Determination of nucleon sigma terms II

Christian Hoelbling
Bergische Universität Wuppertal
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Budapest-Marseille-Wuppertal collaboration
Scalar quark content of the nucleon

Nucleon mass:

\[ M_N = \langle N | T^\mu_\mu | N \rangle - \langle 0 | T^\mu_\mu | 0 \rangle \]

Quark contribution to energy-momentum tensor (lattice regularization):

\[ T^\mu_\mu = \sum_q m_q \bar{q}q + H_U \]

Sigma terms give quark mass contribution towards nucleon mass

\[ \sigma_q = m_q \langle N | \bar{q}q | N \rangle - m_q \langle 0 | \bar{q}q | 0 \rangle \]

Also effective scalar couplings to quarks in nucleons: relevant for DM searches etc.
Feynman-Hellmann theorem

Theorem linking 2-point and 3-point functions

Requirements for lattice theory:
- Transfer matrix on $T = e^{-\bar{\psi}M(U)\psi}$ on gauge config $U$ exists
- Mass term of the form $M_{x,y}(U) = m\delta_{x,y}$

Easy to show that

$$\frac{\partial M_N}{\partial m_q} = m_q\langle N|\bar{q}q|N\rangle - m_q\langle 0|\bar{q}q|0\rangle = \sigma_q$$

Strategy:

$$\frac{\partial M_N}{\partial m_q} = \frac{\partial M_N}{\partial M_P^2} \frac{\partial M_P^2}{\partial m_q}$$

- $\partial M_P^2/\partial m_q$ with physical point staggered data \(\rightarrow\) Lukas Varnhorst
- $\partial M_N/\partial M_P^2$ with 3-HEX clover: this talk
Our Ensembles

- 3-HEX $N_f=4\times1+\text{QED}$
- 2-HEX $N_f=2+1$
- 6-stout $N_f=2+1$

Christian Hoelbling (Wuppertal)
Nucleon quark content

\[(2M_K^2 - M_\pi^2)_{\frac{1}{2}} [\text{MeV}]\]

\[\beta = 3.2\]
\[\beta = 3.3\]
\[\beta = 3.4\]
\[\beta = 3.5\]
Excited state contributions

- Multiple fit ranges
- Per range, keep excited state error constant relative to statistical
  (Assume $\Delta M = 500\text{MeV}$)
- Crosschecked for consistency with excited state fits
Check for random distribution of ensemble fit qualities

KS test of quality of fit cdf

4 plateaux ranges in final analysis
Analysis strategy

Problem:
- Determine \( M_P^2 = M_\pi^2, M_{K\chi}^2 ( = M_K^2 - M_\pi^2 / 2 ) \) dependence of \( M_N \) at physical point

Method:
- Fit \( M_N(M_\pi, M_{K\chi}, L, a) \)
  - Added dedicated FV configs from QCD+QCD ensembles (neutral mesons and baryons extracted)
- Set scale with \( M_N \)
  - Crosscheck with \( M_\Omega \) scale setting
  - No discretization terms at physical point \( \phi \):
    - either \( \alpha a \) or \( a^2 \) times \( (M_\pi^2 - (M_\pi^\phi)^2) \) and \( (M_{K\chi}^2 - (M_{K\chi}^\phi)^2) \)
- Estimate systematic error
Nucleon fit

- $M_\pi \ < \ \{360, 420\}$
- Various Polynomial, Padé and $\chi$PT ansätze
- Spread into systematic error
- $M_N \propto M_0 + cM_\pi$
- bad $Q$ and wrong $M_\Omega$
Nucleon fit

\[ M_N \propto M_0 + cM_\pi \]

\[ M_\pi \text{ MeV} < \{360, 420\} \]

Various Polynomial, Padé and \( \chi \)PT ansätze

Spread into systematic error

\[ M_N \propto M_0 + cM_\pi \]
bad \( Q \) and wrong \( M_\Omega \)
We fit leading effects
\[ \frac{M_X(L) - M_X}{M_X} = c M_\pi^{1/2} L^{-3/2} e^{-M_\pi L} \]

Compatible with \(\chi PT\) expectation: \(c = 36(12)(5)\text{GeV}^{-2}\)

