Nucleon Physics with

All HISQ Fermions

Yin Lin 林胤
University of Chicago/Fermilab
yin01@uchicago.edu

July 26th, 2018

Lattice 2018

Collaborators

Ciaran Hughes, Andreas Kronfeld, Aaron Meyer, James Simone, Alexei Strelchenko

and

Fermilab Lattice & MILC Collaborations

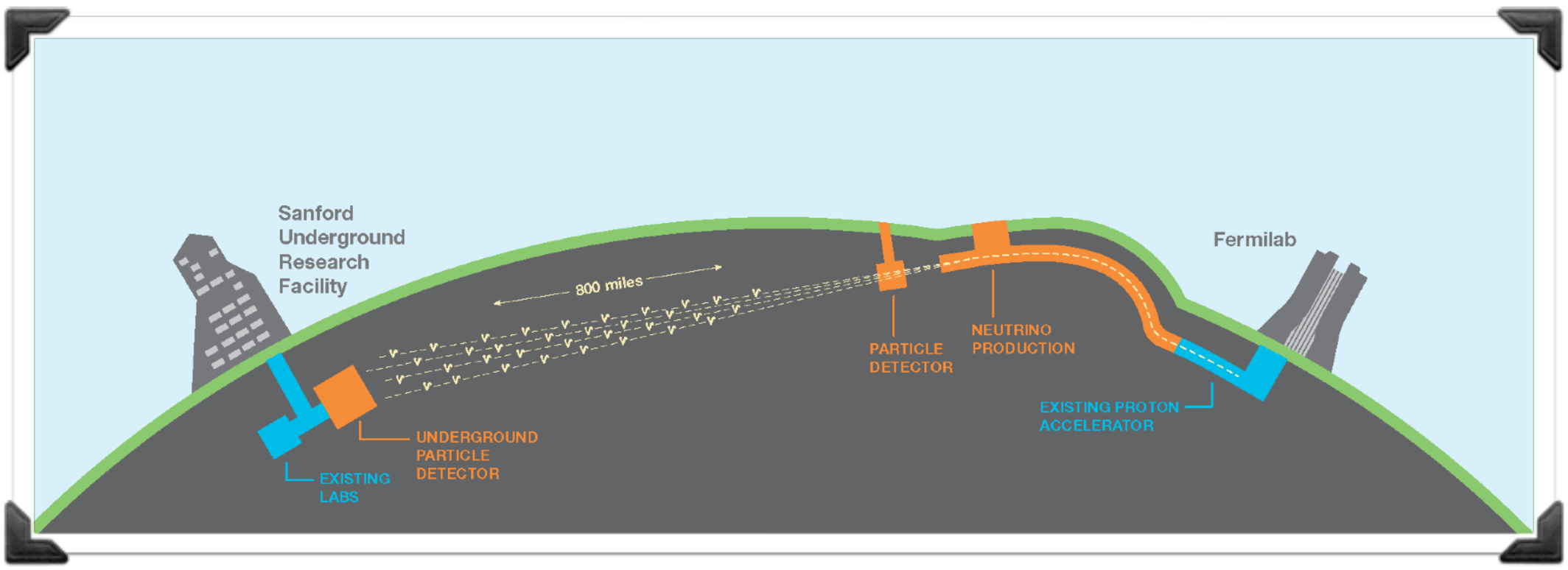
Outline

- Motivation
- Staggered baryon group theory
- Two-point fitting strategy
- Sample two-point fit
- Nucleon mass continuum extrapolation
- Preliminary data

Motivation

Motivation: why nucleon physics?

- **The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments**



[Credit: DUNE Collaboration]

Motivation: why nucleon physics?

- **The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments**

- **Nucleon axial form factor uncertainty is often underestimated by model**
  [Meyer, Betancourt, Gran, Hill arxiv:1603.03048]

Motivation: why nucleon physics?

- **The need of neutrino-nucleon interaction amplitude for neutrino scattering experiments**
- **Nucleon axial form factor uncertainty is often underestimated by model**
  [Meyer, Betancourt, Gran, Hill arxiv:1603.03048]
- **Lattice QCD can provide ab-initio calculation**

Motivation: why all staggered?

- **Baryon correlators suffer from unfavorable S/N**

Motivation: why all staggered?

- **Baryon correlators suffer from unfavorable S/N**
- **Staggered propagator inversion is fast**

Motivation: why all staggered?

- **Baryon correlators suffer from unfavorable S/N**
- **Staggered propagator inversion is fast**
- **2+1+1 Highly Improved Staggered Fermions (HISQ) ensembles at physical pion mass**
  [MILC collaboration arxiv: 1212.4768]

Motivation: why all staggered?

- **Baryon correlators suffer from unfavorable S/N**
- **Staggered propagator inversion is fast**
- **2+1+1 Highly Improved Staggered Fermions (HISQ) ensembles at physical pion mass**
  [MILC collaboration arxiv: 1212.4768]
- **Need to demonstrate controls over remaining fermion doublers from staggered formalism**

Staggered Baryon Group Theory

Staggered Baryon Group Theory

$$S_F = \bar{\chi}(\not{D} + m)\chi$$

- **Baryon operators (at zero momenta) are constructed as irreducible representation (irrep) of geometric time slice (GTS) group [Golterman and Smit 1985]**

Staggered Baryon Group Theory

$$S_F = \bar{\chi}(\not{D} + m)\chi$$

- **Baryon operators (at zero momenta) are constructed as irreducible representation (irrep) of geometric time slice (GTS) group [Golterman and Smit 1985]**
- **GTS includes rotations, spatial inversion, and taste transformations**

Staggered Baryon Group Theory

$$S_F = \bar{\chi}(\not{D} + m)\chi$$

- **Baryon operators (at zero momenta) are constructed as irreducible representation (irrep) of geometric time slice (GTS) group [Golterman and Smit 1985]**
- **GTS includes rotations, spatial inversion, and taste transformations**
- **Three irreps for baryon: 8, 8', and 16**

Staggered Baryon Group Theory

$SU(2)_I \times GTS$	States Mixed	Number of Operators
$\frac{3}{2} \times 8$	$3N + 2\Delta$	5
$\frac{3}{2} \times 8'$	2Δ	2
$\frac{3}{2} \times 16$	$1N + 3\Delta$	4

[Bailey hep-lat/0611023]

- **Isospin 3/2 nucleon-like state has the same properties as isospin 1/2 nucleon in the continuum limit**

