Nucleon EDMs on a Lattice at the Physical Point

Sergey N. Syritsyn,
Stony Brook University & RIKEN / BNL Research Center
together with LHP and RBC collaborations

LATTICE 2018
East Lansing, MI, July 22-28, 2018
Outline

Nucleon Electric Dipole Moments: Introduction
  • Motivation
  • Experimental status & outlook
  • Lattice methodology

Physical point calculations with chiral quarks
  • Form Factors → [T.Izubuchi’s talk, July 27 5:30pm @106 (Hadron Structure)]
  • Electric dipole moments induced by quark chromo-EDM

Studies of $\theta_{QCD}$-induced nucleon EDM
  • Noise reduction with subvolume top.charge sampling
  • Results from $m_\pi \approx 330$ MeV lattices
  • Outlook for physical point calculations
Nucleon Electric Dipole Moments

EDMs are the most sensitive probes of CPv:
- Signals for beyond SM physics \((\text{SM} = 10^{-5} \text{ of the current exp.bound})\)
- Prerequisite for Baryogenesis
- \(\theta_{\text{QCD}}\)-induced EDM: Strong CP problem

\[
\vec{d}_N = d_N \frac{\vec{S}}{S} \quad \mathcal{H} = -\vec{d}_N \cdot \vec{E}
\]

OR
\[
\mathcal{L}_{\text{int}} = e A_{\mu}^{\text{em}} \mathcal{V}_\mu \quad (\text{P,T-even})
\]
\[
+ e A_{\mu}^{\text{em}} A^{\mu} \quad (\text{P,T-odd})
\]

\[
\langle N_{p'} | J^\mu | \bar{N}_p \rangle_{\mathcal{G}_P} = \bar{u}_{p'} \left[ F_1 \gamma^\mu + (F_2 + iF_3 \gamma_5) \frac{\sigma^{\mu\nu}(p' - p)_\nu}{2m_N} \right] u_p
\]

Dirac  
Pauli  
(anom.magnetic)  
Electric dipole
Experimental Outlook

Current nEDM limits:
- $|d_n| < 2.9 \times 10^{-26} \text{ e} \cdot \text{cm}$
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26} \text{ e} \cdot \text{cm}$
  [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity:
- 1–2 years: next best limit?
- 3–4 years: x10 improvement
- 7–10 years: x100 improvement

**Current Limit**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Limit 10^{-28} e cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spallation Source @ORNL</td>
<td>&lt;300</td>
</tr>
<tr>
<td>Ultracold Neutrons @LANL</td>
<td>&lt;5</td>
</tr>
<tr>
<td>PSI EDM</td>
<td>~30</td>
</tr>
<tr>
<td>ILL PNPI</td>
<td>&lt;50 (I), &lt;5 (II)</td>
</tr>
<tr>
<td>Munich FRMII</td>
<td>&lt;5</td>
</tr>
<tr>
<td>RCMP TRIUMF</td>
<td>&lt;50 (I), &lt;5 (II)</td>
</tr>
<tr>
<td>JPARC</td>
<td>&lt;5</td>
</tr>
<tr>
<td>Standard Model (CKM)</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

Other experiments: light nuclei in storage rings, octupole-deformed $^{225}\text{Ra}$, etc
Nucleon EDMs: a Window into New Physics

Effective quark-gluon CPv interactions organized by dimension

\[ \mathcal{L}_{\text{eff}} = \sum_i \frac{c_i}{\Lambda(i)^{d_i-4}} \mathcal{O}_i [d_i] \]

- \( d=4 \) : \( \theta_{\text{QCD}} \)
- \( d=5(6) \) : quark EDM, quark-gluon chromo EDM
- \( d=6 \) : 4-fermion CPv, 3-gluon (Weinberg)

\[ d_{n,p} = d_{n,p}^\theta \theta_{\text{QCD}} + d_{n,p}^{cEDM} c_{cEDM} + \ldots \]

lattice QCD calculations are needed to constrain \( \theta_{\text{QCD}}, c_{cEDM}, \ldots \)

\[ c_i \leftrightarrow d_{n,p} \ ? \]

\[ d_{n,p}^{cEDM} = d_{n,p}^{cEDM} + \ldots \]

\[ F_{3n,p}^n(Q^2) \]

