Transverse spin structure of octet baryons

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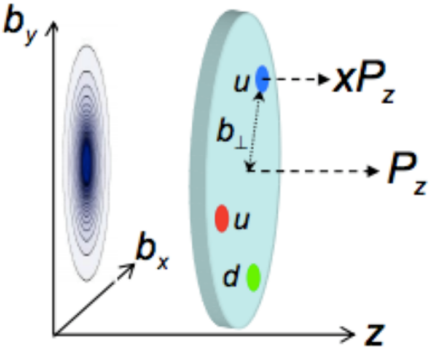
Density Distribution

- Our goal is to understand the internal structure of hadrons
- Specifically we wish to look at the quark density dependence on the hadron and quark spin polarisations
- By utilising the infinite momentum frame, we can interpret the Dirac form factor $F_1(Q^2)$ as a charge distribution by taking a Fourier transform

$$f(b_\perp^2) \equiv \int \frac{d^2\Delta_\perp}{(2\pi^2)} e^{-ib_\perp \cdot \Delta_\perp} f(t = -\Delta_\perp^2)$$

where $\Delta_\perp$ is the transverse momentum transfer

- And so the transverse spin density...



Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

M. Diehl and P. Hagler [hep-ph/0504175]

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1}\, \rho(x, b_\perp, s_\perp, S_\perp)$$
$$= \frac{1}{2}\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2}\triangle_{b_\perp}\tilde{A}_{Tn0}(b_\perp^2)\right)$$
$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{Tn0}(b_\perp^2)\}$$

- Where $b_\perp$ is the distance from the center of momentum
- $s_\perp$ is the transverse spin polarisation of the quarks
- $S_\perp$ is the transverse spin polarisation of the nucleon

Transverse Spin Density Equation

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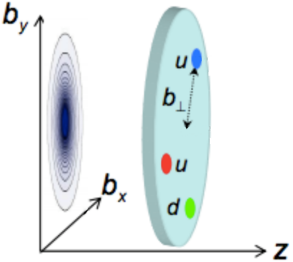

Unpolarised

Transverse Spin Density Equation

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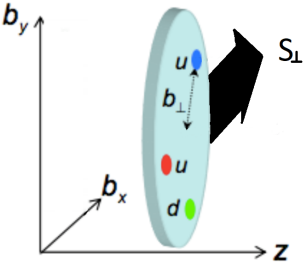

Nucleon Spin Polarisation

Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

M. Diehl and P. Hagler [hep-ph/0504175]

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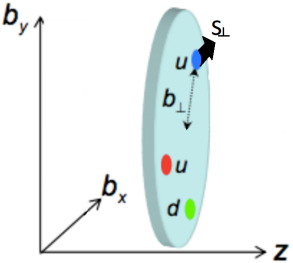


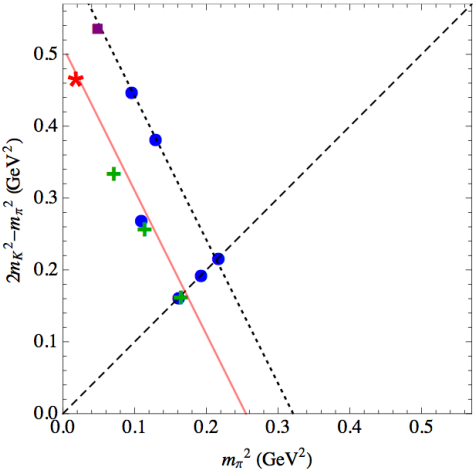
Quark Spin Polarisation

Lattice Parameters

- $N_f = 2+1$ $O(a)$-improved Clover Fermions
- Lattice spacing $a = 0.074fm$
- QCDSF method for tuning the quark masses
  - Keep the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}\left(2m_l^R + m_s^R\right)$$

  - At its physical value $\overline{m}^{R*}$
- Using multiple Lattice volume sizes including $32^3 \times 64$, $48^3 \times 96$



