Test of factorization for the long-distance effects from charmonium in $B \rightarrow Kl^{+}l^{-}$

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Motivation

(1): \( B \rightarrow Kl^+l^- \) is a penguin-induced FCNC process. (GIM and loop suppressed)

(2): Anomaly in experiments:

\[ q^2 < m_{J/\psi}^2 \]

Question: Are the long distance contributions understood?

→ We calculate the corresponding amplitude by lattice formulation.

[D. Du et al. (Fermilab, MILC) 1510.02349]
Charmonium resonance contributions

We focus on the charmonium resonance contribution,

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \sum_{i=1}^{2} (V_{us}^* V_{ub} C_i O_i^u + V_{cs}^* V_{cb} C_i O_i^c) - V_{ts}^* V_{tb} \sum_{i=3}^{10} C_i O_i \right) \]

\[ O_1^c = (\bar{s}_i \gamma_\mu P^- c_j) (\bar{c}_j \gamma_\mu P^- b_i) \]
\[ O_2^c = (\bar{s}_i \gamma_\mu P^- c_i) (\bar{c}_j \gamma_\mu P^- b_j) \]

→ \( O_1^c \) and \( O_2^c \) induce the long-distance contribution, which is often estimated by the factorization approximation. How good is that?
We’d like to calculate the B decay amplitudes on the lattice. Formulation is analogous to the $K \rightarrow \pi ll$ decay amplitudes.

\[ A_\mu (q^2) = \int d^4x \langle \pi(p) | \mathcal{T} [J_\mu(0)H_{\text{eff}}(x)] | K(k) \rangle \]

\[ A_\mu (q^2) = \int d^4x \langle K(k) | \mathcal{T} [J_\mu(0)H_{\text{eff}}(x)] | B(p) \rangle \]

The amplitude is calculable from the integration of 4pt-func.

\[ I_\mu (T_a, T_b, p, k) \approx \int_{t_J-T_a}^{t_J+T_b} dt_H \]

\[ (0 \leq t_J-T_a \leq t_J+T_b \leq t_K) \]
$B \rightarrow Kl^+l^-$ decay amplitudes

$I_{\mu} = \int_{t_J-t_{\alpha}}^{t_J} dt_H + \int_{t_J}^{t_J+t_b} dt_H$

◊ Charm loop would produce contributions like

\[
\int_{0}^{\infty} dt e^{\omega t} e^{-E_{J/\psi} t} = \frac{1}{\omega - E_{J/\psi}} \quad \int_{0}^{0} dt e^{\omega t} e^{E_{J/\psi} t} = \frac{1}{\omega + E_{J/\psi}}
\]

◊ If we focus on $\omega = \sqrt{m_{J/\psi}^2 - q^2} \sim E_{J/\psi}$, $t_H < t_J$ part is dominant.
**Divergence of the amplitude?**

◇ **$K \rightarrow \pi l^+ l^-$ case**

$[N.H. \text{Christ et al. (RBC, UKQCD) 1507.03094}]$

$$I_\mu = \int \limits_{t_J}^{t_H} \frac{dE}{2E} \rho(E) \langle \pi(k) | J_\mu(0) | E, p \rangle \langle E, p | H_{\text{eff}}(0) | K(p) \rangle \left( 1 - e^{[E_K(p) - E]T_a} \right) + (t_J < t_H)$$

Some intermediate states have $E$ lower than $E_K$

→ Since $T_a \rightarrow \infty$, they must be subtracted.

(e.g. $K \rightarrow \pi, \pi \pi, \pi \pi \pi$)
Divergence of the amplitude?

$B \rightarrow Kl^+l^-$ case

$I_\mu = \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle K(k) | J_\mu(0) | E, p \rangle \langle E, p | H_{\text{eff}}(0) | B(p) \rangle}{E_B(p) - E} \left( 1 - e^{[E_B(p) - E]T_a} \right)

\text{if } t_J < t_H$

We restrict ourselves in the setup of

$m_B < m_{J/\psi} + m_K$

→ No divergence.
Amplitude of $B \rightarrow Kl^+l^-$.