Two-point Fits

Two-point Fitting Strategy

- **Fit away correlator contribution by three taste-splitted Δ's and other excitations to focus on nucleon group state**

$$C^{+}_{2pt}(t) = C_{Nucleon}(t) + A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} + C_{radial}(t)$$

Two-point Fitting Strategy

- **Fit away correlator contribution by three taste-splitted Δ's and other excitations to focus on nucleon group state**

$$C^{+}_{2pt}(t) = C_{Nucleon}(t) + \boxed{A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t}} + C_{radial}(t)$$

- **The differences between M_1, M_2, and M_3 are less than 100 MeV ($O(\alpha_s a^2)$). Impossible to resolve them given our statistics which destabilizes our fits**

Two-point Fitting Strategy

- **Fit away correlator contribution by three taste-splitted Δ's and other excitations to focus on nucleon group state**

$$C^{+}_{2pt}(t) = C_{Nucleon}(t) + A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} + C_{radial}(t)$$

- **The differences between M_1, M_2, and M_3 are less than 100 MeV ($O(\alpha_s a^2)$). Impossible to resolve them given our statistics which destabilizes our fits**

- **Solution: replace our fitting model for Δ's**

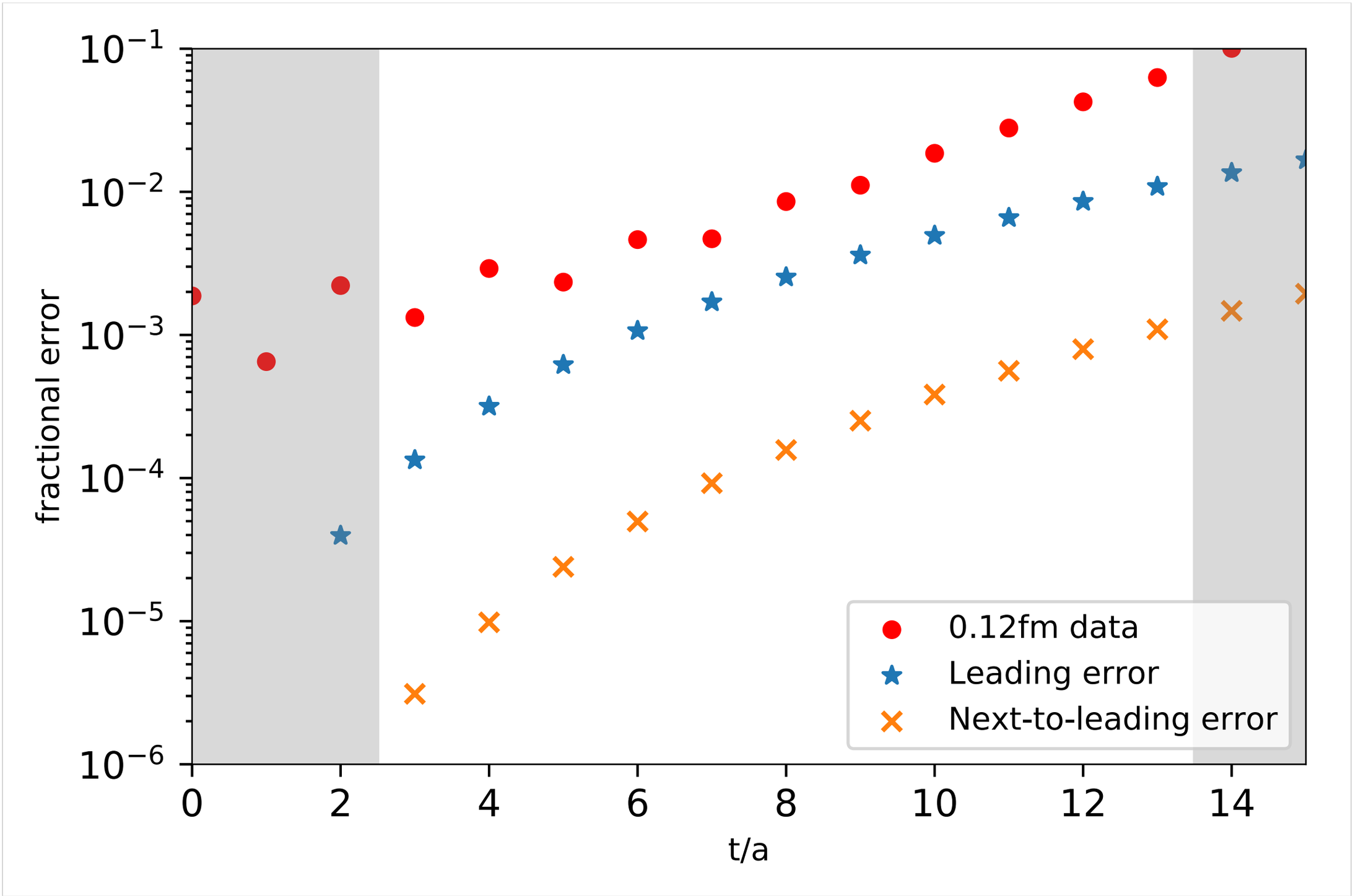
Two-point Fitting Strategy

$$C_\Delta(t) = A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t}$$

$$C'_\Delta(t) = A' e^{-M' t} + B' e^{-(M' + \Delta M')t}$$

Taylor expand both models at M' and we find that leading model error is small comparing to statistical errors

Two-point Fitting Strategy



Two-point Fitting Strategy

- **Four fitting parameters: A’, B’, M', ΔM’**

Two-point Fitting Strategy

- **Four fitting parameters: A’, B’, M', ΔM’**
- **The leading error of $O((\Delta M'_i t)^3)$ is likely overestimated**

Two-point Fitting Strategy

- **Four fitting parameters: A’, B’, M', ΔM’**
- **The leading error of $O((\Delta M'_i t)^3)$ is likely overestimated**
- **This method works for i > 3 taste-splitted states**

Two-point Fitting Strategy

- **Four fitting parameters: A', B', M', ΔM'**
- **The leading error of $O((\Delta M'_i t)^3)$ is likely overestimated**
- **This method works for i > 3 taste-splitted states**
- **We will treat the lowest lying three Δ states in 16 irrep using this two states model (radial excitations are treated as one states)**

