High Luminosity/High Energy LHC perspectives on Taus

Emilie Passemar
Indiana University/Jefferson Laboratory

HL/HE LHC meeting
Fermilab, April 5, 2018

Outline :

1. Introduction and Motivation:

2. Lepton Flavour Violation

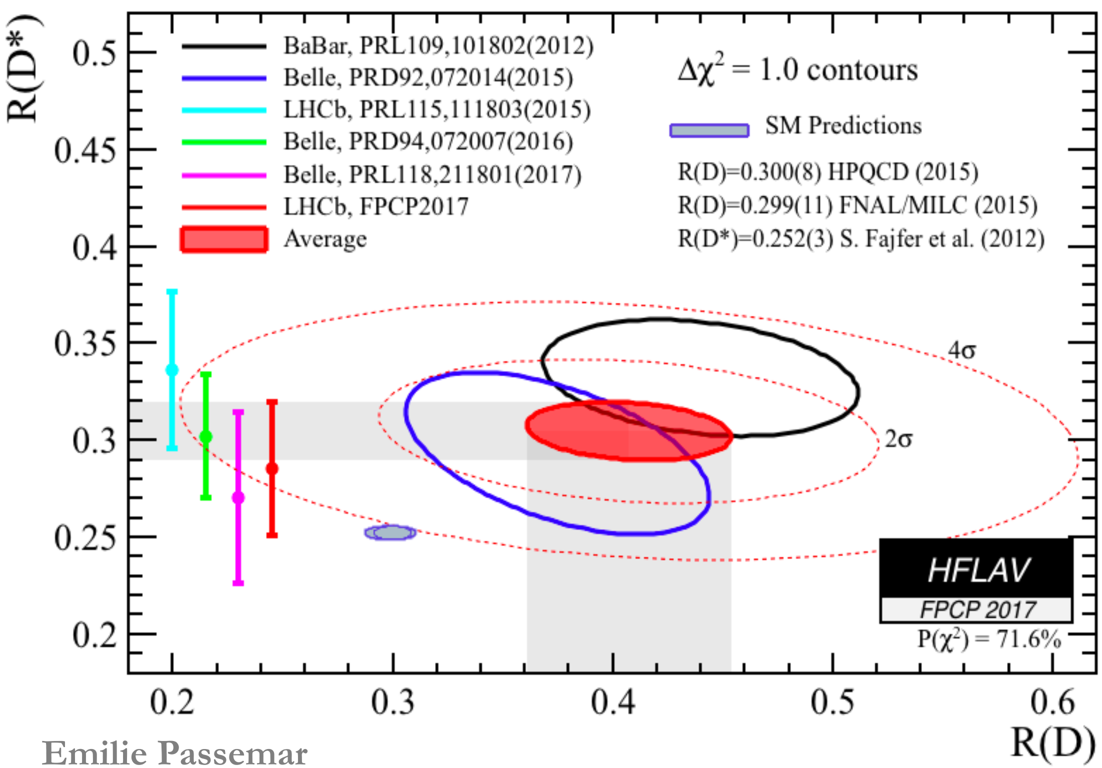
3. Other interesting topics with tau decays

4. Conclusion and outlook

1.1 Quest for New Physics

- New era in particle physics :
  ⇨ (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*

- *Where do we look?* Everywhere! ⇨ search for New Physics with *broad search strategy* given lack of clear indications on the SM-EFT boundaries (*both in energies and effective couplings*)



- Hint from B physics anomalies?

b → c charged currents:

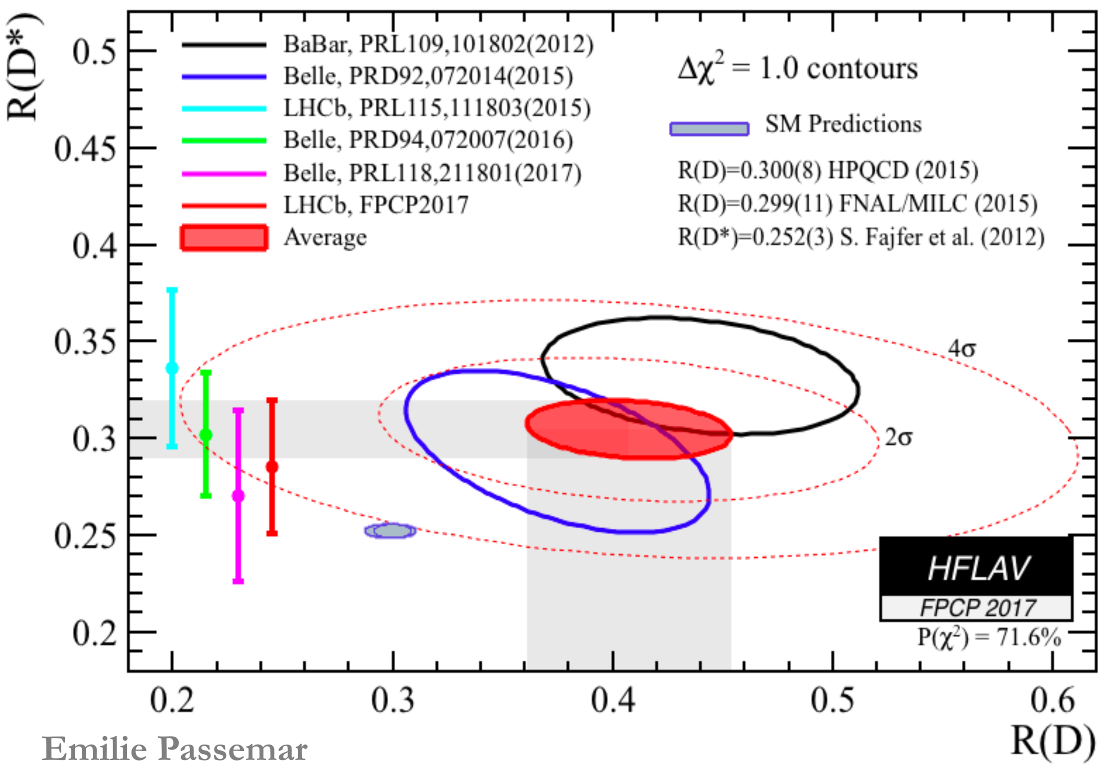
τ vs. light leptons (μ, e) [R(D), R(D*)]

$$\boxed{R(X) = \frac{\Gamma(B \to X\tau\bar{\nu})}{\Gamma(B \to X\ell\bar{\nu})}}$$

1.1 Quest for New Physics

- New era in particle physics :
  ⇨ (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*

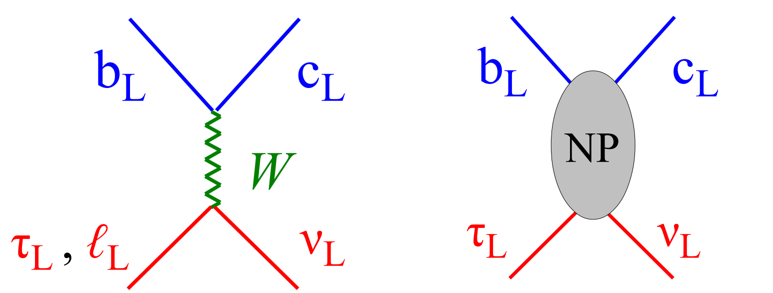
- *Where do we look?* Everywhere! ⇨ search for New Physics with *broad search strategy* given lack of clear indications on the SM-EFT boundaries (*both in energies and effective couplings*)



- Hint from B physics anomalies?

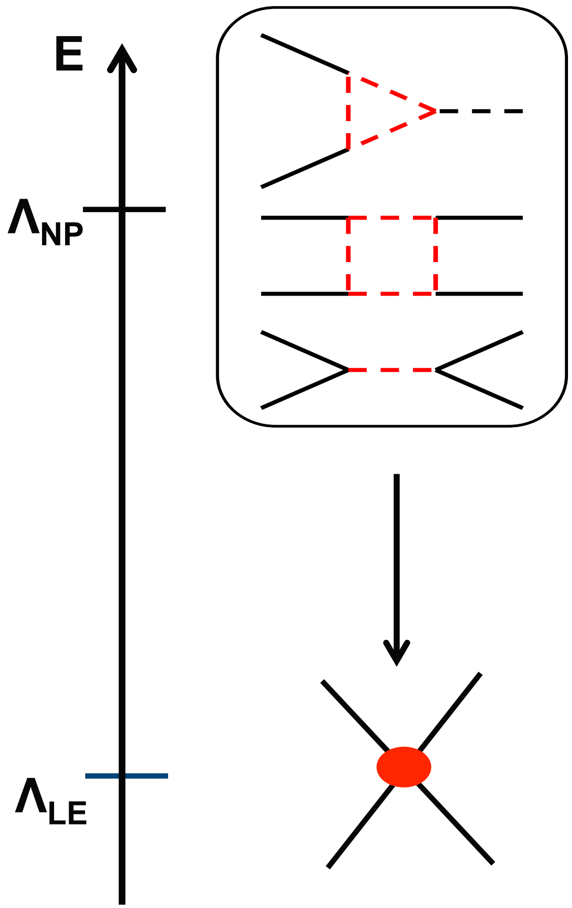
$b \to c$ charged currents:

τ vs. light leptons (μ, e) [R(D), R(D*)]



⇨ Key unique role of *Tau physics*

1.2 τ lepton as a unique probe of new physics



- In the quest of New Physics, can be sensitive to very high scale:
  - Kaon physics: $[\varepsilon_K]$ $\boxed{\frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5 \text{ TeV}}$
  - Tau Leptons: $[\tau \to \mu\gamma]$ $\boxed{\frac{\tau\bar{\mu} f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^2 \text{TeV}}$
- At low energy: lots of experiments e.g., *BaBar*, *Belle*, *BESIII, LHCb* ⇨ important improvements on measurements and bounds obtained and more expected (*Belle II*, *LHCb, ATLAS, CMS*)
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential, e.g. CPV in hadronic Tau decays

⇨ Tau leptons very important to look for *New Physics*!

1.2 τ lepton as a unique probe of new physics

- A lot of progress in tau physics since its discovery on all the items described before ⇨ important experimental efforts from *LEP*, *CLEO, B factories: Babar, Belle,* *BES, VEPP-2M, LHCb, neutrino experiments*,…

  ⇨ More to come from *LHCb, BES, VEPP-2M, Belle II, CMS, ATLAS, HL/HI LHC*

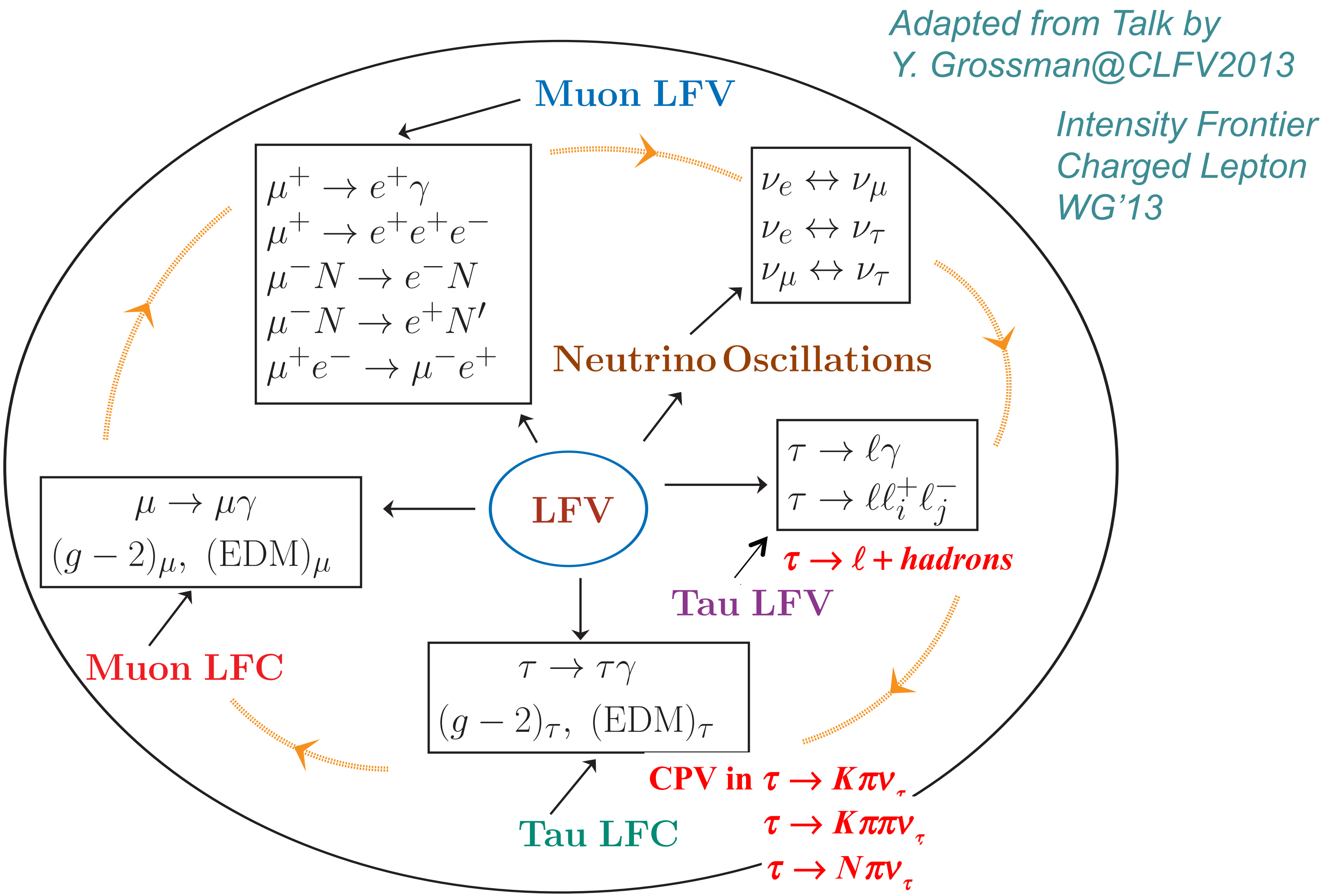
Experiment	Number of τ pairs
LEP	~3x10^5
CLEO	~1x10^7
BaBar	~5x10^8
Belle	~9x10^8
Belle II	~10^{12}

- But τ physics has still potential "*unexplored frontiers*"

  ⇨ deserve future exp. & th. efforts

- In the following, some selected examples

1.3 The Program



2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the SM (m_ν=0)

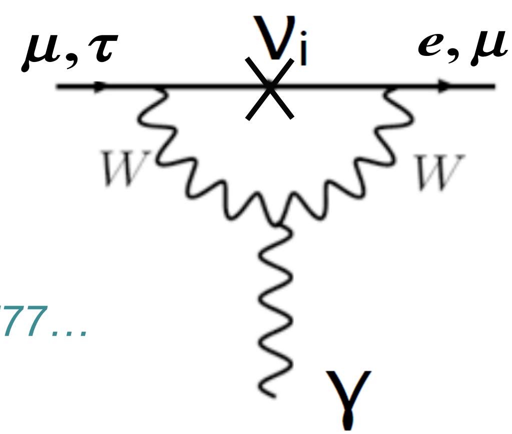
- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br\left(\mu \to e\gamma\right) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br\left(\tau \to \mu\gamma\right) < 10^{-40}\right]$$



- Extremely *clean probe of beyond SM physics*

- In New Physics models: seazible effects
  Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

