Kaon Distribution Amplitude from Lattice QCD

Rui Zhang

$LP^3$ Collaboration

ITP, CAS

Michigan State University

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Overview

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Meson DA in exclusive processes

Meson lightcone distribution amplitudes are important inputs in exclusive processes, such as $B \rightarrow \pi K$, at large momentum transfer $Q^2 \gg \Lambda_{QCD}$, where the scattering amplitude can be factorized into hard parts and soft parts:

$$
< \pi K | Q_i | B > = F_0^{B \rightarrow \pi} T_{K,i}^l f_K \Phi_K + F_0^{B \rightarrow K} T_{\pi,i}^l f_\pi \Phi_\pi + T_{i}^{ll} f_B \Phi_B f_K \Phi_K f_\pi \Phi_\pi
$$
quasi-DA

Instead of calculating the PDA directly, we are actually calculating the quasi-DA in LaMET [Ji, 2013]

\[ \tilde{\phi}_M(x, \mu_R, P_z) = \frac{i}{f_M} \int \frac{dz}{2\pi} e^{-i(x-1)P_z z} \langle M(P)|\bar{\psi}(0)\gamma^z \gamma_5 \Gamma(0, z) \lambda^a \psi(z)|0\rangle \]

after a matching procedure [Ji et al., 2015]

\[ \tilde{\phi}_M(x, \mu_R, P_z) = \int_0^1 dy \ Z_\phi(x, y, \mu, \mu_R, P_z) \phi_M(y, \mu) + O\left( \frac{\Lambda_{QCD}^2}{P_z^2}, \frac{m_M^2}{P_z^2} \right), \]

where the matching kernel \( Z_\phi \) can be expanded to one-loop level as:

\[ Z_\phi(x, y) = \delta(x - y) + \frac{\alpha_s}{2\pi} (Z^{(1)}_\phi(x, y) - \delta(x - y) \int_{-\infty}^{\infty} dx' Z^{(1)}_\phi(x', y)) + O(\alpha_s^2) \]
The observable we compute on lattice is the correlator

\[ \tilde{C}(z, P_z, \tau) = \left\langle \int d^3x e^{i \vec{P} \cdot x} \bar{\psi}(\vec{x}, \tau) \gamma^z \gamma_5 \Gamma(\vec{x}, \vec{x} + z) \lambda^a \psi(\vec{x} + z, \tau) \psi^S(0, 0) \gamma_5 \lambda^a \psi^S(0, 0) \right\rangle, \]

where we use the gauge invariant Gaussian smeared source

\[ \psi^S(x) = \int d^3y e^{-\frac{|x-y|^2}{2\sigma^2} - i \vec{k} \cdot (\vec{x} - \vec{y})} U(x, y) \psi(y), \]

which can be related to the matrix element

\[ \tilde{h}_M(z, P_z) = \langle M(P) | \bar{\psi}(0) \gamma^z \gamma_5 \lambda^a \Gamma(0, z) \psi(z) | 0 \rangle \]

by extracting the ground-state coefficient of the correlator

\[ \tilde{C}(z, P_z, \tau) = \frac{Z_{src} \tilde{h}_M(z, P_z)}{2E_0} e^{-E_0 \tau} + \sum_{i>0} B_i(z, P_z) e^{-E_i \tau} \]
The two-point correlators are obtained by running the chroma program with following parameters:

- Lattice spacing $a = 0.12\, fm$
- $24^3 \times 64$ lattice with $2 + 1 + 1$ flavors of HISQ
- Pion mass $310\, MeV$
- Smeared sources and sinks with smearing mom $k = 0.73P_z$
- Meson momentum $P_z = (4\pi/6\pi/8\pi)/L = (0.77/1.15/1.53)\, GeV$
- 4 source locations
- 967 hypercubic smearing configurations
Matrix Elements

We fit the resulting correlators to the sum of first two terms

$$\tilde{C}(z, P_z, \tau) = A(z, P_z)e^{-E_0\tau} + B(z, P_z)e^{-E_1\tau}$$

Average $\chi^2 = 1.3$.

Normalize the coefficient to obtain

$$h_M(zP_z, P_z) = \frac{\tilde{h}_M(z, P_z)}{P_z f_M} = \frac{A(z, P_z)}{A(0, P_z)}$$

so that $h(0, P_z) = 1$.

