Scale Setting on the MDWF in Gradient Flow HISQ Action With the Omega Baryon

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The Importance of Scale Setting

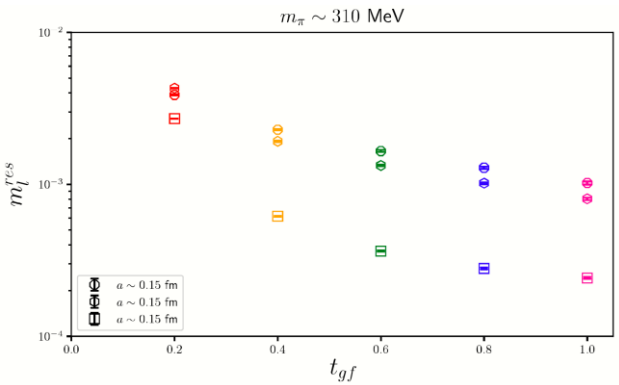
- Necessary for dimensional predictions from Lattice QCD simulations.
- We use ω_0, derived from the Wilson Flow [S. Bornsani et al. JHEP 1209 (2012) 010], to find the lattice spacing.
- We use 10 ensembles generated by MILC [Bazavov et al. PRD82 (2010) [1004.0342], PRD87 (2013) [1212.4768]].
- It's the first time that this action has been used in determining w_0.

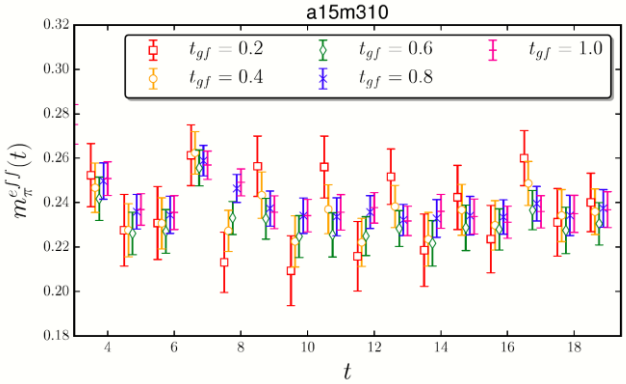
Our Lattice QCD Action

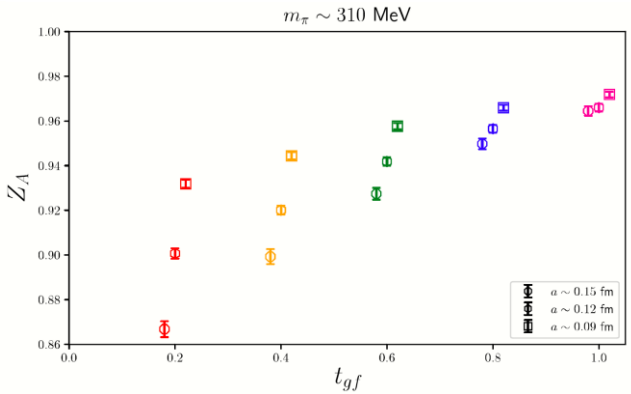
- Möbius Domain Wall Fermions on gradient flowed 2+1+1 HISQ ensembles Berkowitz et al. PRD96 (2017) [1701.07559]
  - Approximate chiral symmetry, many finite lattice spacing operators not allowed
  - Leading discretization errors begin at O(a^2)
- To control the three standard systematics for LQCD calculations, need
  - multiple lattice spacings
  - multiple volumes
  - pion masses at/near the physical pion mass
- The only set of publicly available ensembles which satisfy these criteria are the Nf=2+1+1 Highly Improved Staggered Quark (HISQ: Follana et al. PRD75 (2007) [hep-lat/0610092]) ensembles generated by the MILC Collaboration Bazavov et al. PRD82 (2010) [1004.0342], PRD87 (2013) [1212.4768]
- The DWF on asqtad action (Renner et al. [LHPC] NPPS 140 (2005) [hep-lat/0409130]) was used very successfully: LHPC; NPLQCD; Aubin, Laiho, Van de Water; …
  - Fully developed Mixed-Action EFT: Bar, Bernard, Rupak, Shoresh; Tiburzi; Chen, O'Connell, Van de Water, Walker-Loud; …
  - This motivated us to use an improved version of this action