Ratio of Correlation Functions

We use Three-point functions

$$C_{3pt}(t,\tau;\vec{p},\vec{p'}) = \sum_{s,s'} e^{-E_{p'}(t-\tau)} e^{-E_p \tau} \Gamma_{\beta\alpha}$$

$$\langle\Omega|\chi_\alpha|N(\vec{p'},s')\rangle\langle N(\vec{p'},s')|\mathcal{O}(q)|N(\vec{p},s)\rangle\langle N(\vec{p},s)|\overline{\chi}_\beta|\Omega\rangle$$

and so by constructing a ratio of Two-point and Three-point correlation functions

$$R(t,\tau;\vec{p},\vec{p'}) \approxeq \frac{C_{3pt}(t,\tau;\vec{p'},\vec{p})}{C_{2pt}(t,\tau;\vec{p'},\vec{p})}$$

using Two-point Functions in the form

$$C_{2pt}(t,\vec{p}) = \sum_s e^{-E_p t} \Gamma_{\beta\alpha} \langle\Omega|\chi_\alpha|N(\vec{p},s)\rangle\left\langle N(\vec{p},s)\middle|\overline{\chi}_\beta\middle|\Omega\right\rangle$$

allows us to remove the time dependence and extract matrix elements from the lattice.

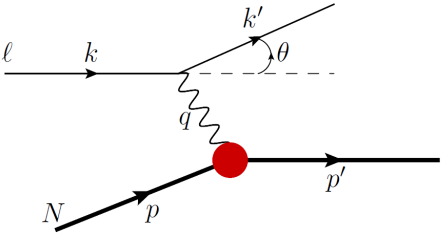
Electromagnetic Form Factors

The Dirac $F_1(Q^2)$ and Pauli $F_2(Q^2)$ form factors are obtained from the decomposition of matrix elements from the electromagnetic current j_μ where

$$\left\langle N(p',s')\middle|j_\mu(q)\middle|N(p,s)\right\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B}F_2(Q^2)\right]u(p,s)$$

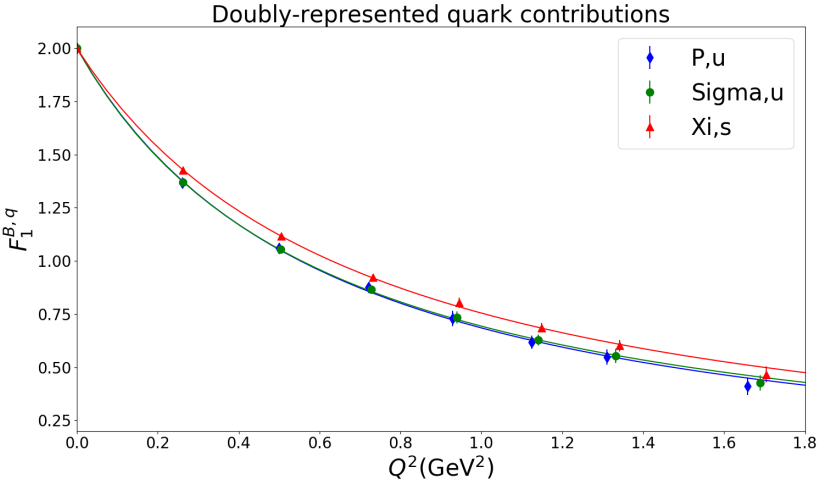
Here

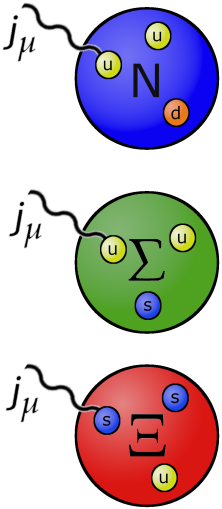
- $u(p,s)$ are Dirac spinors with momentum p and spin polarisation s
- the transfer momentum $q = p' - p$ and $Q^2 = -q^2$
- and the mass of the baryon is m_B.



Electromagnetic Form Factors

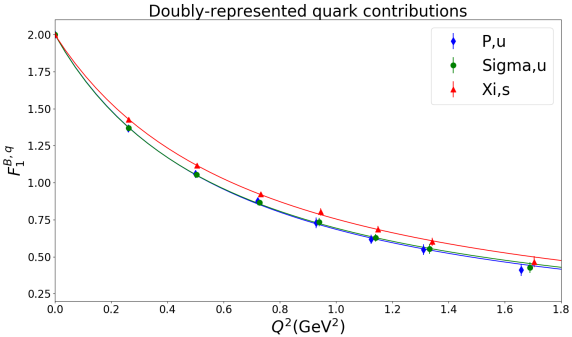
$F_1 = A_{10}$ Dirac Form Factor $\qquad (m_\pi, m_K) = (330, 435) MeV$

