Absorptive and Dispersive CP violation in $D^0 - \overline{D}^0$ mixing

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<u>Plan</u>

Introduction

- Formalism for absorptive and dispersive CPV in $D^0 \overline{D}^0$ mixing
- Indirect CPV phenomenology
- U-spin and Approximate Universality
 - How large is indirect CPV in the SM?
- How large is the current window for New Physics (NP)?
- Prospects for measuring SM indirect CPV

Introduction

- In the SM, CP violation (CPV) in $D^0 \overline{D}^0$ mixing and D decays enters at $O(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$, due to weak phase γ , yielding all 3 types of CPV:
 - direct CPV (dCPV)
 - CPV in pure mixing (CPVMIX): due to interference between dispersive and absorptive mixing amps
 - CPV in the interference of decays with and without mixing (CPVINT)
- Primary interest is in CPVMIX and CPVINT, both of which result from mixing, and which we refer to as "indirect CPV"
 - upper bounds suggest dCPV is already in the SM QCD "brown muck"

- We are interested in the following questions:
 - It was the indirect CP asymmetries in the SM?
 - What is the appropriate minimal parametrization of indirect CPV?
 - It window for new physics (NP)?
 - Can this window be closed at HL-LHCb and Belle-II?
- To answer, we develop the description of CPVINT in terms of generally final state dependent dispersive and absorptive CPV phases ϕ_f^M and ϕ_f^{Γ} for CP conjugate final states f, \bar{f} .

 introduced by Branco, Lavoura, Silva ('99)
- ϕ_f^M and ϕ_f^{Γ} parametrize CPVINT contributions from interference of D^0 decays with and without dispersive mixing, and with and without absorptive mixing
 - These are separately measurable CPV effects

- NP is most likely to appear in dispersive short distance mixing amplitudes
- SM dispersive and absorptive mixing amplitudes are due to long distance off-shell and on-shell intermediate states.
 - subleading $O(V_{cb}V_{ub}/V_{cs}V_{us})$ decay amplitudes $\propto e^{i\gamma}$ yield indirect CPV
- can not currently be calculated from first principles QCD
- meaningful SM estimates of ϕ_f^M , ϕ_f^Γ can be made using $SU(3)_F$ flavor symmetry arguments, yielding
 - a minimal parametrization of indirect CPV: approximate universality
 - estimates of the SM indirect CP asymmetries

Formalism

time-evolution of linear combination $a|D^0\rangle + b|\overline{D}^0\rangle$ follows from Schrodinger equation,

$$i\frac{d}{dt}\begin{pmatrix}a\\b\end{pmatrix} = H\begin{pmatrix}a\\b\end{pmatrix} \equiv (M - \frac{i}{2}\Gamma)\begin{pmatrix}a\\b\end{pmatrix}$$

transition amplitudes

$$\langle D^0 | H | \overline{D^0} \rangle = M_{12} - \frac{i}{2} \Gamma_{12} , \quad \langle \overline{D^0} | H | D^0 \rangle = M_{12}^* - \frac{i}{2} \Gamma_{12}^*$$

• M_{12} is the dispersive mixing amplitude Γ_{12} is the absorptive mixing amplitude

Mass eigenstates $|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$:

 \checkmark mass and width differences expressed in terms of x, y

$$x = \frac{m_2 - m_1}{\Gamma_D}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

introduce three "theoretical" physical mixing parameters

 $x_{12} \equiv 2|M_{12}|/\Gamma_D, \quad y_{12} \equiv |\Gamma_{12}|/\Gamma_D, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$

• ϕ_{12} is the CPV phase responsible for CPVMIX

- CP conserving observables: $|x| = x_{12} + O(CPV^2), |y| = y_{12} + O(CPV^2)$
- Time-evolved meson solutions, for $t \leq \tau_D$: $(D^0(0) = D^0, \overline{D}^0(0) = \overline{D}^0)$ e.g. mixed components

$$\langle \overline{D}^0 | D^0(t) \rangle = e^{-i \left(M_D - i \frac{\Gamma_D}{2} \right) t} \left(e^{-i \delta_M} M_{12}^* - \frac{1}{2} \Gamma_{12}^* \right) t, \dots$$