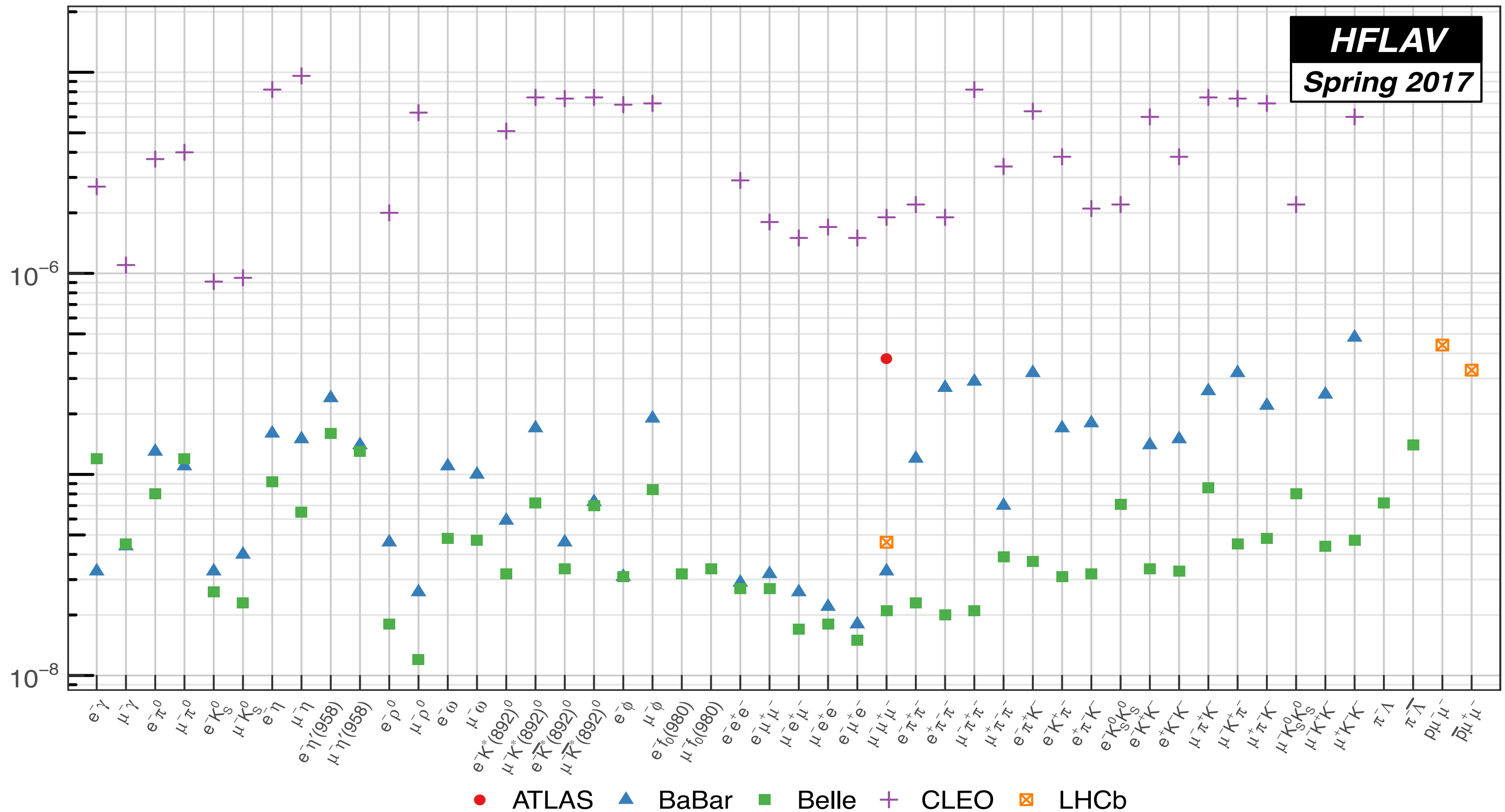
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 Tau LFV

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ $\nwarrow P,\ S,\ V,\ P\bar{P},\ldots$

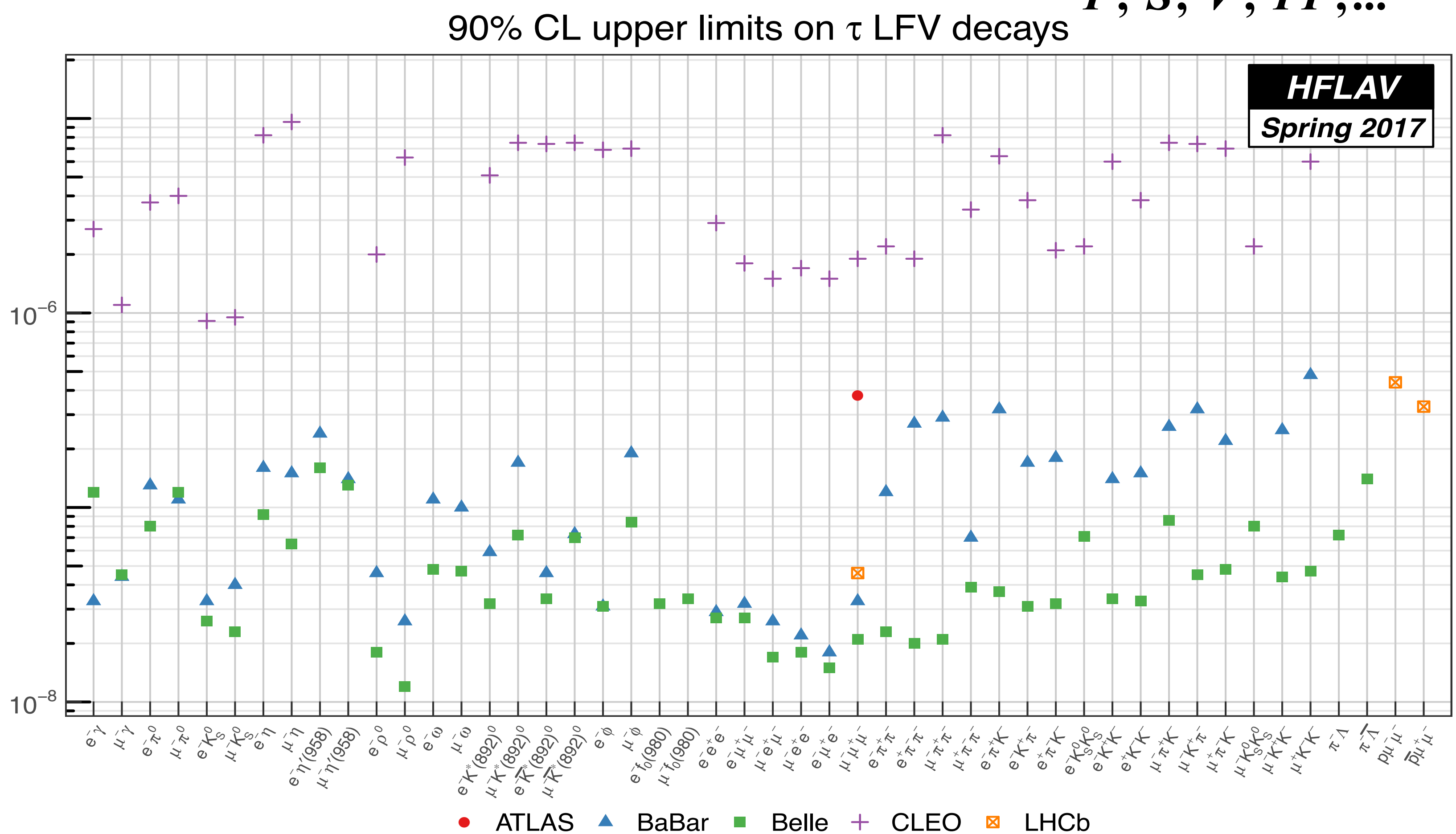
90% CL upper limits on τ LFV decays



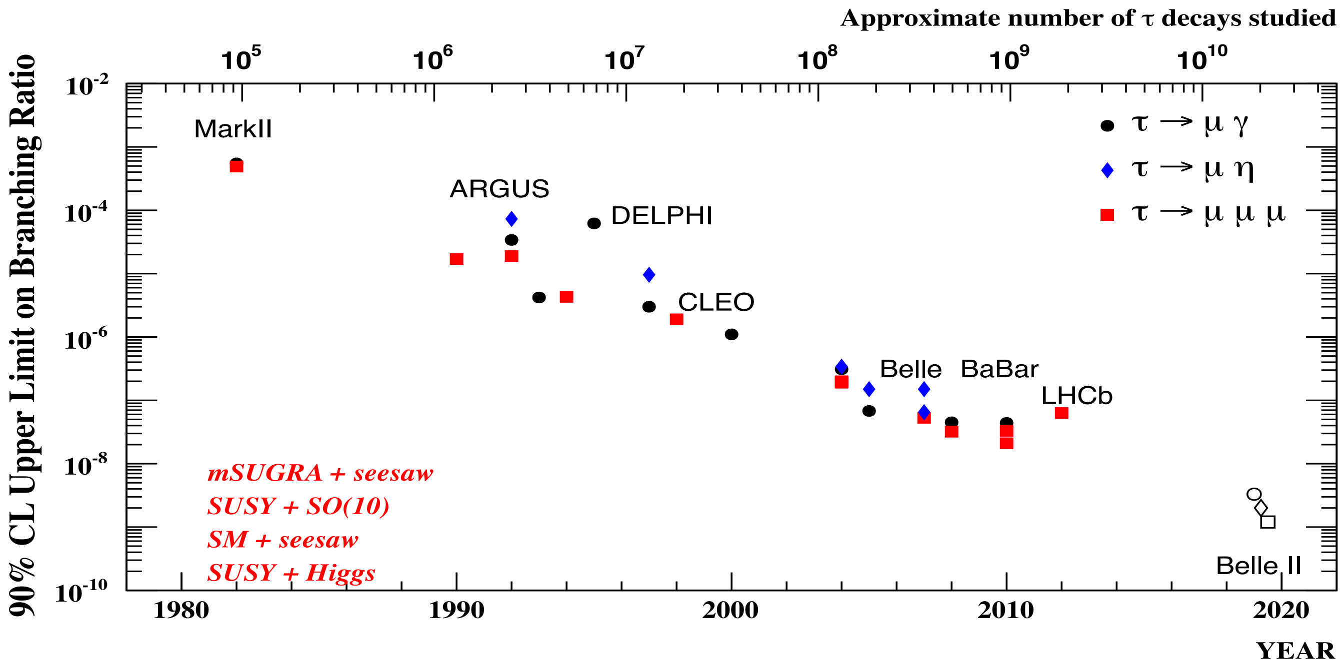
- 48 LFV modes studied at Belle and BaBar

2.2 Tau LFV

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ $P,\ S,\ V,\ P\bar{P},\ldots$

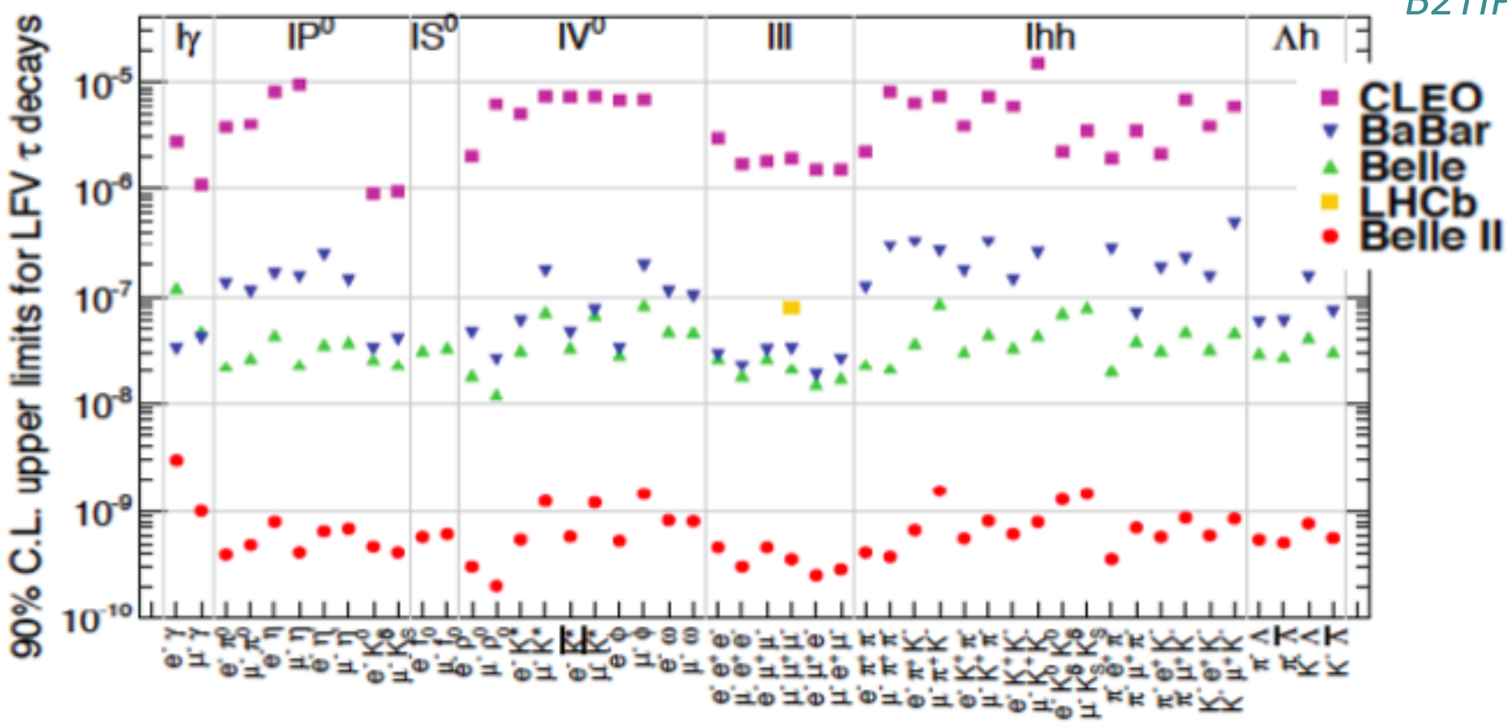


- Expected sensitivity 10^{-9} or better at *LHCb, ATLAS, CMS, Belle II, HL-LHC?*



S. Banerjee’17

B2TIP’18



2.3 Effective Field Theory approach

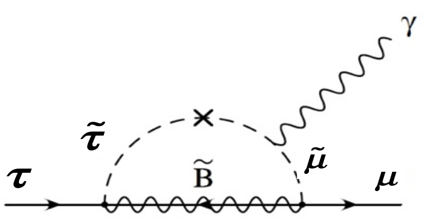
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
    - Dipole:

$$\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$$

e.g.



2.3 Effective Field Theory approach

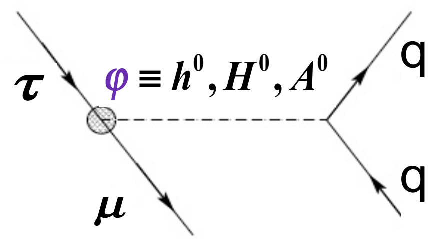
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

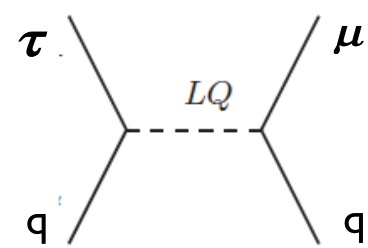
- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\boxed{\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\,\Gamma P_{L,R}\tau\; \bar{q}\Gamma q}$$

e.g.



$\boxed{\Gamma \equiv 1}$



$\boxed{\Gamma \equiv \gamma^\mu}$

.

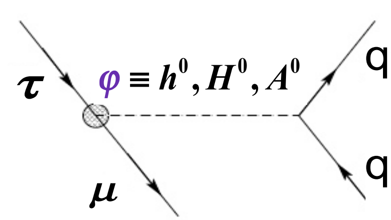
2.3 Effective Field Theory approach

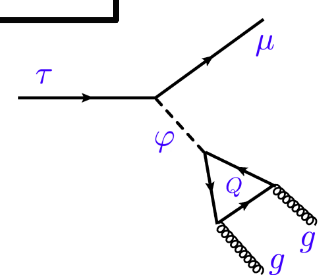
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\; \bar{q}\Gamma q}$
  - Integrating out heavy quarks generates *gluonic operator*

$$\boxed{\frac{1}{\Lambda^2} \bar{\mu} P_{L,R} \tau Q \bar{Q}} \rightarrow \boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau \; G_{\mu\nu}^{a} G_a^{\mu\nu}}$$





2.3 Effective Field Theory approach

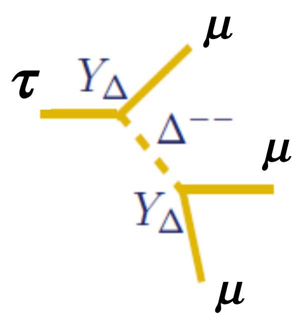
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \dots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

e.g.