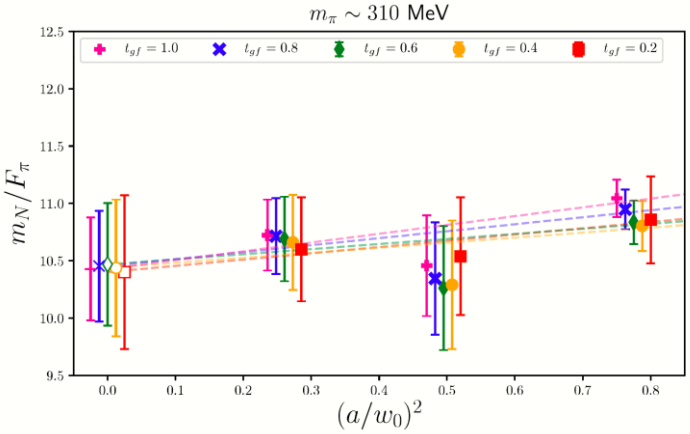
Our Lattice QCD Action

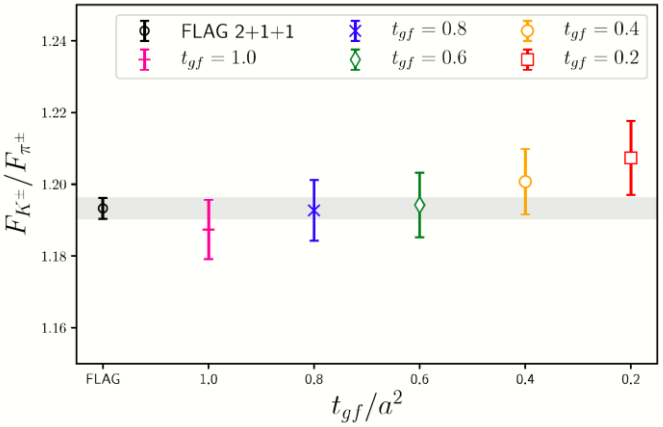
- Möbius Domain Wall Fermions on gradient flowed 2+1+1 HISQ ensembles Berkowitz et al. PRD96 (2017) [1701.07559]
- Gradient Flow smearing of HISQ cfgs more effective at reducing residual chiral symmetry breaking than the HYP smearing used in DWF on asqtad $m_{res} < 0.1\, m_l$ on all ensembles for small-to-moderate L_5 and $M_5 \leq 1.3$











Our Lattice QCD Action

HISQ gauge configuration parameters							valence parameters							
abbr.	$N_{\rm cfg}$	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	$N_{\rm src}$	L_5/a	aM_5	b_5	c_5	$am_l^{\rm val.}$	$\sigma_{\rm smr}$	$N_{\rm smr}$
a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

MDWF pion mass tuned to taste-5 HISQ pion mass within 1-2% - ensuring the unitary limit is recovered in the continuum.

additional HISQ ensembles generated @ LLNL

available to interested parties

Effective Mass, Correlation Functions, & Overlap Factors

We get effective mass by the typical equation:

$$m_{eff} = \frac{1}{\tau} \ln\left(\frac{C(t)}{C(t+\tau)}\right).$$

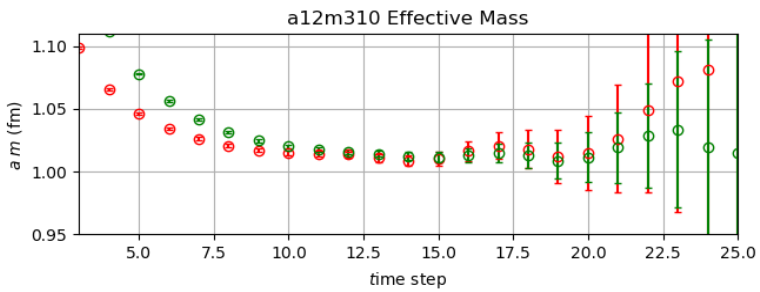
Taking the long time limit we get the ground state of the effective overlap factors