• $\delta_M = \pi/2$ is a CP-even "dispersive strong phase", originating from the time derivative. Contributes to strong phase differences in indirect CPV

The dispersive and absorptive CPV phases ϕ_f^M, ϕ_f^Γ

CPVINT observables for CP-conjugate final states $f = \overline{f}$:

$$\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|} \frac{A_f}{\overline{A}_f} = \eta_f^{CP} \left| \frac{A_f}{\overline{A}_f} \right| e^{i \phi_f^M}, \quad \lambda_f^{\Gamma} \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|} \frac{A_f}{\overline{A}_f} = \eta_f^{CP} \left| \frac{A_f}{\overline{A}_f} \right| e^{i \phi_f^{\Gamma}}.$$

with the decay amplitudes $A_f = \langle f | \mathcal{H} | D^0 \rangle$, $\bar{A}_f = \langle f | \mathcal{H} | \bar{D}^0 \rangle$

pairs of CPVINT observables for non-CP conjugate final states $f \neq \overline{f}$:

$$\lambda_{f}^{M} \equiv \frac{M_{12}}{|M_{12}|} \frac{A_{f}}{\overline{A}_{f}} = \left| \frac{A_{f}}{\overline{A}_{f}} \right| e^{i(\phi_{f}^{M} - \Delta_{f})}, \quad \lambda_{f}^{\Gamma} \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|} \frac{A_{f}}{\overline{A}_{f}} = \left| \frac{A_{f}}{\overline{A}_{f}} \right| e^{i(\phi_{f}^{\Gamma} - \Delta_{f})},$$
$$\lambda_{\bar{f}}^{M} \equiv \frac{M_{12}}{|M_{12}|} \frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} = \left| \frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} \right| e^{i(\phi_{f}^{\Gamma} + \Delta_{f})}, \quad \lambda_{\bar{f}}^{\Gamma} \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|} \frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} = \left| \frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} \right| e^{i(\phi_{f}^{\Gamma} + \Delta_{f})}.$$

• Δ_f is the strong phase difference between \overline{A}_f and A_f , and between $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$

relation to the CPVMIX phase: $\phi_{12} = \arg(M_{12}/\Gamma_{12}) = \phi_M^f - \phi_\Gamma^f$

Hadronic $D^0(t)$ and $\overline{D}^0(t)$ decay amplitudes sum over contributions with and without mixing,

$$A(D^{0}(t) \to f) = \langle \overline{D}^{0} | D^{0}(t) \rangle \overline{A}_{f} + \langle D^{0} | D^{0}(t) \rangle A_{f},$$

$$A(\overline{D}^{0}(t) \to f) = \langle D^{0} | \overline{D}^{0}(t) \rangle A_{f} + \langle \overline{D}^{0} | \overline{D}^{0}(t) \rangle \overline{A}_{f}.$$

• Time-dependent decay rates given in terms of CPVINT observables $\lambda_{f,\bar{f}}^M$, $\lambda_{f,\bar{f}}^\Gamma$

 $igstarrow \phi^M_{12}$ and ϕ^Γ_{12} are the CPV phase differences between mixed and unmixed decay amps

The strong phase differences are sum of $\delta^M = \pi/2$ and $\pm \Delta_f$ (dispersive mixing), $\pm \Delta_f$ (absorptive mixing)



In SM Cabibbo favored/ doubly Cabibbo suppressed decays (CF/DCS) the CPVINT phases are universal, e.g. $D^0 \rightarrow K\pi, K^*\pi,...$

$$\phi^M_{\text{cfds}} \equiv \phi^M_f, \quad \phi^\Gamma_{\text{cfds}} \equiv \phi^\Gamma_f, \quad f \in \text{CF/DCS}$$

- Iso true under the well motivated assumption that CF/DCS decays do not contain NP weak phases,
 - Solution NP with non-negligible direct CPV in DCS/CF decays, which evades ϵ_K bounds, must be very exotic or tuned Bergmann, Nir
- In SM singly Cabibbo suppressed decays (SCS), e.g. $D^0 \rightarrow \pi^+\pi^-$, K^+K^- ,... the CPVINT phases have small final state dependence due to the subleading QCD penguin decay amplitudes.

