Determination of nucleon sigma terms II

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Scalar quark content of the nucleon

Nucleon mass:

$$M_N = \langle N | T^\mu_\mu | N \rangle - \langle 0 | T^\mu_\mu | 0 \rangle$$

Quark contribution to energy-momentum tensor (lattice regularization):

$$T^\mu_\mu = \sum_q m_q \bar{q} q + H_U$$

Sigma terms give quark mass contribution towards nucleon mass

$$\sigma_q = m_q \langle N | \bar{q} q | N \rangle - m_q \langle 0 | \bar{q} q | 0 \rangle$$

Also effective scalar couplings to quarks in nucleons: relevant for DM searches etc.

Feynman-Hellmann theorem

Theorem linking 2-point and 3-point functions

Requirements for lattice theory:

- Transfer matrix on $T = e^{-\bar{\Psi} M(U) \Psi}$ on gauge config U exists
- Mass term of the form $M_{x,y}(U) = m\delta_{x,y}$

Easy to show that

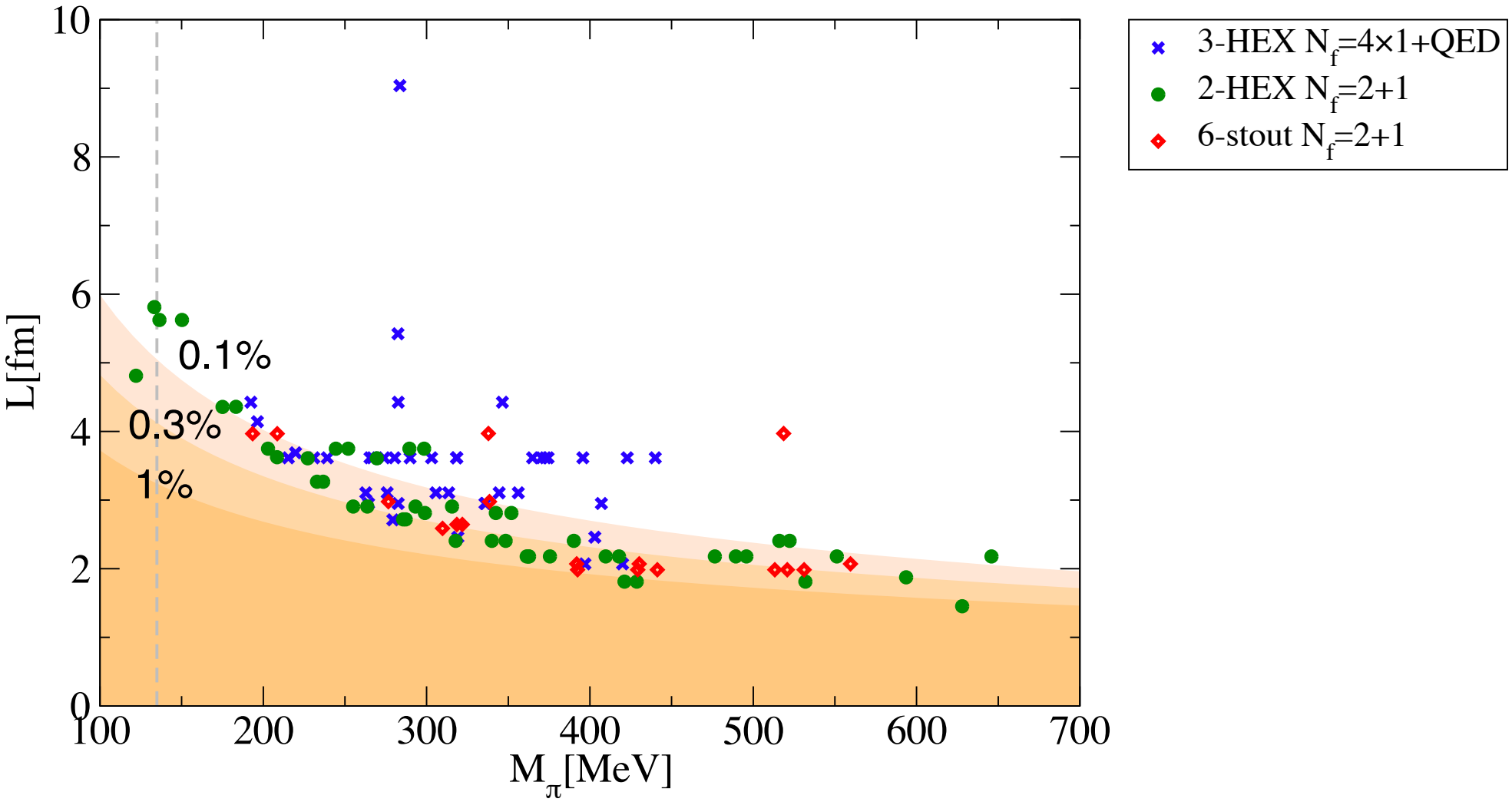
$$\frac{\partial M_N}{\partial m_q} = m_q \langle N|\bar{q}q|N\rangle - m_q \langle 0|\bar{q}q|0\rangle = \sigma_q$$

