



Nucleon form factors on a $(10.8\text{fm})^4$ lattice at the physical point in 2+1 flavor QCD

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Plan of talk

- PACS Collaboration Members
- “PACS10” Configs with Physical Volume over $(10 \text{ fm})^4$
- Improvements from Previous Work (arXiv:1807.03974)
- Results for Form Factors
 - Vector Current
 - Axial Vector Current
 - Generalized Goldberger-Treiman Relation
 - ⇒ Pseudoscalar Density
- Summary



PACS Collaboration Members

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“PACS10” Configs @ $\beta=1.82$ in 2+1 Flavor QCD

arXiv:1807.06237

- Wilson-clover quark action + Iwasaki gauge action
- Stout smearing with $\alpha=0.1$ and $N_{\text{smear}}=6$
- NP $C_{\text{SW}}=1.11$ determined by SF
- $\beta=1.82 \Rightarrow a^{-1}=2.33$ GeV
- **Lattice size= $128^4 \Rightarrow (10.8 \text{ fm})^3$ spatial volume**
- Hopping parameters: $(\kappa_{\text{ud}}, \kappa_{\text{s}})=(0.126117, 0.124902)$
 $\Rightarrow m_{\pi} \approx 135$ MeV, $m_{\pi}L \approx 7.5$
- Simulation algorithm
 - (MP)²DDHMC w/ active link for ud quarks, RHMC for s quark
 - Block size= $16 \times 16 \times 8 \times 64$
 - MP parameters: $(\rho_1, \rho_2)=(0.9997, 0.9940)$
 - Multi-time scale integrator: $(N_0, N_1, N_2, N_3, N_4)=(8, 2, 2, 2, 22)$
 - trajectory length: $\tau=1$
 - $N_{\text{RHMC}}=8$, $[F_{\text{min}}, F_{\text{max}}]=[0.00025, 1.85]$
 - Chronological inverter guess for IR parts
 - Solver: mixed precision nested BiCGStab



Measurement Details with Plateau Method (1)

2-pt correlator

$$\begin{aligned} C_{XS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}) \\ = \frac{1}{4} \text{Tr} \{ \mathcal{P}_+ \langle N_X(t_{\text{sink}}, \mathbf{p}) \bar{N}_S(t_{\text{src}}, -\mathbf{p}) \rangle \} \end{aligned}$$

3-pt correlator

$$\begin{aligned} C_{O,\alpha}^{\mathcal{P}_k}(t, \mathbf{p}', \mathbf{p}) \\ = \frac{1}{4} \text{Tr} \{ \mathcal{P}_k \langle N(t_{\text{sink}}, \mathbf{p}') J_\alpha^O(t, \mathbf{q}) \bar{N}(t_{\text{src}}, -\mathbf{p}) \rangle \} \end{aligned}$$

Ratio of 3-pt to 2-pt correlators

$$\mathcal{R}_{O,\alpha}^k(t, \mathbf{p}', \mathbf{p}) = \frac{C_{O,\alpha}^{\mathcal{P}_k}(t, \mathbf{p}', \mathbf{p})}{C_{SS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}')} \sqrt{\frac{C_{LS}(t_{\text{sink}} - t, \mathbf{p}) C_{SS}(t - t_{\text{src}}, \mathbf{p}') C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}')}{C_{LS}(t_{\text{sink}} - t, \mathbf{p}') C_{SS}(t - t_{\text{src}}, \mathbf{p}) C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p})}}$$



Measurement Details with Plateau Method (2)

- AMA is used to gain high statistical precision
- $O(100)$ measurements/config $\Rightarrow O(10^3 \sim 10^4)$ measurements so far
- 9 choices for spatial momenta:
 $\vec{n} = (1,0,0), (1,1,0), (1,1,1), (2,0,0), (2,1,0), (2,1,1), (2,2,0), (3,0,0), (2,2,1)$
minimum mom = $2\pi/L \sim 0.115$ GeV thanks to $L = 10.8$ fm
- Lattice size = $128^4 \Rightarrow (10.8 \text{ fm})^3$ spatial volume allows small q^2 region
- Exp smeared src/sink operators for 2-pt and 3-pt functions
- **Src-sink separation: $t_{\text{sink}} - t_{\text{src}} = 10, 12, 14, 16$ (~ 1.35 fm)**
- $Z_A = 0.9650(68)(95)$, $Z_V = 0.95153(76)(1487)$ in SF scheme
PoS(LATTICE2015)271



How Large Spatial Size is Necessary?