The more familiar general CPV observables

CPVMIX :
$$\left| \frac{q}{p} \right| - 1$$

CPVINT : $\phi_{\lambda_f} = \arg\left(\frac{q}{p}\frac{\overline{A}_f}{A_f}\right)$, for $f = \overline{f}$

Relation to absorptive and dispersive CPVINT phases ($\phi_{12}=\phi_f^M-\phi_f^\Gamma$)

$$\begin{split} \left| \frac{q}{p} \right| - 1 &= \frac{x_{12} \, y_{12} \sin \phi_{12}}{x_{12}^2 + y_{12}^2} \, \left[1 + O(\sin \phi_{12}) \right] \\ \\ \tan 2\phi_{\lambda_f} &= - \left(\frac{x_{12}^2 \sin 2\phi_f^M + y_{12}^2 \sin 2\phi_f^\Gamma}{x_{12}^2 \cos 2\phi_f^M + y_{12}^2 \cos 2\phi_f^\Gamma} \right). \end{split}$$

same number of CPV quantities in each description

Indirect CPV phenomenology

CPV requires non-trivial CPV "weak phase" differences (ϕ_w) and CP conserving "strong phase" differences (δ_s) between interfering amplitudes

 \Rightarrow CP asymmetries $\propto \sin \delta_{\rm s} \, \sin \phi_{\rm w}$

this dependence is manifest in the absorptive/dispersive CPV phase formalism

Examples:

The CPVMIX "wrong sign" semileptonic CP asymmetry

$$a_{\rm SL} \equiv \frac{\Gamma(D^0(t) \to \ell^- X) - \Gamma(\overline{D^0}(t) \to \ell^+ X)}{\Gamma(D^0(t) \to \ell^- X) + \Gamma(\overline{D^0}(t) \to \ell^+ X)},$$

$$= \frac{2x_{12} y_{12} \sin \delta^M \sin \phi_{12}}{x_{12}^2 + y_{12}^2 - 2\cos \phi_{12} \cos \delta^M}$$

$$= \frac{2x_{12} y_{12}}{x_{12}^2 + y_{12}^2} \sin \phi_{12}.$$

Inote the importance of the dispersive "strong phase" $\delta^M = \pi/2$

- Itime-dependent CP asymmetries in SCS decays to CP conjugate final states ($f = \bar{f}$), e.g. $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$
- to good approximation, the decay widths take the exponential forms.

$$\Gamma(D^0(t) \to f) = |A_f|^2 \exp[-\hat{\Gamma}_{D^0 \to f} \tau], \quad \Gamma(\overline{D^0}(t) \to f) = |\bar{A}_f|^2 \exp[-\hat{\Gamma}_{\overline{D^0} \to f} \tau]$$

$$\begin{split} \text{CP asymmetry :} \quad \Delta Y_f &\equiv \frac{\hat{\Gamma}_{\overline{D}{}^0 \to f} - \hat{\Gamma}_{D^0 \to f}}{2} \\ &= \eta^f_{CP} \left(-x_{12} \sin \phi^M_f \sin \delta^M + a^d_f \, y_{12} \cos \phi^\Gamma_f \right) \\ &= \eta^f_{CP} \left(-x_{12} \sin \phi^M_f + a^d_f \, y_{12} \right), \end{split}$$

• ΔY_f depends on ϕ_f^M , but not ϕ_f^{Γ} :

- If or *f* = *f*, no strong phase difference between *A_f*, *A_f*. Thus, the only available CP-even strong phase is $\delta^M = \pi/2$ ⇒ asymmetry purely dispersive in origin!
- up to subleading dCPV contribution (second term), where

$$a_f^d = 1 - \left| \bar{A}_f / A_f \right| = -2r_f \sin \delta_f \, \sin \phi_f \,.$$

