Exclusive Channel Study of the Muon HVP

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Introduction
  ▶ Motivation from Experiment
  ▶ Tensions in Experiment

Computation
  ▶ Lattice Parameters
  ▶ GEVP Study

Results
  ▶ Correlation Function Reconstruction
  ▶ (Improved) Bounding Method

Conclusions/Outlook
Introduction
### Pieces of Muon $g - 2$ Theory Prediction

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value $\times 10^{10}$</th>
<th>Uncertainty $\times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>11 658 471.895</td>
<td>0.008</td>
</tr>
<tr>
<td>EW</td>
<td>15.4</td>
<td>0.1</td>
</tr>
<tr>
<td>HVP LO</td>
<td>692.5</td>
<td>2.7</td>
</tr>
<tr>
<td>HVP NLO</td>
<td>-9.84</td>
<td>0.06</td>
</tr>
<tr>
<td>HVP NNLO</td>
<td>1.24</td>
<td>0.01</td>
</tr>
<tr>
<td>Hadronic light-by-light</td>
<td>10.5</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Total SM prediction</strong></td>
<td><strong>11 659 181.7</strong></td>
<td><strong>3.8</strong></td>
</tr>
<tr>
<td>BNL E821 result</td>
<td>11 659 209.1</td>
<td>6.3</td>
</tr>
<tr>
<td>Fermilab E989 target</td>
<td></td>
<td>$\approx 1.6$</td>
</tr>
</tbody>
</table>

Experiment-Theory difference is $27.4(7.3) = 3.7\sigma$ tension!

[Blum et al., (2018)]
Tensions in Experiment

R-ratio data for $ee \rightarrow \pi\pi$ exclusive channel, $\sqrt{s} = 0.6 - 0.9$ GeV region
Tension between most precise measurements
Other measurements not precise enough to favor one over the other

Avoid tension by computing precise lattice-only estimate of $a_{\mu}^{HVP}$
Use lattice QCD to inform experiment, resolve discrepancy

[Davier, KEK (2018)]
Interplay between R-ratio, Lattice

\[ a_{\mu}^{\text{cont.}} = \int_0^\infty dt \ K(t) C^{\text{cont.}}(t) \]
\[ a_{\mu}^{\text{latt.}} = \sum_t w_t C^{\text{latt.}}(t) \]
\[ C^{\text{cont.}}(t) = \int_0^\infty d\sqrt{s} \ s \ R(s) \ e^{-\sqrt{s} t} \]
\[ C^{\text{latt.}}(t) = \sum_n |\Omega| V_{\mu} |n\rangle |^2 e^{-E_n t} \]

\[ w_t \text{ from Bernecker, H. Meyer: 1107.4388 [hep-lat]} \]

R-Ratio, Lattice precise in complimentary regions
Lattice uncertainty dominated by long-distance region
\[ \implies \text{need to address long-distance region to reduce lattice uncertainty} \]

Precisely determine \( E_n \) and \( \langle \Omega | V_{\mu} |n\rangle \) from exclusive \( \pi\pi \) study
Use those to approximate \( C^{\text{latt.}}(t) \) for large \( t \)
Computation
Computed on 2 + 1 flavor Möbius Domain Wall Fermions for valance and sea, $M_\pi$ at physical value on all ensembles

All results in this talk on one coarse ensemble:

- $a \approx 0.20 \text{ fm} \approx (1.015 \text{ GeV})^{-1}$,
- $24^3 \times 64 (4.8 \text{ fm})$

Extending program to three other ensembles:

- 2 ensembles on same volume - volume dependence (see C. Lehner’s talk)
- multiple lattice spacings - continuum extrapolation
Distillation

Phys.Rev.D 80, 054506 (0905.2160 [hep-lat])

Eigenvectors of (spin-diagonal) Laplacian operator used to construct projection matrices (\(M \to \infty\) gives identity)

\[
\mathcal{P}_{t;xy}^{ab} = \sum_{i=0}^{M-1} \langle x|i^a_t\rangle \langle i^b_t|y \rangle
\]

Inserting distillation projection matrices smears quarks in bilinear

\[
\sum_a \tilde{Q}^a(z) \Gamma Q^a(z) \to \sum_{xycab} \tilde{Q}^a(x) \mathcal{P}_{t;xz}^{ac} \Gamma \mathcal{P}_{t;zy}^{cb} Q^b(y)
\]

\[
= \sum_{xycab} \tilde{Q}^a(x) f^{ac}(x - z) \Gamma f^{cb}(z - y) Q^b(y)
\]

Propagators contracted with eigenvectors at source & sink creates “perambulator” objects

\[
M_{t,\beta\alpha}^{ii} = \sum_{xy} \sum_{ab} \langle j^b_t|y \rangle \left( D_{yx,\beta\alpha}^{ba} \right)^{-1} \langle x|i^a_0 \rangle
\]

Perambulators stitched together to form desired \(N\)-point correlation functions

\[\implies\] ideal for creating \(2\pi \to 2\pi\) correlation functions
Fit Procedure

Operators in $I = 1$ $P$-wave channel

Local vector current operator:

Local $O_0 = \sum_x \bar{\psi}(x)\gamma_\mu \psi(x)$, $\mu \in \{1, 2, 3\}$

Three $2\pi$ operators with $O_{1,2,3}$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$

$$O_n = \left| \sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi \cdot \vec{z}}\gamma_5 f(z-y)\psi(y) \right|^2$$

Correlators arranged in a $4 \times 4$ symmetric matrix:

<table>
<thead>
<tr>
<th>$\otimes$</th>
<th>$O_0$</th>
<th>$O_1$</th>
<th>$O_2$</th>
<th>$O_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_0$</td>
<td>$C^{(2)}_\rho$</td>
<td>$C^{(3)}_\rho \rightarrow \pi \pi$</td>
<td>$C^{(3)}_\rho \rightarrow \pi \pi$</td>
<td>$C^{(3)}_\rho \rightarrow \pi \pi$</td>
</tr>
<tr>
<td>$O_1$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
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</tr>
<tr>
<td>$O_3$</td>
<td>$C^{(4)}_{\pi \pi \rightarrow \pi \pi}$</td>
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</table>

Extra operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (2, 0, 0)$ to estimate excited state systematics

Generalized EigenValue Problem (GEVP) to estimate overlaps & energies

$$C(t) V = C(t + \delta t) V \Lambda(\delta t) ; \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t} , \quad V_{im} \propto \langle \Omega | O_i | m \rangle$$

Reconstruct exponential dependence of local vector correlation function

$$C^{latt.}_{ij}(t) = \sum_{n}^{N} \langle \Omega | O_i | n \rangle \langle n | O_j | \Omega \rangle e^{-E_n t}$$

In practice, only finite $N$ necessary to model correlation function
Scatter points from solving GEVP at fixed $\delta t$

$$C(t) \ V = C(t + \delta t) \ V \ \Lambda(\delta t), \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Black lines are from fit ansatz: $f_i(t) = E_i + \alpha e^{-(E_N-E_i)t}$

Overlaps picked to have approximately same contamination from excited states

Bands are extracted spectrum/overlaps ($= E_i$), with excited state systematics

Systematics estimated from difference between 4- and 5-operator GEVP basis.
Correlation Function Reconstruction

**GEVP results to reconstruct long-distance behavior of local vector correlation function needed to compute connected HVP**

**Explicit reconstruction good estimate of correlation function at long-distance, missing excited states at short-distance**

**More states \(\implies\) better reconstruction, can replace \(C(t)\) at shorter distances**
Improved Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on $a^{HVP}_{\mu}$

$$\tilde{C}(t; t_{\text{max}}, E) = \begin{cases} 
C(t) & t < t_{\text{max}} \\
C(t_{\text{max}}) e^{-E(t-t_{\text{max}})} & t \geq t_{\text{max}}
\end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

Lower bound: $E = \log[\frac{C(t_{\text{max}})}{C(t_{\text{max}}+1)}]$

Good control over lower states in spectrum with exclusive reconstruction:

Replace $C(t) \rightarrow C(t) - \sum_{n}^{N} |c_n|^2 e^{-E_n t}$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\implies$ Smaller overall contribution from neglected states

Add back contribution from reconstruction after bounding correlator
Bounding Method

(See talk by C. Lehner)

No bounding method:
Bounding method $t_{\text{max}} = 2.1$ fm, no reconstruction:

$$a_{\mu}^{\text{HVP}} = 577(31)$$
$$a_{\mu}^{\text{HVP}} = 566.8(9.0)$$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
Could expect 10 – 20% systematic errors
Bounding Method

\[ \sum_{t=0}^{T/2} w_t \hat{C}(t) \]

(See talk by C. Lehner)

PRELIMINARY

No bounding method:
Bounding method \( t_{\text{max}} = 2.1 \) fm, no reconstruction: \( a_{\mu}^{HVP} = 577(31) \)
Bounding method \( t_{\text{max}} = 1.7 \) fm, 1 state reconstruction: \( a_{\mu}^{HVP} = 566.8(9.0) \)

Bounding method \( t_{\text{max}} = 1.6 \) fm, 2 state reconstruction: \( a_{\mu}^{HVP} = 559.5(3.8) \)

Very large lattice spacing: \( a^{-1} = 1.015 \) GeV, finite volume effects
Could expect 10 – 20% systematic errors
Bounding Method

(See talk by C. Lehner)

No bounding method:
Bounding method $t_{\text{max}} = 2.1$ fm, no reconstruction:
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Bounding method $t_{\text{max}} = 1.6$ fm, 2 state reconstruction:

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
Could expect 10 – 20% systematic errors

$$a_{\mu}^{\text{HVP}} = 577(31)$$
$$a_{\mu}^{\text{HVP}} = 566.8(9.0)$$
$$a_{\mu}^{\text{HVP}} = 561.5(4.5)$$
$$a_{\mu}^{\text{HVP}} = 559.5(3.8)$$
Outlook and Conclusions
Summary

- $g - 2$ is an interesting and exciting topic to work on!
- Tensions in experimental $ee \rightarrow \pi \pi$ data make independent study of exclusive channels valuable
- Lattice QCD is a first principles method capable of accessing necessary matrix elements
- Additional studies using correlated fits, additional ensembles in progress
- Study of exclusive channels able to significantly reduce statistical uncertainty on an all-lattice computation of muon HVP
  \[ \Rightarrow \text{expect lattice-only calculation with precision comparable to R-ratio by 2020} \]
- Part of ongoing lattice study to address all lattice systematics in HVP computation
Thanks

Computing time support from many sources:

▶ ANL
▶ BNL
▶ Oak Forest
▶ Hokusai
▶ USQCD
▶ XSEDE

Lots of data to analyze, lots of work ahead of us!

Thank you for your attention!
Backup
Full program of computations to reduce uncertainties:

Reduce statistical uncertainties on light connected contribution

Compute QED contribution

Improve lattice spacing determination

Finite volume and continuum extrapolation study
Distillation Smearing Visualization

Free-field Laplacian in 2-dimensions, $24^2$ volume
More evecs, better ability to localize

9 evecs (57 equiv), $\sum_i p_i^2 \leq 2$

13 evecs (99 equiv), $\sum_i p_i^2 \leq 4$

21 evecs (171 equiv), $\sum_i p_i^2 \leq 5$