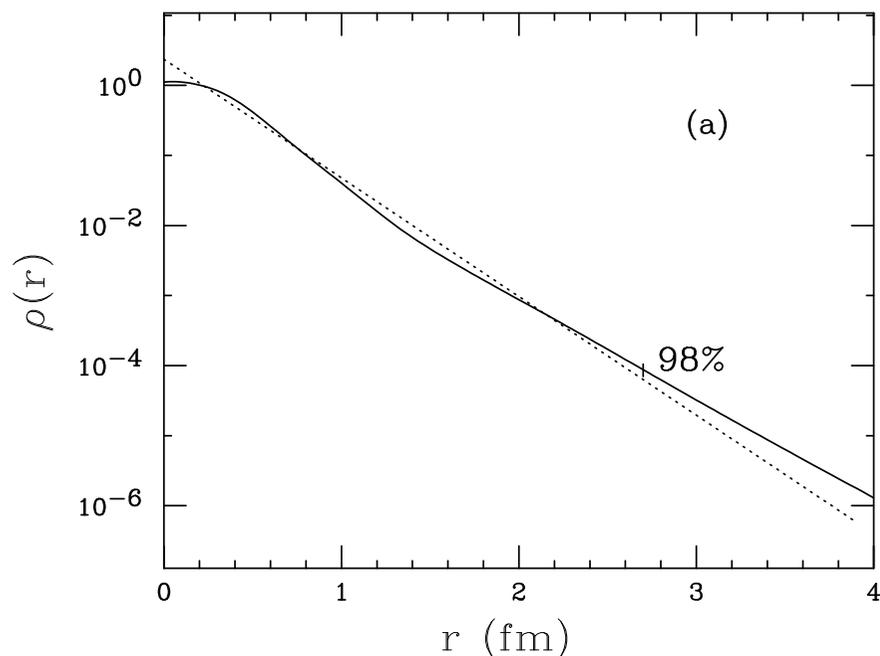
Sick, Atoms 6(2018)2

Charge RMS radius

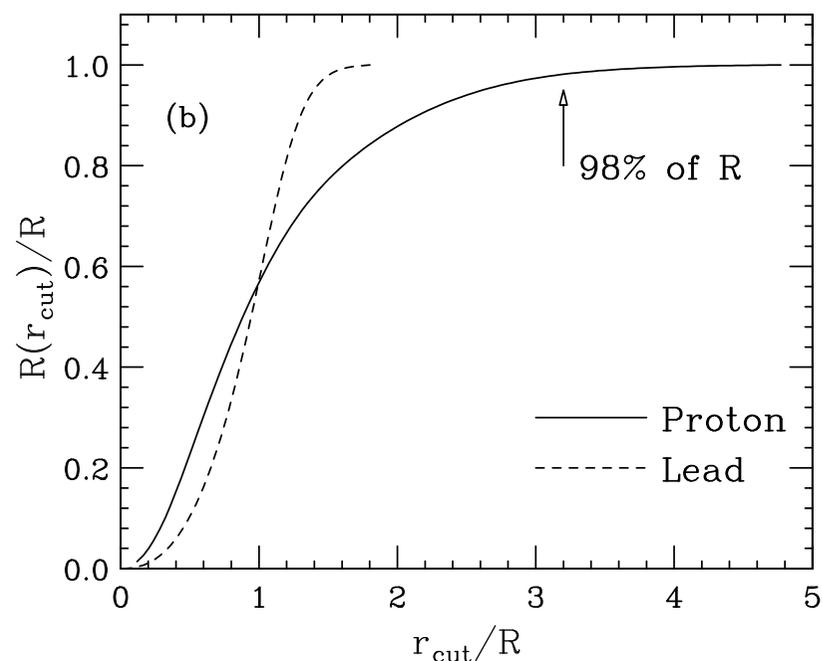
$$R^2 \equiv \int_0^{\infty} \rho(r) r^4 4\pi dr,$$

$$R(r_{cut}) = \left[\int_0^{r_{cut}} \rho(r) r^4 dr / \int_0^{\infty} \rho(r) r^4 dr \right]^{1/2}$$

Charge density



$R(r_{cut})$



Integration up to $r_{cut}=2.7\text{fm} \Rightarrow$ Only 98% of charge RMS radius