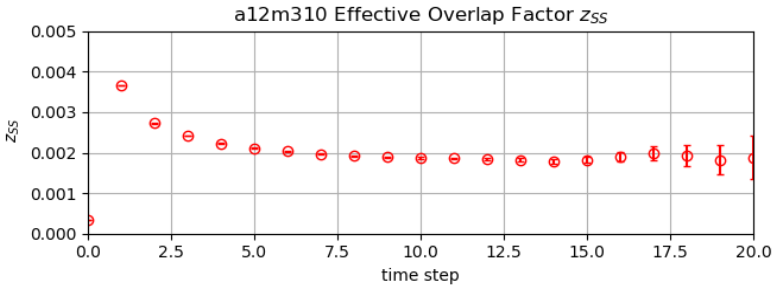
$$\mathrm{C_{SS}(t)} = \sum_{n=0}^{\infty} \{ Z_{S,n}^2 \quad e^{-E_n t} \} \quad \rightarrow \quad C_{SS} = Z_{S,0}^2 \, e^{-E_0 t} \qquad \rightarrow Z_{S,0} = \sqrt{C_{SS} \, e^{E_0 t}}$$

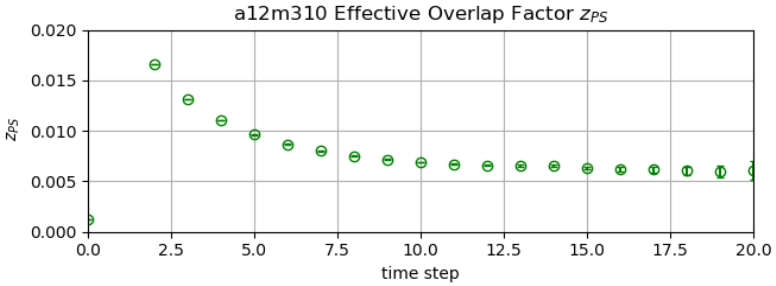
$$\mathrm{C_{PS}(t)} = \sum_{n=0}^{\infty} \{ Z_{P,n} \, Z_{S,n} \; e^{-E_n t} \} \rightarrow \quad C_{SS} = \mathrm{Z_{P,0}} \, \mathrm{Z_{S,0}} \, e^{-E_0 t} \rightarrow Z_{P,0} = \sqrt{\frac{C_{PS}^2 \; e^{E_0 t}}{C_{SS}}} \, .$$

Bayesian Constrained Curve Fitting

- We need to choose the center and standard deviation for the priors $E_n, Z_{S,n}, \& Z_{P,n}$.
- Method for E_n:
  - $E_0 \approx \mathrm{m_{platue}} \pm \mathrm{m_{platue}} * 0.1$
  - E_n gaps = 2 * $m_\pi \pm m_\pi$
- Method for $Z_{S,n}$:
  - $Z_{S,0} \approx \mathrm{Z_{S-platue}} \pm \mathrm{Z_{S-platue}} * 0.5$
  - Z_S gaps $\approx Z_{S,0} * c \pm (Z_{S,0} * c) * 100$ where $c \in [0.5, 1]$.
- Method for $Z_{P,n}$:
  - $Z_{P,0} \approx \mathrm{Z_{P-platue}} \pm \mathrm{Z_{P-platue}} * 0.5$
  - Z_P gaps $= 0 \pm 0.1$