Two-point Data

$a \approx$ (fm)	$(L/a)^3 \times (T/a)$	am_l	am_s	am_c	$N_{\rm conf}$	$N_{\rm tsrc}$	$N_{\rm meas}$
0.15	$32^3 \times 48$	0.002426	0.0673	0.8447	3224	2	6448
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	952	2	1904
0.09	$64^3 \times 96$	0.0012	0.0363	0.432	940	1	940

- **2+1+1 MILC HISQ ensembles with valence HISQ quarks [MILC collaboration arxiv: 1212.4768]**

Two-point Data

$a \approx$ (fm)	$(L/a)^3 \times (T/a)$	am_l	am_s	am_c	$N_{\rm conf}$	$N_{\rm tsrc}$	$N_{\rm meas}$
0.15	$32^3 \times 48$	0.002426	0.0673	0.8447	3224	2	6448
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	952	2	1904
0.09	$64^3 \times 96$	0.0012	0.0363	0.432	940	1	940

- **2+1+1 MILC HISQ ensembles with valence HISQ quarks [MILC collaboration arxiv: 1212.4768]**
- **Three lattice spacings at physical light-quark masses**

Two-point Data

$a \approx$ (fm)	$(L/a)^3 \times (T/a)$	am_l	am_s	am_c	$N_{\rm conf}$	$N_{\rm tsrc}$	$N_{\rm meas}$
0.15	$32^3 \times 48$	0.002426	0.0673	0.8447	3224	2	6448
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	952	2	1904
0.09	$64^3 \times 96$	0.0012	0.0363	0.432	940	1	940

- 2+1+1 MILC HISQ ensembles with valence HISQ quarks [MILC collaboration arxiv: 1212.4768]
- Three lattice spacings at physical light-quark masses
- All ensembles are Coulomb gauge fixed

16 Irrep Correlators Fit

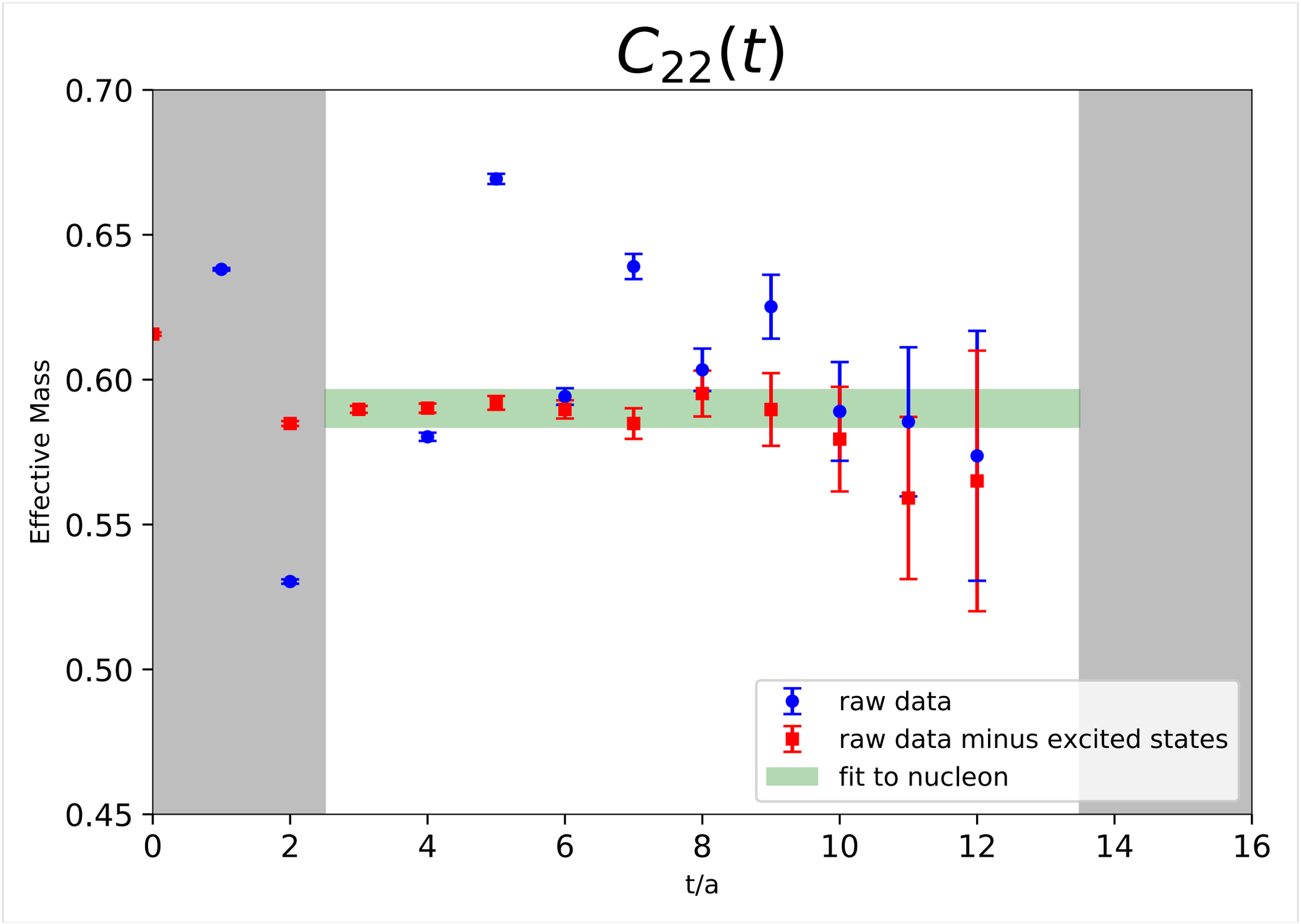
$$C_{ij}(t) = C^{+}_{ij}(t) + C^{-}_{ij}(t)$$

$$C^{+}_{ij}(t) = \sum_{n=0}^{N_E} \langle n^{+}|(\mathcal{O}_i^{\mathrm{src}})^{\dagger}|\Omega\rangle \langle\Omega|\mathcal{O}_j^{\mathrm{snk}}|n^{+}\rangle \left(e^{-M_{n^+}t} - (-1)^t e^{-M_{n^+}(T-t)}\right)$$

$$C^{-}_{ij}(t) = \sum_{n=0}^{N_O} \langle n^{-}|(\mathcal{O}_i^{\mathrm{src}})^{\dagger}|\Omega\rangle \langle\Omega|\mathcal{O}_j^{\mathrm{snk}}|n^{-}\rangle \left(e^{-M_{n^-}(T-t)} - (-1)^t e^{-M_{n^-}t}\right)$$