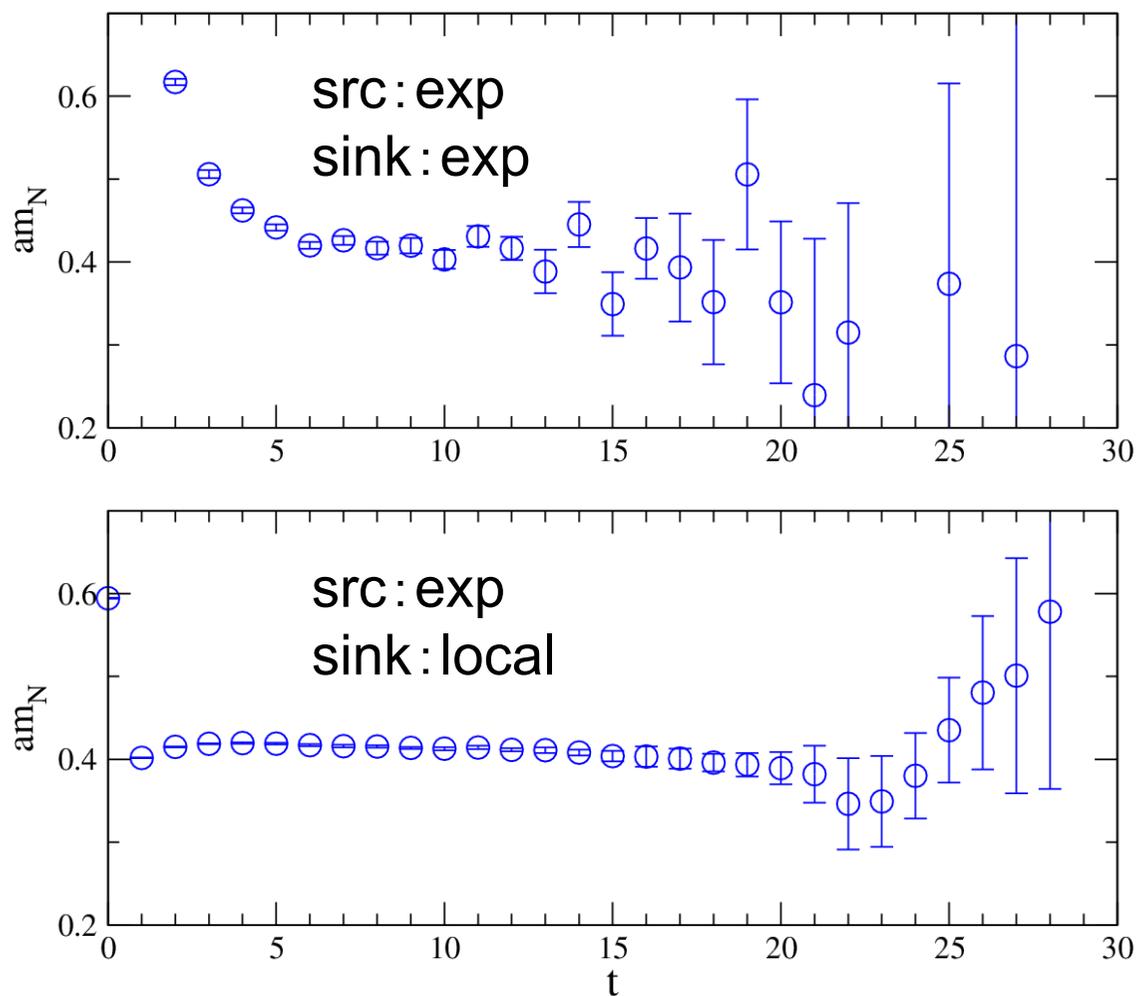


Improvements from Our Previous Work

	Lattice 2018	arXiv:1807.03974
Volume	128 ⁴ (10.8 fm) ⁴	96 ⁴ (8.1 fm) ⁴
Minimum q ²	0.013 GeV ²	0.024 GeV ²
m _π	135 MeV (physical)	146 MeV
Measurement to increase statistics	w/ AMA	w/o AMA
t _s = t _{sink} - t _{src} dependence	t _s = 10, 12, 14, 16	t _s = 15



Nucleon Effective Mass

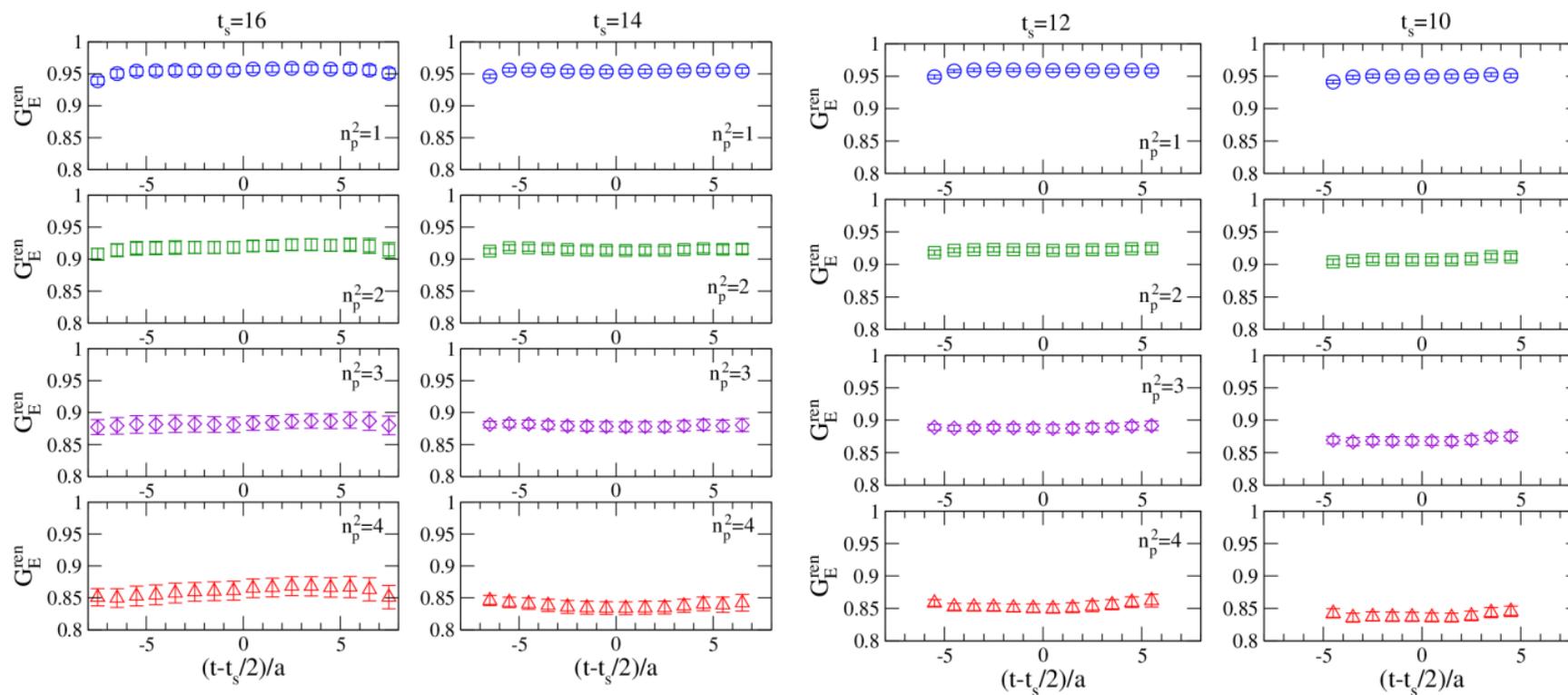


Good plateau is observed from small time slice



Isvector Electric Form Factor (1)

Ratio of 3-pt to 2-pt correlators as a function of t (location of V)