CP-odd Nucleon Structure on a Lattice

\[ \langle \mathcal{O} \ldots \rangle_{CP} = \langle \mathcal{O} \ldots \rangle_{CP-even} - i\theta \langle Q \cdot \mathcal{O} \ldots \rangle_{CP-even} + O(\theta^2) \]

\( CP \) coupling \( CP \) operator: G\( \tilde{G} \), cEDM, G\( G \tilde{G} \)(Weinberg), etc

\textit{CPv interaction induces a chiral phase in fermion fields:}

\[ \langle \text{vac} | N | p, \sigma \rangle_{CP} = e^{i\alpha \gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma} \]

To determine \( F_{2,3} \) correctly, one has to use positive-parity spinors

\[ \langle N_{p'} | \bar{q} \gamma^\mu q | N_p \rangle_{CP} = \bar{u}_{p'} \left[ F_1 \gamma^\mu + (F_2 + iF_3 \gamma_5) \frac{i\sigma^{\mu\nu}(p' - p)_\nu}{2m_N} \right] u_p , \quad \text{with} \quad \gamma_4 u = +u \quad \bar{u} \gamma_4 = +\bar{u} \]

Prior to 2017, lattice determinations of EDM were subject to large bias from \( F_{2,3} \) mixing

\[ "F_3" \approx [F_3]_{\text{true}} - 2\alpha [F_2]_{\text{true}} \]

\[ "d_{n,p}" \approx [d_{n,p}]_{\text{true}} - 2\alpha \frac{\kappa_{n,p}}{2m_N} \]
Quark Chromo-EDM on a Lattice

\[ \mathcal{L}_{c\text{EDM}} = \sum_{q=u,d} \frac{\tilde{\delta}q}{2} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \right] q \]

- dim-5 operator: \( O(a^{-2}) \) mixing with dim-3 pseudoscalar density
  \( \Rightarrow \) evaluate & subtract p,nEDM induced by PS density \( P = \bar{q} \gamma_5 q \)
  [T.Bhattacharya et al, 1502.07325]

- Chiral symmetry is important:
  \( O(a) \) clover term in, e.g., Wilson fermion action \( \equiv \) chromo-magnetic DM
  \[ \mathcal{L}_{\text{clover}} = a^2 \frac{c}{4} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \right] q \]

In presence of CPv, condensate is realigned \( q \rightarrow e^{i\gamma_5 \Omega} q \)
so that \( \langle \text{vac} | \mathcal{L}_m + \mathcal{L}_{\text{CP}} | \pi^a \rangle = 0 \)

leading to mixing (chromo)EDM \( \leftrightarrow \) (chromo)MDM:
\[ \delta \mathcal{L}_{c\text{EDM}} = \delta(\bar{q} [\tilde{D}_q G_{\mu\nu} \sigma^{\mu\nu} \gamma_5] q) = \bar{q} \left[ \{\Omega, \tilde{D}_q\} G_{\mu\nu} \sigma^{\mu\nu} \right] q \sim \delta \mathcal{L}_{c\text{MDM}} \]
Quark-Gluon EDM: Insertions of dim-5 Operators

\[ \mathcal{L}^{(5)} = \sum_q \bar{d}_q \bar{q} (G \cdot \sigma) \gamma_5 q \]

\[ \langle N(y) \tilde{N}(0) \int d^4x \bar{q}(G \cdot \sigma) \gamma_5 q \rangle \]

**This work:** Only quark-connected insertions

\[ B_u(\sigma \bar{G}) C_u \]
\[ \bar{E}_u(\sigma \bar{G}) F_u \]
\[ G_{u \tau}(\sigma \bar{G}) F_u \]
\[ G_{d \tau}(\sigma \bar{G}) F_d \]

**In future:** Single- and double-disconnected diagrams
(contribute to isosinglet cEDM, mix with \( \theta \)-term)

\[ \delta_u \rightarrow d_u \]
\[ \delta_d \rightarrow d_d \]
Nucleon Vector (Sachs) Form Factors

\[ G_E = F_1 - \frac{Q^2}{4m_N^2} F_2 \]

\[ G_M = F_1 + F_2 \]

See [T. Izubuchi's talk, Fri 5:30pm @106 (Hadron Structure)]