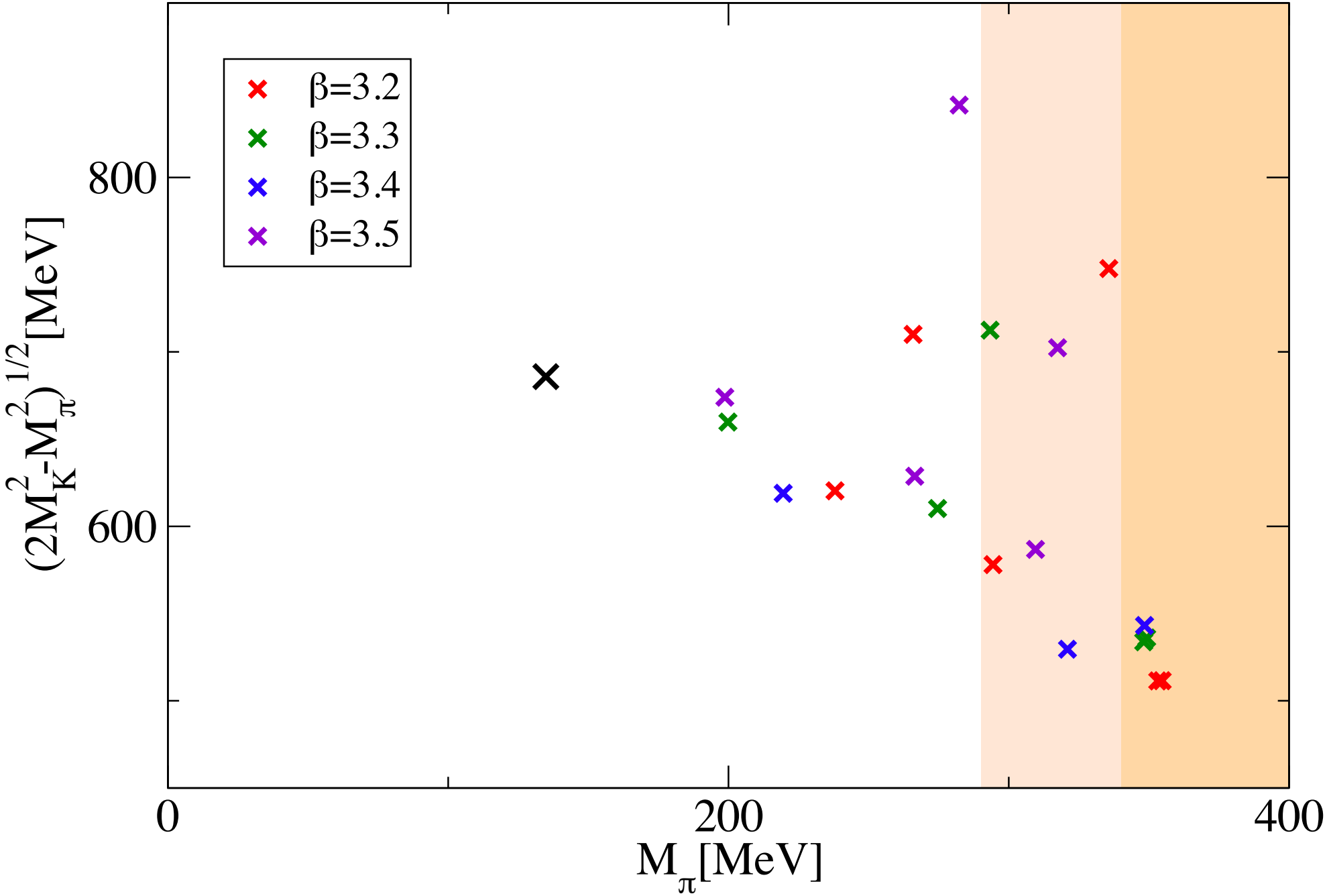
Strategy:

$$\frac{\partial M_N}{\partial m_q} = \frac{\partial M_N}{\partial M_P^2} \frac{\partial M_P^2}{\partial m_q}$$

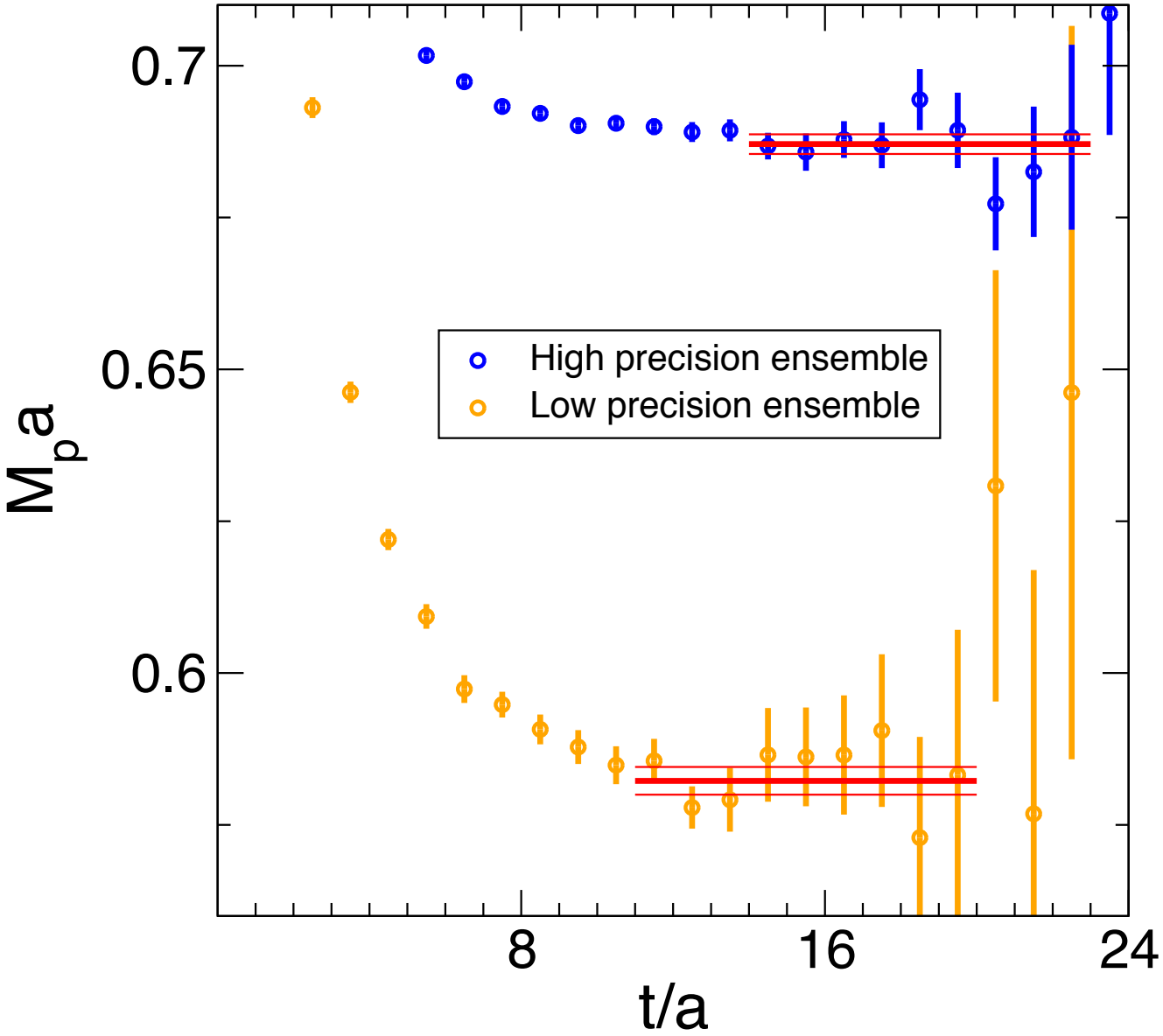
- $\partial M_P^2 / \partial m_q$ with physical point staggered data ➛Lukas Varnhorst
- $\partial M_N / \partial M_P^2$ with 3-HEX clover: this talk