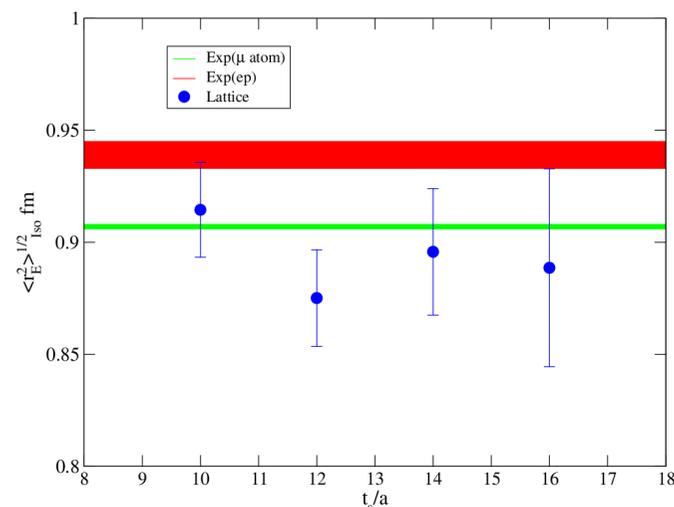
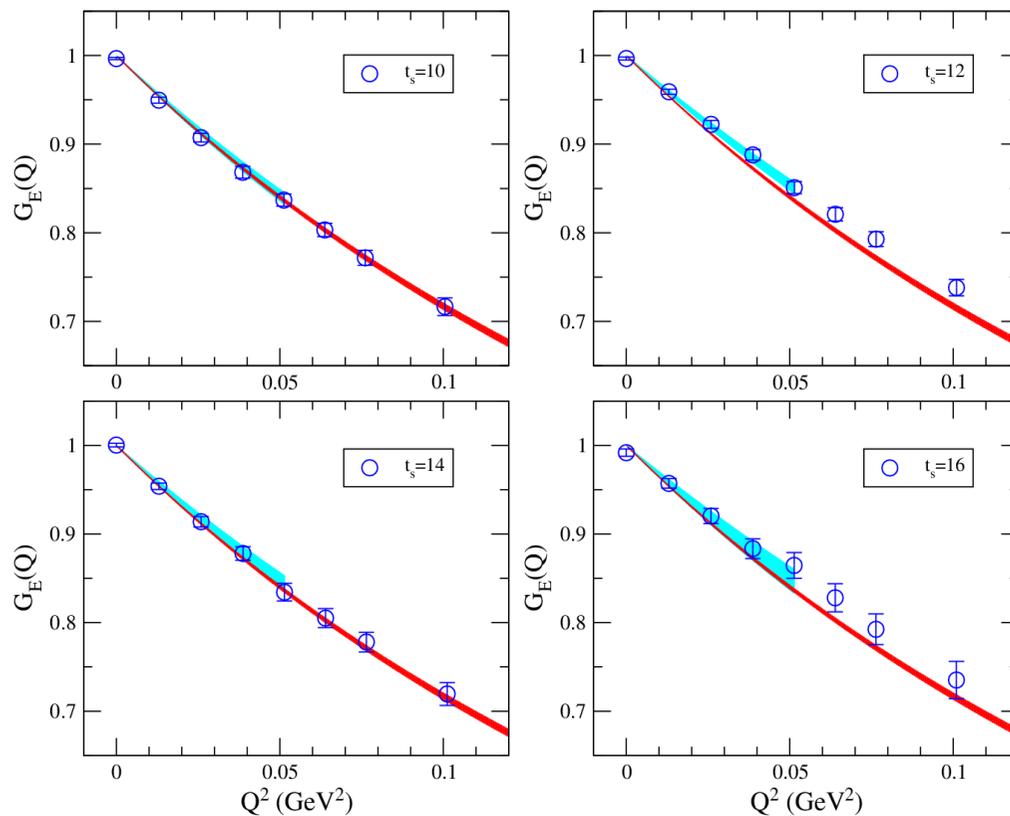
Good plateau for $t_s=10, 12, 14, 16$



Isovector Electric Form Factor (2)

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4m_N^2} F_2(q^2)$$

$$\langle r_E^2 \rangle = -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0}$$



