Nucleon form factors on a (10.8fm)$^4$ lattice at the physical point in 2+1 flavor QCD

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July 26, 2018 @ MSU
Plan of talk

- PACS Collaboration Members
- “PACS10” Configs with Physical Volume over (10 fm)$^4$
- Improvements from Previous Work (arXiv:1807.03974)
- Results for Form Factors
  - Vector Current
  - Axial Vector Current
  - Generalized Goldberger-Treiman Relation
    $\Rightarrow$ Pseudoscalar Density
- Summary
PACS Collaboration Members

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K.-I. Ishikawa, Hiroshima

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“PACS10” Configs @ $\beta=1.82$ in 2+1 Flavor QCD

- Wilson-clover quark action + Iwasaki gauge action
- Stout smearing with $\alpha=0.1$ and $N_{\text{smear}}=6$
- NP $C_{SW}=1.11$ determined by SF
- $\beta=1.82 \Rightarrow a^{-1}=2.33$ GeV
- Lattice size=$128^4 \Rightarrow (10.8 \text{ fm})^3$ spatial volume
- Hopping parameters: $(\kappa_{ud},\kappa_s)=(0.126117,0.124902) \Rightarrow m_\pi \approx 135 \text{ MeV}, m_\pi L \approx 7.5$

- Simulation algorithm
  - $(\text{MP})^2\text{DDHMC}$ w/ active link for ud quarks, RHMC for s quark
  - Block size=$16 \times 16 \times 8 \times 64$
  - MP parameters: $(\rho_1,\rho_2)=(0.9997,0.9940)$
  - Multi-time scale integrator: $(N_0,N_1,N_2,N_3,N_4)=(8,2,2,2,22)$
  - Trajectory length: $\tau=1$
  - $N_{\text{RHMC}}=8, [F_{\text{min}},F_{\text{max}}]=[0.00025,1.85]$
  - Chronological inverter guess for IR parts
  - Solver: mixed precision nested BiCGStab
Measurement Details with Plateau Method (1)

2-pt correlator

\[ C_{XS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}) = \frac{1}{4} \text{Tr} \left\{ \mathcal{P}_+ \langle N_X(t_{\text{sink}}, \mathbf{p}) \tilde{N}(t_{\text{src}}, -\mathbf{p}) \rangle \right\} \]

3-pt correlator

\[ C_{O,\alpha}^P(t, \mathbf{p}', \mathbf{p}) = \frac{1}{4} \text{Tr} \left\{ \mathcal{P}_k \langle N(t_{\text{sink}}, \mathbf{p}') J^O_{\alpha}(t, \mathbf{q}) \tilde{N}(t_{\text{src}}, -\mathbf{p}) \rangle \right\} \]

Ratio of 3-pt to 2-pt correlators

\[ R_{O,\alpha}^k(t, \mathbf{p}', \mathbf{p}) = \frac{C_{O,\alpha}^P(t, \mathbf{p}', \mathbf{p})}{C_{SS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}') \sqrt{C_{LS}(t_{\text{sink}} - t, \mathbf{p}) C_{SS}(t - t_{\text{src}}, \mathbf{p}') C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}')}} \]

\[ \sqrt{C_{LS}(t_{\text{sink}} - t, \mathbf{p}') C_{SS}(t - t_{\text{src}}, \mathbf{p}) C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p})}. \]
Measurement Details with Plateau Method (2)

- AMA is used to gain high statistical precision
- O(100) measurements/config ⇒ O(10^3~10^4) measurements so far
- 9 choices for spatial momenta:
  \( \vec{n} = (1,0,0),(1,1,0),(1,1,1),(2,0,0),(2,1,0),(2,1,1),(2,2,0),(3,0,0),(2,2,1) \)
  minimum mom=\(2\pi/L\)\(\sim 0.115 \text{ GeV} \) thanks to \(L=10.8 \text{ fm} \)
- Lattice size=\(128^4 \) ⇒ (10.8 fm)^3 spatial volume allows small q^2 region
- Exp smeared src/sink operators for 2-pt and 3-pt functions
- Src-sink separation: \(t_{\text{sink}} - t_{\text{src}} = 10, 12, 14, 16 \ (\sim 1.35 \text{ fm}) \)
- \(Z_A = 0.9650(68)(95), Z_V = 0.95153(76)(1487) \) in SF scheme

PoS(LATTICE2015)271
How Large Spatial Size is Necessary?