Our Ensembles

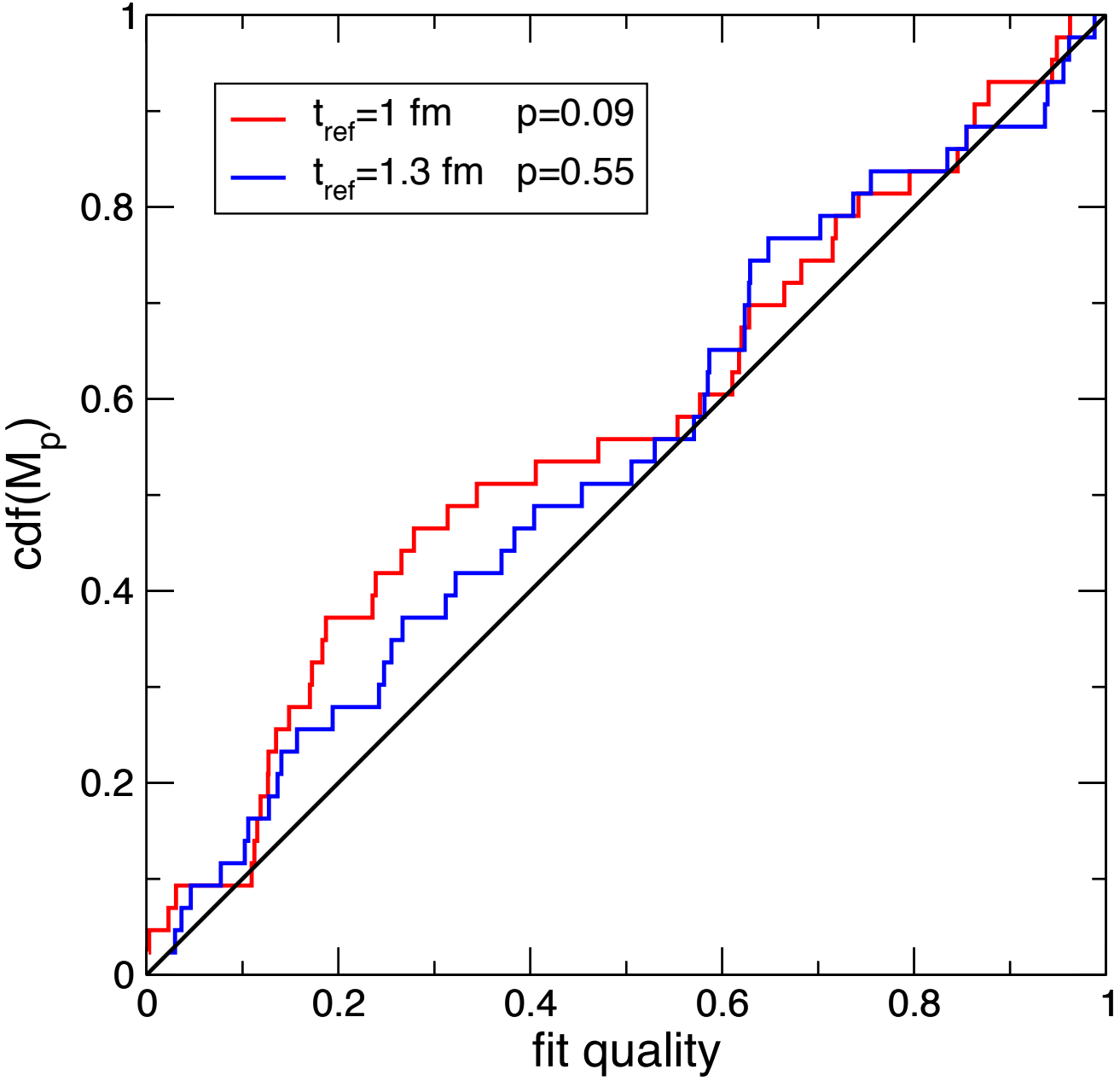



β=3.2
β=3.3
β=3.4
β=3.5
800
600
$(2M_K^2-M_\pi^2)^{1/2}$[MeV]
0
200
400
M_π[MeV]

Excited state contributions



- Multiple fit ranges
- Per range, keep excited state error constant relative to statistical (Assume $\Delta M = 500\text{MeV}$)
- Crosschecked for consistency with excited state fits



- Check for random distribution of ensemble fit qualities
- KS test of quality of fit cdf
- 4 plateaux ranges in final analysis