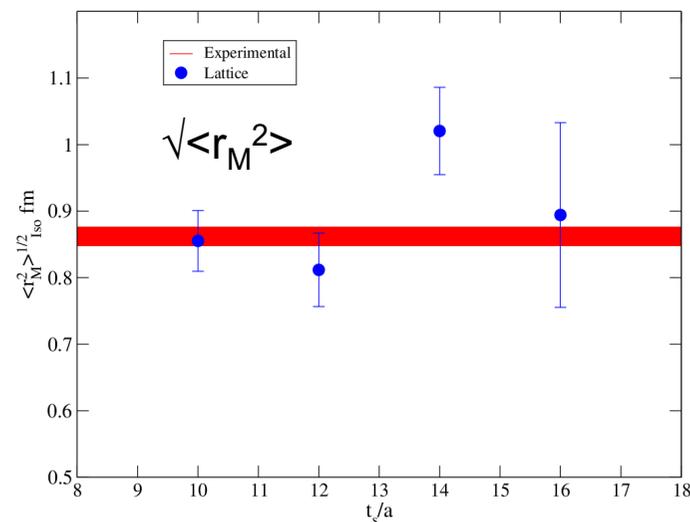
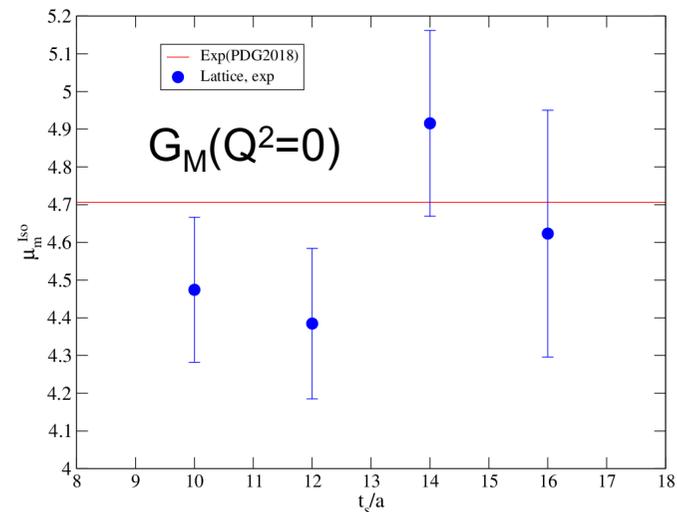
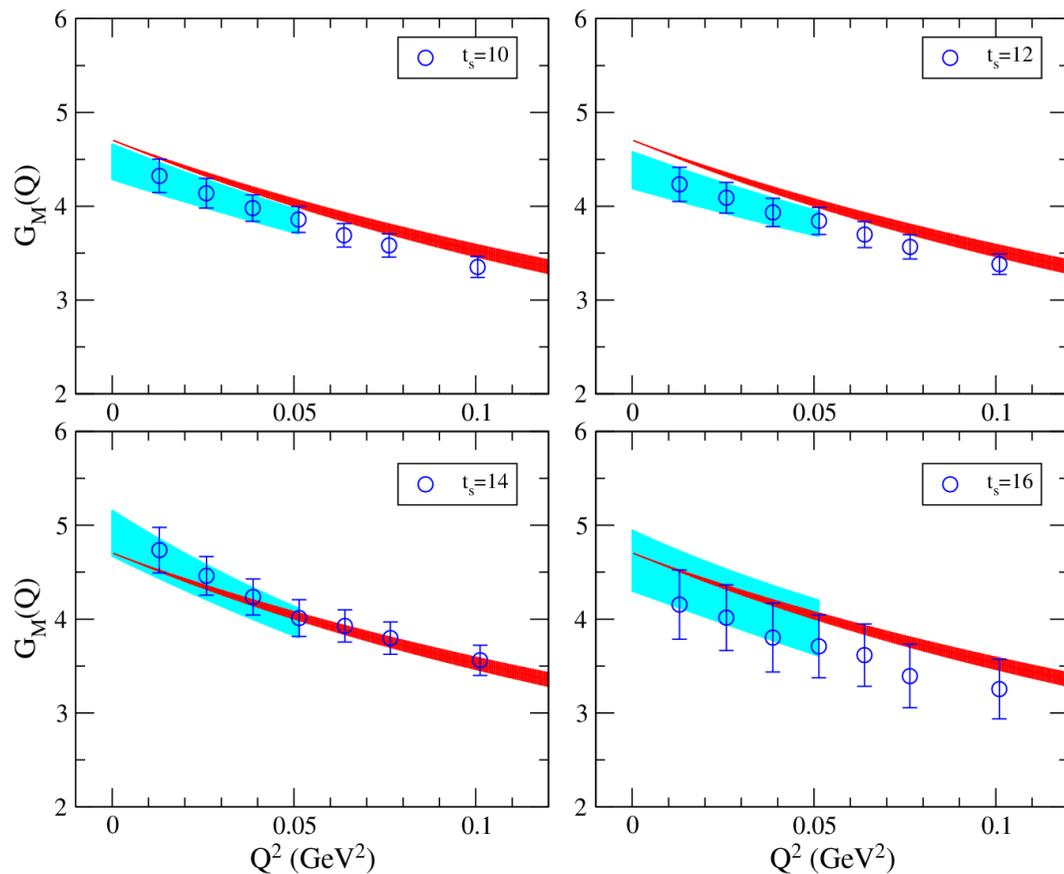
Seems to prefer μ H experiment