- **Simultaneous fit to 9 correlators using Bayesian methodology**
- **16 irrep: 1N, 3Δ → 1N, 2Δ' for the even parity channel**

16 Irrep Correlators Fit



Nucleon Continuum Extrapolation

Nucleon Mass Continuum Extrapolation

$$M(a) = M_{phy}\left(1 + b\alpha_s(\Lambda_{QCD}a)^2 + c(\Lambda_{QCD}a)^4\right)$$

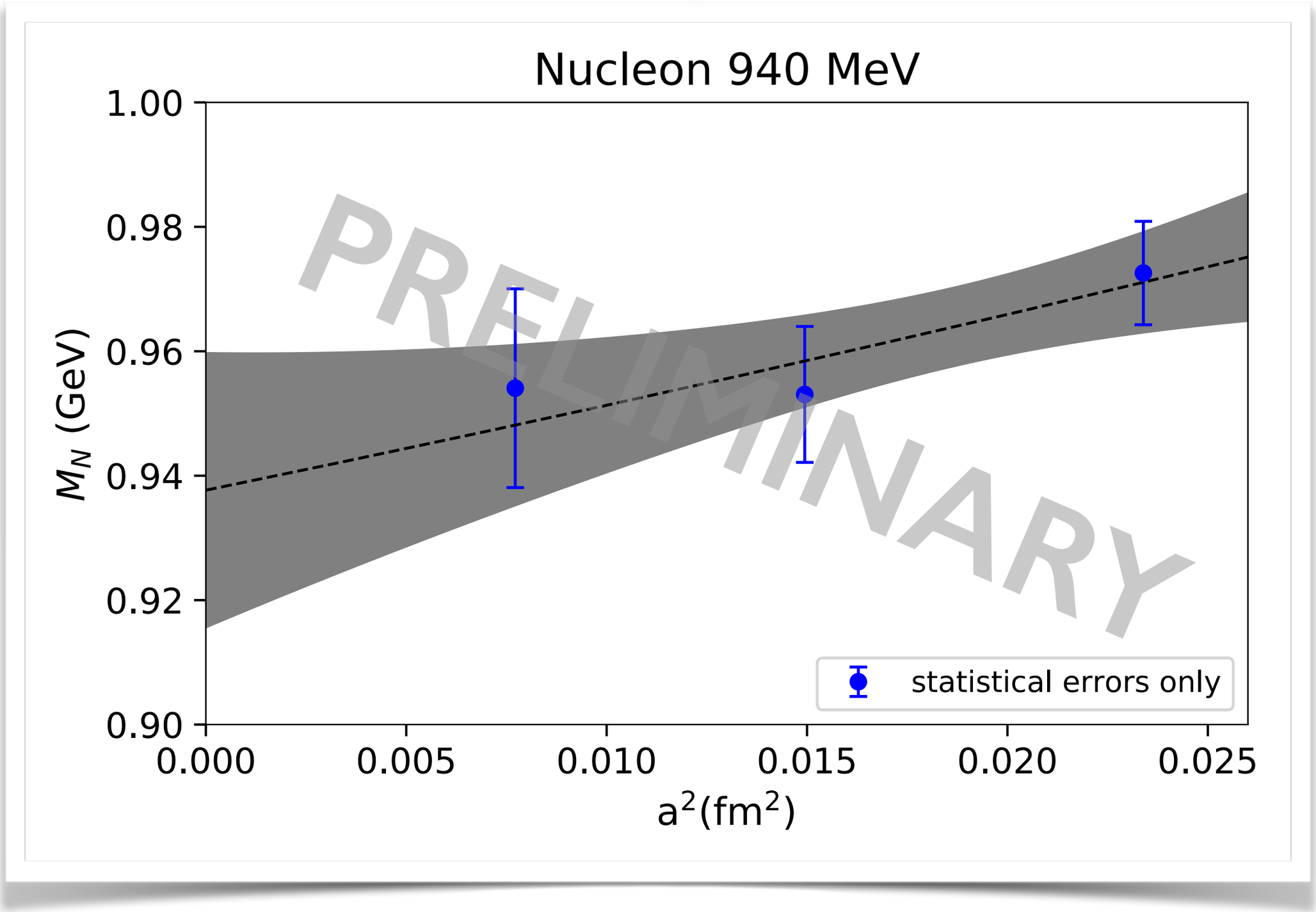
- **Three lattice spacing: 0.15, 0.12, and 0.09fm**

Nucleon Mass Continuum Extrapolation

$$M(a) = M_{phy}\left(1 + b\alpha_s(\Lambda_{QCD}a)^2 + c(\Lambda_{QCD}a)^4\right)$$

- **Three lattice spacing: 0.15, 0.12, and 0.09fm**
- **Quadratic Bayesian fit to 3 parameters: M_{phy}, b, and c**