Analysis strategy

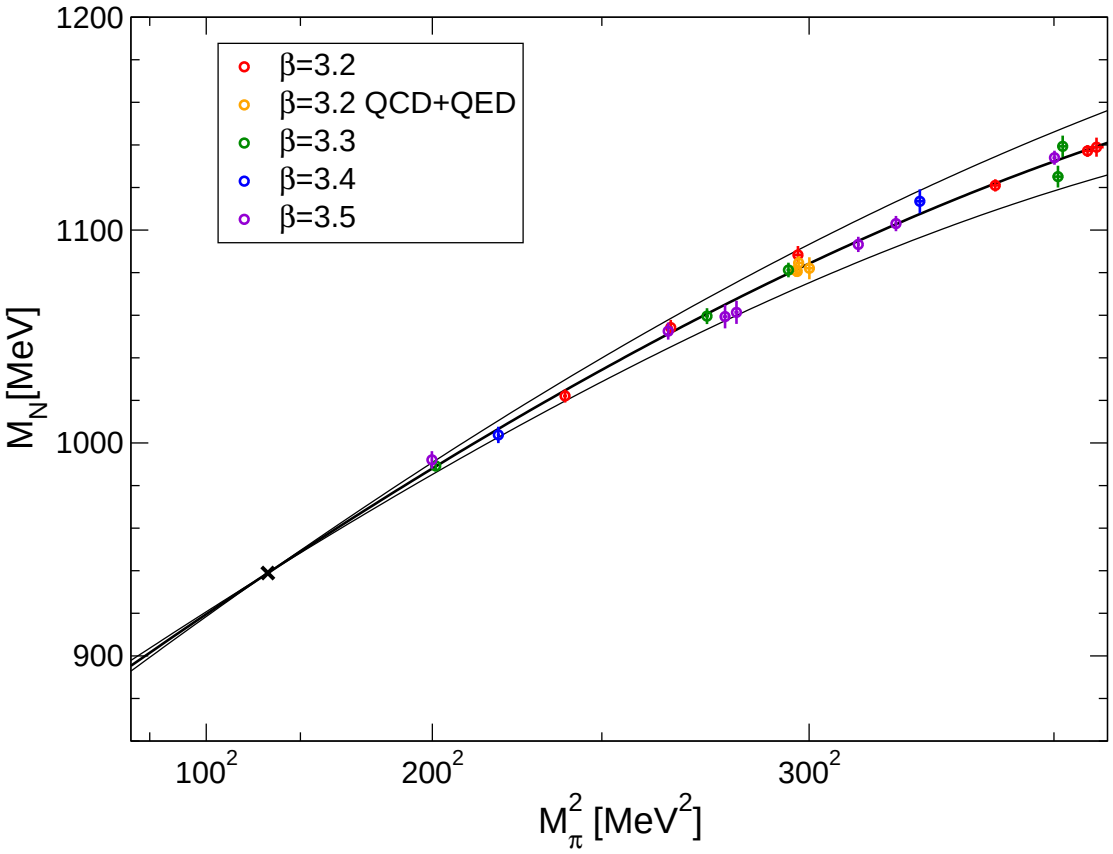
Problem:

- Determine $M_P^2 = M_\pi^2, M_{K_\chi}^2 (= M_K^2 - M_\pi^2/2)$ dependence of M_N at physical point

Method:

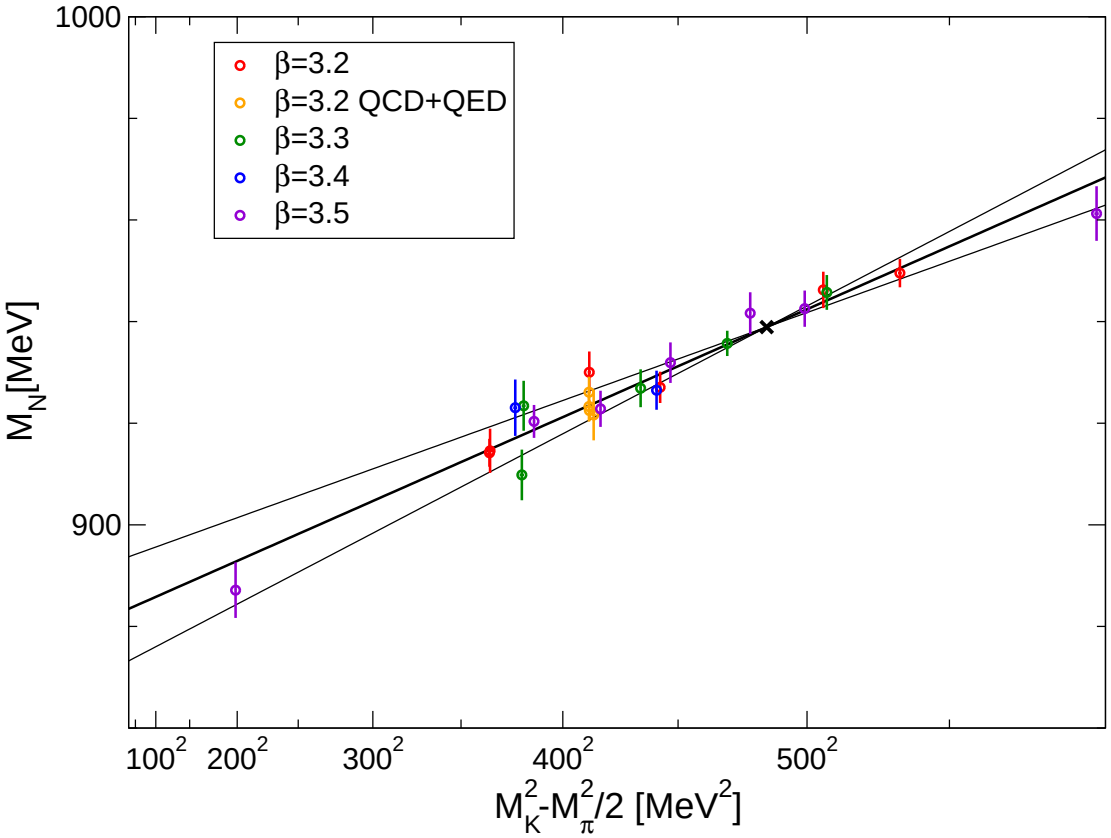
- Fit $M_N(M_\pi, M_{K_\chi}, L, a)$
  - Added dedicated FV configs from QCD+QCD ensembles (neutral mesons and baryons extracted)
- Set scale with M_N
  - Crosscheck with M_Ω scale setting
  - No discretization terms at physical point ϕ: either αa or a^2 times $(M_\pi^2 - (M_\pi^\phi)^2)$ and $(M_{K_\chi}^2 - (M_{K_\chi}^\phi)^2)$
- Estimate systematic error