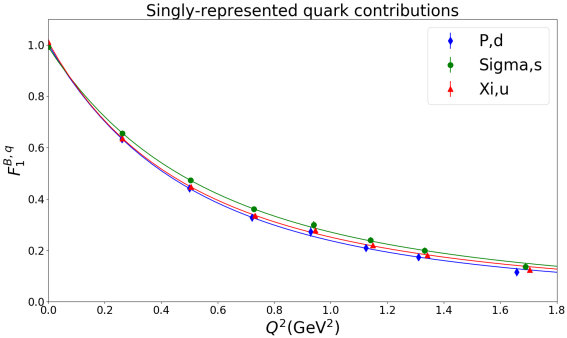




Electromagnetic Form Factors

$F_1 = A_{10}$ Dirac Form Factor





Transverse Spin Density Equation

A reminder of the equation and required form factors

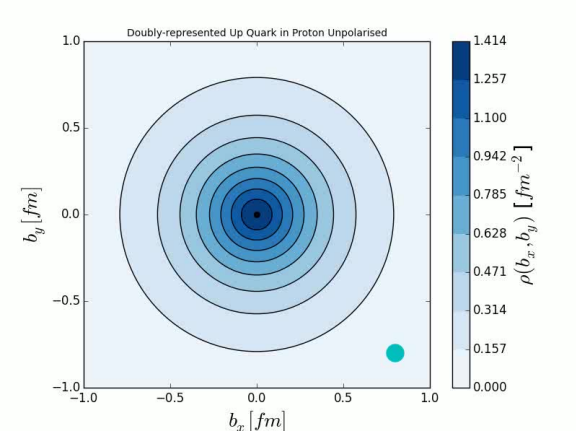
$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp}\tilde{A}_{T10}(b_\perp^2)\right)$$

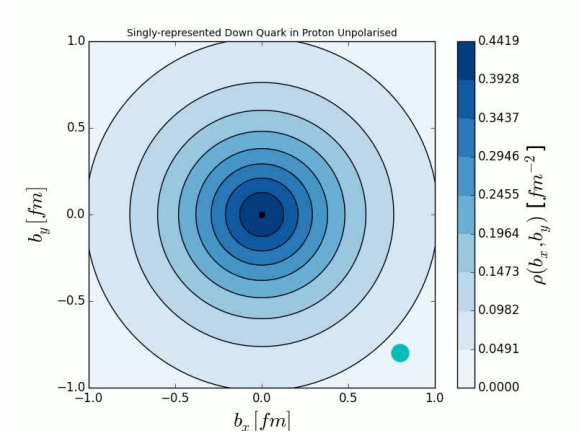
$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{10}(b_\perp^2) + s_\perp^i \overline{B}'_{T10}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T10}(b_\perp^2)\}$$

Unpolarised

Unpolarised Quark Densities



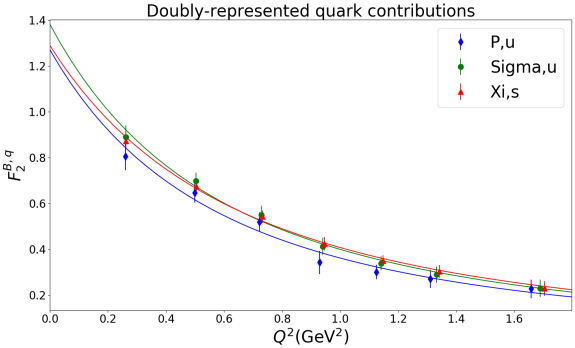
Doubly represented unpolarised up quark in the unpolarised proton.

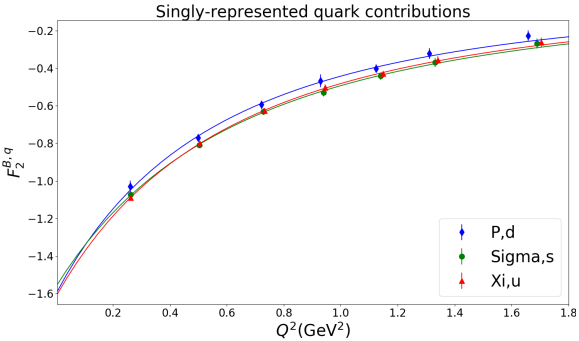


Singly represented unpolarised down quark in the unpolarised proton.