\Rightarrow Possibility to distinguish two experimental values



Isvector Magnetic Form Factor

$$G_M(q^2) = F_1(q^2) + F_2(\hat{q}^2)$$

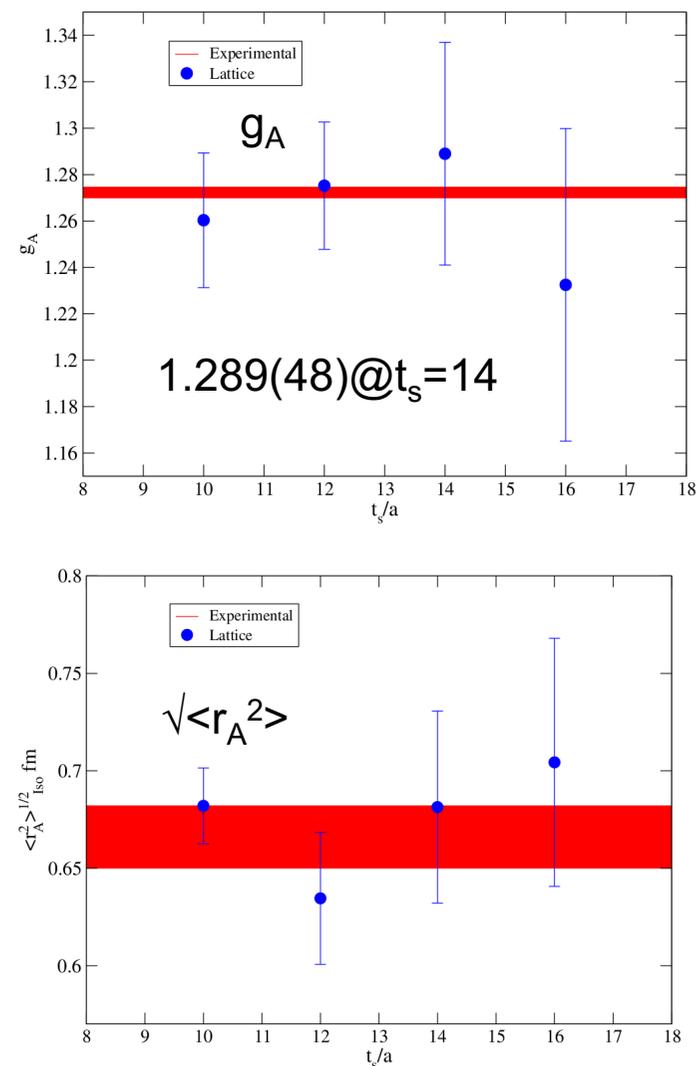
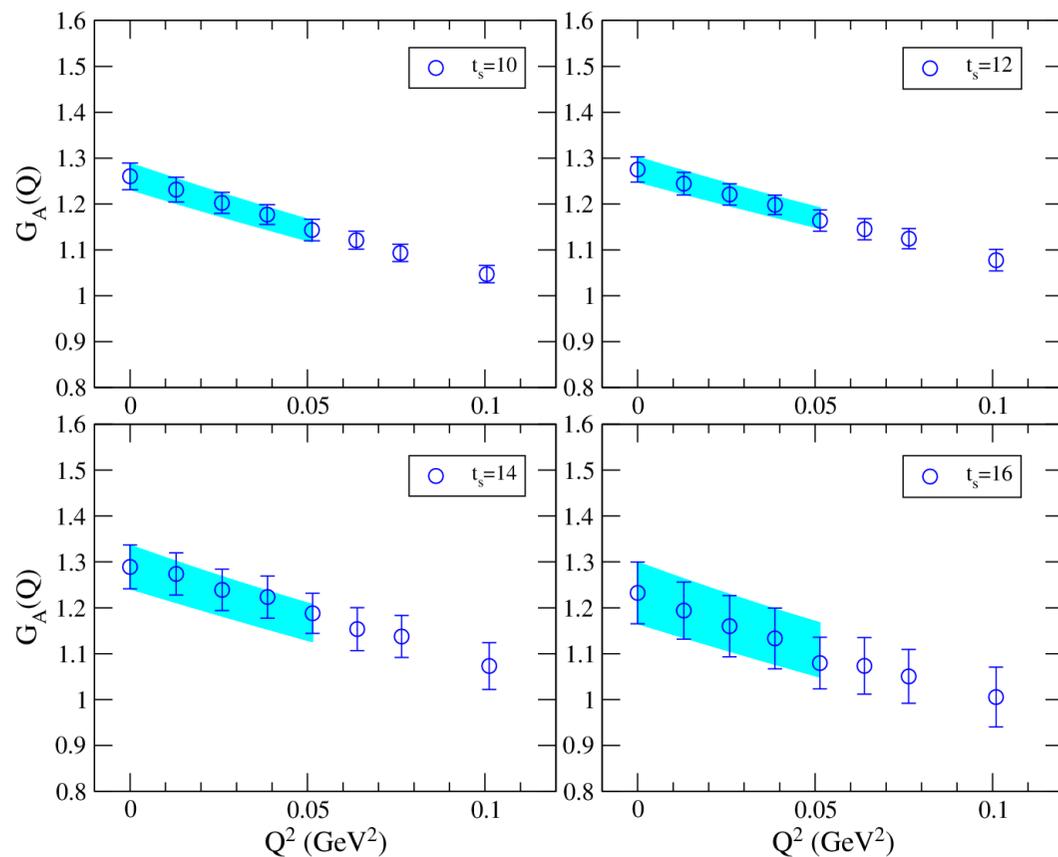