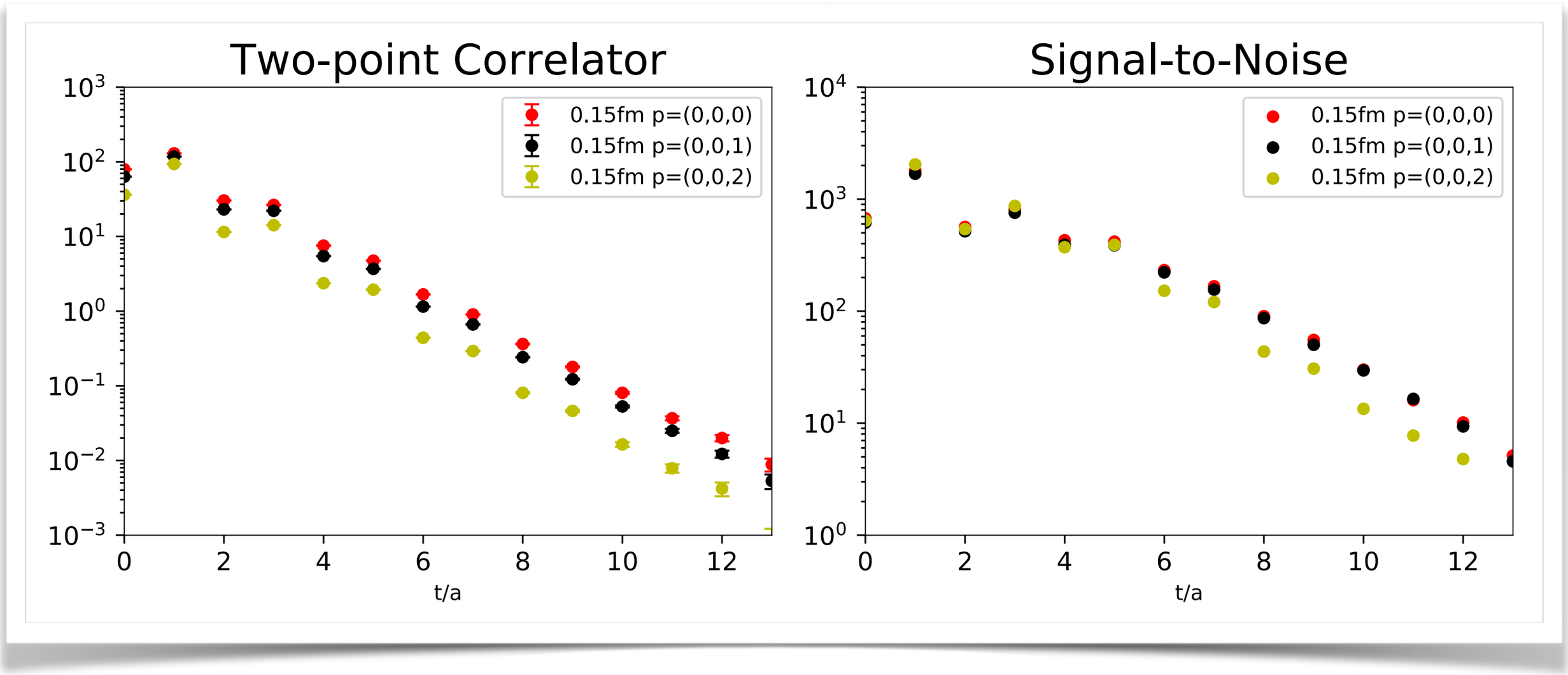
Nucleon Mass Continuum Extrapolation



$$M_{phy} = 938 \pm 22\ MeV$$

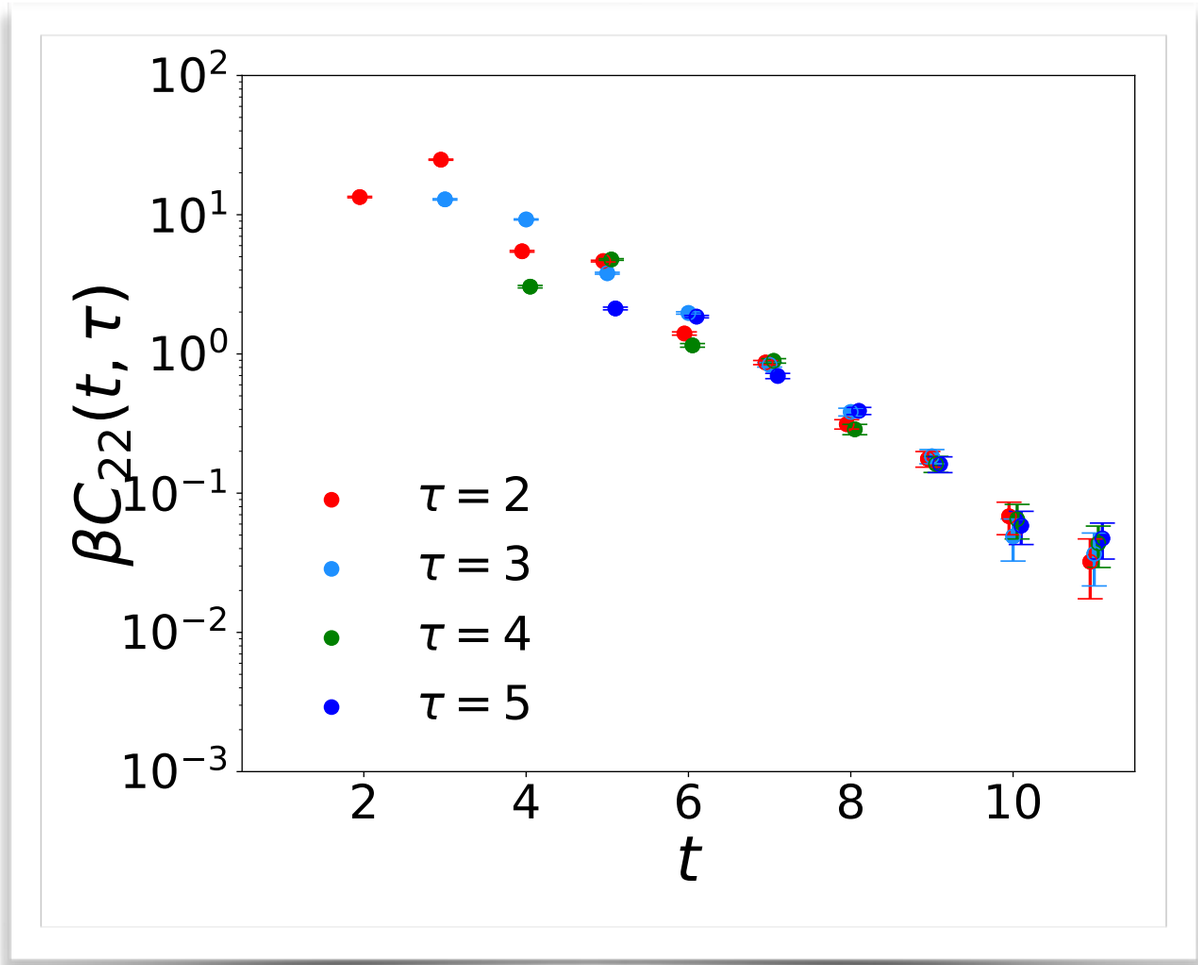
Preliminary Data

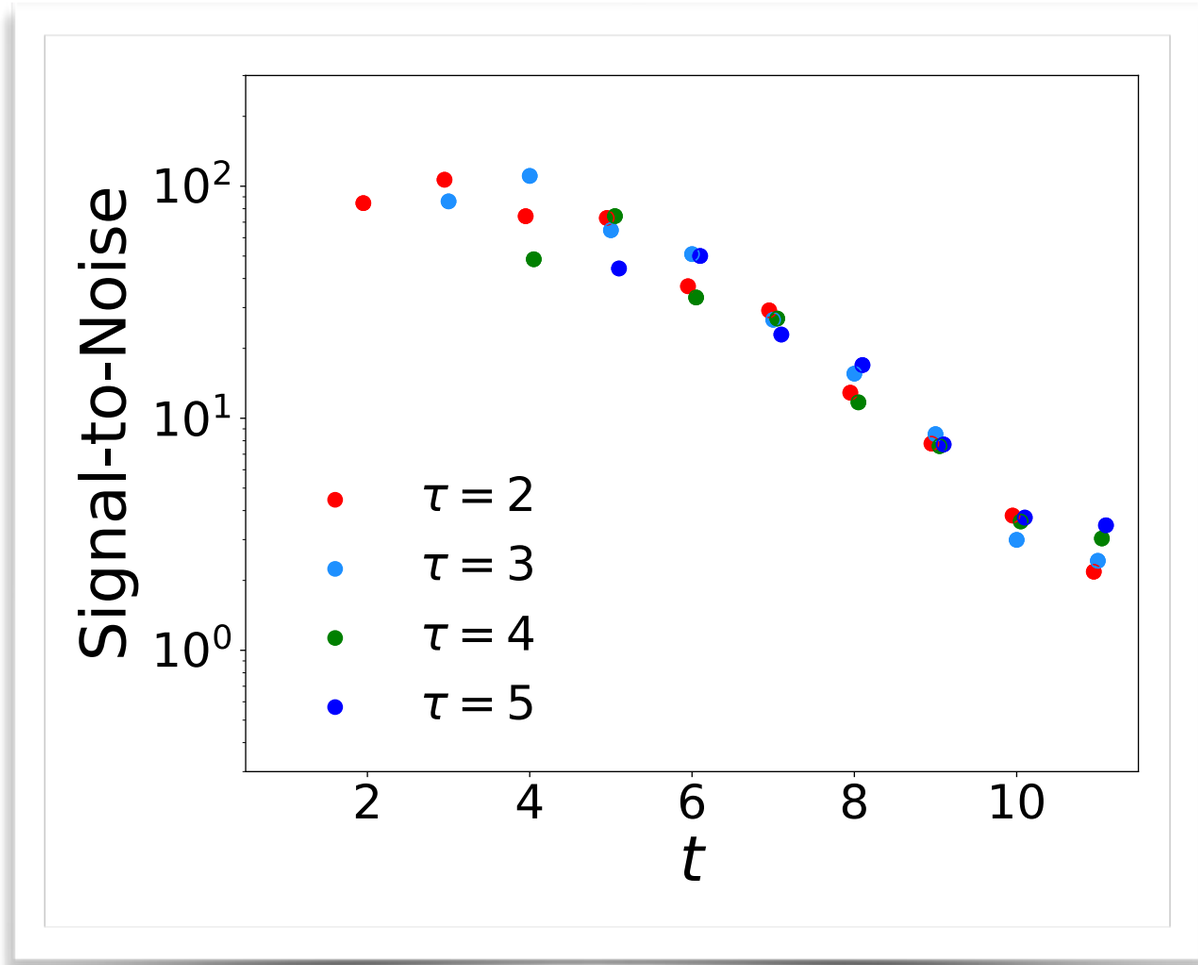
Non-zero Momentum Two-point



(a=0.15fm, one lattice momentum = 230 MeV)

Zero Momentum Three-point





(a=0.15fm, Az-Az current)

[Credit: Aaron Meyer]

Conclusion

- New proven two-point fitting strategy for taste-splitted states
- Continuum extrapolated to physical nucleon mass
- Many more analyses are underway (axial charge, non-zero momenta two-point and three-point)

Backup Slides

Two-point Fitting Model

$$\left|\frac{\delta C_\Delta(t)}{C_\Delta(t)}\right| \equiv \left|\frac{C_\Delta(t) - C'_\Delta(t)}{C_\Delta(t)}\right| = \left|\frac{1}{6}\left(\frac{\sum_{i=1}^{3}\sum_{j=1}^{3} A_i A_j((\Delta M'_i)^2(\Delta M'_j)^2 - \Delta M'_i(\Delta M'_j)^3)}{\sum_{i=1}^{3}\sum_{j=1}^{3} A_i A_j \Delta M'_i}\right) t^3 + O(\Delta(M'_i)^4 t^4)\right|$$

- **$\Delta M'_i$ is the difference between M' and M_i**
- **This will match the new model to our old model exactly up to second order in the Taylor expansion**
