \sum \sum \sum C^{(r)}_{ABC} \chi_A(x, t) \chi_B(y, t) \chi_C(z, t)
\]
Nominal fit for $a = 0.12\text{fm}$ ensemble with $N_E=10$ and $N_O = 10$

Simultaneous fit to all 9 correlators using Bayesian method

Identical large prior widths centered at zero for all overlap factors, $\langle \Omega | O_{snk}^{\text{snk}} | n^\pm \rangle$ and $\langle n^\pm | (O_{\text{src}}^i)^\dag | \Omega \rangle$

Fit from $t_{\text{min}}=3$ to $t_{\text{max}}$ such that percent error $< 10%$

Use $C_{22}(t)$ to demonstrate quality of fit (full correlators fit in backup slides)
16 Irrep Correlators Fit
### 16 Irrep Correlators Fit

<table>
<thead>
<tr>
<th>Parity</th>
<th>Prior (MeV)</th>
<th>Posterior (MeV)</th>
<th>Abs. (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0^{+}$</td>
<td>+</td>
<td>970(50)</td>
<td>953(11)</td>
</tr>
<tr>
<td>$\Delta M_1^{+}$</td>
<td>+</td>
<td>250(100)</td>
<td>300(48)</td>
</tr>
<tr>
<td>$\Delta M_2^{+}$</td>
<td>+</td>
<td>60(50)</td>
<td>68(44)</td>
</tr>
<tr>
<td>$\Delta M_3^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>329(149)</td>
</tr>
<tr>
<td>$\Delta M_4^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>383(183)</td>
</tr>
<tr>
<td>$\Delta M_5^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>384(185)</td>
</tr>
<tr>
<td>$\Delta M_6^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>392(190)</td>
</tr>
<tr>
<td>$\Delta M_7^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>399(194)</td>
</tr>
<tr>
<td>$\Delta M_8^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>399(194)</td>
</tr>
<tr>
<td>$\Delta M_9^{+}$</td>
<td>+</td>
<td>400(200)</td>
<td>399(194)</td>
</tr>
<tr>
<td>$\Delta M_0^{-}$</td>
<td>−</td>
<td>1375(100)</td>
<td>1347(42)</td>
</tr>
<tr>
<td>$\Delta M_1^{-}$</td>
<td>−</td>
<td>200(100)</td>
<td>183(54)</td>
</tr>
<tr>
<td>$\Delta M_2^{-}$</td>
<td>−</td>
<td>400(200)</td>
<td>225(83)</td>
</tr>
<tr>
<td>$\Delta M_3^{-}$</td>
<td>−</td>
<td>400(200)</td>
<td>334(152)</td>
</tr>
<tr>
<td>$\Delta M_4^{-}$</td>
<td>−</td>
<td>400(200)</td>
<td>384(185)</td>
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<td>$\Delta M_5^{-}$</td>
<td>−</td>
<td>400(200)</td>
<td>390(188)</td>
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</tbody>
</table>
Check 1: States stability

Nominal fit
Check 2: $t_{\text{min}}$ stability

**Even Parity**

**Odd Parity**

Nominal fit
Check 3: Prior stability