2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R}\tau\ \bar{q}\Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\ G_{\mu\nu}^a G_a^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R}\tau\ \bar{\mu}\, \Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

2.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part:
  form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

2.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau\to 3\mu$	$\tau\to\mu\gamma$	$\tau\to\mu\pi^+\pi^-$	$\tau\to\mu K\bar{K}$	$\tau\to\mu\pi$	$\tau\to\mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- Form factors for $\tau\to\mu(e)\pi\pi$ determined using *dispersive techniques*
- Hadronic part:

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13
Celis, Cirigliano, E.P.'14

$$\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(Lorentz\ \text{struct.}\right)^i_\mu F_i(s)}$$

with

$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

- 2-channel unitarity condition is solved with I=0 S-wave ππ and KK scattering data as input

$$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$$n = \pi\pi, K\bar{K}$$

2.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.5 Handles

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state: $\boxed{\dfrac{dBR(\tau \to \mu\mu\mu)}{d\sqrt{s}}}$

- Benchmarks:
  - Dipole model: $C_D \neq 0$, $C_{else} = 0$
  - Scalar model: $C_S \neq 0$, $C_{else} = 0$
  - Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
  - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

2.6 Model discriminating of BRs

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

Benchmark

2.6 Model discriminating of BRs

- Studies in specific models

Buras et al.'10

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 0.1$	$0.06\ldots 2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.02\ldots 0.04$	$0.03\ldots 1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots 2$	~ 5	$0.3\ldots 0.5$	$1.5\ldots 2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots 1.6$	~ 0.2	$5\ldots 10$	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$0.08\ldots 0.15$	$10^{-12}\ldots 26$

⇨ Disentangle the *underlying dynamics* of NP

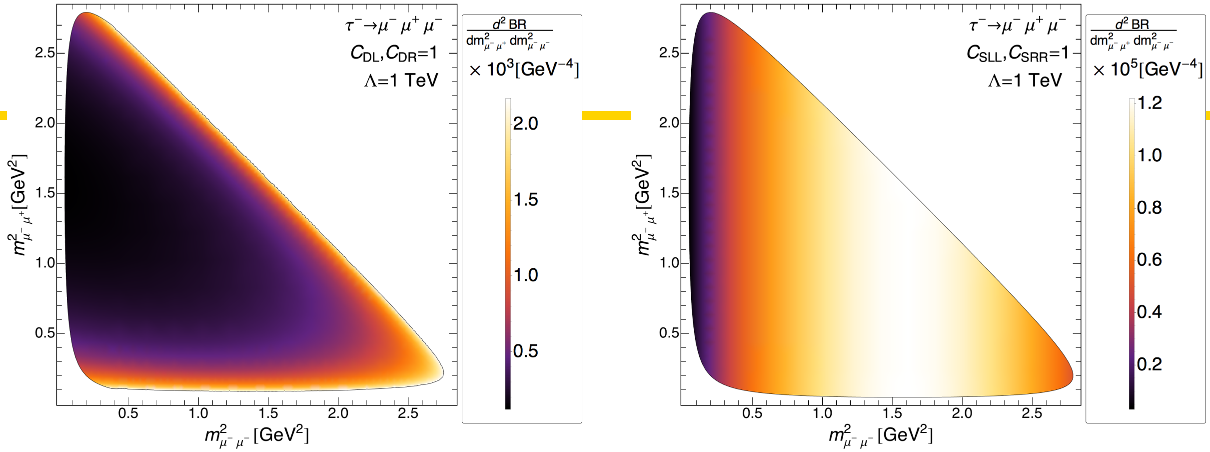


Figure 3: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of $C_{DL,DR} = 1$ (left) and $C_{SLL,SRR} = 1$ (right), taking $\Lambda = 1$ TeV in both cases. Colors denote the density for $d^2BR/(dm^2_{\mu^-\mu^+} dm^2_{\mu^-\mu^-})$, small values being represented by darker colors and large values in lighter ones. Here $m^2_{\mu^-\mu^+}$ represents m^2_{12} or m^2_{23}, defined in Sec. 3.1.*

Dassinger, Feldman, Mannel, Turczyk' 07
Celis, Cirigliano, E.P.'14

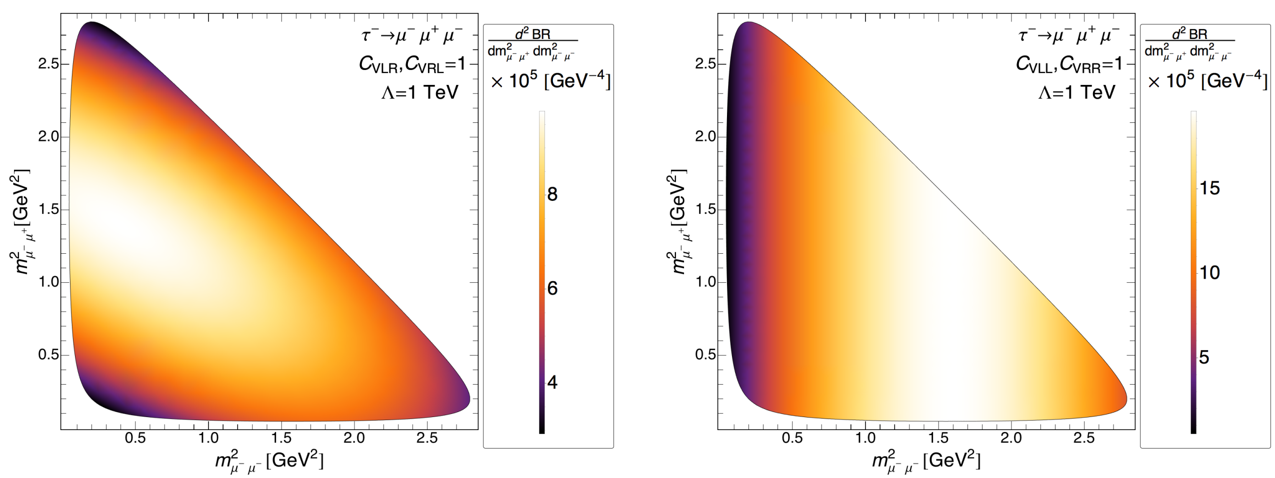


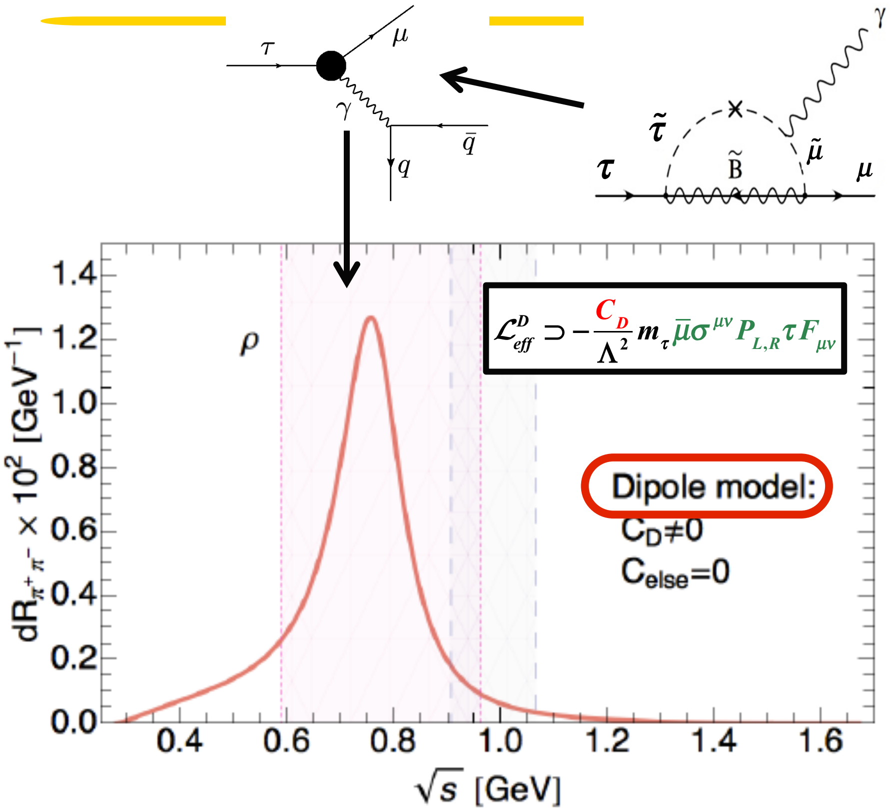
Figure 4: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of $C_{VRL,VLR} = 1$ (left) and $C_{VLL,VRR} = 1$ (right), taking $\Lambda = 1$ TeV in both cases. Colors are defined as in Fig. 3.*

Angular analysis with polarized taus

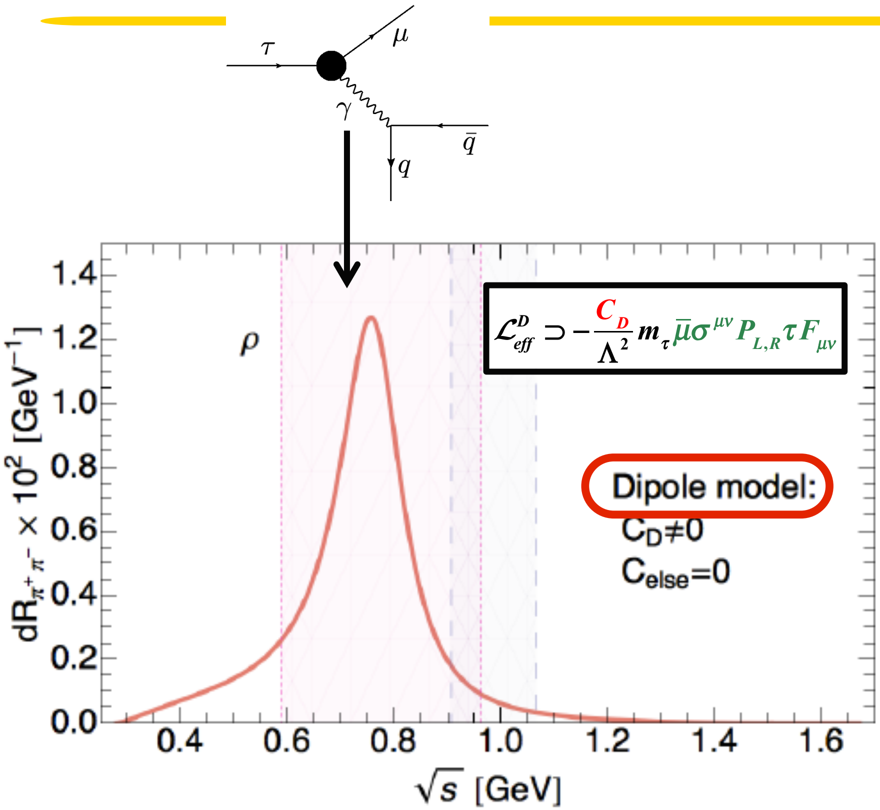
Dassinger, Feldman, Mannel, Turczyk' 07

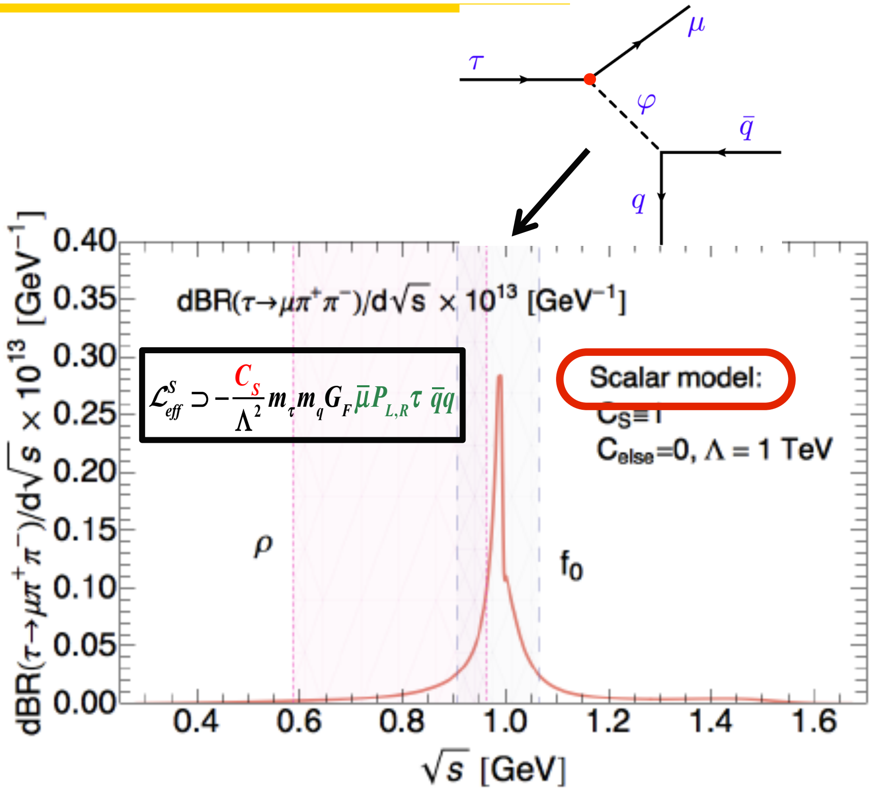
2.7 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays

Celis, Cirigliano, E.P.'14

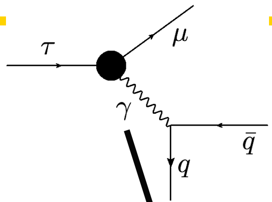


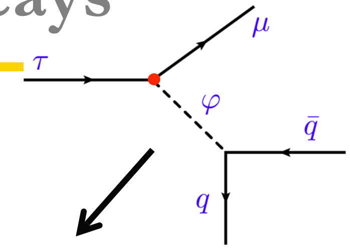
2.7 Discriminating power of τ→μ(e)ππ decays