Nucleon fit



- $\frac{M_\pi}{\text{MeV}} < \{360, 420\}$
- Various Polynomial, Padé and χPT ansätze
- Spread into systematic error
- $M_N \propto M_0 + cM_\pi$
  bad Q and wrong M_Ω

Nucleon fit



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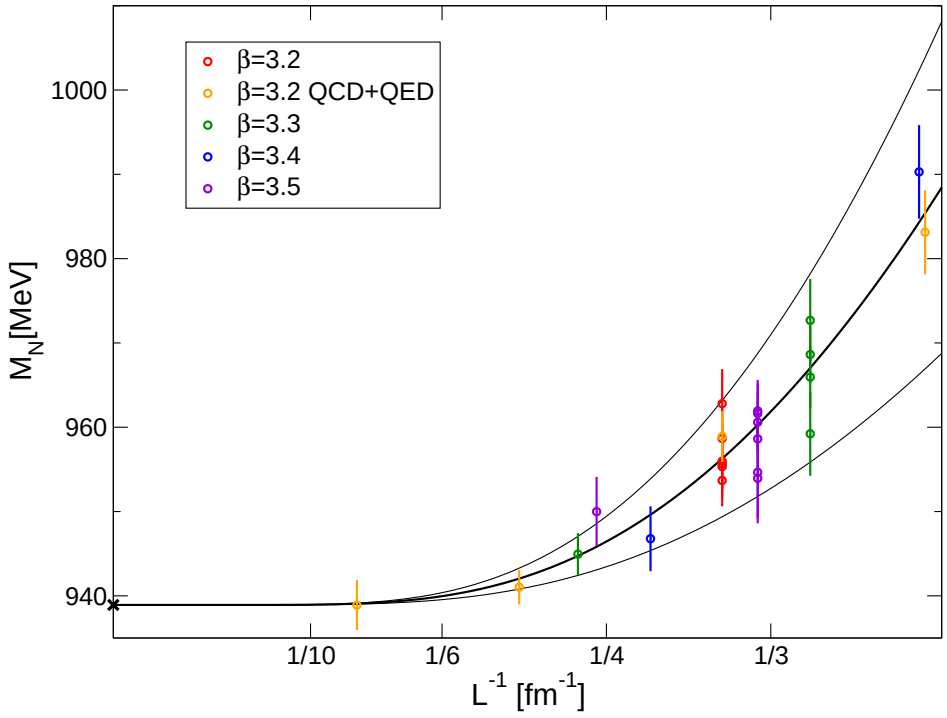
Finite volume effects



- We fit leading effects $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2}L^{-3/2}e^{-M_\pi L}$
- Compatible with χPT expectation: $c = 36(12)(5)\text{GeV}^{-2}$

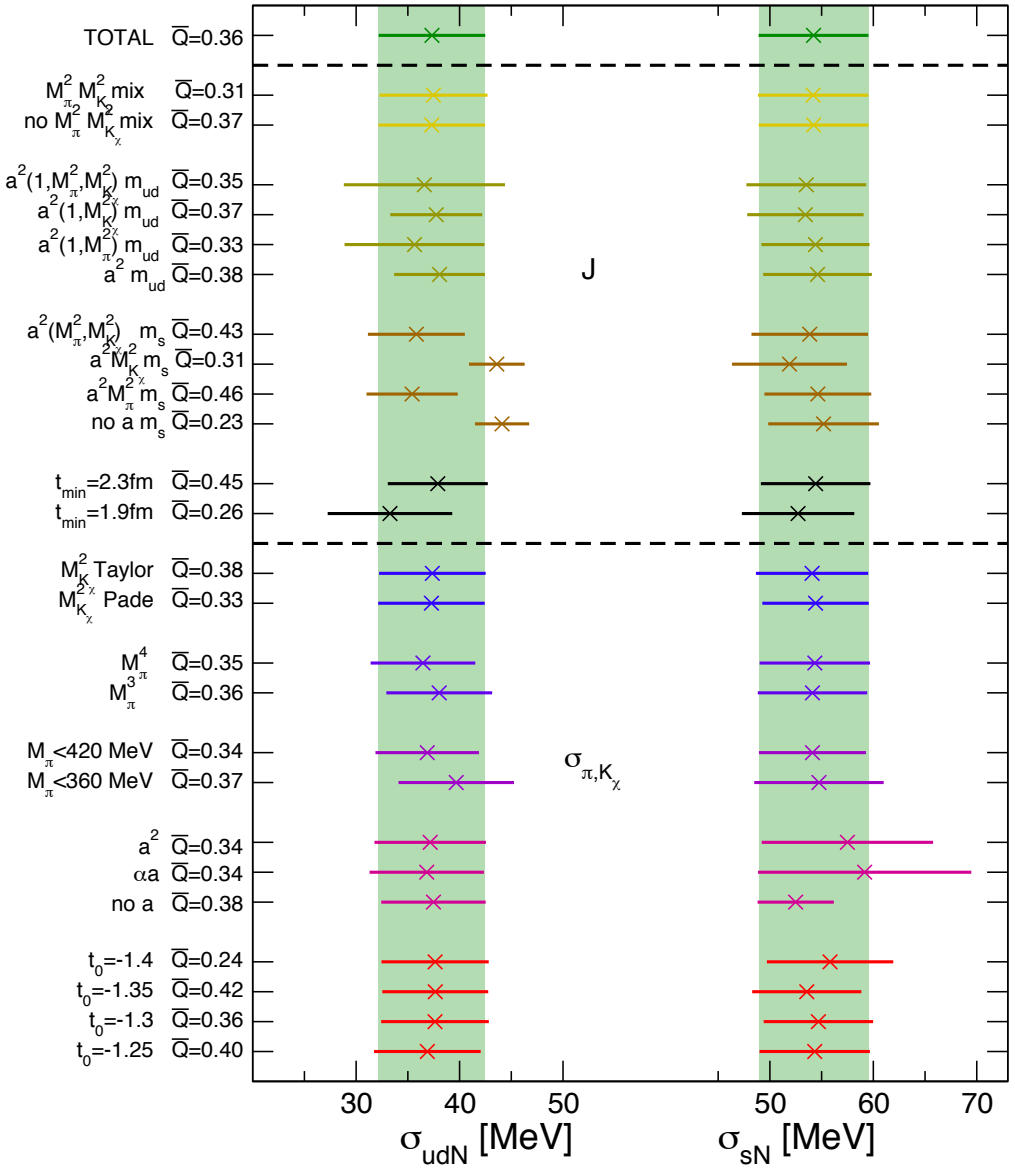
(Colangelo et. al., 2010)