Electromagnetic Form Factors

$F_2 = B_{10}$ Pauli Magnetic Form Factor





Transverse Spin Density Equation

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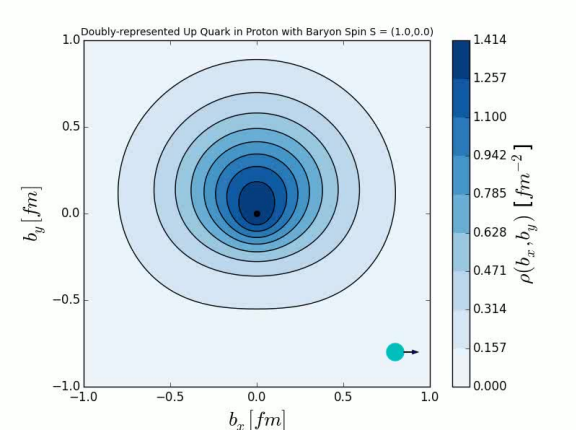
$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\triangle_{b_\perp}\tilde{A}_{T10}(b_\perp^2)\right)$$

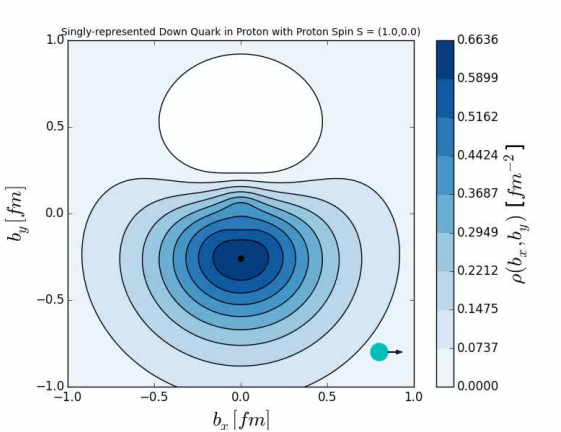
$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{10}(b_\perp^2) + s_\perp^i \overline{B}'_{T10}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T10}(b_\perp^2)$$

Hadron Spin Polarisation

Proton Quark Densities with Nucleon spin polarisation



Doubly represented up quark in the proton with polarised Nucleon spin.



Singly represented down quark in the proton with polarised Nucleon Spin.

Transverse Spin Density Equation

A reminder of the equation and required form factors

$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp}\tilde{A}_{Tn0}(b_\perp^2)\right)$$

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Quark Spin Polarisation

Tensor Form Factors

Similar to the electromagnetic form factor, we calculate the tensor form factors using a new insertion operator $i\sigma_{\mu\nu}$

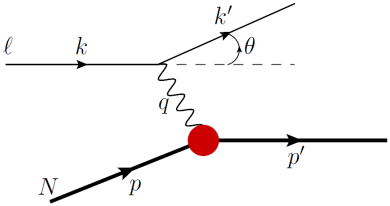
[hep-lat/0507001]

$$\left\langle N(p',s')\left|\overline{\psi}(0)i\sigma^{\mu\nu}\psi(0)\right|N(p,s)\right\rangle =$$

$$\bar{u}(p',s')\left[i\sigma^{\mu\nu}A_{T10}(Q^2)+\frac{\bar{P}^{[\mu}\Delta^{\nu]}}{m^2}\tilde{A}_{T10}(Q^2)+\frac{\gamma^{[\mu}\bar{P}^{\nu]}}{2m}B_{T10}(Q^2)\right]u(p,s)$$