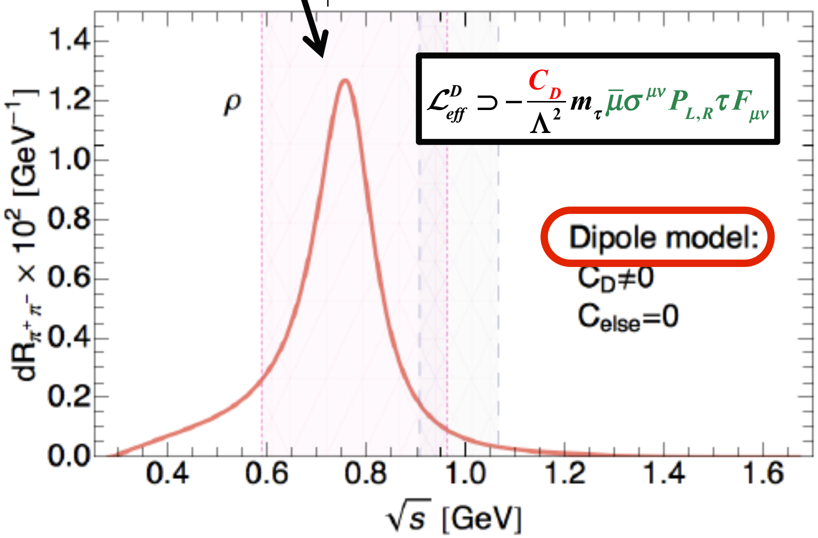


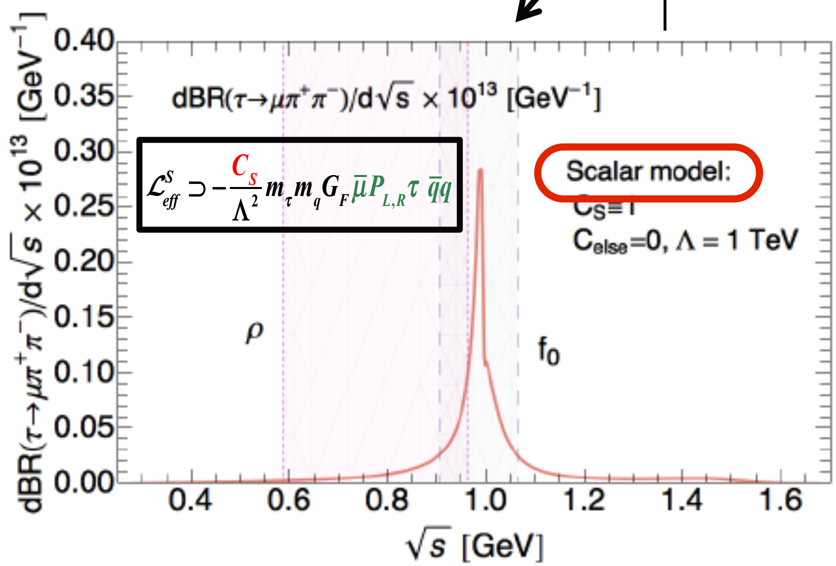


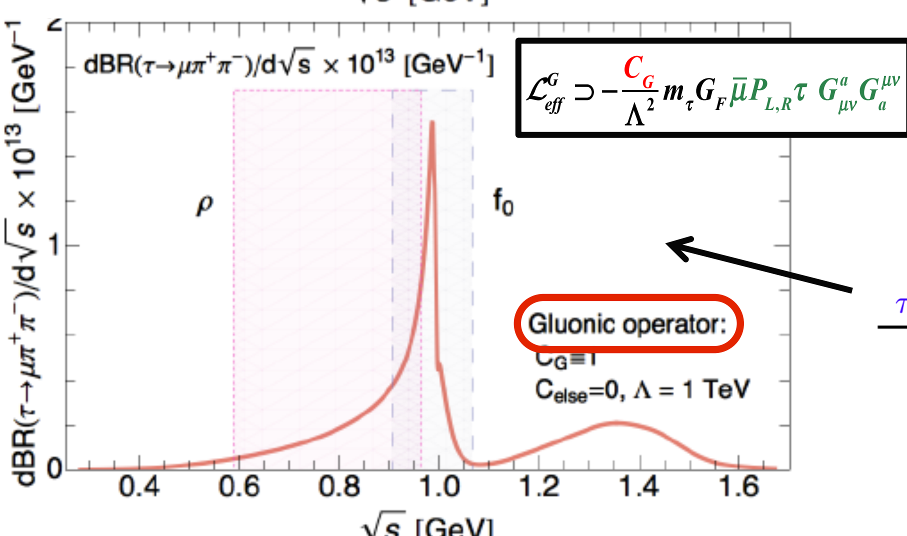
2.7 Discriminating power of τ→μ(e)ππ decays



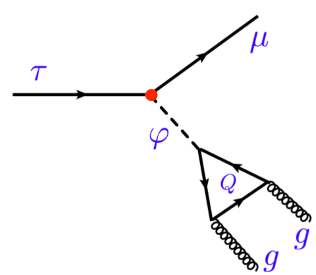








Different distributions according to the *operator!*



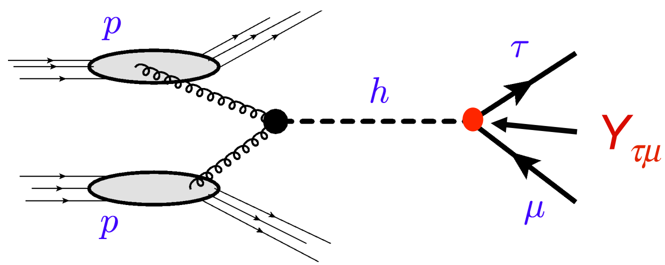
2.8 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

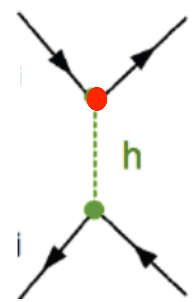
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

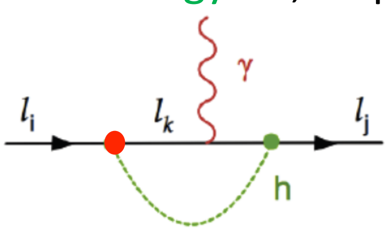
- High energy : LHC

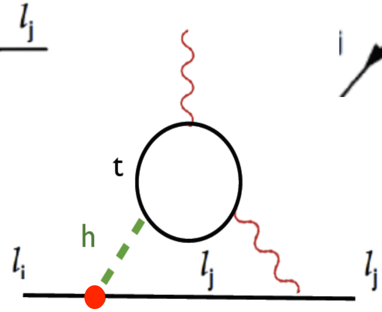


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







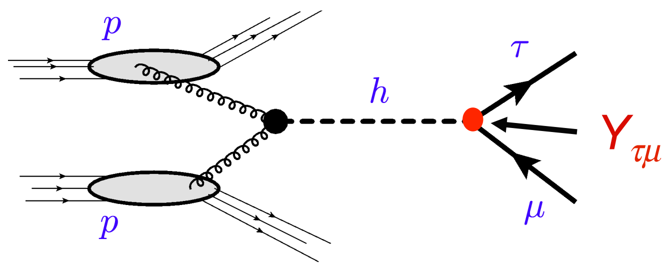
2.8 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H} \Rightarrow \boxed{-Y_{ij}\left(\bar{f}_L^i f_R^j\right)h}$$

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

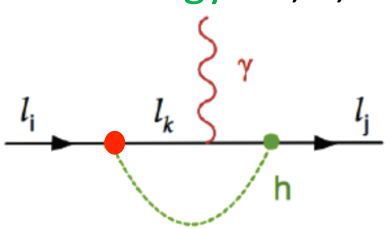
- High energy : LHC

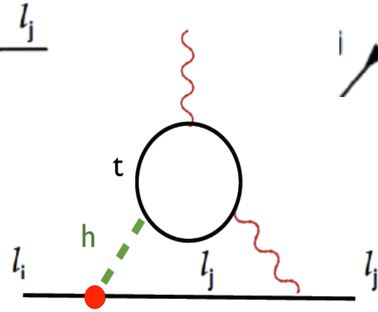


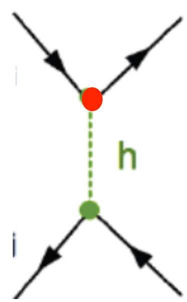
Hadronic part treated with perturbative QCD

Reverse the process

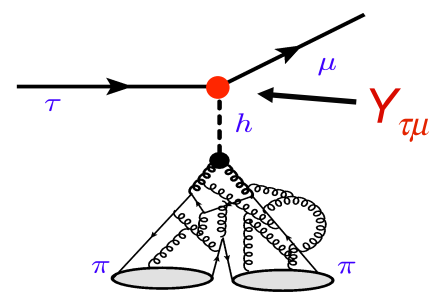
- Low energy : D, S, G operators







+



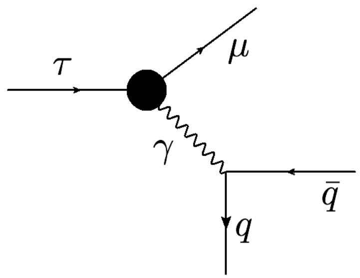
Hadronic part treated with non-perturbative QCD

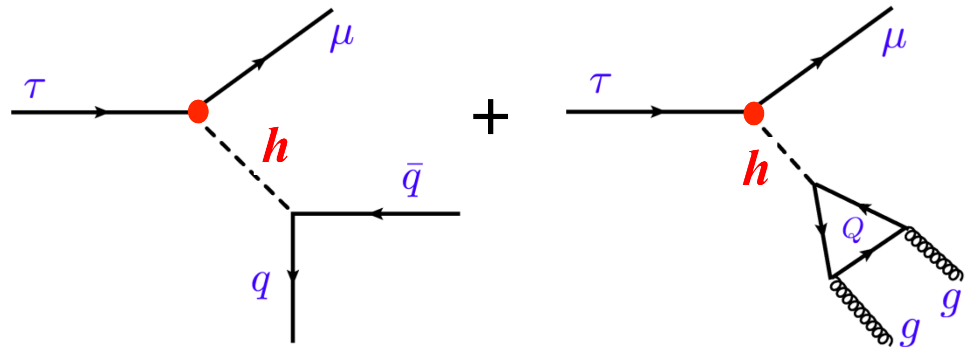
Constraints in the τμ sector

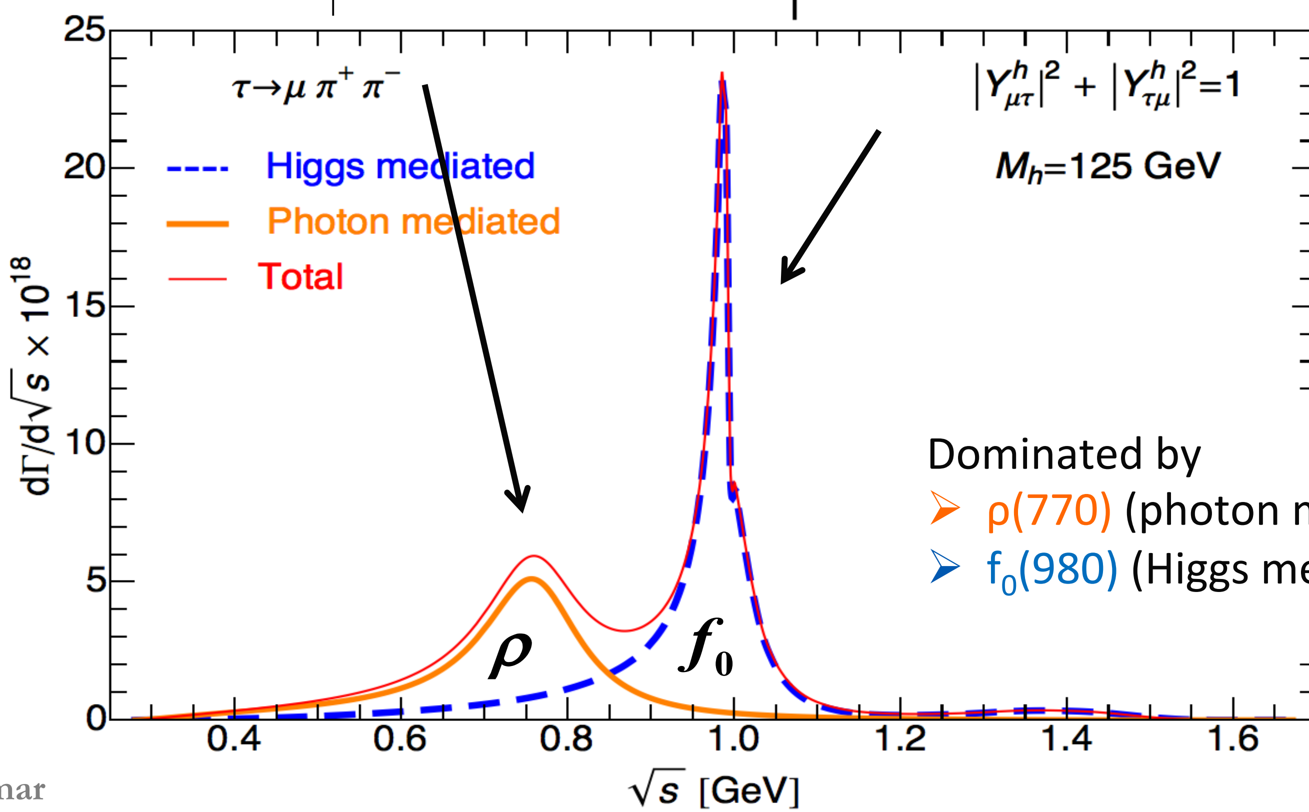
- At low energy

Cirigliano, Celis, E.P.'14

  - $\tau \to \mu\pi\pi$:

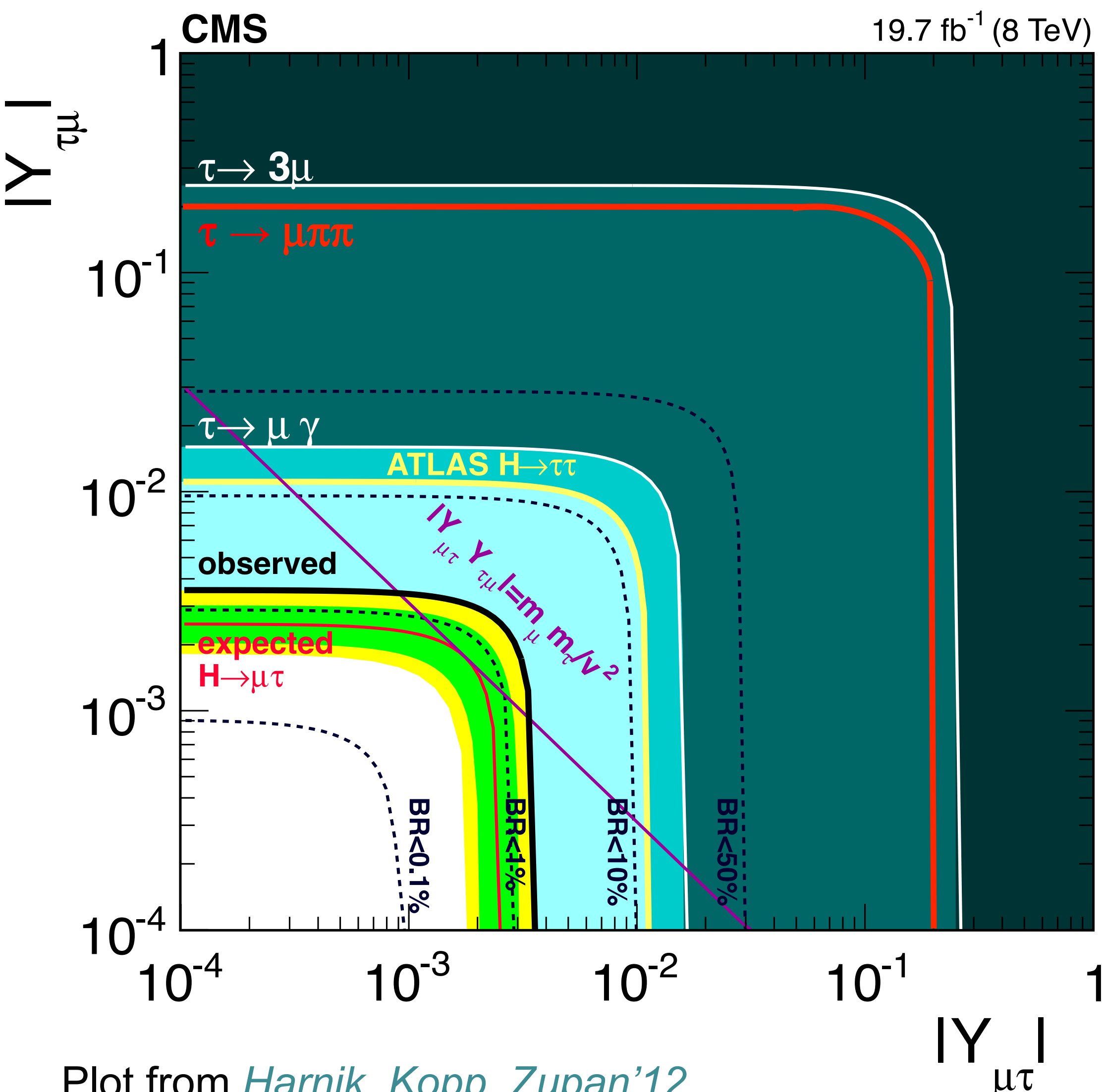






Dominated by
- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ➡ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ➡ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