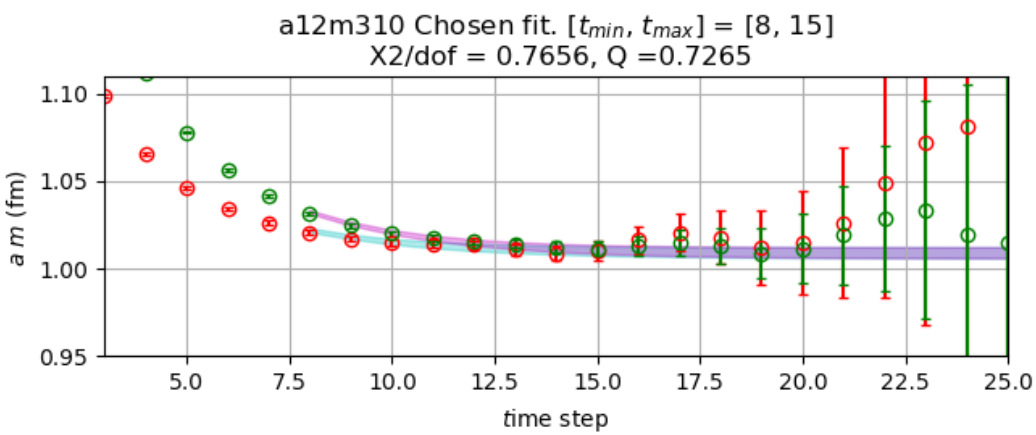


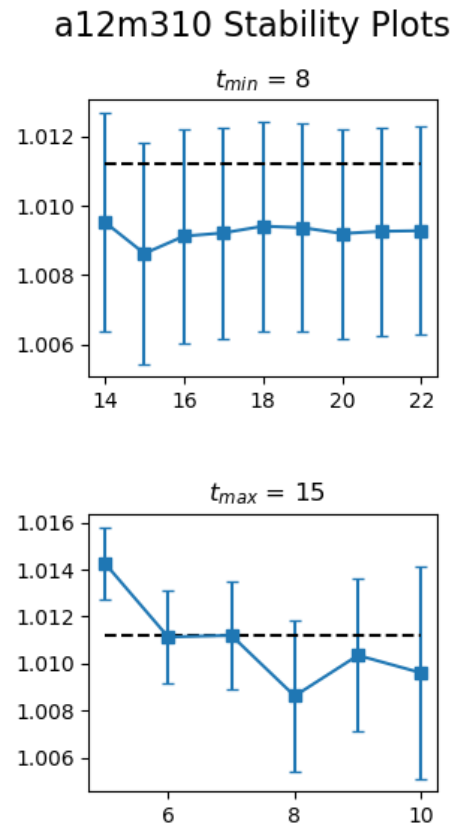




Stability Plots & Choosing a Fit

- For a09 ensembles:
  - $[t_{min}, t_{max}] = [10,20]$.
- For a12 and a15 ensembles:
  - $t_{min} \in [6,8]$.
  - $t_{max} = 15$.





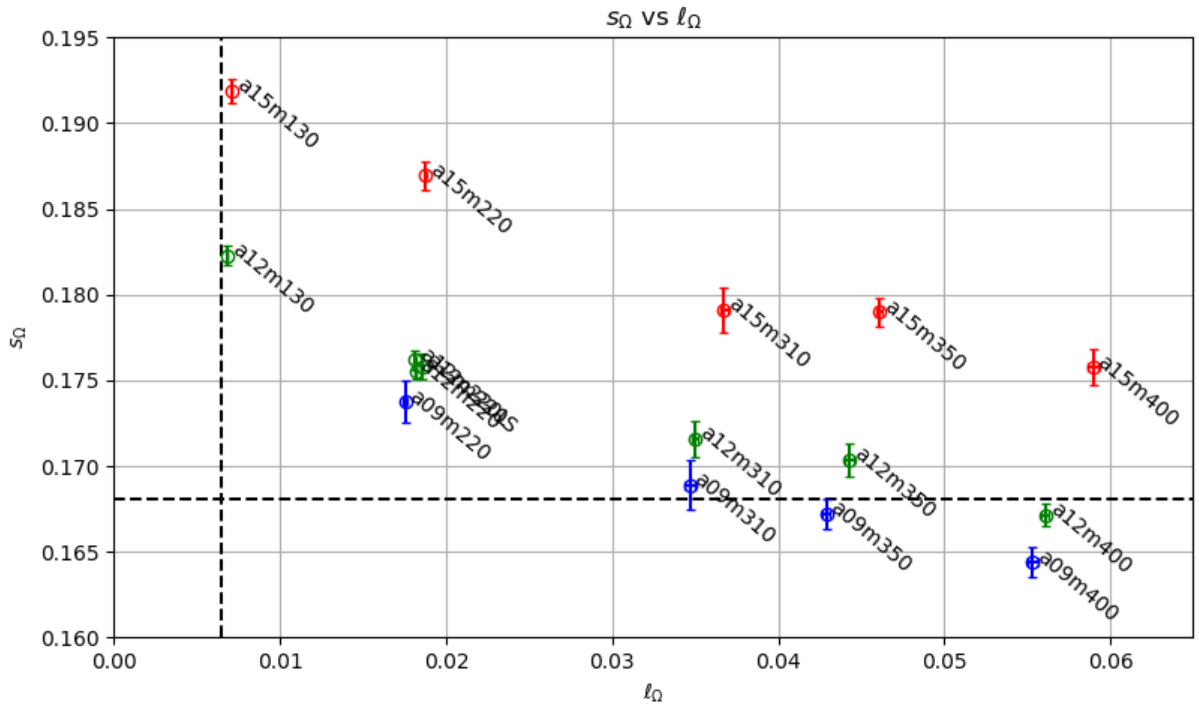
Using w_0 as the scale

- Previous calculations:

Collaboration	N_f	$\sqrt{t_0}$ (fm)	$\Delta\sqrt{t_0}/\sigma$	w_0 (fm)	$\Delta w_0/\sigma$
MILC	2+1+1	$0.1416(1)(^{+8}_{-5})$	...	$0.1714(2)(^{+15}_{-12})$	...
HPQCD [30]	2+1+1	0.1420(8)	+0.4	0.1715(9)	+0.1
ETMC* [31]	2+1+1	...	...	0.1782	...
HotQCD [33]	2+1	...	...	0.1749(14)	+1.8
BMW [5]	2+1	0.1465(21)(13)	+1.9	0.1755(18)(04)	+1.7
QCDSF-UKQCD* [32]	2+1	0.153(7)	+1.6	0.179(6)	+1.2
ALPHA* [29]	2	0.1535(12)	+8.3	0.1757(13)	+2.2

[A. Bazavov et. al. Phys. Rev. D 93, 094510 (2016)]

s_Ω vs ℓ_Ω



Where $s_\Omega = \frac{2m_k^2 - m_\pi^2}{m_\Omega^2}$ and $\ell_\Omega = \frac{m_\pi^2}{m_\Omega^2}$ [Huey-Wen Lin et al. Phys.Rev.D79:034502,2009].

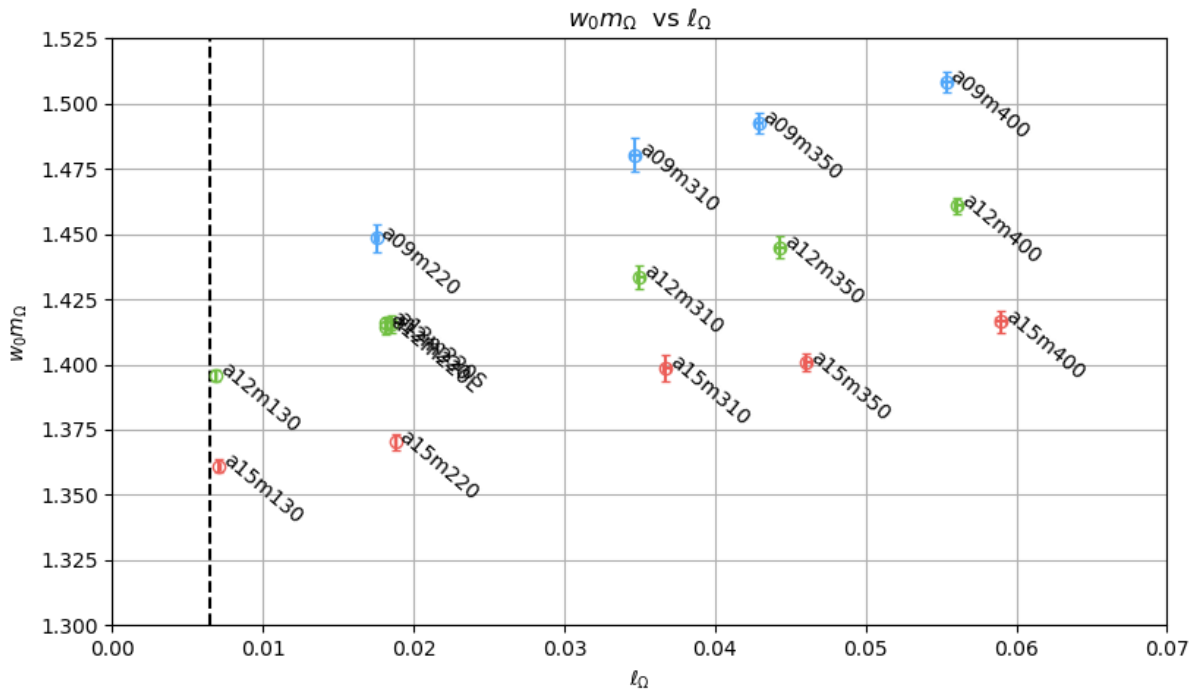
Fit Function Derivation

- We construct a fit function that has strange and light dependence and Taylor expand about the continuum and chiral limits to order $\ell_\Omega^2, \quad s_\Omega^2, \quad \epsilon_a^4$