Charge RMS radius

\[ R^2 \equiv \int_0^\infty \rho(r) \, r^4 \, 4\pi \, dr, \]

\[ R(r_{cut}) = \left[ \int_0^{r_{cut}} \rho(r) \, r^4 \, dr \Bigg/ \int_0^\infty \rho(r) \, r^4 \, dr \right]^{1/2} \]

Charge density

Integration up to \( r_{cut} = 2.7 \text{fm} \) ⇒ Only 98% of charge RMS radius
## Improvements from Our Previous Work

<table>
<thead>
<tr>
<th></th>
<th>Lattice 2018</th>
<th>arXiv:1807.03974</th>
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<tbody>
<tr>
<td>Volume</td>
<td>128⁴ (10.8 fm)⁴</td>
<td>96⁴ (8.1 fm)⁴</td>
</tr>
<tr>
<td>Minimum (q^2)</td>
<td>0.013 GeV²</td>
<td>0.024 GeV²</td>
</tr>
<tr>
<td>(m_{ππ})</td>
<td>135 MeV (physical)</td>
<td>146 MeV</td>
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<td>Measurement to</td>
<td>w/ AMA</td>
<td>w/o AMA</td>
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<tr>
<td>increase statistics</td>
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<tr>
<td>(t_s =</td>
<td>t_{sink} - t_{src}</td>
<td>) dependence</td>
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Nucleon Effective Mass

Good plateau is observed from small time slice
Isovector Electric Form Factor (1)

Ratio of 3-pt to 2-pt correlators as a function of $t$ (location of $V$)

Good plateau for $t_s = 10, 12, 14, 16$
Isovector Electric Form Factor (2)

\[ G_E(q^2) = F_1(q^2) - \frac{q^2}{4m_N^2} F_2(q^2) \]

Seems to prefer µH experiment
⇒ Possibility to distinguish two experimental values

\[ \langle r^2_E \rangle = -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \]

Table 2: Results of (isovector) electric rms obtained from various fits. Experimental values of isovector mean square radius is given by
\[ \langle r^2 \rangle = \langle r^2_p \rangle - \langle r^2_n \rangle \]
with \( \langle r^2_n \rangle = 0 \).

For proton mean square radius, there are two values; \( \langle r^2_p \rangle = 0.88611(926) \text{ fm}^2 \) from ep scattering and \( \langle r^2_p \rangle = 0.82316(22) \text{ fm}^2 \) from µ-H atom spectroscopy.
Isovector Magnetic Form Factor

\[ G_M(q^2) = F_1(q^2) + F_2(q^2) \]

Consistent with \( \mu_M \) and \( \sqrt{\langle r_M^2 \rangle} \) within 2\( \sigma \) error
Axial Form Factor

Two form factors $F_A$ and $F_P$ for axial vector current

$g_A$ is consistent with experiment being independent of $t_s$

$\sqrt{<r_A^2>}$ is also consistent with experiment
Induced Pseudoscalar Form Factor $F_P$

$Q^2$ dependence of $2M_N F_P$

Clear $t_s$ dependence for $F_P \Rightarrow$ Excited state contributions

※ ChPT analysis by Bär, Wed·14:00[HIS]
Generalized Goldberger-Treiman (GT) Relation

$F_A$ and $F_P$ are not independent

$$2M_N F_A(q^2) - q^2 F_P(q^2) = 2\hat{m} G_P(q^2)$$

⇒ Check Generalized GT relation with $G_P$
Pseudoscalar Form Factor $G_P(1)$

Ratio of 3-pt to 2-pt correlators as a function of t (location of P)

Mound like shape $\Rightarrow$ Signal of excited state contributions
Pseudoscalar Form Factor $G_P$ (2)

**Q² dependence of $G_P$**

Generalized GT relation

$$m_{AWTI} = \frac{2 M_N F_{ren}^A(q^2) - q^2 F_{ren}^P(q^2)}{2 G_P(q^2)}$$

**$t_s$ dependence of $m_{AWI}(GT)$**

$m_{AWI}(GT)$ becomes closer to $m_{AWI}(PCAC)$ for larger $t_s$
Summary

• 2+1 flavor QCD simulation at the physical point on (10.8 fm)$^4$ lattice
  - Large spatial volume allows investigation at small $Q^2$ region
• $t_s$ dependence is systematically investigated
  - $G_E$, $G_M$ and $F_A$ show no $t_s$ dependence
  - Clear $t_s$ dependence is observed for $F_P$ and $G_P$
• Results for $G_E$, $G_M$ and $F_A$ are consistent with experiment including $g_A$
• Violation of Generalized GT relation diminishes as $t_s$ increases
BACKUP