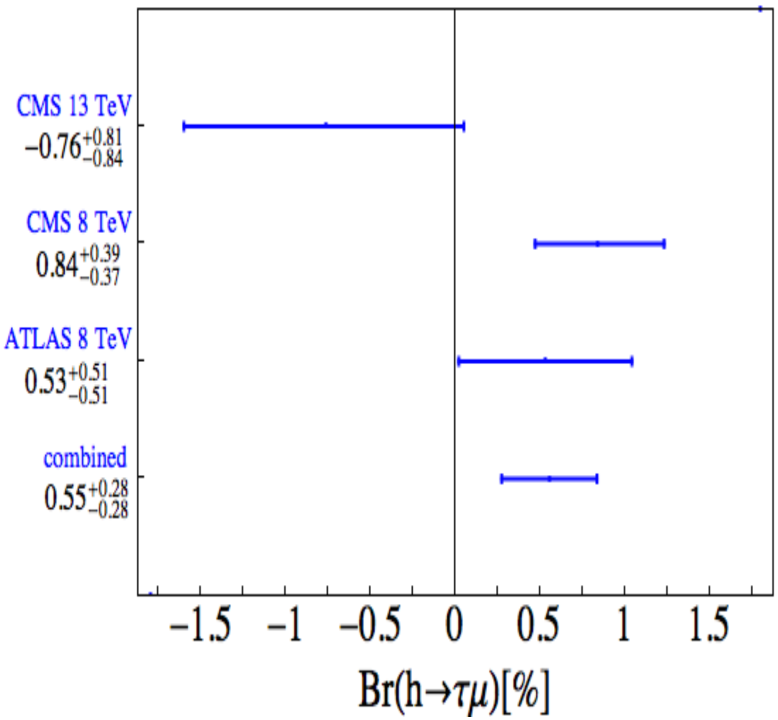
➡

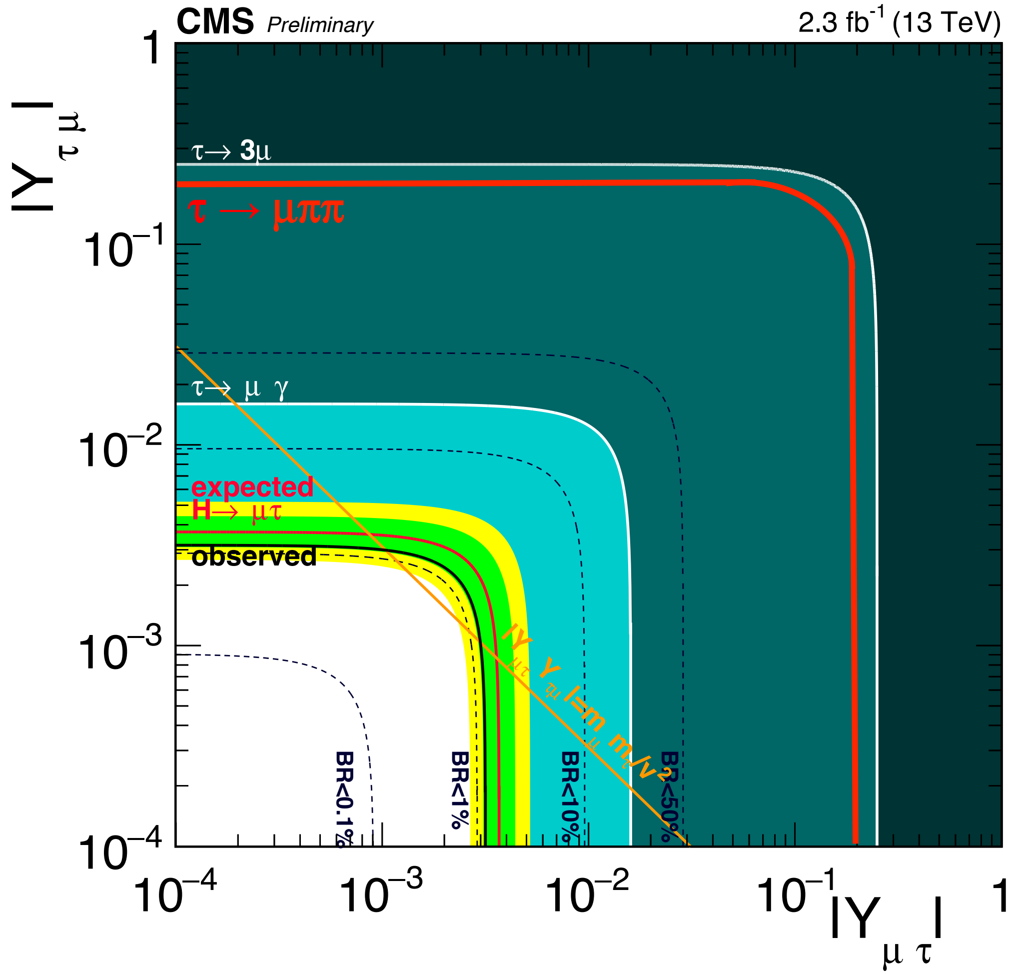
$$BR(\tau \to \mu\gamma) < 2.2\times10^{-9}$$

$$BR(\tau \to \mu\pi\pi) < 1.5\times10^{-11}$$

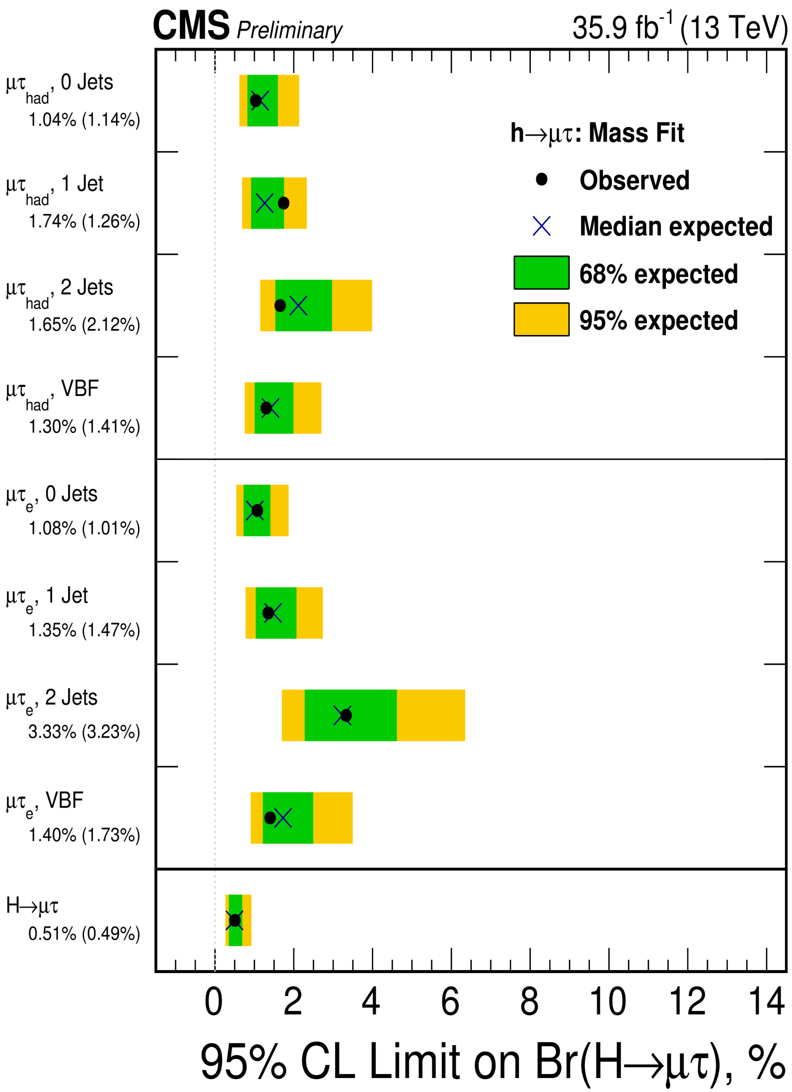
Hint of New Physics in h → τμ?

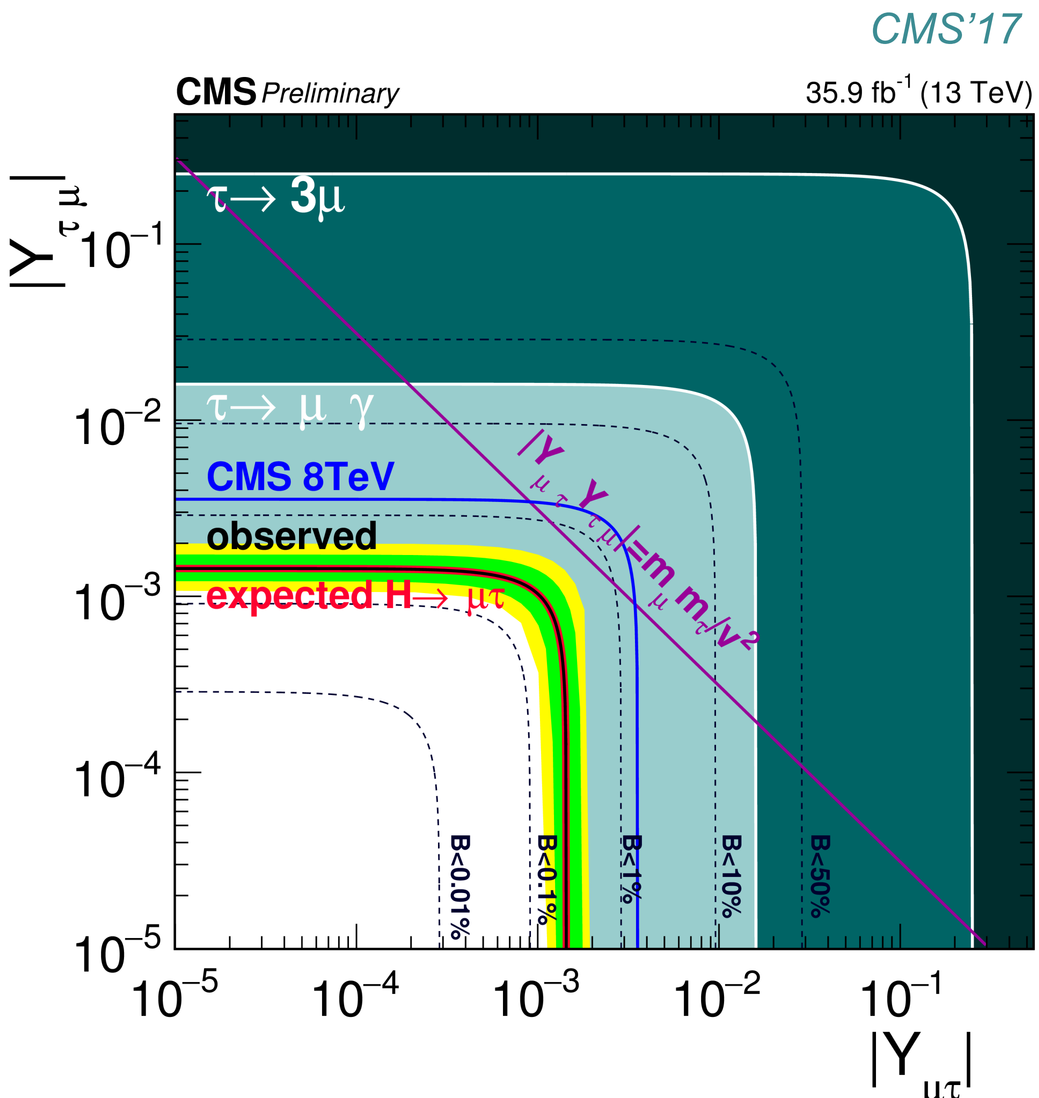
CMS'16





Hint of New Physics in h → τμ?

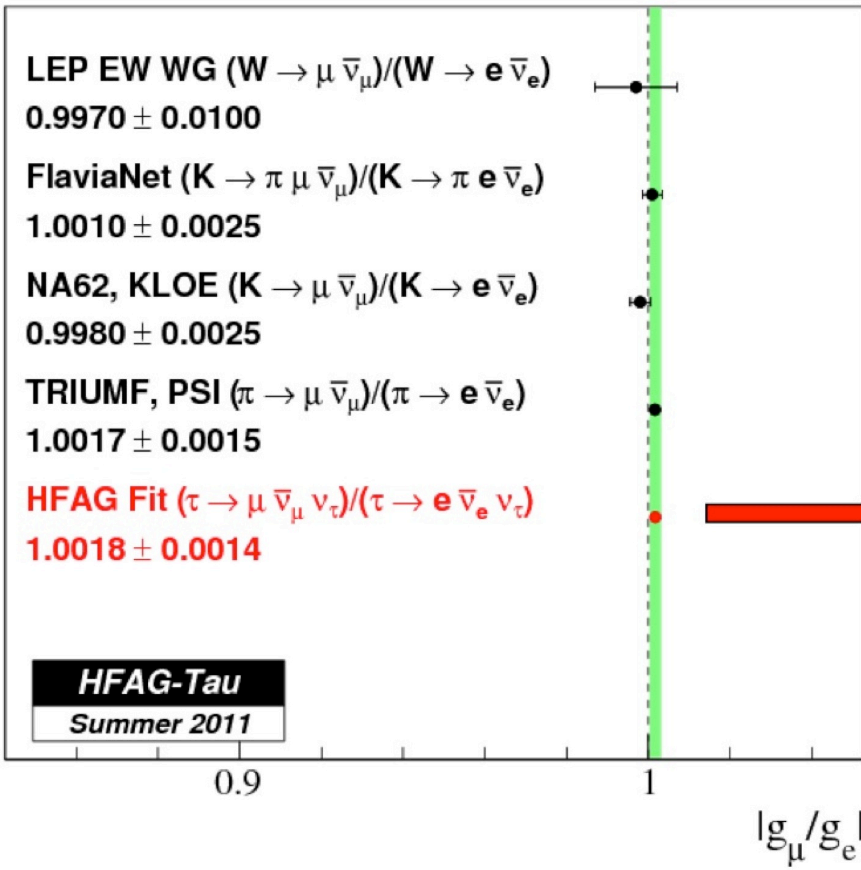


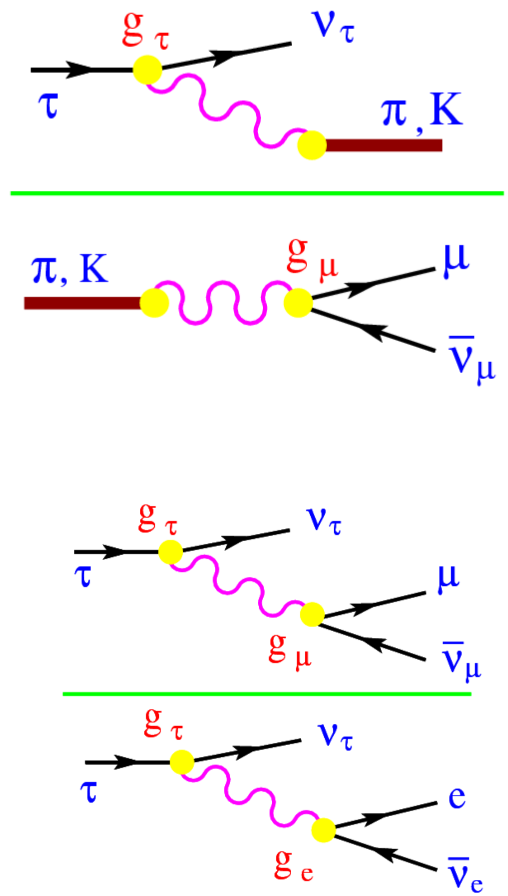


3. Other interesting topics with tau decays

3.1 Lepton Universality

- Test of μ/e universality:

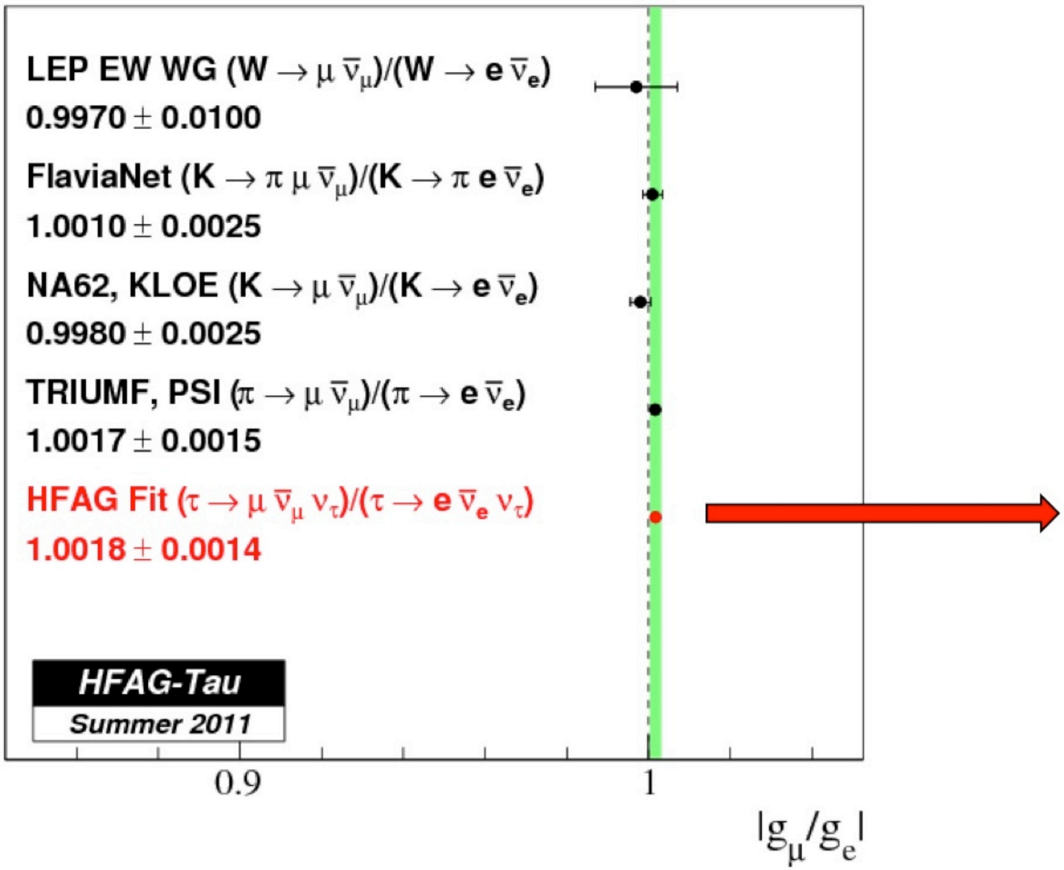


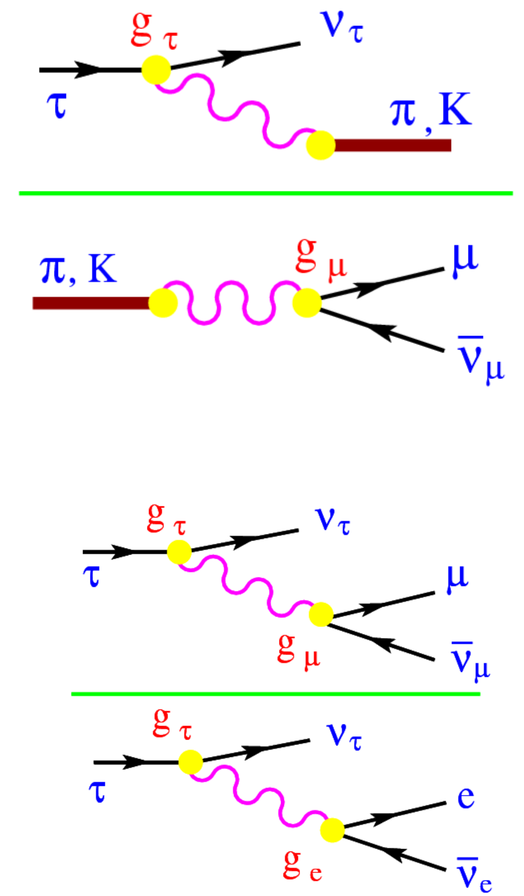


- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)

3.1 Lepton Universality

- Test of μ/e universality:

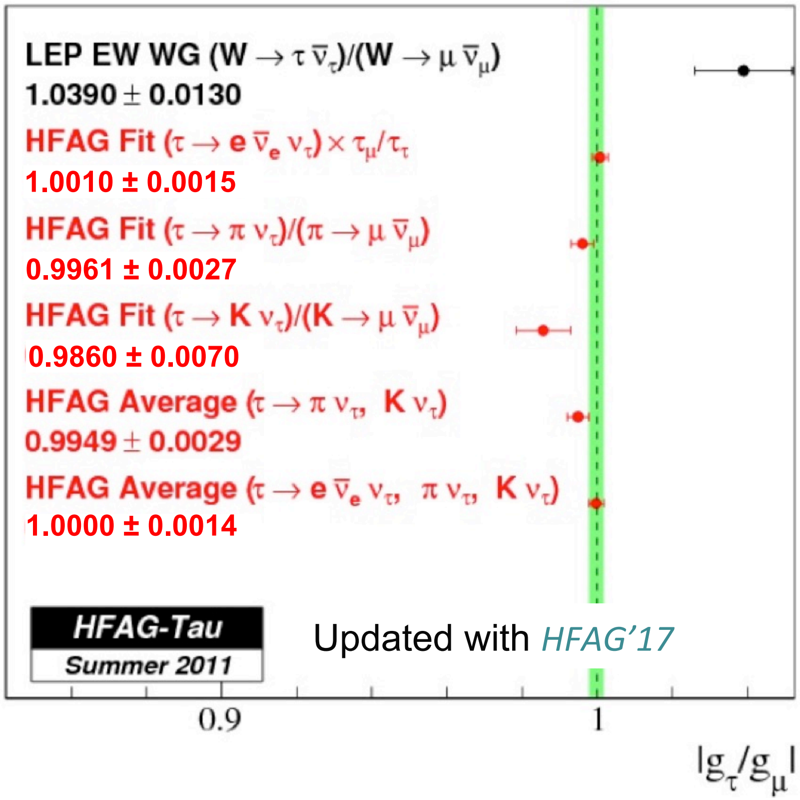


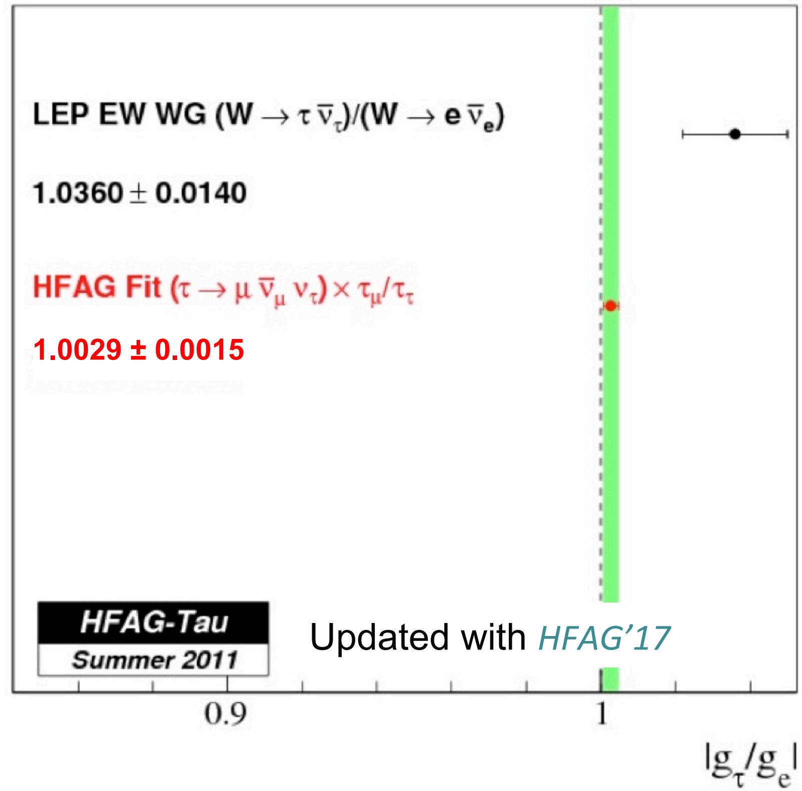


- Tested at *0.14%* from Tau leptonic Brs! (0.28% in Z decays)
- What about the *third family*?

3.1 Lepton Universality

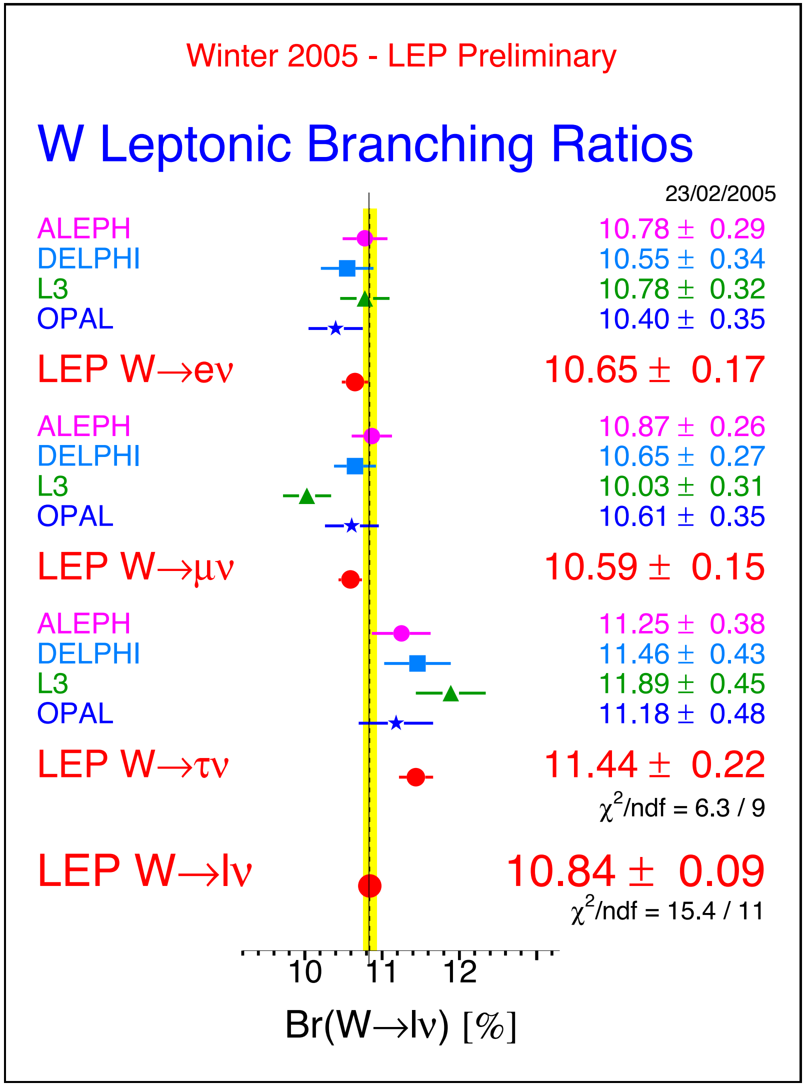
- What about the *third family*?





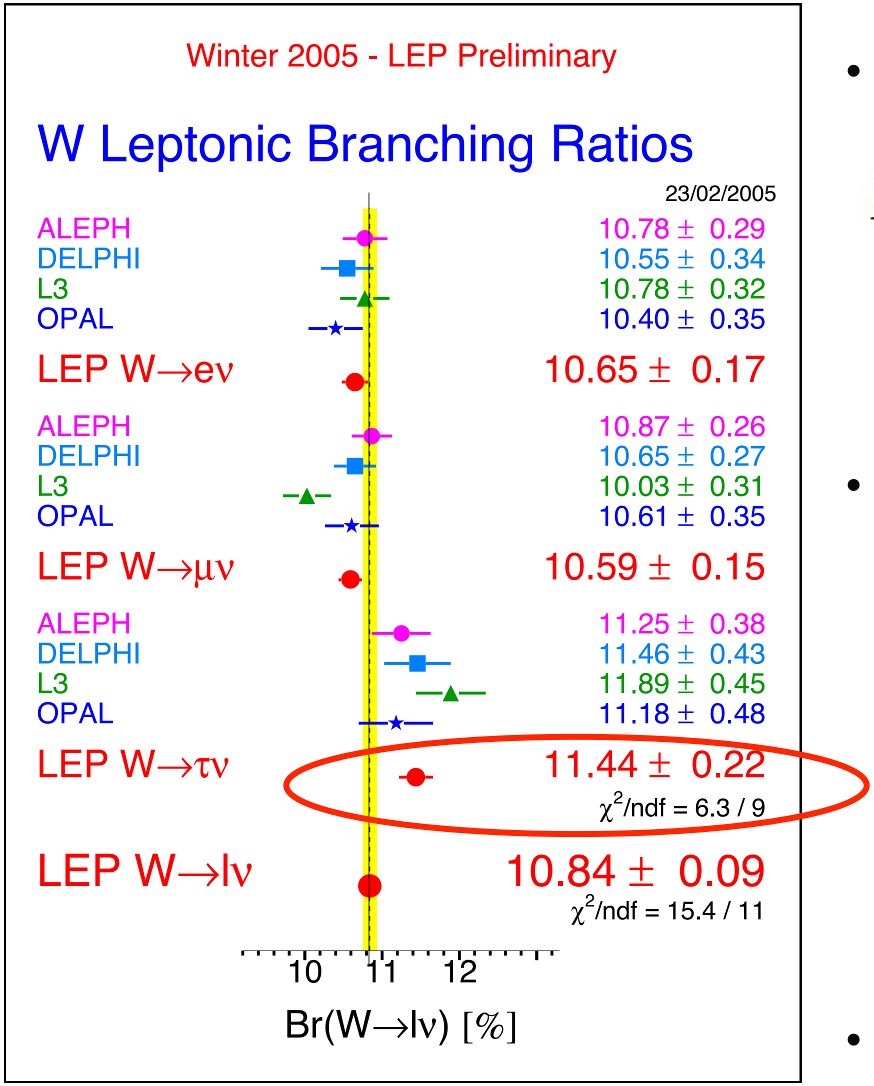
- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays ⇨ *See talks in this morning Flavour session*

3.1 Lepton Flavour Universality anomaly $W \to \tau\nu_\tau$



- Old LEP anomaly

3.1 Lepton Flavour Universality anomaly $W \to \tau\nu_\tau$



- Old LEP anomaly

$$R^W_{\tau\ell} = \frac{2\,\mathrm{BR}\,(W \to \tau\,\bar{\nu}_\tau)}{\mathrm{BR}\,(W \to e\,\bar{\nu}_e) + \mathrm{BR}\,(W \to \mu\,\bar{\nu}_\mu)} = 1.077(26)$$

2.8σ away from SM!

- New physics?