From the integration of the 4-point correlators, we can extract the amplitude after taking $T_{a,b} \rightarrow \infty$ limit.

$$I_\mu (T_a, T_b, p, k) = - \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle K(k) | J_\mu(0) | E, p \rangle \langle E, p | H_{\text{eff}}(0) | B(p) \rangle}{E_B(p) - E} \left( 1 - e^{[E_B(p) - E]T_a} \right)$$

$$+(t_J < t_H)$$

$$A_\mu (q^2) = \int d^4x \langle K(k) | T [J_\mu(0) H_{\text{eff}}(x)] | B(p) \rangle$$

$$A_\mu(q^2) = -i \lim_{T_{a,b} \rightarrow \infty} I_\mu(T_a, T_b, k, p)$$
Factorization method for $B \rightarrow Kl^+l^-$ decay
Assume that long-range gluon exchange can be ignored. We test this assumption by the lattice calculation.

\[
\left\langle P_K \right| J_{\nu}^{\bar{c}c} (\bar{c}_i \gamma_\mu P_c c_i)(\bar{s}_j \gamma_\mu P_b b_j) \left| P_B \right\rangle = \frac{1}{(\text{Vol.})} \left\langle 0 \right| J_{\nu}^{\bar{c}c} J_{\mu}^{\bar{c}c} \left| 0 \right\rangle \left\langle P_K \right| V_\mu \left| P_B \right\rangle
\]

→ We test this assumption by the lattice calculation.
Factorization

Factorizable operator $O_F$ and non-factorizable operator $O_{NF}$

Fierz trans.

\[ O_F^{(1)} = (\bar{c}_i \gamma_\mu P_c c_i) (\bar{s}_j \gamma_\mu P_b b_j) \]
\[ O_{NF}^{(8)} = (\bar{c}_i [T^a]_{ij} \gamma_\mu P_c c_j) (\bar{s}_k [T^a]_{kl} \gamma_\mu P_b b_l) \]

\[ O_1^c = O_F^{(1)} \]
\[ O_2^c = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)} \]

Assume non-factorizable operator $O_{NF}^{(8)}$ could be ignored

\[ \rightarrow \text{We test this assumption } O_2^c = \frac{1}{3} O_1^c. \]

\[ K \rightarrow \pi\pi \text{ case, Lattice. } O_2^l \simeq -0.7 O_1^l \]

[P.A. Boyle et al. (RBC, UKQCD) 1212.1474]
Preliminary result for the test of factorization
**Current status**

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<th>am_c</th>
<th>am_b</th>
<th>am_π</th>
<th>aE_K</th>
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Mobius domain-wall fermion with 2+1 flavor.

- **up and down mass same as strange.**

- **“bottom” mass:** \( m_b = \{(1.25^2)m_c, (1.25^4)m_c\} \)

- **Finite momentum in the final state** \( \mathbf{k} = \frac{2\pi}{L}(1, 0, 0) \)
4-point functions

\[ \Gamma_\mu^{(4)} (t_H, t_J, p, k) = \int d^3x d^3y e^{-i\mathbf{q} \cdot \mathbf{y}} \left\langle \phi_K (t_K, \mathbf{k}) T [J_\mu (t_J, \mathbf{y}) H_{\text{eff}} (t_H, \mathbf{x})] \phi_B^\dagger (0, \mathbf{p}) \right\rangle \]
4-point functions

\[ \left\langle O_{1}^{c} \right\rangle = \frac{\left\langle O_{1} \right\rangle}{(\text{Fact.})} \]
4-point functions

\[ \frac{\langle O_2^c \rangle}{\langle O_1^c \rangle} = \frac{1}{3} \]

Factorization = \[ \frac{1}{3} \]
TO DO LIST

We have to…

(1): determine the lattice renormalization constants.

(2): Input more realistic momentum.

\[ E_B(0) = E_{J/\psi}(k) + E_K(k) \quad \Rightarrow \quad k \geq \frac{2\pi}{L}(2, 2, 2) \]

(3): Input or extrapolate to physical quark masses.

(4): Complete integration and taking limit to extract amplitude.
Summary

- We study the charmonium contribution to $B \to Kl^+l^-$ by the lattice calculation.

- $B \to Kl^+l^-$ is calculable analogously to $K \to \pi l^+l^-$ for lighter bottom quark masses.

- As a first step, we study the accuracy of the factorization approximation.

- Sizable non-factorizable contribution is observed in the long-distance region.
4 point functions  \[ a = 2.45 \text{ GeV} \]
4 point functions \( a = 2.45 \) GeV