- **The leading error comes in at the third order**

More On Staggered Group Theory

Wilson Mesons

J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

TABLE III: Continuum spins subduced into lattice irreps $\Lambda(\dim)$.

[arXiv:1004.4930]

Staggered Baryons

$$(\tfrac{1}{2}, \mathbf{572_M}) \to 3(\mathbf{10_S}, \mathbf{8}) \oplus (\mathbf{10_S}, \mathbf{16}) \oplus 5(\mathbf{8_M}, \mathbf{8}) \oplus 3(\mathbf{8_M}, \mathbf{16}) \oplus 3(\mathbf{1_A}, \mathbf{8}) \oplus (\mathbf{1_A}, \mathbf{16}) \quad (10a)$$

$$(\tfrac{3}{2}, \mathbf{364_S}) \to 2(\mathbf{10_S}, \mathbf{8}) \oplus 2(\mathbf{10_S}, \mathbf{8'}) \oplus 3(\mathbf{10_S}, \mathbf{16}) \oplus (\mathbf{8_M}, \mathbf{8}) \oplus (\mathbf{8_M}, \mathbf{8'}) \oplus 4(\mathbf{8_M}, \mathbf{16}) \oplus (\mathbf{1_A}, \mathbf{16}). \quad (10b)$$

[hep-lat/0611023]

A_1 mixes J=0 and J=4 states

(10_S,16) mixes one nucleon and three Δ's

Source and Sink Operators Construction

- **Sink operators: Exact projection onto lattice irrep with appropriate gauge links**

$$\mathcal{O}^{snk}(t) = N \sum_{x_i \in even} \sum_{A,B,C} C^{(r)}_{ABC} \chi_A(x,t) U(x+A, x+B; t) \chi_B(x,t) U(x+A, x+C; t) \chi_C(x,t)$$