Physical point DWF \( N_f=2+1 \) 48\(^3\)x96, \( a=0.114 \) fm
Parity Mixing: cEDM and pseudoscalar(*)

\[ N_\delta = \epsilon^{abc} u_\delta^a (u^a T C \gamma_5 d^c) \]

\[
\langle N(t) \bar{N}(0) \rangle_{CP} = \frac{-i \phi + m_N e^{2i \alpha_5 \gamma_5}}{2m_N} e^{-E_N t}
\]

\[ \hat{\alpha}_5 = \frac{\alpha_5}{d} = -\frac{\text{ReTr}[T^+ \gamma_5 \cdot C_{2pt}^{CP}(t)]}{\text{ReTr}[T^+ \cdot C_{2pt}^{CP}(t)]}, \quad t \to \infty \]

(Physical point 
DWF \(N_f=2+1\) 
48\(^3\)x96, \(a=0.114\) fm 

(flavor labels for the proton \(uud\))

(*)connected-only, bare cEDM and PS operators
Proton & Neutron EDM Form Factors (*)

Proton, u-cEDM

Neutron, u-cEDM

Proton, d-cEDM

Neutron, d-cEDM

Proton, u-PS

Neutron, u-PS

Proton, d-PS

Neutron, d-PS

Physical point
DWF $N_f=2+1$
$48^3 \times 96$, $a=0.114$ fm

(*) connected-only, bare cEDM and PS operators
Neutron EDM from Isovector Quark cEDM

Outlook for cEDM-induced p,nEDM

- Renormalization & mixing subtractions : work underway using position-space scheme
- Flavor-dependent CPv from cEDM : disconnected diagrams are required, will be challenging due to noise and mixing with $\theta_{QCD}$ term
θ_{QCD}\text{-induced nEDM} : Status

Correction to previous results:

\[ [F_3]_{\text{true}} = "F_3" + 2\alpha F_2 \]

<table>
<thead>
<tr>
<th>( m_\pi ) [MeV]</th>
<th>( m_N ) [GeV]</th>
<th>( F_2 )</th>
<th>( \alpha )</th>
<th>( \tilde{F}_3 )</th>
<th>( F_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>373</td>
<td>1.216(4)</td>
<td>-1.50(16)^a</td>
<td>-0.217(18)</td>
<td>-0.555(74)</td>
</tr>
<tr>
<td>( n )</td>
<td>530</td>
<td>1.334(8)</td>
<td>-0.560(40)</td>
<td>-0.247(17)^b</td>
<td>-0.325(68)</td>
</tr>
<tr>
<td>( p )</td>
<td>530</td>
<td>1.334(8)</td>
<td>0.399(37)</td>
<td>-0.247(17)^b</td>
<td>0.284(81)</td>
</tr>
<tr>
<td>( n )</td>
<td>690</td>
<td>1.575(9)</td>
<td>-1.715(46)</td>
<td>-0.070(20)</td>
<td>-1.39(1.52)</td>
</tr>
<tr>
<td>( n )</td>
<td>605</td>
<td>1.470(9)</td>
<td>-1.698(68)</td>
<td>-0.160(20)</td>
<td>0.60(2.98)</td>
</tr>
<tr>
<td>( n )</td>
<td>465</td>
<td>1.246(7)</td>
<td>-1.491(22)^c</td>
<td>-0.079(27)^d</td>
<td>-0.375(48)</td>
</tr>
<tr>
<td>( n )</td>
<td>360</td>
<td>1.138(13)</td>
<td>-1.473(37)^c</td>
<td>-0.092(14)^d</td>
<td>-0.248(29)</td>
</tr>
</tbody>
</table>

After removing the spurious contribution,

- no lattice signal for θ_{QCD}-induced nEDM ⇔ \( d_n \) is very small
- no more conflict with phenomenology values or \( m_q \) scaling
θ-Term Noise Reduction for EDM

Variance of lattice θ-induced nEDM signal \( \sim (Volume)^{4d} \):

\[
d_N \sim \langle Q \cdot (N J_{\mu} \tilde{N}) \rangle
\]

Top. charge \( Q \sim \int_{V_4} (G \tilde{G}) \), with \( \langle |Q|^2 \rangle \sim V_4 \)