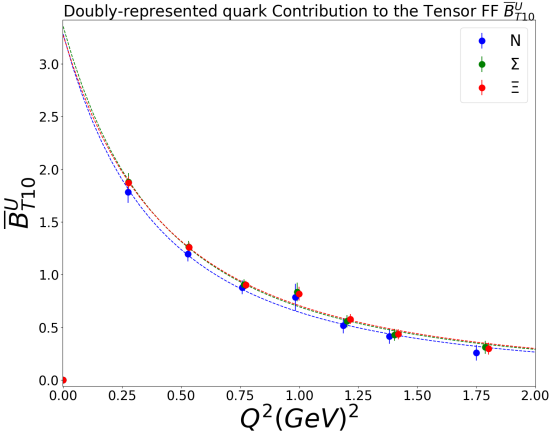
where here

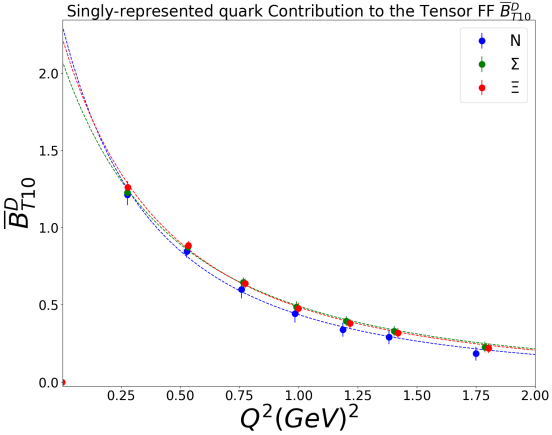
- $\gamma^{[\mu}\bar{P}^{\nu]} \equiv \gamma^{\mu}\bar{P}^{\nu} - \gamma^{\nu}\bar{P}^{\mu}$
- $\Delta = p' - p$, $\bar{P} = \frac{p'+p}{2}$
- $i\sigma^{\mu\nu} = i\gamma^{\mu}\gamma^{\nu}$



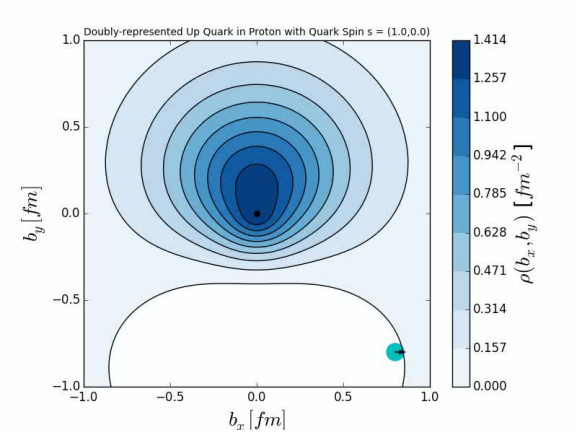
Tensor Form Factors

$$\overline{B}_{T10}(Q^2) \approx 2\tilde{A}_{T10} + B_{T10}$$

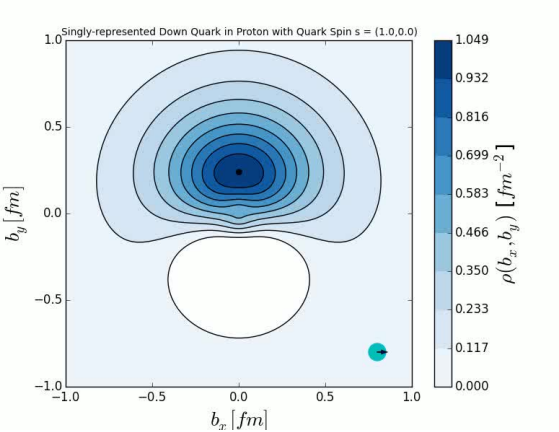




Proton Quark Densities with Quark spin polarisation



Doubly represented up quark in the proton with polarised Quark spin.

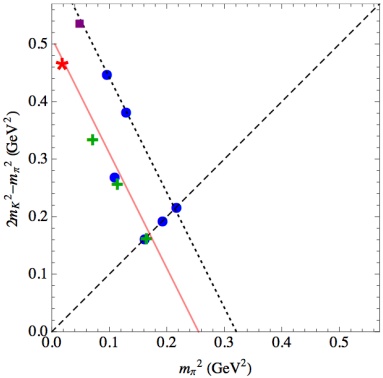


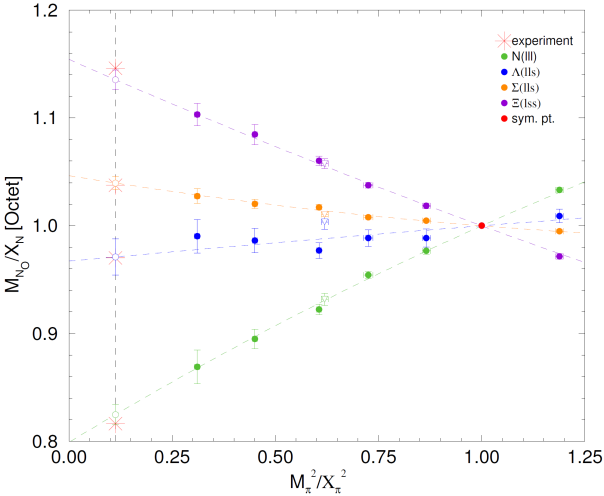
Singly represented down quark in the proton with polarised Quark Spin.