Consistent with μ_M and $\sqrt{\langle r_M^2 \rangle}$ within 2σ error



Axial Form Factor

Two form factors F_A and F_P
for axial vector current

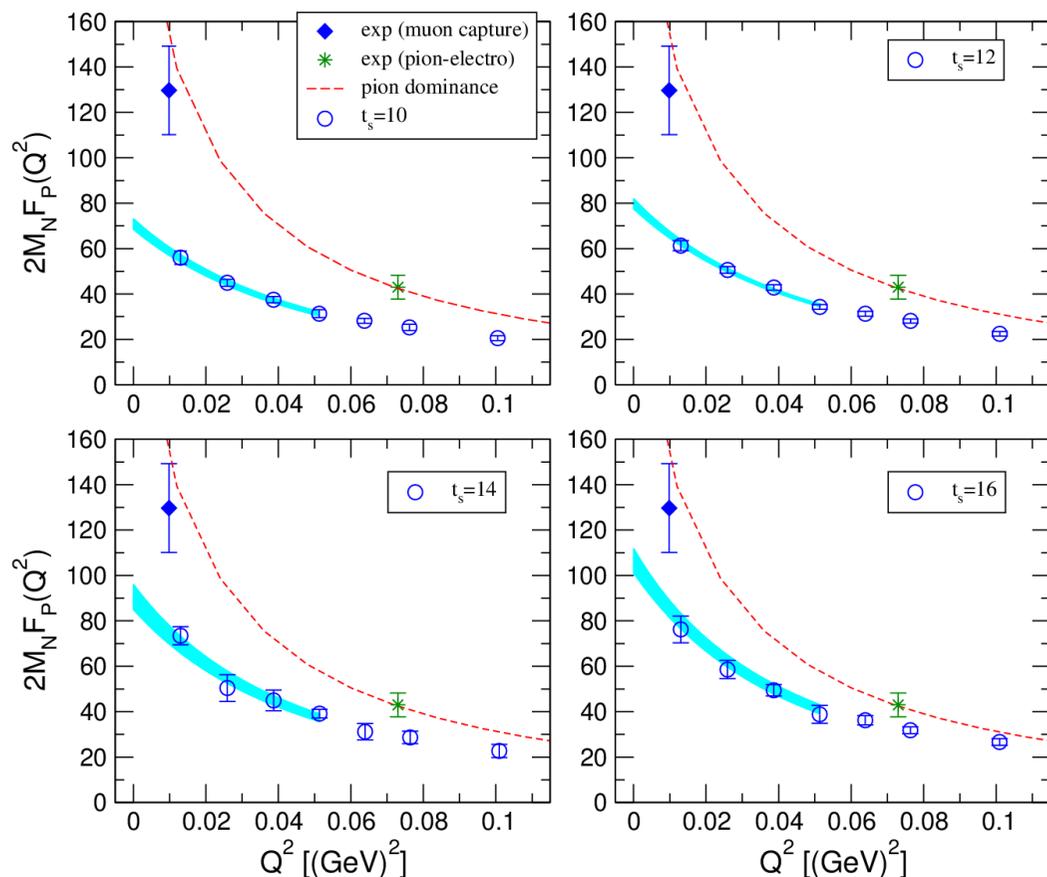


g_A is consistent with experiment being independent of t_s
 $\sqrt{\langle r_A^2 \rangle}$ is also consistent with experiment

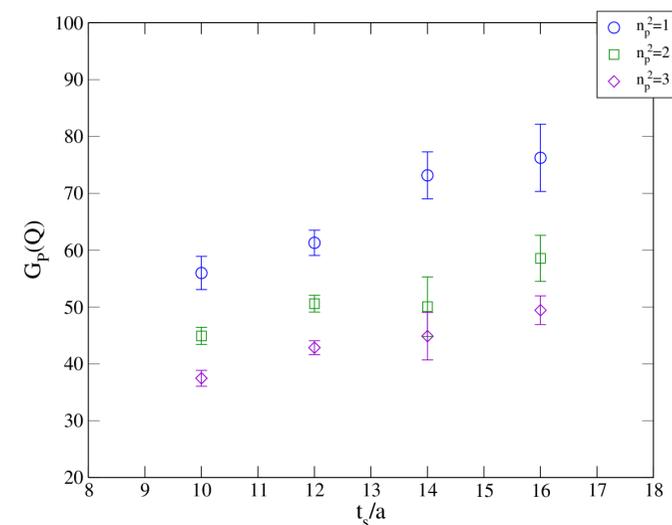


Induced Pseudoscalar Form Factor F_P

Q^2 dependence of $2M_N F_P$



t_s dependence of F_P



Clear t_s dependence for $F_P \Rightarrow$ Excited state contributions

✂ ChPT analysis by Bär, Wed • 14:00[HIS]



Generalized Goldberger-Treiman (GT) Relation

F_A and F_P are not independent

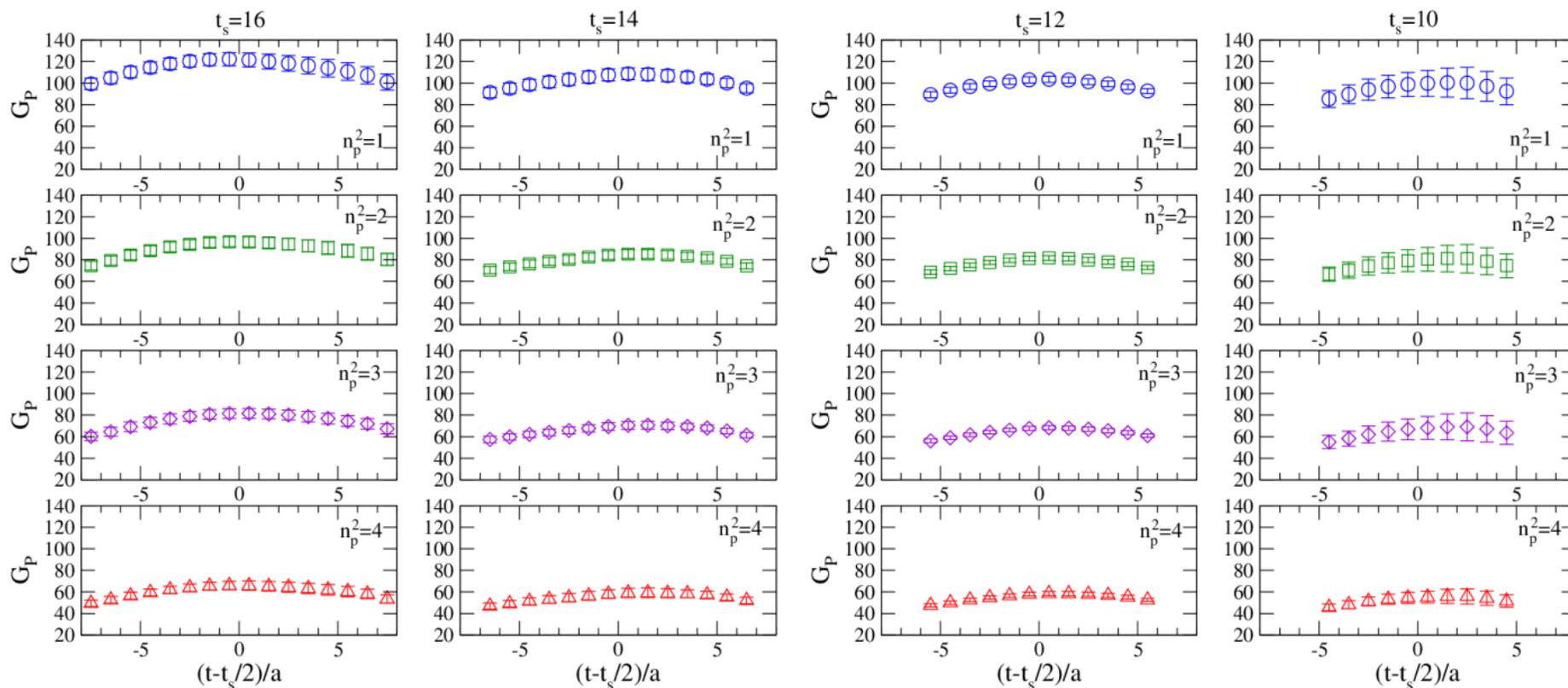
$$2M_N F_A(q^2) - q^2 F_P(q^2) = 2\hat{m} G_P(q^2)$$