Systematic error treatment

One conservative strategy for systematics:(BMWc 2008, BMWc 2014)

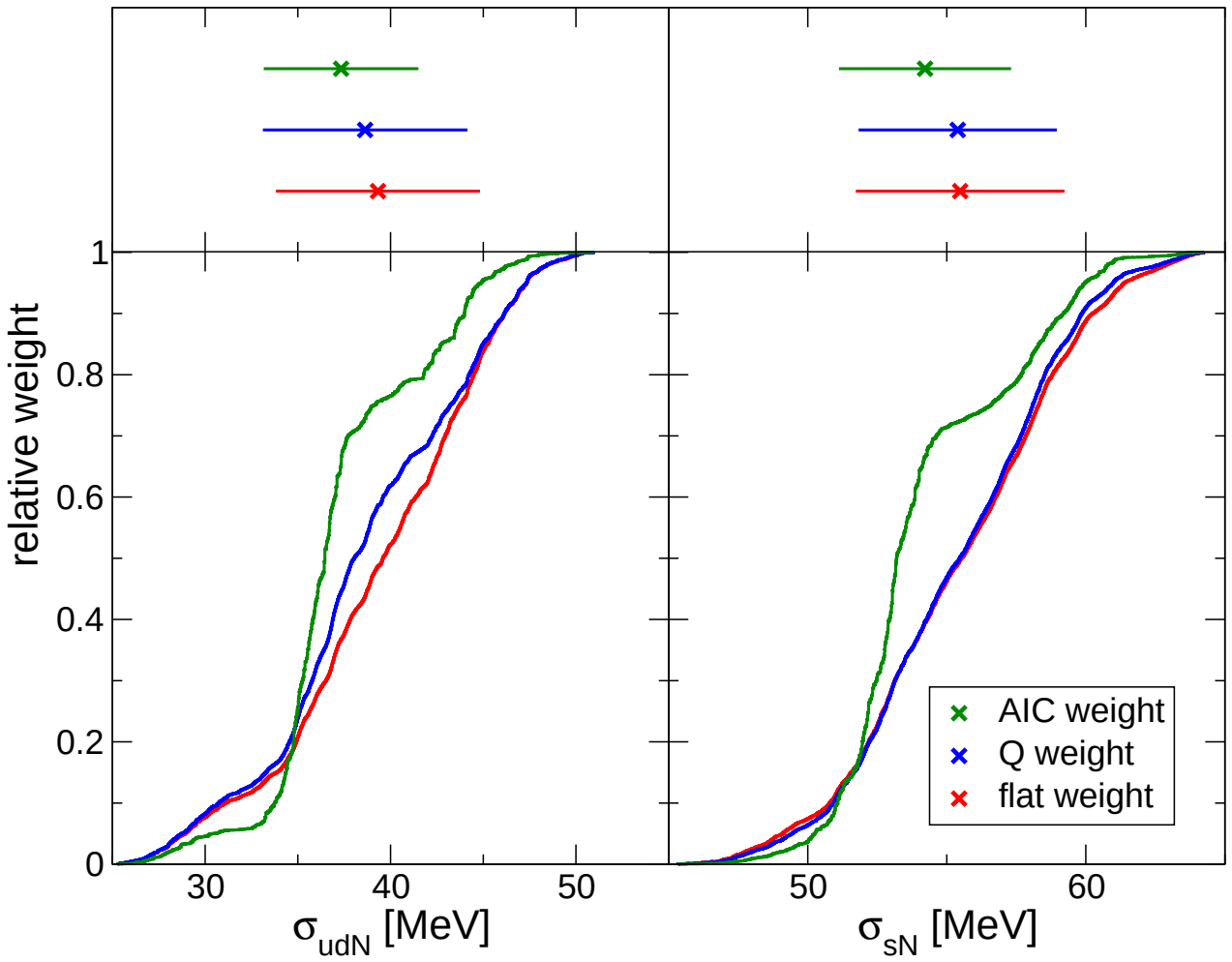
- Identify all higher order effects you have to neglect
- For each of them:
  - Repeat the entire analysis treating this one effect differently
  - Add the spread of results to systematics
- Important:
  - Do not do suboptimal analyses
  - Do not double-count analyses
- Error sources considered:
  - Plateaux range (Excited states)
  - M_π, M_{K_χ} interpolations/extrapolations
  - Cuts on maximal M_π
  - Continuum extrapolation