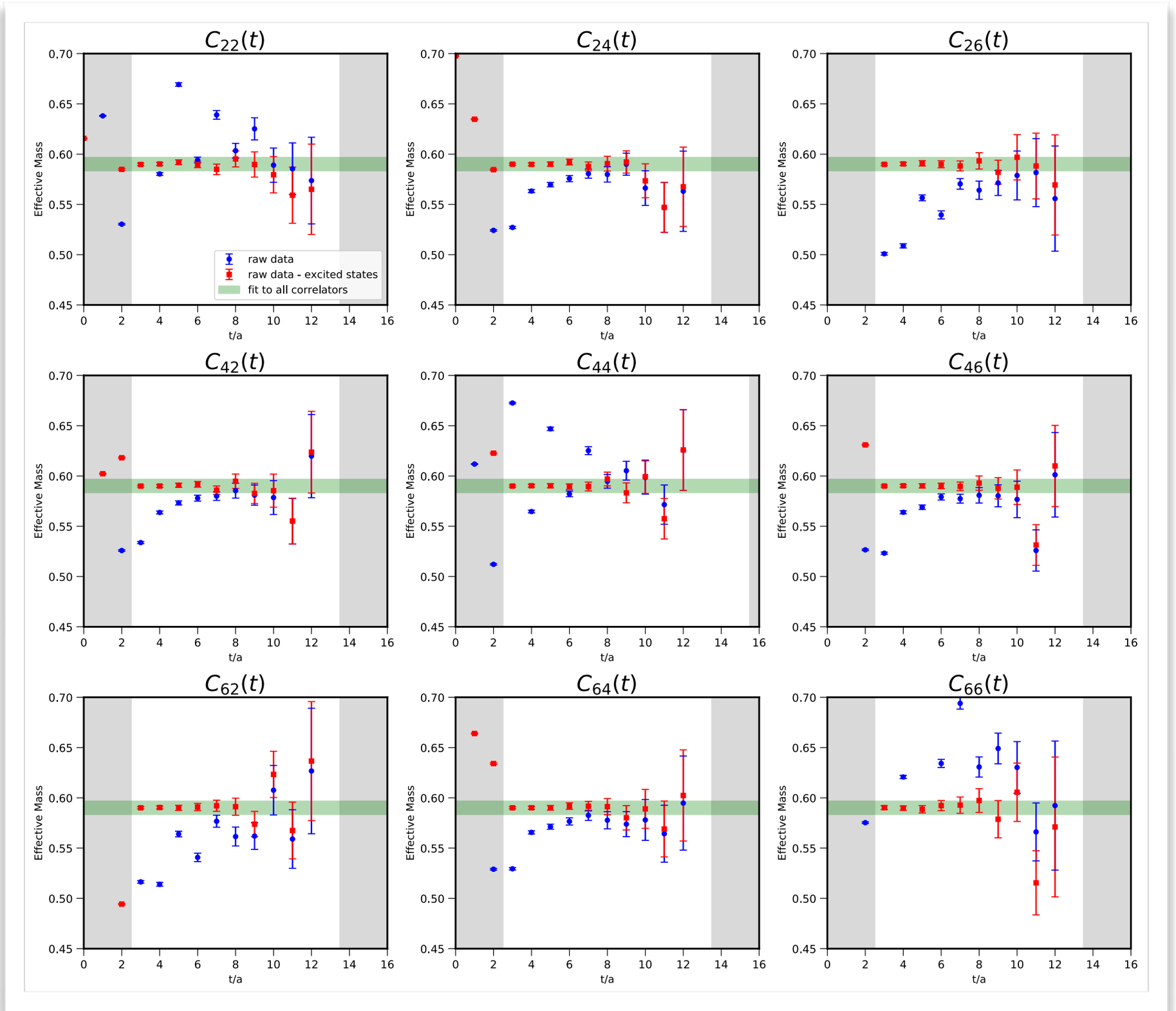
- **Source operators: Approximate projection onto lattice irrep without gauge links → need gauge fixing**

$$\mathcal{O}^{src}(t) = N \sum_{x_i \in even} \sum_{z_i \in even} \sum_{y_i \in even} \sum_{A,B,C} C^{(r)}_{ABC} \chi_A(x,t) \chi_B(y,t) \chi_C(z,t)$$

16 Irrep Correlators Fit Details

- **Nominal fit for a = 0.12fm ensemble with N_E=10 and N_O = 10**
- **Simultaneous fit to all 9 correlators using Bayesian method**
- **Identical large prior widths centered at zero for all overlap factors,** $\langle\Omega|\mathcal{O}_i^{\rm snk}|n^\pm\rangle$ **and** $\langle n^\pm|(\mathcal{O}_i^{\rm src})^\dagger|\Omega\rangle$
- **Fit from t_{min}= 3 to t_{max} such that percent error < 10%**
- **Use $C_{22}(t)$ to demonstrate quality of fit (full correlators fit in backup slides)**

