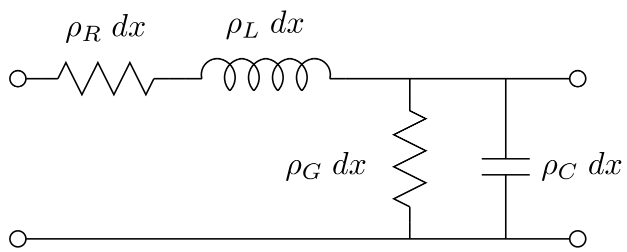
Transmission Lines

Transmission Line Equivalent Circuit



ρ_R : resistance/length
ρ_L : inductance/length
ρ_G : conductance/length
ρ_C : capacitance/length

Coupled 1st Order Differential Equations

$$\frac{\partial V(x)}{\partial x} = -(\rho_R + j\omega\rho_L)$$
$$\frac{\partial I(x)}{\partial x} = -(\rho_G + j\omega\rho_C)$$

Uncoupled 2nd Order Differential Equations

$$\frac{\partial^2 V(x)}{\partial x^2} = \gamma^2 V(X)$$
$$\frac{\partial^2 I(x)}{\partial x^2} = \gamma^2 I(x)$$
$$\gamma = \sqrt{(\rho_R + j\omega_L)(G + j\omega_C)}$$

Differential Equation Solutions

$$V(x) = V_{(+)}e^{-\gamma x} + V_{(-)}e^{+\gamma x}$$
$$I(x) = \frac{1}{Z_0}\left(V_{(+)}e^{-\gamma x} + V_{(-)}e^{+\gamma x}\right)$$

$V_{(+)}$, $V_{(-)}$ positive and negative propagating waves

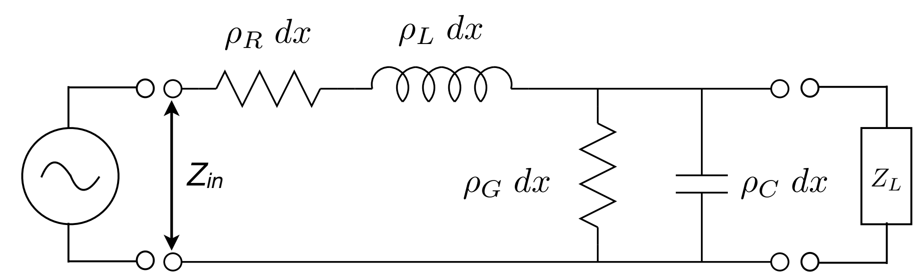
Wave propagation velocity

$$v = \frac{1}{\sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}}}$$

Open Circuit Impedance

general	low ω limit	high ω limit
$Z = \sqrt{\frac{\rho_R + j\omega\rho_L}{\rho_G + j\omega\rho_C}}$	$Z_{\text{DC}} = \sqrt{\frac{\rho_R}{\rho_G}}$	$Z_0 = \sqrt{\frac{\rho_L}{\rho_C}}$

Closed Equivalent Circuit



Closed Circuit Input Impedance

$$Z_{\text{in}}(\ell) = Z_0\frac{Z_L + jZ_0\tanh(\gamma L)}{Z_0 + jZ_L\tanh(\gamma L)}$$

Max Power Transmission (not max efficiency)

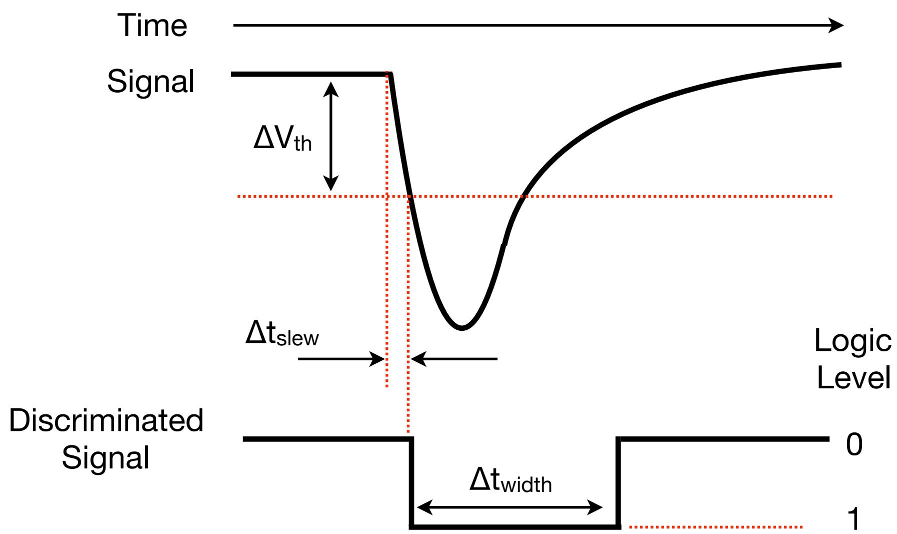
$$Z_{\text{in}} = Z_{\text{in}}^*$$
$$Z_L = Z_0$$

Signal Reflection Coefficient at Load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Logic and Triggering