Systematic error



- Total 6144 analyses:
- 64 variations of matrix J:
  - 4 m_{ud} continuum terms
  - 4 m_s continuum terms
  - 2 plateaux ranges
- 96 variations of σ_{π,K_χ}
  - 2 M_{K_χ} fit forms
  - 2 M_π fit forms
  - 2 M_π cuts
  - 3 continuum terms
  - 4 plateaux ranges
- Other variations crosschecked: no further relevant terms found

Systematic error



- Total 6144 analyses
- Difference: higher order effects
- Draw cdf of results
- Different weights possible
- Crosscheck agreement

From the effective Hamiltonean

$$H = H_{\text{iso}} + \frac{\delta m}{2} \int d^3x (\bar{d}d - \bar{u}u)$$

we obtain (with $\delta m = m_d - m_u$ and normalization $\langle N|N\rangle = 2M_N$)

$$\Delta_{QCD} M_N = \frac{\delta m}{2M_p} \langle p|\bar{u}u - \bar{d}d|p\rangle$$

which, together with

$$\sigma^p_{u/d} = \left(\frac{1}{2} \mp \frac{\delta m}{4m_{ud}}\right) \sigma^p_{ud} + \left(\frac{1}{4} \mp \frac{m_{ud}}{2\delta m}\right) \frac{\delta m}{2M_p} \langle p|\bar{d}d - \bar{u}u|p\rangle$$

gives ($r = m_u/m_d$)

$$\sigma^{p/n}_u = \left(\frac{r}{1+r}\right) \sigma^N_{ud} \pm \frac{1}{2} \left(\frac{r}{1-r}\right) \Delta_{QCD} M_N + O(\delta m^2, m_{ud}\delta m)$$

$$\sigma^{p/n}_d = \left(\frac{1}{1+r}\right) \sigma^N_{ud} \mp \frac{1}{2} \left(\frac{1}{1-r}\right) \Delta_{QCD} M_N + O(\delta m^2, m_{ud}\delta m)$$

Preliminary results

Mesonic σ terms:

$$\sigma_\pi^N = 42.0(1.3)(1.4)\text{MeV} \qquad \sigma_{K_\chi}^N = 50.9(3.3)(2.8)\text{MeV}$$

Nucleon mass in $SU(2)$ and $SU(3)$ chiral limit:

$$M_{N\chi}^{SU(2)} = 895.7(1.4)(1.9)\text{MeV} \qquad M_{N\chi}^{SU(3)} = 848.1(3.5)(3.3)\text{MeV}$$

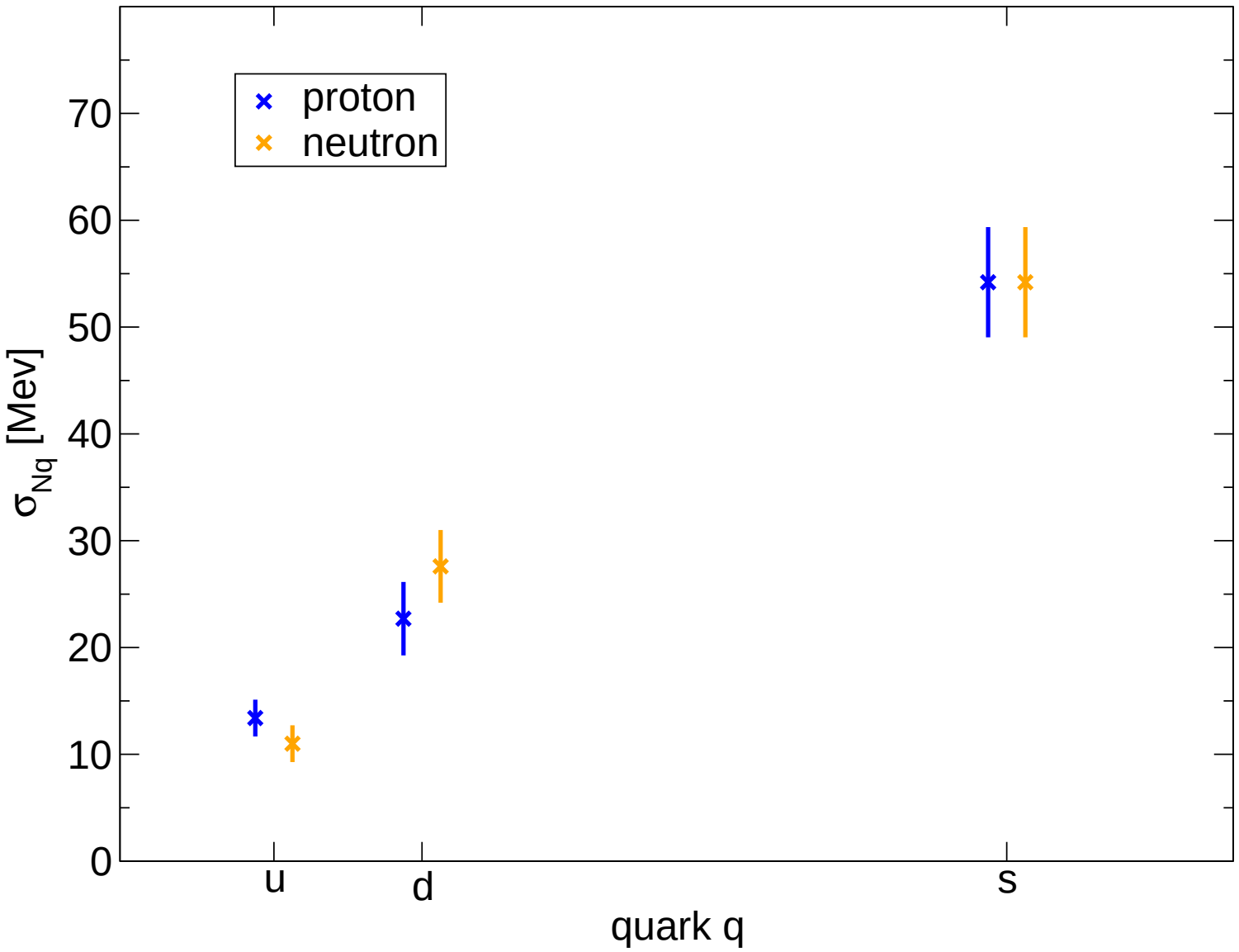
Quark σ terms with staggered mixing matrix:

$$\sigma_{ud}^N = 37.3(3.0)(4.2)\text{MeV} \qquad \sigma_s^N = 54.2(4.3)(3.1)\text{MeV}$$