$$w_0 m_\Omega(\ell_\Omega, s_\Omega, \epsilon_a) = w_0 m_0 + \ell_\Omega c_1 + s_\Omega c_2 + \epsilon_a^2 c_3 + \ell_\Omega^2 c_4 + s_\Omega^2 c_5 + \ell_\Omega s_\Omega c_6 + \epsilon_a^4 c_7 + \ell_\Omega \epsilon_\mathrm{a}^2 c_8 + s_\Omega \epsilon_\mathrm{a}^2 c_9 + \ell_\Omega^2 \ln(\ell_\Omega) \ ,$$

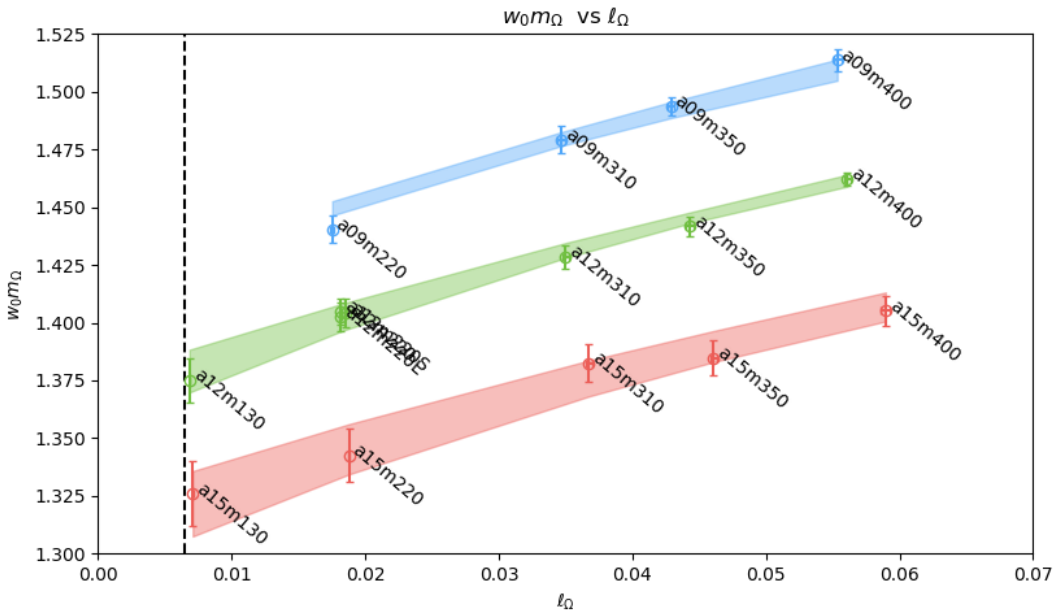
where $\ell_\Omega = \frac{m_\pi^2}{m_\Omega^2}$, $s_\Omega = \frac{2m_k^2 - m_\pi^2}{m_\Omega^2}$, and $\epsilon_\mathrm{a} = \frac{a}{\sqrt{4\,\pi}\, w_0}$.

$w_0 m_\Omega$ vs l_Ω



Where $\ell_\Omega = \left(\frac{m_\pi^2}{m_\Omega^2}\right)$.