■ CF/DCS decays for $f \neq \bar{f}$, e.g. $D^0 \to K^{\pm}\pi^{\mp}$: the wrong sign $D^0(t) \to f$ and $\overline{D}^0 \to f$ decay widths expressed as

$$\Gamma(D^0(t) \to \bar{f}) = e^{-\tau} |A_f|^2 \left(R_f^+ + \sqrt{R_f^+} c_f^+ \tau + c_f'^+ \tau^2 \right) ,$$

$$\Gamma(\overline{D}^0(t) \to f) = e^{-\tau} |\bar{A}_{\bar{f}}|^2 \left(R_f^- + \sqrt{R_f^-} c_f^- \tau + c_f'^- \tau^2 \right) ,$$

and R_f^{\pm} are the DCS to CF ratios $R_f^+ = |A_{\bar{f}}/A_f|^2$, $R_f^- = |\bar{A}_f/\bar{A}_{\bar{f}}|^2$

Inear time dependence yields the CPVINT asymmetry (assuming no NP weak phases in CF/DCS)

$$\Delta c_f = x_{12} \sin \phi_{\text{cfds}}^M \cos \Delta_f - y_{12} \sin \phi_{\text{cfds}}^\Gamma \sin \Delta_f$$

• the $\cos \Delta_f$ and $\sin \Delta_f$ dependence originates from the strong phase differences $\Delta_f - \delta^M$ (dispersive), and Δ_f (absorptive)

U-spin decomposition and Approximate Universality

using CKM unitarity ($\lambda_i = V_{ci} V_{ui}^*$)

$$\Gamma_{12} = -\sum_{i,j=d,s} \lambda_i \lambda_j \Gamma_{ij} = \frac{(\lambda_s - \lambda_d)^2}{4} \Gamma_5 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2} \Gamma_3 + \frac{\lambda_b^2}{4} \Gamma_1$$

$$M_{12} = -\sum_{i,j=d,s} \lambda_i \lambda_j \Gamma_{ij} = \frac{(\lambda_s - \lambda_d)^2}{4} M_5 + \frac{(\lambda_s - \lambda_d) \lambda_b}{2} M_3 + \frac{\lambda_b^2}{4} M_1$$

• the Γ_i , M_i are $\Delta U_3 = 0$ elements of U-spin multiplets. They enter at different orders in ϵ , which characterizes $SU(3)_F$ breaking. Nominally, $\epsilon = O(0.2)$.

 $\Gamma_5 = \Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd} \sim (\bar{s}s - \bar{d}d)^2 \Rightarrow \Delta U = 2 \ (5 \text{ plet}) \Rightarrow O(\epsilon^2), \ CF/DCS/SCS$

 $\Gamma_3 = \Gamma_{ss} - \Gamma_{dd} \sim (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) \Rightarrow \Delta U = 1 \ (3 \text{ plet}) \Rightarrow O(\epsilon), \text{ SCS}$

- **9** Γ_5 , M_5 dominate and yield ΔM , $\Delta \Gamma$, or y_{12} , x_{12}
- $\delta\Gamma_{12} \propto \Gamma_3$, $\delta M_{12} \propto M_3 \Rightarrow \text{CPV via } \gamma = \arg(\lambda_b)$
- neglect $O(\lambda_b^2)$ effects of Γ_1 , M_1

define a pair of theoretical CPV phases ϕ_5^M , ϕ_5^{Γ} , with respect to the dominant $(\Delta U = 2)$ direction in the complex mixing plane $\propto (\lambda_s - \lambda_d)^2$,

$$\phi_5^{\Gamma} \equiv \arg\left(\frac{\Gamma_{12}}{\Gamma_{12}^{\Delta U=2}}\right) \approx \operatorname{Im}\left(\frac{2\lambda_b}{\lambda_s - \lambda_d}\frac{\Gamma_3}{\Gamma_5}\right) \sim \left|\frac{\lambda_b}{\theta_c}\right| \sin\gamma \times \frac{1}{\epsilon}$$

and similarly for ϕ_5^M

for "nominal" U-spin breaking,

$$\epsilon \sim 0.2 \quad \Rightarrow \quad \phi_{12}^{\Gamma} \sim \phi_{12}^{M} \sim 3 \times 10^{-3}$$