SU(3)-Flavour Symmetry Breaking

Fan plot of the baryon octet mass spectrum

[arXiv:1212.2564 [hep-lat]]

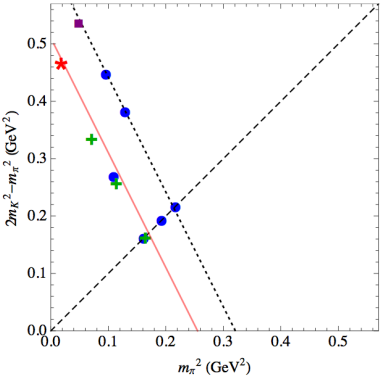


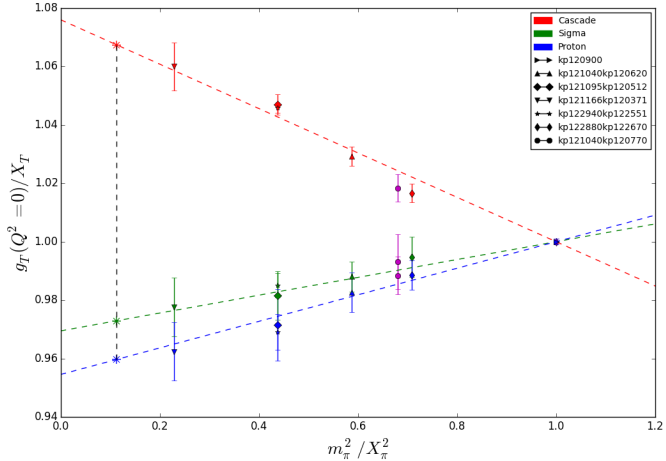


SU(3)-Flavour Symmetry Breaking

Fan plot of generalised tensor form factor A_{T10} at $Q^2 = 0$

[arXiv:1212.2564 [hep-lat]]

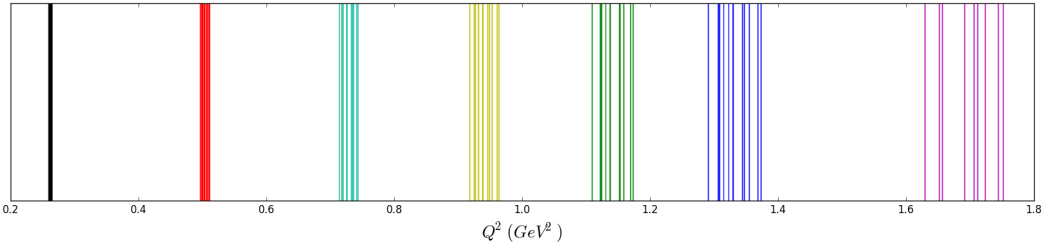




Physical Expansion

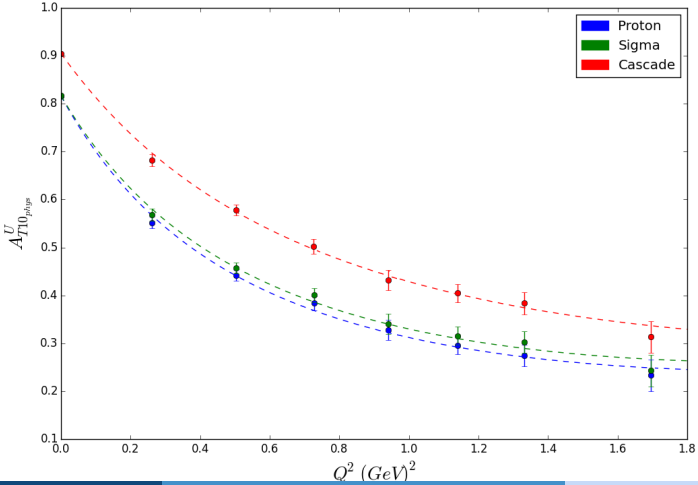
Binning

- The 4-momentum Q^2 is dependent on M_B
- We bin the Q^2 values from each ensemble into separate bins and take an average of each bin
- Using this average we then shift each ensemble to fit the average Q^2 value such that we can compare and create fan plots for each Q^2 .