Construct the fit $\omega_0 m_\Omega(\epsilon_a, \ell_\Omega, s_\Omega^*)$

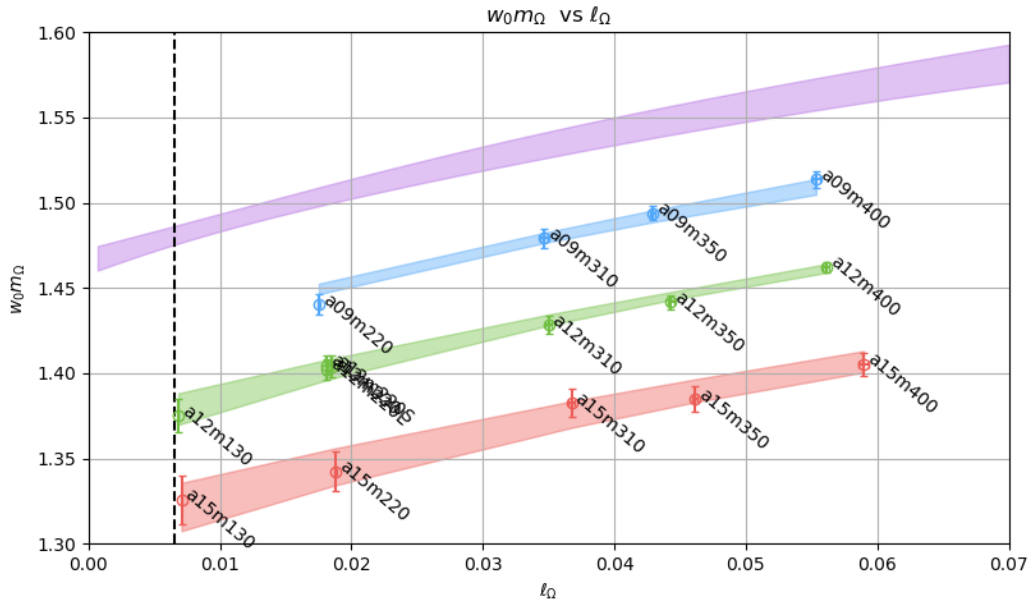


Data rescaled by $\frac{\omega_0 m_\Omega(\epsilon_a, \ell_\Omega, s_\Omega^*)}{\omega_0 m_\Omega(\epsilon_a, \ell_\Omega, s_\Omega)}$ where $s_\Omega^* =$ physical point.

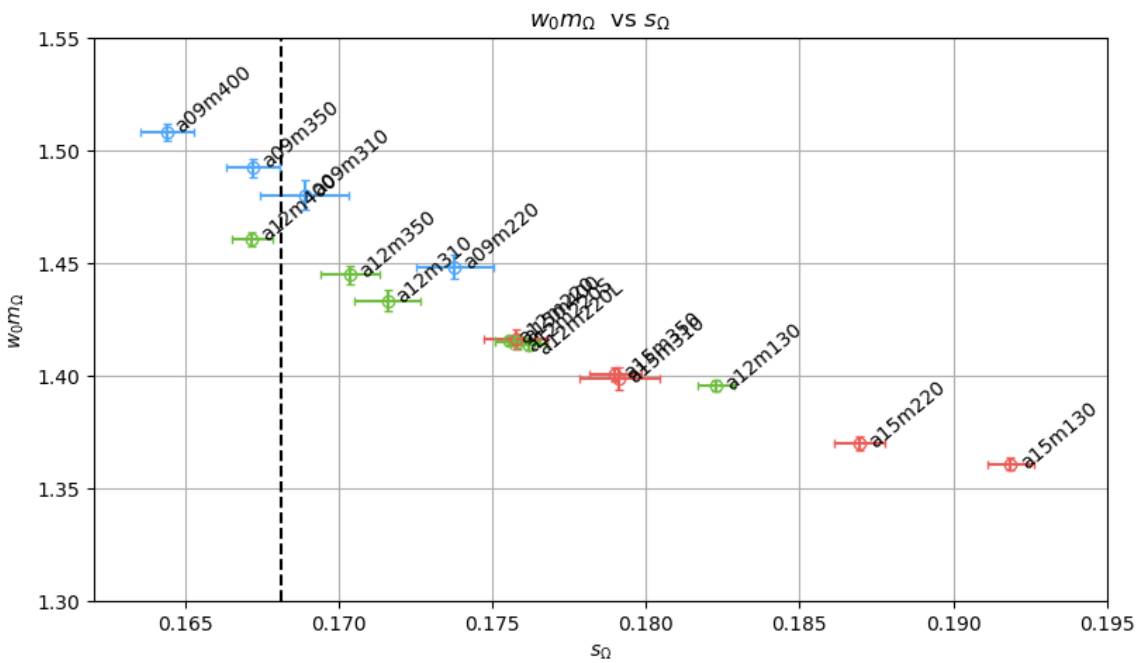
Continuum Limit Extrapolation

$$w_0 = \frac{w_0\, m_\Omega}{m_\Omega^{PDG}}$$

```
w0(a=0) = 0.17473(65)  fm

Calculated with w0(a=0):
a_09 = 0.08920(33)  fm
a_12 = 0.12294(45)  fm
a_15 = 0.15383(58)  fm

w0s calculated on ensembles:
w0(a09) = 0.16836(71)  fm
w0(a12) = 0.1626(11)   fm
w0(a15) = 0.1558(17)   fm

Calculated with w0s a09, a12, & a15:
a_09 = 0.08595(37)  fm
a_12 = 0.11443(76)  fm
a_15 = 0.1372(15)   fm
```