$$\phi_5 \equiv \arg\left(\frac{q}{p}\frac{1}{\Gamma_{12}^{\Delta U=2}}\right)$$

- How large is the final state dependence in ϕ_f^M , ϕ_f^{Γ} , and ϕ_{λ_f} compared with our theoretical phases?
 - Define misalignment between the general phases and the "theoretical" phases

$$\delta\phi_f \equiv \phi_f^{\Gamma} - \phi_5^{\Gamma} = \phi_f^M - \phi_5^M = \phi - \phi_{\lambda_f}$$

CF/DCS decays with no NP weak phases: misalignment is known and negligible, i.e. $\delta \phi_f = O(\lambda_b^2/\theta_c^2)$

$$\Rightarrow \phi_f^{\Gamma} = \phi_5^{\Gamma}, \quad \phi_f^M = \phi_5^M, \quad \phi_{\lambda_f} = \phi_5, \quad f \in \mathrm{cf/dcs}$$

• $\delta \phi_f$ is related to direct CPV: $\delta \phi_f = A_{CP}^{\text{dir}}(D \to f) \cot \delta$, where δ is the strong phase difference in A_{CP}^{dir}

● $D^0 \to K^+K^-, \pi^+\pi^-$: $A_{CP}^{dir} \lesssim O(10^{-3}) \Rightarrow \delta \phi_f \lesssim O(10^{-3})$

⇒ small misalignment compared to expected Bellell/LHCb sensitivity

U-spin argument: in the SM, $\phi_5^{\Gamma} = O(1/\epsilon)$, due to $O(\epsilon^2)$ cancelation in $\Gamma_5 \approx \Gamma_{12}$, but for SCS decays, $\delta \phi_f = O(1)$ in U-spin breaking,

$$\Rightarrow \quad \frac{\delta \phi_f}{\phi_5^{\Gamma}} = O(\epsilon) \text{ in SCS } D^0 \text{ decays}$$

yielding parametric suppression of misalignment relative to ϕ_5^{Γ}

- We conclude that in the SM, relative to the theoretical phases ϕ_5^M , ϕ_5^{Γ} , and ϕ_5 , the final state dependence in ϕ_f^M , ϕ_f^{Γ} , and ϕ_{λ_f} , respectively, is
 - subleading in $SU(3)_F$ and negligible compared to the expected LHC-b/Belle-2 sensitivity, in SCS decays
 - entirely negligible in CF/DCS decays
- Thus, a single pair of dispersive and absorptive phases suffices to parametrize all indirect CPV effects, which we can identify with our theoretical phases ϕ_5^{Γ} , ϕ_5^M . (The more familiar CPVINT phases ϕ_{λ_f} can be replaced with the single phase ϕ_5 , combined with 1 |q/p|)

-we refer to this fortunate circumstance as approximate universality

Approximate universality generalizes beyond the SM under the following conservative assumptions about subleading decay amplitudes containing new weak phases:

- they can be neglected in CF/DCS decays
- In SCS decays their magnitudes are similar to, or smaller than the SM QCD penguin amplitudes, as already hinted at by the experimental bounds on $A_{CP}^{dir}(K + K?, +?)$
- These assumptions can ultimately be tested by future direct CPV measurements

How large is the current window for NP?

we have estimated that $\phi_5^M \sim \phi_5^\Gamma \sim 3 \times 10^{-3}$ for nominal U-spin breaking, $\epsilon \sim 0.2$.

$$\Rightarrow \phi_{12} = \phi_5^M - \phi_5^\Gamma \sim 3 \times 10^{-3}$$

- a more sophisticated U-spin breaking analysis of ϕ_5^{Γ} , which can be improved with more data on D^0 decays, yields a similar result, $\phi_5^{\Gamma} \lesssim 0.005$
- the tightest upper bounds on ϕ_{12} are obtained in the "superweak limit" (Ciuchini et al '07; Grossman, Perez, Nir '09; A.K., Sokoloff '09):
 - neglect subleading decay weak phases in indirect CPV