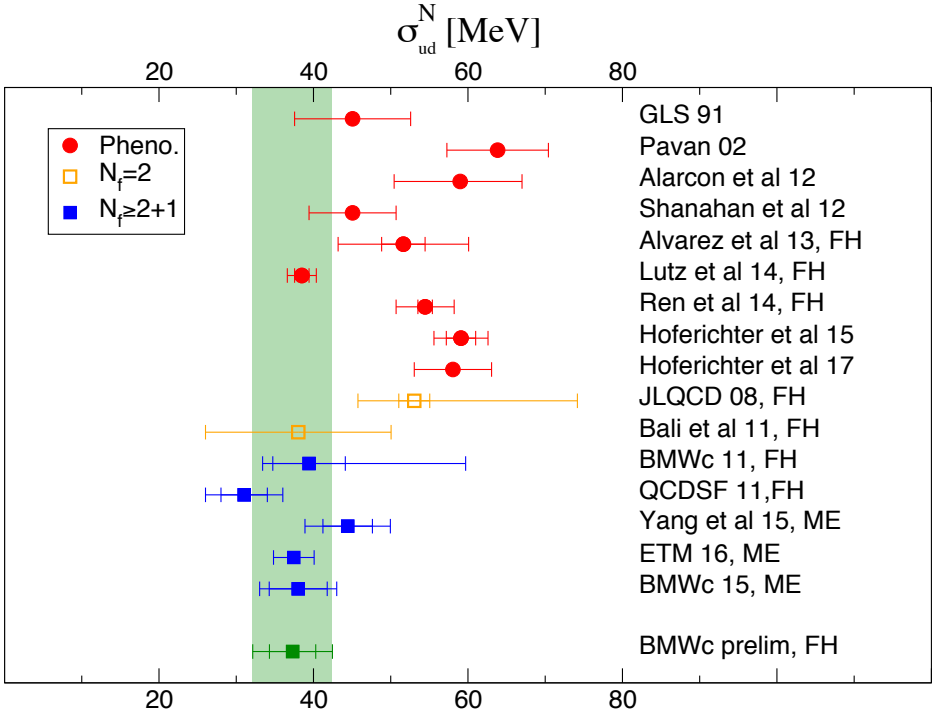
With $\Delta_{QCD} M_N = 2.52(17)(24)\text{MeV}$ from (BMWc 2014)

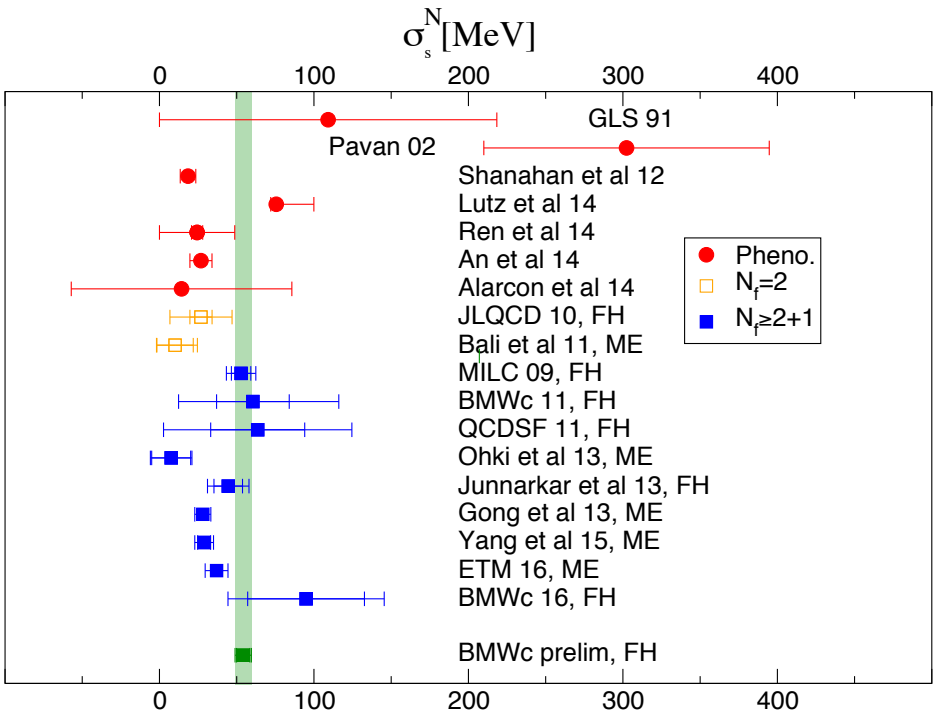
$$\sigma_u^p = 13.4(1.0)(1.4)\text{MeV} \qquad \sigma_d^p = 22.7(2.1)(2.8)\text{MeV}$$
$$\sigma_u^n = 11.0(1.0)(1.4)\text{MeV} \qquad \sigma_d^n = 27.6(2.0)(2.8)\text{MeV}$$

PRELIMINARY results



COMPARISON





Compatible with old results

Tension with Hoferichter et. al. 15,17