We also checked the 3-term fit results. They’re consistent with our 2-term fit, thus we can safely exclude the excited-state effect here.
Dispersion Relation

The dispersion relations for $\pi$, $K$ and $\eta_s$ (with the connected diagram contribution only). The lines are $E^2(P_z) = m^2 + \hat{P}_z^2$, with $\hat{P}_z = 2/a \sin(P_z a/2)$, which are satisfied within two sigmas of the statistical uncertainties.
Bare quasi-DA ME

The kaon bare quasi-DA matrix elements. The dashed lines are the asymptotic forms. The bare results for pion and $\eta_s$ are quite similar to the kaon’s.
Renormalization

The gauge-invariant quark Wilson line operator contributes to power divergences. It can be renormalized multiplicatively in the coordinate space:

\[ \tilde{O}_r(z) = \bar{\psi}(z) \Gamma W(z, 0) \psi(0) = Z_\psi, Z e^{-\delta m |z|} (\bar{\psi}(z) \Gamma W(z, 0) \psi(0))^R \]

[Ji et al., 2017; Green et al., 2017; Ishikawa et al., 2017] where \( \delta m \) captures the linear power divergence, and \( Z \) is a logarithmic renormalization constant. The power divergence has to be nonperturbatively renormalized.
The $\delta m$ can be determined by computing the $q - \bar{q}$ static potential $V(r) = \frac{c_{-1}}{r} + c_0 + c_1 r$

where $c_0 = \frac{c_{0,1}}{a} + c_{0,2}$, $\delta m = -\frac{c_{0,1}}{2a} = 0.154(2)/a = 225(3) \text{MeV}$. The improved quasi-DA is [Zhang et al., 2017]

$$\tilde{\phi}_M^{\text{imp}}(x, P_z) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-i(x-1)zP_z + \delta m|z|} P_z h_M(z, P_z).$$
Matching and mass corrections

Final DAs are obtained by applying the one-loop matching kernel

\[
\phi_{M}^{\text{imp,match}}(x, P_z) \approx \phi_{M}^{\text{imp}}(x, P_z) - \frac{\alpha_s}{2\pi} \int_{-\infty}^{\infty} dy \left[ Z_{\phi}^{(1)}(x, y) \phi_{M}^{\text{imp}}(y, P_z) - Z_{\phi}^{(1)}(y, x) \phi_{M}^{\text{imp}}(x, P_z) \right]
\]

and then the mass corrections to the improved DAs. The dashed line is the asymptotic form, the green band is DA without mass correction.

![Graph showing the comparison of different DAs](image)

\[ P_z = 1.53\,\text{GeV} \]
Improved kaon DA

We then obtain the kaon distribution amplitudes for $P_z = (0.77/1.15/1.53) \text{GeV}$, with statistical errors only. The purple dashed line is the asymptotic form.
Improved pion DA

the pion distribution amplitudes for $P_z = (0.77/1.15/1.53)$ GeV, with statistical errors only. The purple dashed line is the asymptotic form. The $\eta_s$ result is similar, with smaller errors.
SU(3) relations

It was shown in ChPT that the DAs satisfied the SU(3) relation

\[
\phi_{K^+}(x, \mu) - \phi_{K^-}(x, \mu) = \phi_{K^0}(x, \mu) - \phi_{\bar{K}^0}(x, \mu) \propto m_s - m_{u/d},
\]

\[
\phi_{\pi}(x, \mu) + 3\phi_{\eta}(x, \mu) - 2\phi_{K^+}(x, \mu) - 2\phi_{K^-}(x, \mu) = \mathcal{O}(m^2_q),
\]

[Chen and Stewart, 2004] where the \(\phi_\eta\) can be obtained by

\[
\phi_\eta = (2\phi_{\eta s} + \phi_\pi)/3.
\]

Thus we can compare the two magnitudes

\[
\delta_{SU(3),1} = (\phi_{K^-} - \phi_{K^+})/2 = \mathcal{O}(m_q),
\]

\[
\delta_{SU(3),2} = (\phi_\pi + \phi_{\eta s} - \phi_{K^+} - \phi_{K^-})/4 = \mathcal{O}(m^2_q).
\]
SU(3) relations

\[ \frac{1}{2} \left( \phi_K - \phi_K^+ \right) \]

\[ (3 \phi_{\eta} + \phi_{\pi}^+ - 2 \phi_K^+ - 2 \phi_K^-) \]
Summary

- We compute the quasi-DA of pion, kaon and $\eta_s$ on lattice;
- Applied the $\delta m$ counterterm renormalization, one loop matching kernel and mass corrections;
- Supported the $SU(3)$ relation predicted by ChPT.
- Future study: smaller lattice spacing, larger volume, physical pion mass.
The End
- In an OPE the leading order Wilson coefficient has an ambiguity from perturbation which requires higher order power corrections to cancel it.

- Our renormalization is done non-perturbatively, so there is no renormalon ambiguity.

- The perturbative matching could have renormalon ambiguity, but its size is the same order as the (twist-4) power correction.
disconnected diagrams for $\eta_s$

- The disconnected diagram is $O((m_s - \bar{m})^2)$ suppressed because there are two fermion loops.
- The error caused by the different values of ground-state energy $E_0$ is reduced when $P_z$ increases, and is negligible at our momentum.
- The $\eta_0$ contribution is suppressed by a mixing factor $\sin\theta \sim 0.08$ times a factor of $(m_s - \bar{m})$