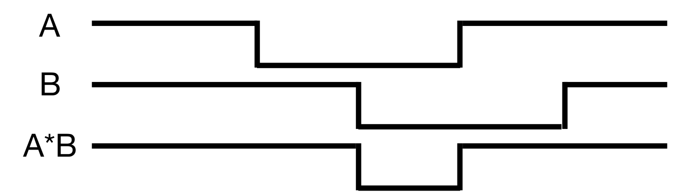
Discrimination (1 bit ADC)



Width may be fixed or “time over threshold”

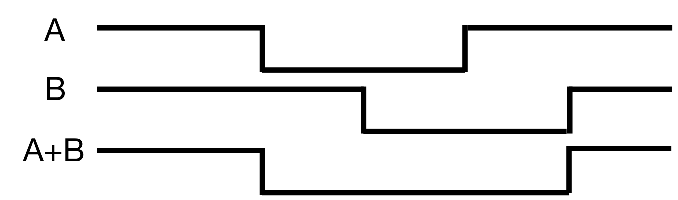
Logic Gate Behavior

AND (Coincidence)

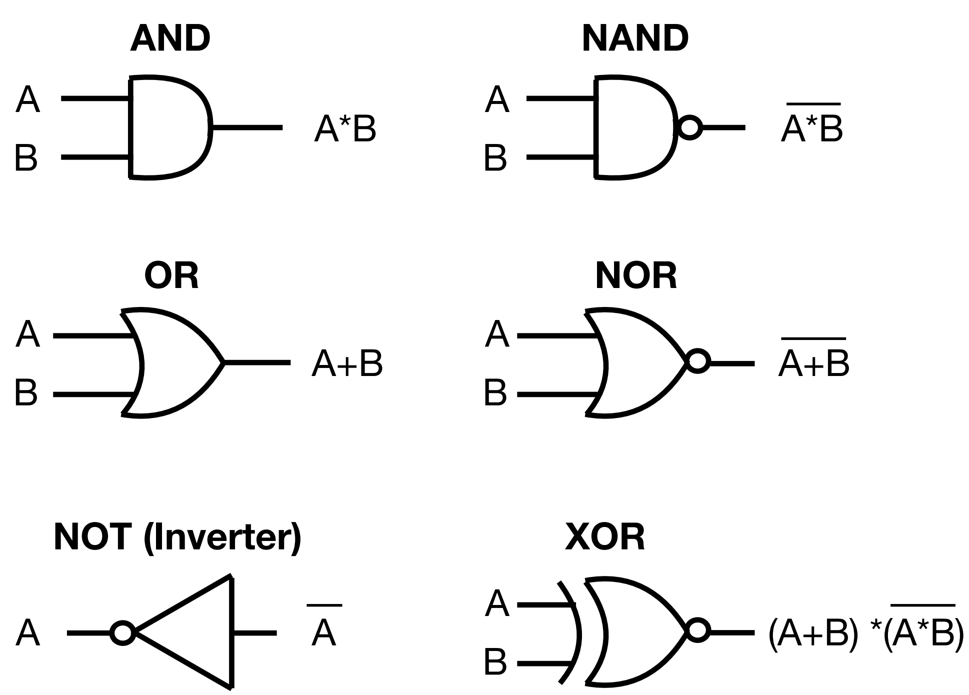


Width may be coincidence duration or fixed

OR (Fan-In)

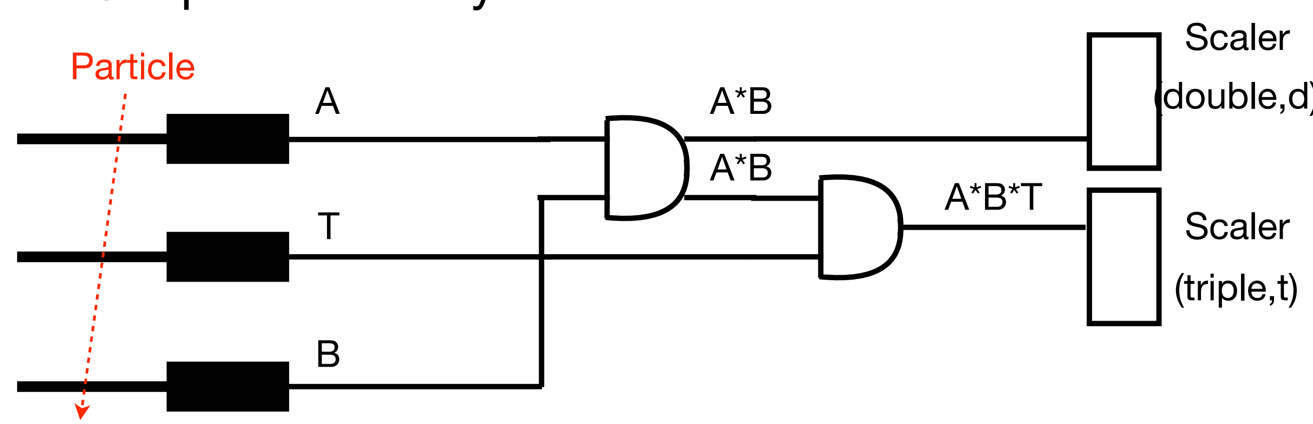


Logic Gate Symbols



Gates may have any number of inputs and outputs

Simple Efficiency Measurement



Efficiency and its uncertainty

$$\epsilon = \frac{N_t}{N_d} \qquad \sigma_\epsilon = \sqrt{\frac{\epsilon(1-\epsilon)}{N_d}}$$