  Some models:

  Li & Ma'05, Park'06, Dermisek'08

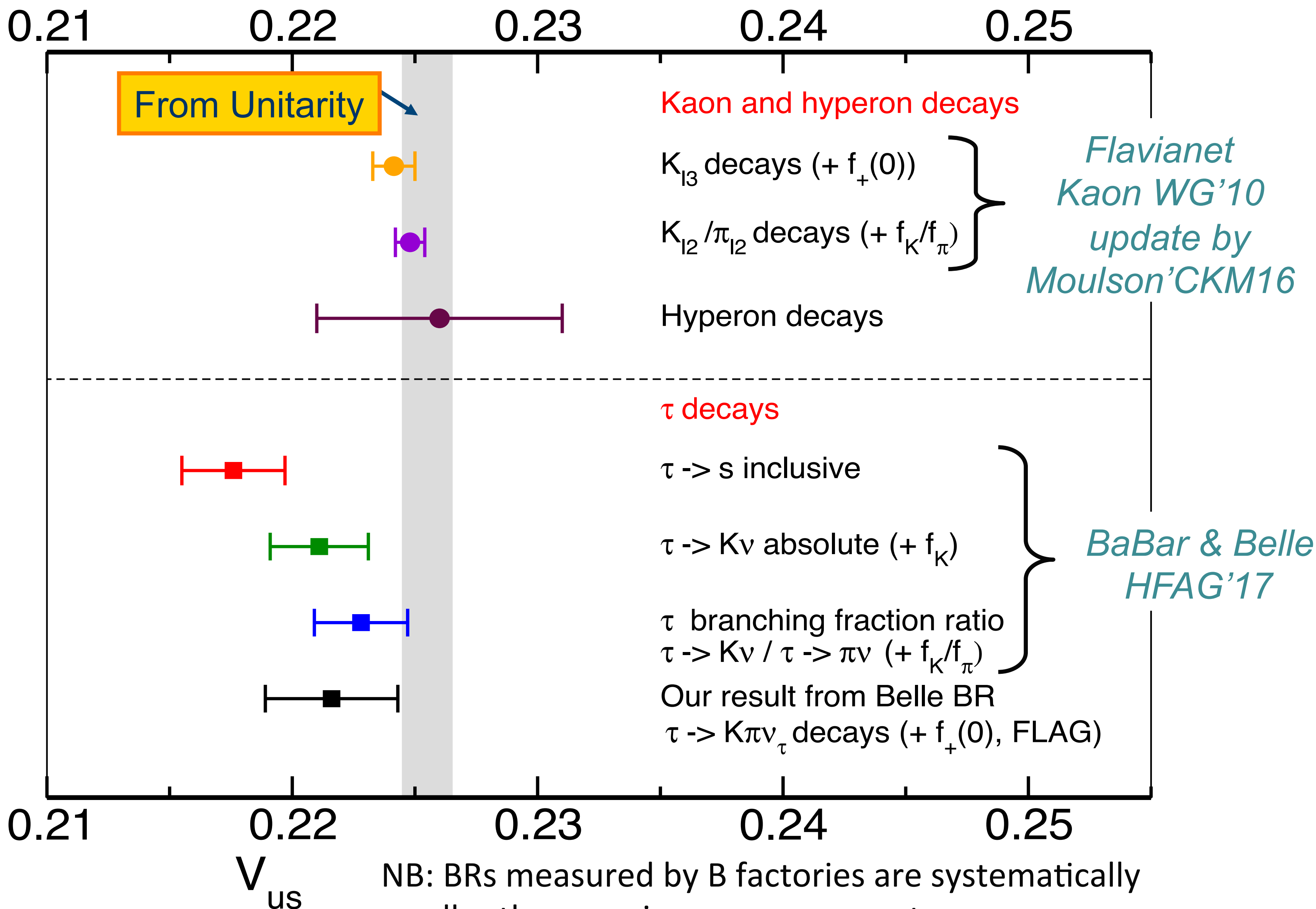
  Try to explain with SM EFT approach with $[U(2)xU(1)]^5$ flavour symmetry

  ⇨ Very difficult to explain without modifying any other observables

  Filipuzzi, Portoles, Gonzalez-Alonso'12

- Would be great to have another measurement by LHC

3.2 V_{us} determination



NB: BRs measured by B factories are systematically smaller than previous measurements

3.2 V_{us} determination

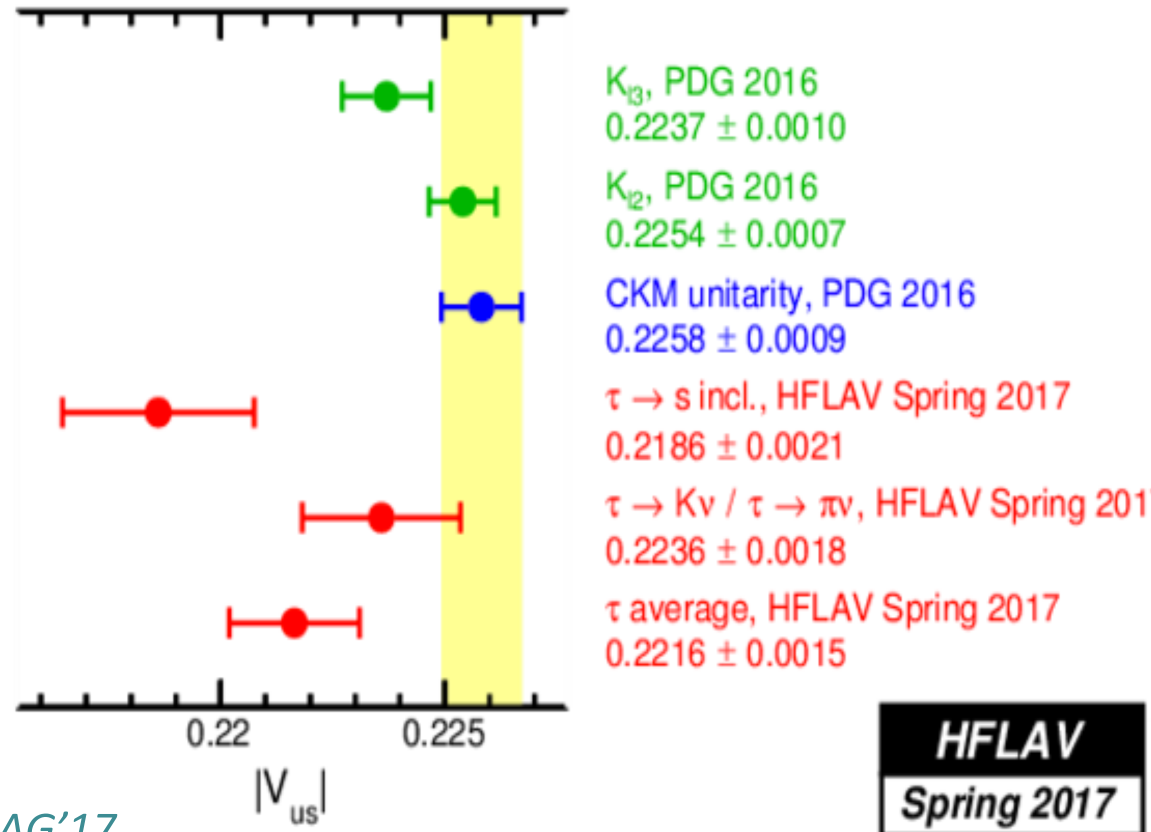
- Longstanding inconsistencies between inclusive τ and kaon decays in extraction of V_{us}
- Inclusive τ decays:

$$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$

Gamiz et al'07, Maltman'11



$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFAG'17

$$R_{\tau,S} = 0.1633(28)$$

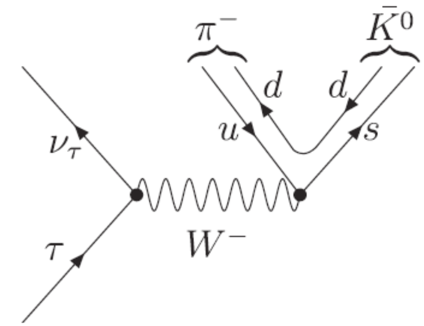
$$R_{\tau,NS} = 3.4718(84)$$

$$|V_{ud}| = 0.97417(21)$$

$$|V_{us}| = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}$$

3.1σ away from unitarity!

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

  $$= |p|^2 - |q|^2 \quad \boxed{\approx (0.36 \pm 0.01)\%}$$ in the SM

  Bigi & Sanda'05
  Grossman & Nir'11

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$$\boxed{\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)}$$

- Experimental measurement : *BaBar'11*

  $$\boxed{A_{Q\,\mathrm{exp}} = \left(-0.36 \pm 0.23_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%} \Rightarrow \boxed{2.8\sigma}$$ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

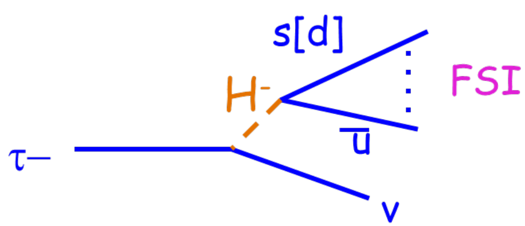
  Grossman & Nir'11

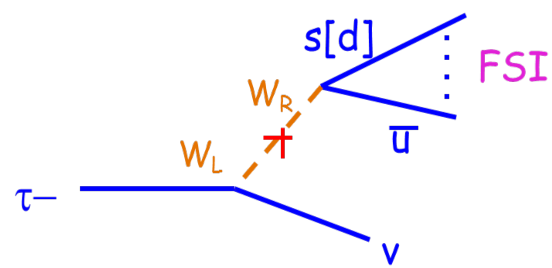
  $$\boxed{A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)}} = \left(-0.54 \pm 0.14\right)\%$$

  Belle, Babar, CLEO, FOCUS

3.3 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14, Cirigliano, Crivellin, Hoferichter'17*)?





Bigi'Tau12

Very difficult to explain!

- Need to investigate how large can be the prediction in realistic new physics models: it looks like *a tensor interaction* can explain the effect but in conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17

⇨ light BSM physics?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha'14
Cirigliano, Crivellin, Hoferichter'17

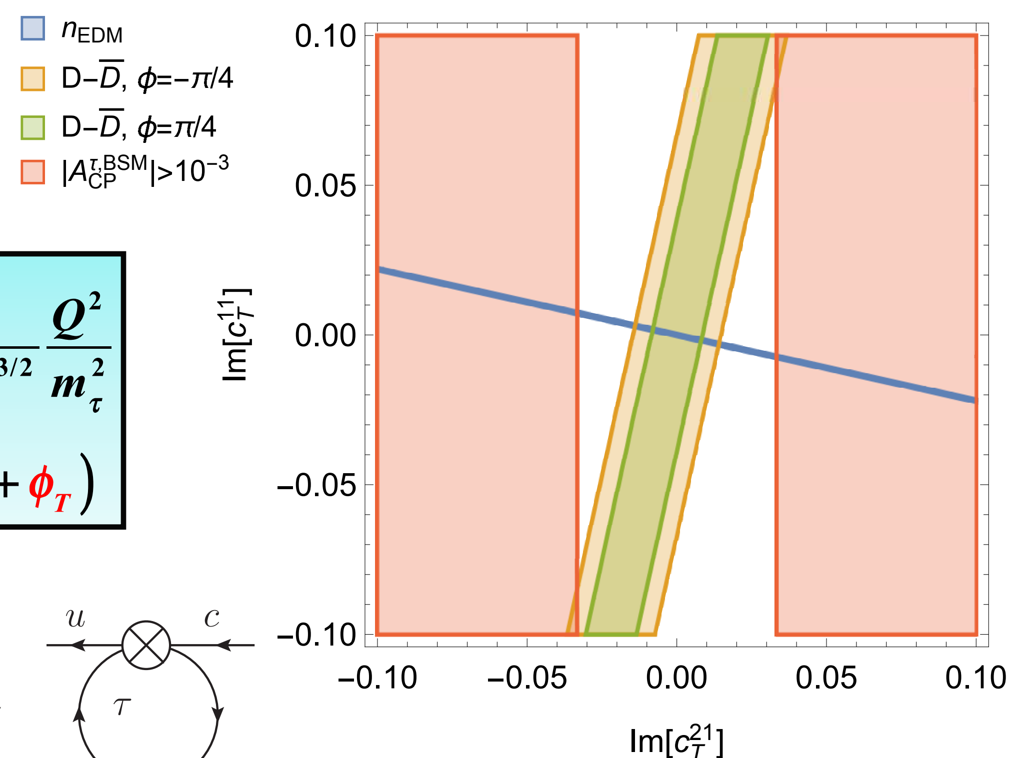
- We need a tensor interaction to get some interference:

$$\boxed{\mathcal{H}_T^{eff} \equiv G'(\bar{s}\sigma_{\mu\nu}u)(\bar{\nu}_\tau(1+\gamma_5)\sigma^{\mu\nu}\tau)} \quad \text{with} \quad G' = \frac{G_F}{\sqrt{2}}C_T, \quad C_T = |C_T|e^{i\phi_T}$$

- When integrating the interference term between vector and tenson does not vanish:

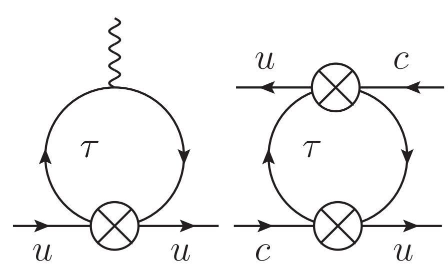
$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma_{SM}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$

$$\boxed{\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2\theta_C \frac{m_\tau^3}{32\pi^3}\left(\frac{m_\tau^2 - Q^2}{m_\tau^2}\right)^2 \frac{q_1^3}{(Q^2)^{3/2}}\frac{Q^2}{m_\tau^2} \times |C_T||F_V(s)||F_T(s)|\cos(\delta_T(s) - \delta_V(s) + \phi_T)}$$



In conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17



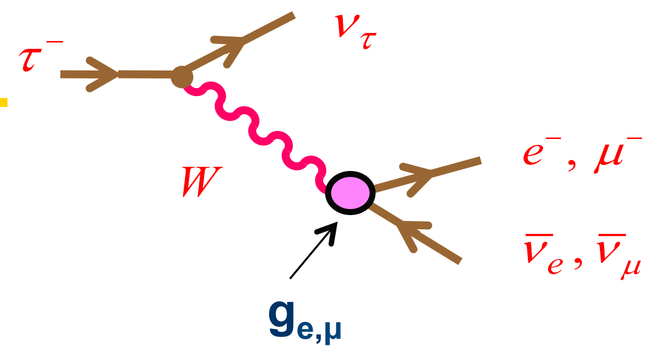
4. Conclusion and outlook

Conclusion and outlook

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier
- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*
- Charged leptons and in particular *tau physics* offer an important spectrum of possibilities:
  - ➢ LFV measurement has SM-free signal
  - ➢ Several interesting anomalies: LFU, Vus, CPV in $\tau \to K\pi\nu_\tau$
  - ➢ Progress towards a better knowledge of hadronic uncertainties
  - ➢ New physics models usually strongly correlate the flavours sectors
  - ➢ Important experimental activities: Belle, BaBar, LHCb, ATLAS, CMS and more to come: Belle II, HL LHC, etc
- A lot of interesting physics remains to be done in the Tau sector!