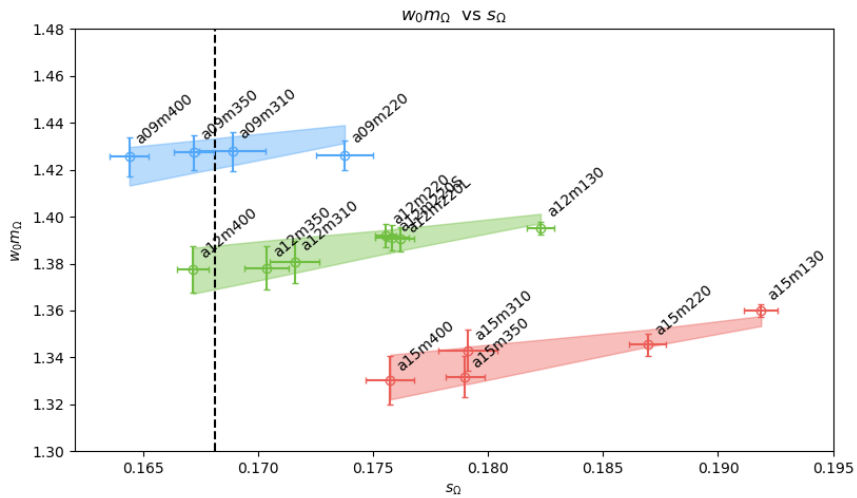


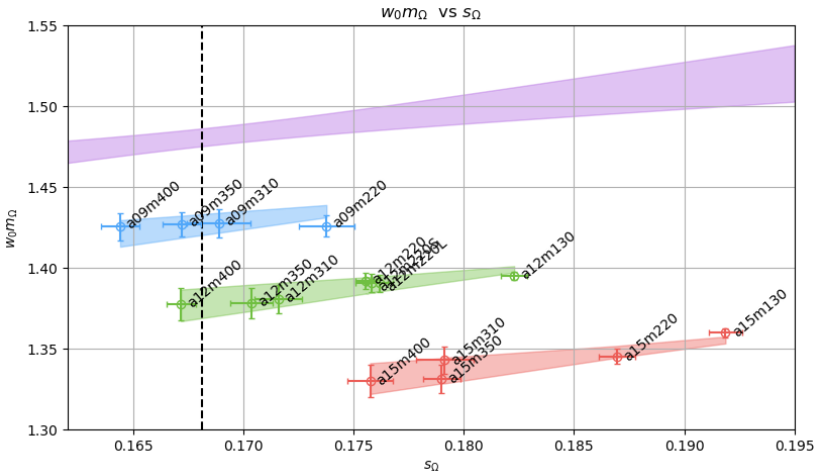
$w_0 m_\Omega$ vs s_Ω



Where $s_\Omega = \frac{2m_k^2 - m_\pi^2}{m_\Omega^2}$. This needs rescaling.

Construct the fit $\omega_0 m_\Omega(\epsilon_a, \ell^*_\Omega, s_\Omega)$ and Extrapolate to the Continuum Limit





Data rescaled by $\dfrac{\omega_0 m_\Omega(\epsilon_a, \ell^*_\Omega, s_\Omega)}{\omega_0 m_\Omega(\epsilon_a, \ell_\Omega, s_\Omega)}$

```
w0(a=0) = 0.17473(65)  fm
```

where $l^*_\Omega =$ physical point.

Comparison with Other Calculations

Collaboration	N_f	$\sqrt{t_0}$ (fm)	$\Delta\sqrt{t_0}/\sigma$	w_0 (fm)	$\Delta w_0/\sigma$
MILC	2+1+1	$0.1416(1)(^{+8}_{-5})$	...	$0.1714(2)(^{+15}_{-12})$	...
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Plus this work: CALLAT	2+1+1	...	...	0.17473(65)	$\pm$ systematic

*

*[A. Bazavov et. al. Phys. Rev. D 93, 094510 (2016)]

Stuff to Do Next

- Mass of the Proton.
- Apply the scale to get observables in other projects.

Acknowledgments

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Questions