16 Irrep Correlators Fit



16 Irrep Correlators Fit

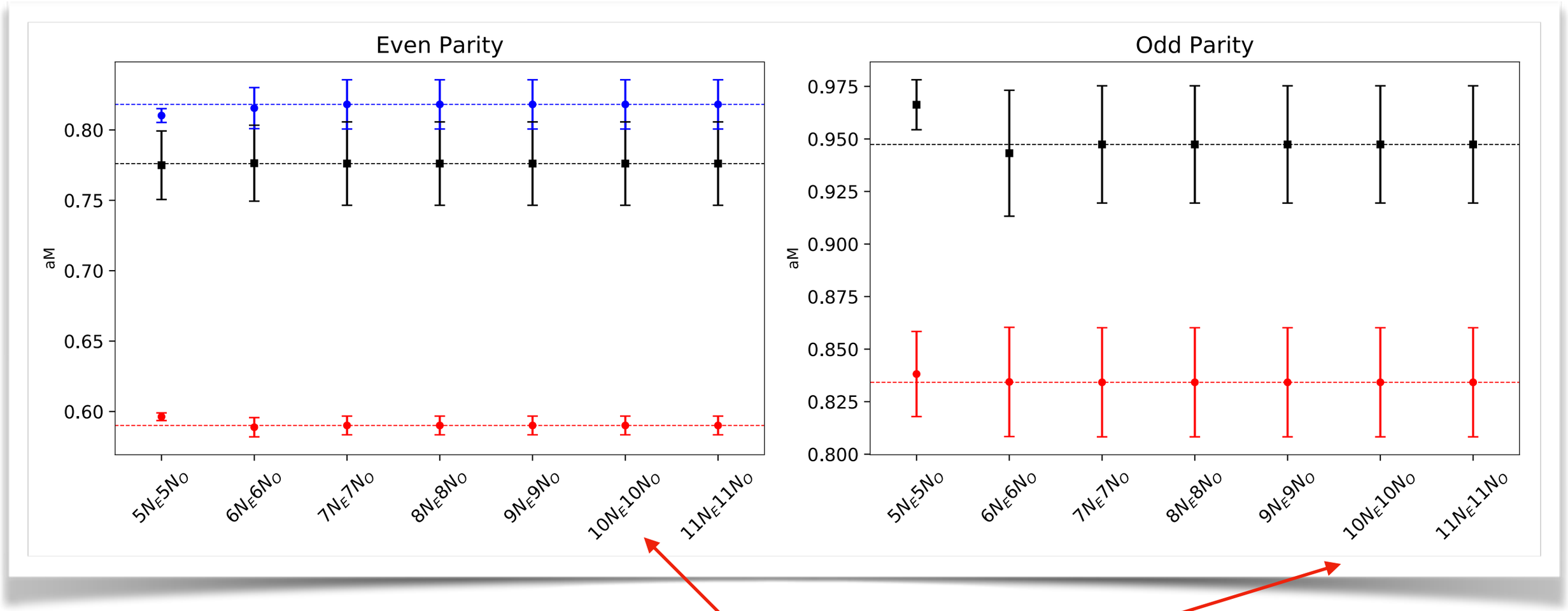
N-like

Δ'-like

	Parity	Prior (MeV)	Posterior (MeV)	Abs. (MeV)
M_0^+	$+$	970(50)	953(11)	953(11)
ΔM_1^+	$+$	250(100)	300(48)	1253(48)
ΔM_2^+	$+$	60(50)	68(44)	1321(28)
ΔM_3^+	$+$	400(200)	329(149)	1649(153)
ΔM_4^+	$+$	400(200)	383(183)	2033(236)
ΔM_5^+	$+$	400(200)	384(185)	2417(299)
ΔM_6^+	$+$	400(200)	392(190)	2809(353)
ΔM_7^+	$+$	400(200)	399(194)	3208(402)
ΔM_8^+	$+$	400(200)	399(194)	3606(446)
ΔM_9^+	$+$	400(200)	399(194)	4005(487)
ΔM_0^-	$-$	1375(100)	1347(42)	1347(42)
ΔM_1^-	$-$	200(100)	183(54)	1529(45)
ΔM_2^-	$-$	400(200)	225(83)	1754(90)
ΔM_3^-	$-$	400(200)	334(152)	2088(161)
ΔM_4^-	$-$	400(200)	384(185)	2472(241)
ΔM_5^-	$-$	400(200)	390(188)	2862(301)
ΔM_6^-	$-$	400(200)	399(194)	3260(358)
ΔM_7^-	$-$	400(200)	399(194)	3659(407)
ΔM_8^-	$-$	400(200)	399(194)	4058(451)
ΔM_9^-	$-$	400(200)	399(194)	4456(491)

16 Irrep Correlators Fit: Stability Tests

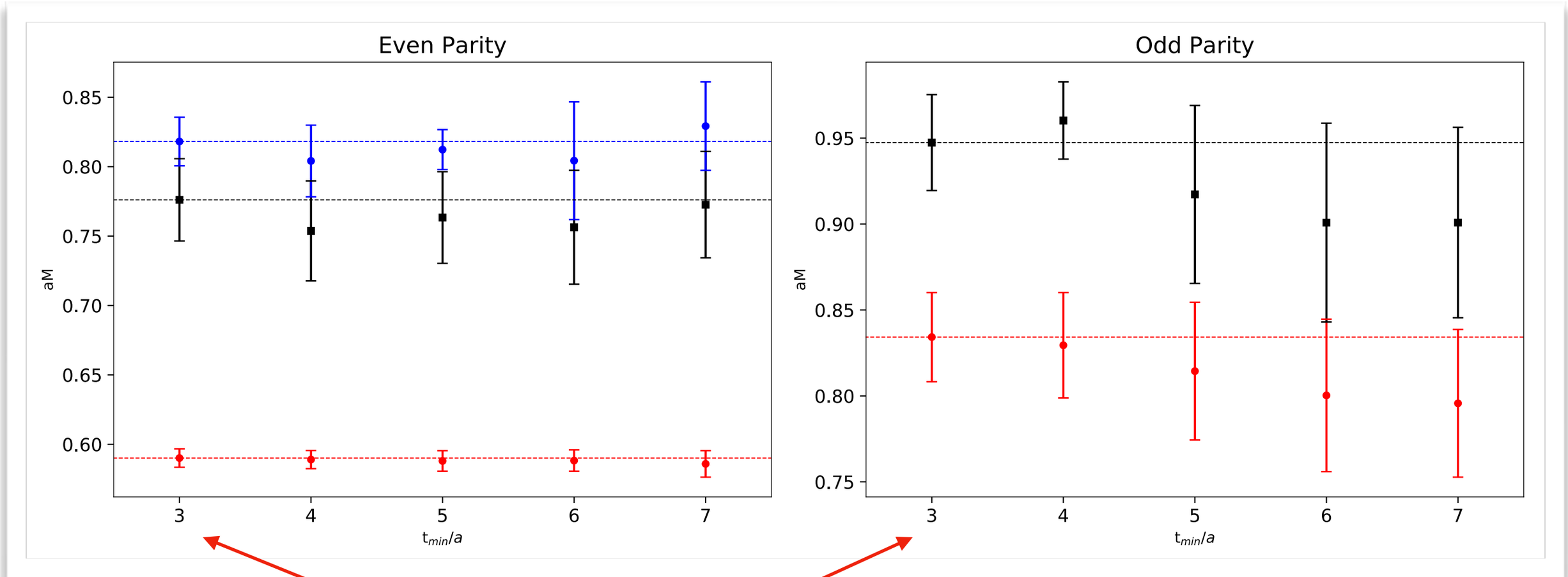
Check 1: States stability



Nominal fit

16 Irrep Correlators Fit: Stability Tests

Check 2: t_{min} stability



Nominal fit

16 Irrep Correlators Fit: Stability Tests

Check 3: Prior stability