⇒ Check Generalized GT relation with G_P



Pseudoscalar Form Factor $G_P(1)$

Ratio of 3-pt to 2-pt correlators as a function of t (location of P)

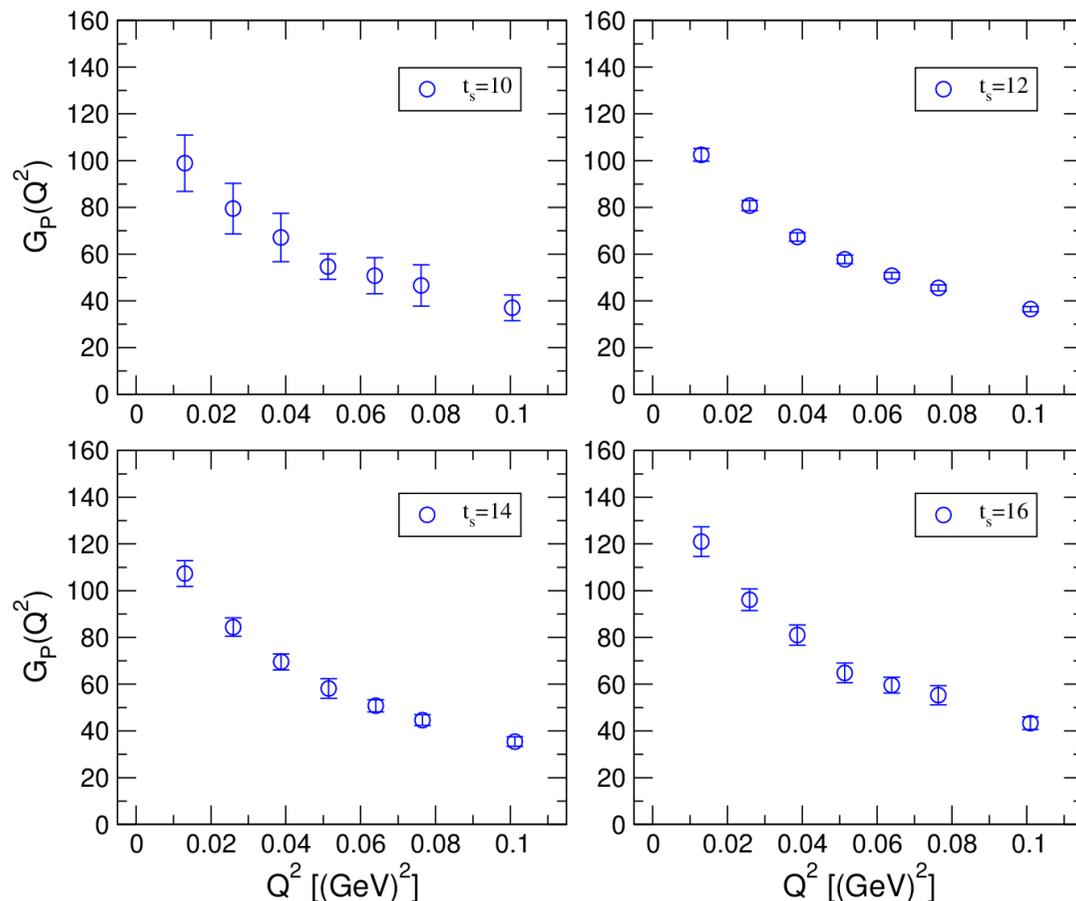


Mound like shape \Rightarrow Signal of excited state contributions



Pseudoscalar Form Factor G_P (2)

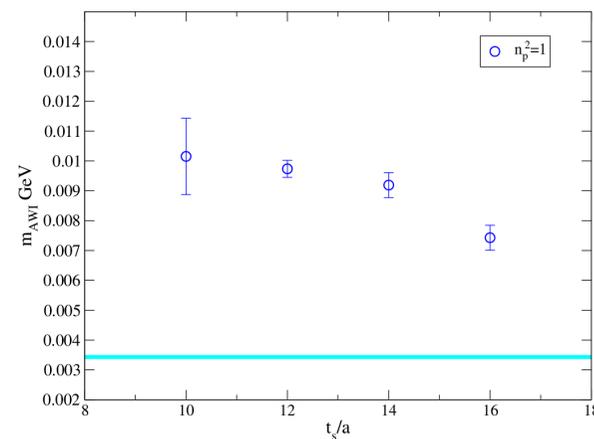
Q^2 dependence of G_P



Generalized GT relation

$$m_{\text{AWTI}} = \frac{2M_N F_A^{\text{ren}}(q^2) - q^2 F_P^{\text{ren}}(q^2)}{2G_P(q^2)}$$

t_s dependence of $m_{\text{AWI}}(\text{GT})$



$m_{\text{AWI}}(\text{GT})$ becomes closer to $m_{\text{AWI}}(\text{PCAC})$ for larger t_s



Summary

- 2+1 flavor QCD simulation at the physical point on $(10.8 \text{ fm})^4$ lattice
 - Large spatial volume allows investigation at small Q^2 region
- t_s dependence is systematically investigated
 - G_E , G_M and F_A show no t_s dependence
 - Clear t_s dependence is observed for F_P and G_P
- Results for G_E , G_M and F_A are consistent with experiment including g_A
- Violation of Generalized GT relation diminishes as t_s increases



BACKUP