(Colangelo et. al., 2010)
One conservative strategy for systematics: (BMWc 2008, BMWc 2014)

- Identify all higher order effects you have to neglect
- For each of them:
  - Repeat the entire analysis treating this one effect differently
  - Add the spread of results to systematics

Important:
- Do not do suboptimal analyses
- Do not double-count analyses

Error sources considered:
- Plateaux range (Excited states)
- $M_\pi, M_{K\chi}$ interpolations/extrapolations
- Cuts on maximal $M_\pi$
- Continuum extrapolation
Systematic error

- Total 6144 analyses:
  - 64 variations of matrix $J$:
    - 4 $m_{ud}$ continuum terms
    - 4 $m_s$ continuum terms
    - 2 plateaux ranges
  - 96 variations of $\sigma_{\pi,K}\chi$
    - 2 $M_{K\chi}$ fit forms
    - 2 $M_\pi$ fit forms
    - 2 $M_\pi$ cuts
    - 3 continuum terms
    - 4 plateaux ranges
- Other variations crosschecked: no further relevant terms found
Systematic error

- Total 6144 analyses
- Difference: higher order effects
- Draw cdf of results
- Different weights possible
- Crosscheck agreement
From the effective Hamiltonian

\[ H = H_{\text{iso}} + \frac{\delta m}{2} \int d^3x (\bar{d}d - \bar{u}u) \]

we obtain (with \( \delta m = m_d - m_u \) and normalization \( \langle N|N\rangle = 2M_N \))

\[ \Delta_{QCD}M_N = \frac{\delta m}{2M_p} \langle p|\bar{u}u - \bar{d}d|p\rangle \]

which, together with

\[ \sigma_{u/d}^p = \left( \frac{1}{2} \mp \frac{\delta m}{4m_{ud}} \right) \sigma_{ud}^p + \left( \frac{1}{4} \pm \frac{m_{ud}}{2\delta m} \right) \frac{\delta m}{2M_p} \langle p|\bar{d}d - \bar{u}u|p\rangle \]

gives \( r = m_u/m_d \)

\[ \sigma_{u}^p/n = \left( \frac{r}{1 + r} \right) \sigma_{ud}^N \pm \frac{1}{2} \left( \frac{r}{1 - r} \right) \Delta_{QCD}M_N + O(\delta m^2, m_{ud}\delta m) \]

\[ \sigma_{d}^p/n = \left( \frac{1}{1 + r} \right) \sigma_{ud}^N \pm \frac{1}{2} \left( \frac{1}{1 - r} \right) \Delta_{QCD}M_N + O(\delta m^2, m_{ud}\delta m) \]
Preliminary results

Mesonic $\sigma$ terms:

\[
\sigma^N_\pi = 42.0(1.3)(1.4)\text{MeV} \quad \quad \sigma^N_K = 50.9(3.3)(2.8)\text{MeV}
\]

Nucleon mass in $SU(2)$ and $SU(3)$ chiral limit:

\[
M^{SU(2)}_{N\chi} = 895.7(1.4)(1.9)\text{MeV} \quad \quad M^{SU(3)}_{N\chi} = 848.1(3.5)(3.3)\text{MeV}
\]

Quark $\sigma$ terms with staggered mixing matrix:

\[
\sigma^N_{ud} = 37.3(3.0)(4.2)\text{MeV} \quad \quad \sigma^N_s = 54.2(4.3)(3.1)\text{MeV}
\]

With $\Delta_{QCD}M_N = 2.52(17)(24)\text{MeV}$ from (BMWc 2014)

\[
\sigma^p_u = 13.4(1.0)(1.4)\text{MeV} \quad \quad \sigma^p_d = 22.7(2.1)(2.8)\text{MeV}
\]

\[
\sigma^n_u = 11.0(1.0)(1.4)\text{MeV} \quad \quad \sigma^n_d = 27.6(2.0)(2.8)\text{MeV}
\]
PRELIMINARY results

\[ \sigma_{Nq} \text{[Mev]} \]

- \text{proton}
- \text{neutron}

\text{quark q: u, d, s}
COMPARISON

Compatible with old results  Tension with Hoferichter et. al. 15,17

\[ \sigma_{ud}^N \text{ [MeV]} \]

\[ \sigma_{s}^N \text{ [MeV]} \]