Constrain Q sum to the fiducial volume

- in time around current, \( |t_Q - t_J| < \Delta t \) [E.Shintani et al (2015)]
- in time around source, \( |t_Q - t_{source}| < \Delta t \) [J. Dragos, talk on Tue]
- 4-d sphere around sink, \( |x_Q - x_{sink}| < R \) [K.-F.Liu et al, (2017)]:

Proper treatment of nucleon parity mixing is critical for correct determination of \( F_3 \)

\[ N^{(+)} \rightarrow \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)} \]
\[ N^{(-)} \rightarrow \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)} \]

\( \Rightarrow \) constrain time and space differently:

4d "cylinder" \( V_Q : |\vec{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q \)
Tests on $m_\pi=330$ MeV Lattices: Parity Mixing

- $N_f=2+1$ Domain Wall (RBC/UKQCD) $24^3\times64$ $a = 0.114$ fm
- 1400 configs * (64sloppy+1exact) samples $\rightarrow$ 89.6k stat.
- Top. charge with 5-loop improved $G\bar{G}$ [P. de Forcrand et al '97] on Wilson-flowed ($t=8a^2$) gauge links [M. Luscher, 1006.4518]

parity mixing angle $\alpha_N$ as a function of $r_Q$, $\Delta t_Q$

- convergence at $r_Q \approx 16a$, $\Delta t_Q \approx 8a$
Tests on $m_\pi=330$ MeV Lattices: EDM(Form Factor)

$r_Q = 8a$

$Q^2 [GeV^2]$
chiral fermions, $m_\pi = 330$ MeV [this work]

$|2m_n d_n| = |F_{3n}(0)| \approx 0.05 \cdot \theta$

Wilson fermions, $m_\pi = 360$ MeV [Guo et al 2015] after correction

$|2m_n d_n| = |F_{3n}(0)| \lesssim 0.06 \cdot \theta$

best guess for the physical point with $|d_n| \sim m_q \sim (m_\pi)^2$

$\implies$ phys. point

$|F_{3n}(0)| \approx 0.01 \cdot \theta, \ |d_n| \approx 0.001 \cdot \theta \ e \ fm$

$|F_{3n}^{\text{phys}}(0)| \sim O(10^{-2}) \theta, \ |d_n| \sim O(10^{-3}) \ e \ fm \ \theta$
Physical point: $\theta_{QCD}$-induced Parity Mixing $\alpha_N$

Parity-mixing angle from constrained Q sum

Reassuring results for noise reduction at the physical point

- time region $\Delta t_Q \gtrsim 8a \approx 1.2$ fm
- spatial region $r_Q \gtrsim 20a \approx 2.3$ fm

Physical point
DWF $N_f=2+1$
$48^3 \times 96$, $a=0.114$ fm

$48^3 \times 96$ $m_\pi=139$ MeV (PRELIMINARY)
Physical point: $\theta_{\text{QCD}}$-induced EDFF $F_3$

EDFF $F_3$ from constrained Q sum (the most aggressive Q cuts)

- $33k$ lattice samples, $\sim 30$ M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$ constraint

Need $x30..100$ more statistics to constrain $|F_{3n}| \approx 0.01$:
$\theta$-nEDM remains difficult at the physical point

Physical point DWF $N_f=2+1$
$48^3 \times 96, a=0.114$ fm

$48^3 \times 96, m_\pi=139$ MeV (PRELIMINARY)
Nucleon EDM: Summary

- Encouraging physical-point results for nucleon EDM induced by quark chromo-EDM
  - ~20% stochastic uncertainty for quark cEDM-induced EDM
  - Renormalization & mixing subtractions are underway
  - Full flavor dependence will require disconnected diagrams & $\theta_{QCD}$-term

- Clear signal for $\theta_{QCD}$-induced nEDM at $m_\pi = 330$ MeV
  - Variance-reduction for Q sampling is essential
  - Physical $|d_{n,p}| \approx 10^{-3}$ e fm values are in agreement with phenomenology

- Constraining $\theta_{QCD}$-induced nEDM at the physical point will be challenging
  - $O(300-1000)$ M core*hours may be required even with variance reduction
  - Shall look for alternative methods: dynamical $\theta$-therm?