Direct lattice-QCD calculation of pion valence quark distribution

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Lattice parton physics project (LP³)
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Introduction

- Pion plays a fundamental role in QCD
  - Lightest quark-antiquark bound state
  - Goldstone boson associated with dynamical chiral symmetry breaking
  - Explains the flavor asymmetry in the nucleon quark sea

- Its parton structure mainly from Drell-Yan data on $\pi N$ scattering
  - Soft gluon resummation renders $q_v^\pi$ softer at large $x, \sim (1 - x)^2$ [Aicher, Schäfer and Vogelsang, PRL 10']

  - Consistent with perturbative QCD [Farrar and Jackson, PRL 79', Berger and Brodsky, PRL 79'] and Dyson-Schwinger
  - Equation [Hecht, Roberts and Schmidt, PRC 01']
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- Quark models favor a linear dependence $(1 - x)$ at large $x$
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    [Detmold, Melnitchouk, Thomas, PRD 03']
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- It can shed more light on pion parton structure if its computational potential can be extended beyond that
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Large momentum effective theory (LaMET)

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum
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- Parton physics usually formulated in terms of light-cone quantization [Dirac]
  - light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Example: [Collins and Soper, NPB 82']

$$q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\cdot P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp \left( -ig \int_{0}^{\xi^-} d\eta^- A^+(\eta^-) \right) \psi(0)|P\rangle$$
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- However, it was originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities
  $$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$
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However, it was originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

$$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$


- Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice
- A finite but large $P_z$ already offers a good approximation, where (leading) frame-dependence can be removed through a factorization procedure
Pion PDF from LaMET

- **Pion PDF**

  \[ q_f^\pi (x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda \cdot P} \langle \pi(P) | \overline{\psi}_f (\lambda n) \gamma^5 \Gamma (\lambda n, 0) \psi_f (0) | \pi(P) \rangle \]

  \( P^\mu = (p_0, 0, 0, p_z), n^\mu = (1, 0, 0, -1) / \sqrt{2} \)

- **Pion quasi-PDF** [Ji, PRL 13']

  \[ \tilde{q}_f^\pi (x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda \cdot \tilde{n}} \langle \pi(P) | \overline{\psi}_f (\lambda \tilde{n}) \gamma^5 \Gamma (\lambda \tilde{n}, 0) \psi_f (0) | \pi(P) \rangle \]

  \( \tilde{n}^\mu = (0, 0, 0, -1), \tilde{\gamma} = \gamma^t \) can also be replaced by \( \gamma^t \)

- **Nonperturbative renormalization of quasi-PDF** [Ji, JHZ and Zhao, PRL 18', Ishikawa, Ma, Qiu and Yoshida, PRD 17', Green, Jansen and Steffens, 17']

  \[ \tilde{h}_R (\lambda \tilde{n}) = Z_1 Z_2 e^{\delta m \lambda} \tilde{h} (\lambda \tilde{n}) \]

  \( \delta m \) can be calculated from Wilson loop corresponding to static quark-antiquark potential
Pion PDF from LaMET

- **Pion PDF**

\[
q_f^\pi(x) = \int \frac{d\lambda}{4\pi} e^{-ix\lambda n \cdot P} \langle \pi(P) | \bar{\psi}_f(\lambda n) \gamma^\mu \Gamma(\lambda n, 0) \psi_f(0) | \pi(P) \rangle
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\[
\tilde{h}_R(\lambda \tilde{n}) = Z^{-1}(\lambda \tilde{n}, p_z^R, 1/a, \mu_R) \tilde{h}(\lambda \tilde{n})
\]

- **RI/MOM** [Stewart and Zhao, PRD 17’, Alexandrou et al, NPB 17’, LP3, PRD 17’]

\[
Z(\lambda \tilde{n}, p_z^R, 1/a, \mu_R) = \frac{\text{Tr}[\bar{\psi} \sum_s \langle p, s | \bar{\psi}_f(\lambda \tilde{n}) \gamma^\mu \Gamma(\lambda \tilde{n}, 0) \psi_f(0) | p, s \rangle]}{\text{Tr}[\bar{\psi} \sum_s \langle p, s | \bar{\psi}_f(\lambda \tilde{n}) \gamma^\mu \Gamma(\lambda \tilde{n}, 0) \psi_f(0) | p, s \rangle_{\text{tree}}]} \bigg|_{p^2 = -\mu_R^2, p_z = p_z^R}
\]
Pion PDF from LaMET

- **Pion PDF**

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- \( \tilde{n}^\mu = (0, 0, 0, -1), \tilde{\gamma}_t = \gamma^t \) can also be replaced by \( \gamma^t \)

- **Factorization [Ji, PRL 13’, Xiong, Ji, JHZ and Zhao, PRD 14’, Stewart and Zhao, PRD 18’, Ma and Qiu, 14’ & PRL 18’]**

\[
\tilde{q}_{v,R}^\pi (x, \tilde{n} \cdot P, \tilde{\mu}) = \int_0^1 \frac{dy}{y} C \left( \frac{x}{y}, \frac{\tilde{\mu}}{\mu}, \frac{\mu}{y \tilde{n} \cdot P} \right) q_{v,R}^\pi (y, \mu) + \mathcal{O} \left( \frac{m_\pi^2}{(\tilde{n} \cdot P)^2}, \frac{\Lambda_{QCD}^2}{(\tilde{n} \cdot P)^2} \right)
\]

- \( q_{u,v}^\pi (x) = q_{u}^\pi (x) - q_{d}^\pi (x) = q_{u}^\pi (x) - q_{d}^\pi (x) \) due to isospin symmetry
Other proposals

- They all share the same property of computing correlations at spacelike separations

- Current-current correlation functions
  - [Liu and Dong, PRL 94’]
  - [Detmold and Lin, PRD 06’]
  - [Braun and Müller, EPJC 08’]
  - [Davoudi and Savage, PRD 12’]
  - [Chambers et al., PRL 17’]

- Lattice cross sections
  - [Ma and Qiu, 14’ & PRL 17’]

- Ioffe-time /pseudo-distribution
  - [Radyushkin, PRD 17’]
Results on pion valence quark PDF

- Renormalized matrix element

LP3, 1804.01483,
\( m_\pi \approx 310 \text{ MeV}, a = 0.12 \text{ fm}, L \approx 3 \text{ fm} \)
Results on pion valence quark PDF

- One-loop matching effect

- Matching has a sizeable effect, and cannot be ignored, as was done in [Xu, Chang, Roberts and Zong, PRD 18'], where they observed that for $P_z \geq 2$ GeV, by further increasing pion momentum the quasi-PDF shrinks to the physical region very slowly.

LP3, 1804.01483, $m_\pi \approx 310$ MeV, $a = 0.12$ fm, $L \approx 3$ fm
Results on pion valence quark PDF

- Pion momentum dependence

LP3, 1804.01483,

\[ m_\pi \approx 310 \text{ MeV}, a = 0.12 \text{ fm}, L \approx 3 \text{ fm} \]
Results on pion valence quark PDF

- Final result

LP3, 1804.01483, $m_\pi \approx 310$ MeV, $a = 0.12$ fm, $L \approx 3$ fm
Summary and outlook

- **Large momentum effective theory** opens a new door for *ab initio* studies of hadron structure

- It has been applied to computing dynamical properties of hadrons, like nucleon PDFs, meson PDFs & DAs, and yields encouraging results

- Systematic studies of uncertainties or artifacts are required:
  - Physical pion mass
  - Continuum extrapolation
  - Finite volume effects
  - Discretization effects
  - Higher-order matching
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