Physical Form Factors

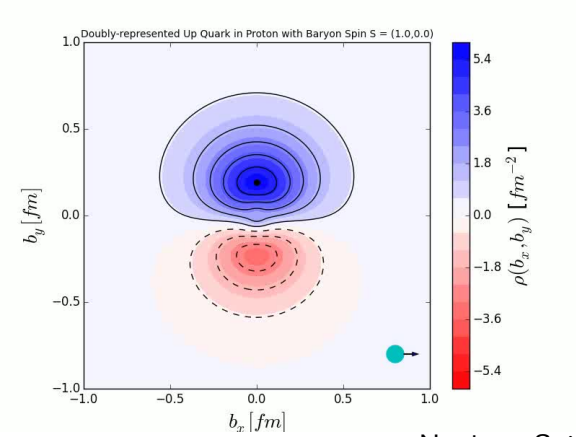
A_{T10} First Tensor form factor



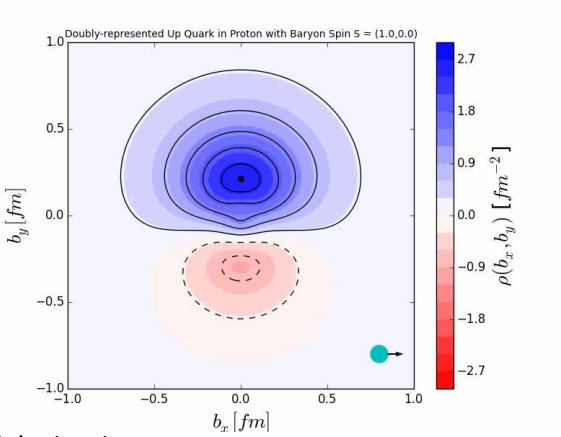
Comparing Baryon Spin Densities

Difference between the doubly represented up quarks in the Proton and Sigma

Up Quarks in Proton



Up Quarks in Sigma



Nucleon Spin Polarisation

Comparing Baryon Spin Densities

Allowing both $S_\perp$ and $s_\perp$ to be non-zero

Transverse Spin Density equation

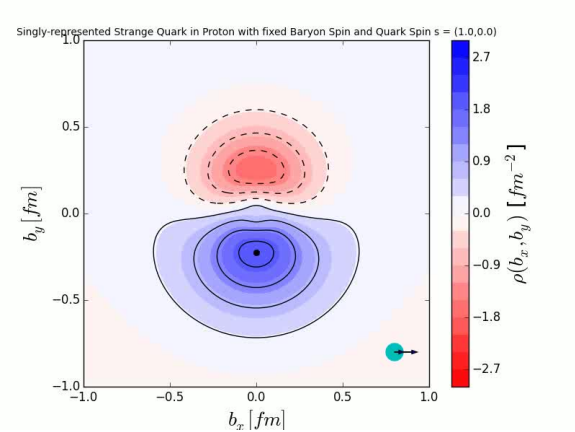
$$\begin{aligned}\rho(b_\perp, s_\perp, S_\perp) &= \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp) \\ &= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp}\tilde{A}_{T10}(b_\perp^2)\right) \\ &+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{10}(b_\perp^2) + s_\perp^i \overline{B}'_{T10}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{Tn0}(b_\perp^2)\}\end{aligned}$$

$$f' = \frac{\partial}{\partial b^2} f, \qquad f'' = \left(\frac{\partial}{\partial b^2}\right)^2 f, \qquad \Delta_b f = 4\frac{\partial}{\partial b^2}\left(b^2 \frac{\partial}{\partial b^2}\right) f$$

Comparing Baryon Spin Densities

Allowing both $S_\perp$ and $s_\perp$ to be non-zero

Fixed Nucleon Spin varying Quark Spin



Down Quark in the Proton

Conclusion

- Electromagnetic and tensor form factor results
- Transverse spin density results
- SU(3)-flavour symmetry breaking expansion
- Physical results and spin-spin density distributions

Thank you for Listening