 $\Rightarrow \phi_{12} \neq 0$ would be purely dispersive, entirely due to short-distance NP in M_{12} (short distance NP is negligible)

$$\phi_{12} = \phi_5^M, \quad \phi_5^\Gamma = 0, \quad (\phi_{\lambda_f} \to \phi_5)$$

⇒ only one CPV phase ϕ_{12} controls all indirect CPV. Therefore superweak fits to CPV data are highly constrained (1 - |q/p|) and ϕ_5 are related)

FIT superweak fits to ϕ_{12} :

HFAG : $\phi_{12} = 0.00 \pm 0.03 (1\sigma)$, [-0.07, +0.08] (95%c.l.) UTfit : $\phi_{12} = 0.01 \pm 0.05 (1\sigma)$, [-0.10, +0.15] (95%c.l.)

- \square comparing with U-spin based estimate of ϕ_{12} , current CPV measurements
 - $\Rightarrow O(10)$ window for NP

Final HFAG superweak fit for ϕ and |q/p| at 1σ ,

$$\phi = 0.00 \pm 0.01 \text{ [rad]}, \ |q/p| = 1.002 \pm 0.014.$$

in superweak limit

$$\tan 2\phi_5 = -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_5^M$$
$$\tan \phi_5 \approx \left(1 - \left|\frac{q}{p}\right|\right) \frac{x}{y}$$

Prospects for measuring SM indirect CPV

- lacksquare fit mixing data to ϕ^{Γ}_5 and ϕ^{M}_5
 - in practice, equivalent to "traditional" two parameter fit: for ϕ_5 , |q/p|
 - Iess constrained than superweak: current HFAG errors increase by O(10) compared to superweak fit.
 - LHCb/Belle-II improved sensitivity will help overcome this
- useful to consider the approximate universality relation

$$\tan 2(\phi + \phi_5^{\Gamma}) \approx -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_{12}$$

• our rough U-spin based estimate $\phi_5^{\Gamma} \sim 0.003$ and the current ≈ 0.01 HFAG 1σ error on ϕ are not far apart. Going forward, the LHS \Rightarrow already must move beyond superweak to two - parameter fits

apologies for not checking updated projections

■ projected Belle-II errors at 50 ab⁻¹ on x (%), y (%), |q/p| (%), and ϕ (mrad) from $D^0 \to K_s \pi^+ \pi^-$ alone, and on ΔY_f (%) from $D^0 \to K^+ K^-, \pi^+ \pi^-$ (Belle2-NOTE-PH-2015-002):

 $0.11(x), 0.05(y), 7.2(|q/p|), 72(\phi), 0.04(\Delta Y_f)$

If old projections for LHCb upgrade, based on 50 fb⁻¹ for q/p| from $a_{sl}(D^0 \to K\mu\nu)$; x, y, ϕ from $D^0 \to K_s \pi^+ \pi^-$, and ΔY_f from $D^0 \to K^+ K^-, \pi^+ \pi^-$ (1208.3355)

 $0.015(x), \ 0.010(y), \ 1.0(|q/p|), \ 52(\phi), \ 0.004(\Delta Y_f).$

illustration of the potential reach in ϕ_{12}^M and ϕ_{12}^{Γ} : the above LHCb errors, using the error correlation matrix of the present Belle-II measurements, and the central values x = 0.35%, y = 0.58%, yields the 95% CL errors

$$\delta \phi_{12}^M = \pm 34 \; [\text{mrad}], \; \delta \phi_{12}^\Gamma = \pm 17 \; [\text{mrad}].$$

- Naively halving the errors, for HE-LHCb with 300 fb⁻¹ approaches the SM level, but perhaps factor of 2 too large
- ideas for using a binned model independent Dalitz Plot analysis at LHCb, e.g. in $D^0 \rightarrow K_s \pi^+ \pi^-$ could further reduce the errors (1209.0172, C. Thomas, G. Wilkinson)
- an HFAG type fit to all possible measurements will also help
- However, an LHCb at the HE-LHC would be most welcome!