5. Back-up

3.1 Lepton Universality



- The leptonic decay width:

$$\Gamma(\tau \to \nu_\tau\, l\, \bar{\nu}_l) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\, f(m_l^2/m_\tau^2)\, \left(1 + \delta_{RC}\right)$$

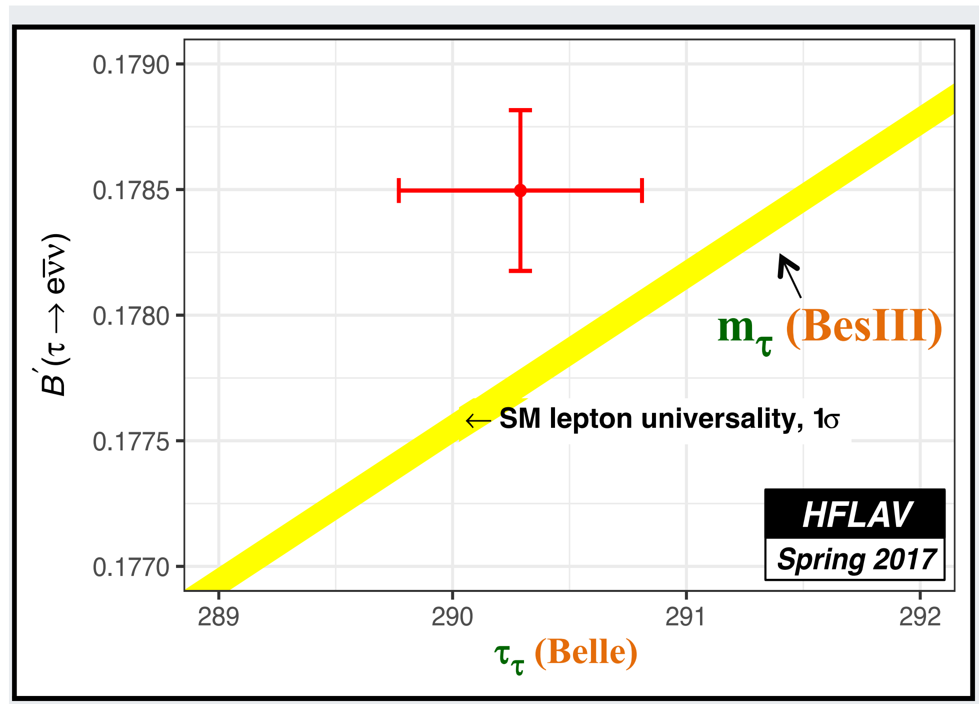
Experimental inputs:

$\Gamma(\tau_{l3})$ Rates with well-determined treatment of radiative decays

- Branching ratios
- Tau lifetimes

Inputs from theory: *Marciano'88*

δ_{RC} Radiative corrections



- Test of μ/e universality:

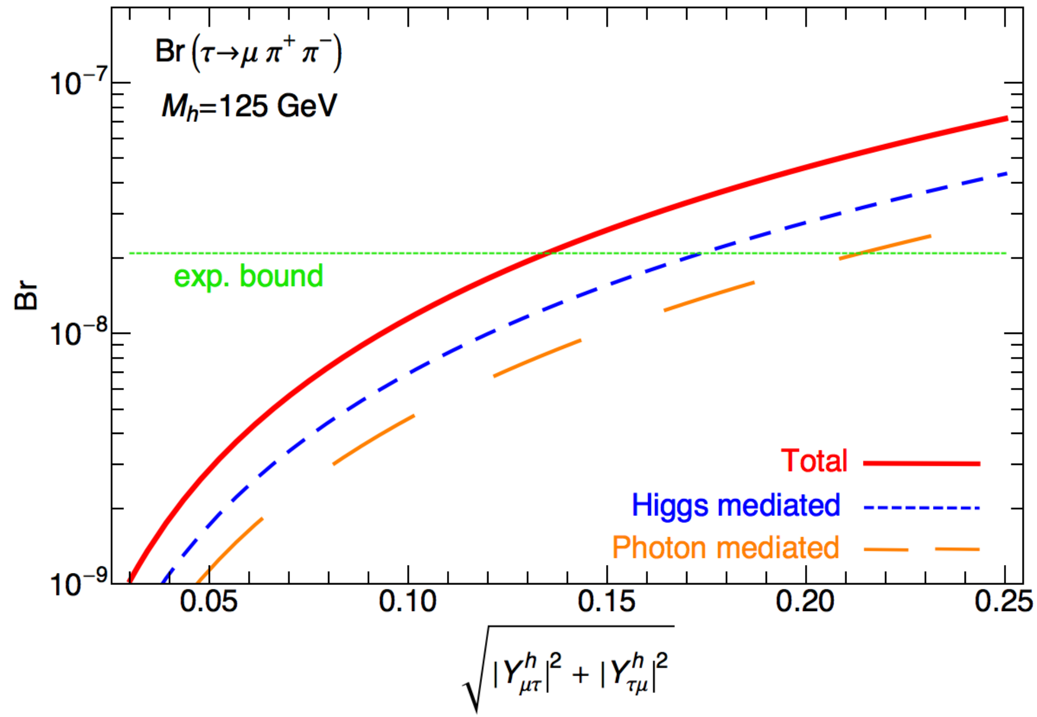
$$\left(B_\mu / B_e\right)_{\exp} = 0.9761 \pm 0.0028$$

Non-BF: 0.9725 ± 0.0039

BaBar '10: 0.9796 ± 0.0039

$$\boxed{B_e^{univ} = (17.818 \pm 0.0022)}$$

3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13}$$

Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

?

Less stringent but more robust handle on LFV Higgs couplings

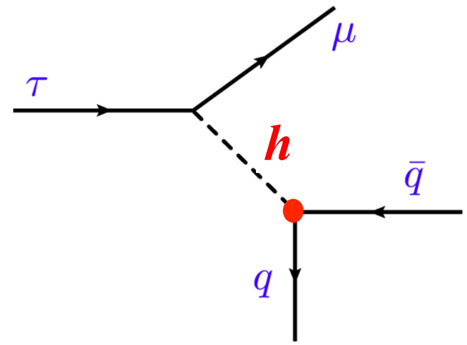
Belle'08'11'12 except last from *CLEO'97*

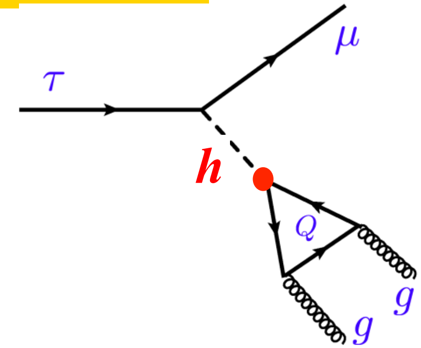
3.5 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!





- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

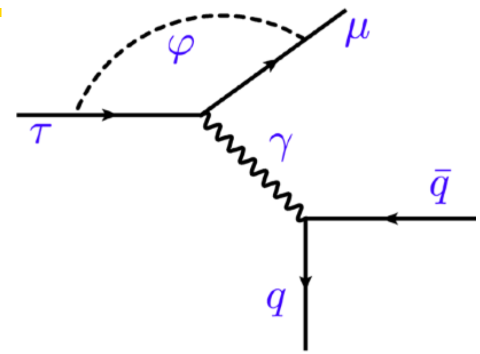
below present experimental limits!

- If discovered ⇨ among other things *upper limit* on $Y_{u,d,s}$*!*

⇨ Interplay between high-energy and low-energy constraints!

3.1 Constraints from $\tau \rightarrow \mu\pi\pi$

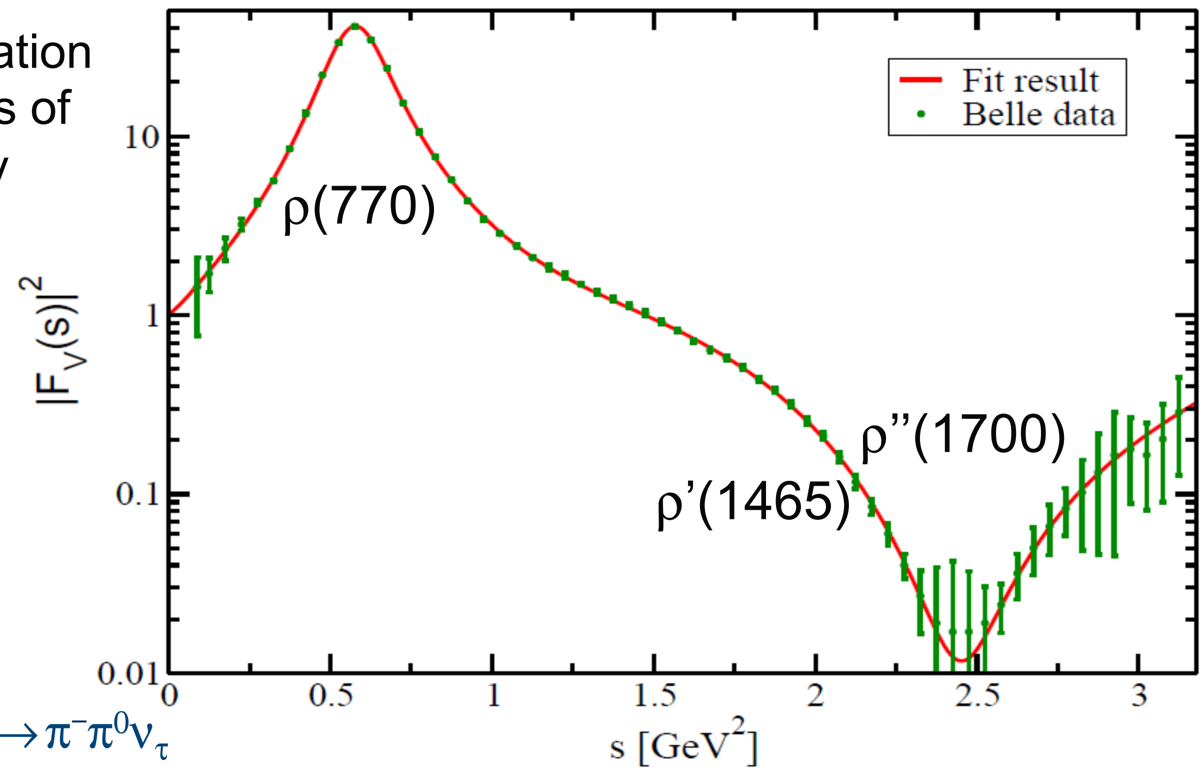
- Photon mediated contribution requires the pion vector form factor:



$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) | \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) | 0 \rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

Gasser, Meißner´91
Guerrero, Pich'97
Oller, Oset, Palomar´01
Pich, Portolés ´08
Gómez Dumm&Roig'13
...



- Determined from a fit to the Belle data on $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

Celis, Cirigliano, E.P.'14

Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements
    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)
  - Theoretically: Dispersive parametrization for $F_V(s)$

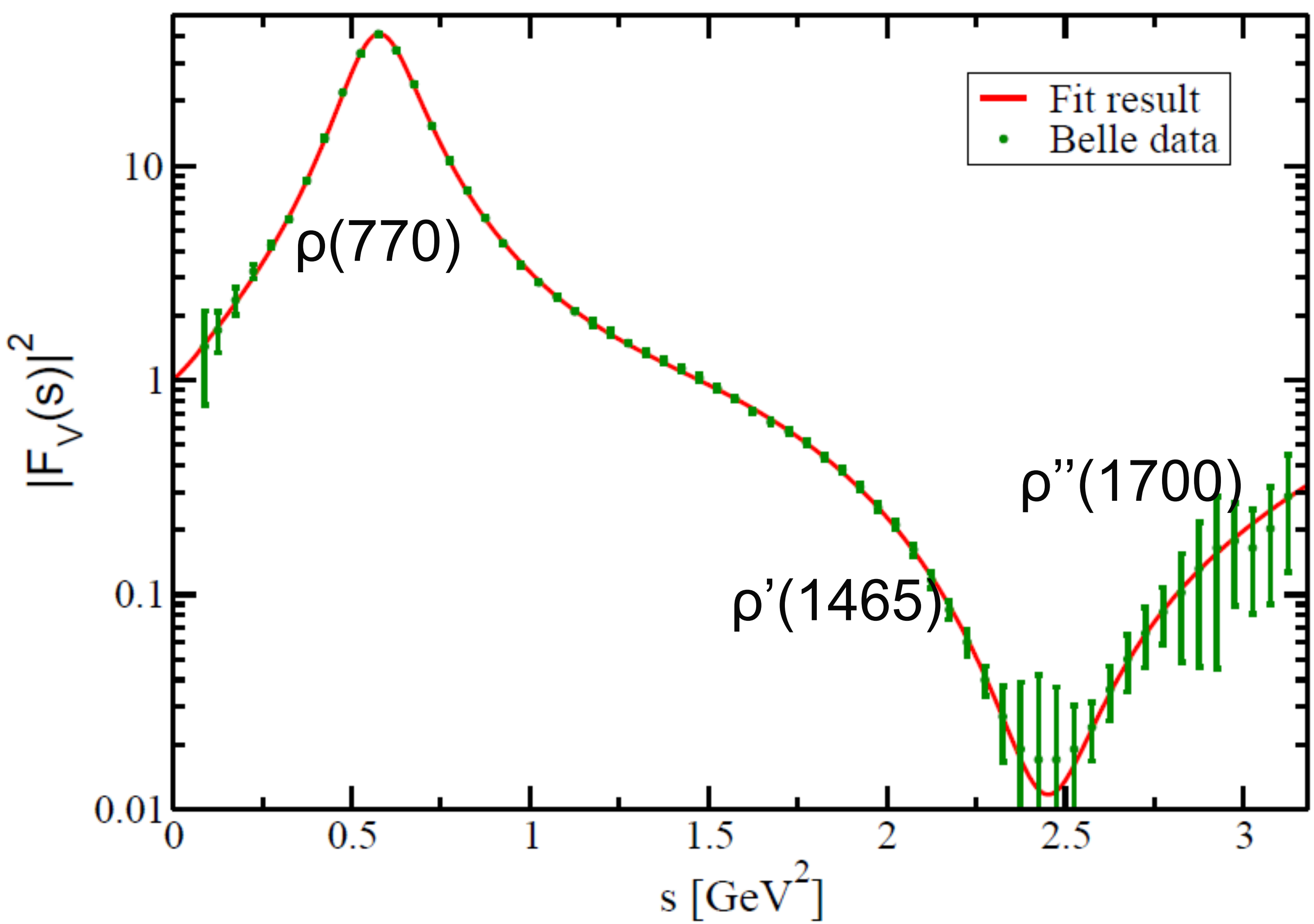
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda'_V \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

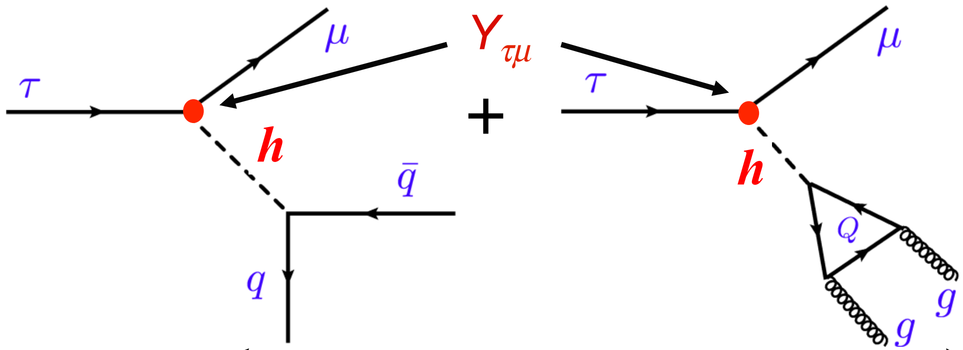
Determination of $F_V(s)$

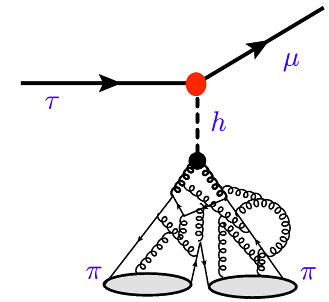


Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange





$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\left\langle \pi^+\pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s)$$

$$\left\langle \pi^+\pi^- \left| \theta^\mu_\mu \right| 0 \right\rangle \equiv \theta_\pi(s)$$

Voloshin'85

$$\left\langle \pi^+\pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2}$$

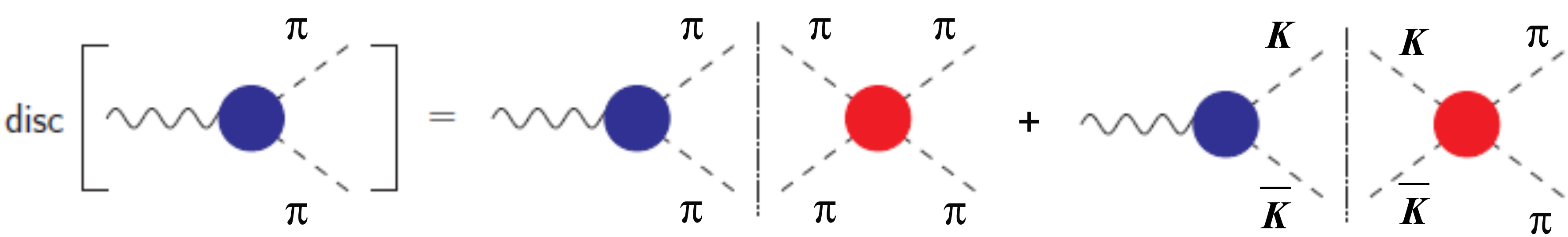
$$f\left(y_q^h\right)$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs ⇨ *Coupled channel analysis*
  up to $\sqrt{s}\sim$1.4 GeV
  Inputs: I=0, S-wave $\pi\pi$ and $K\bar{K}$ data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:

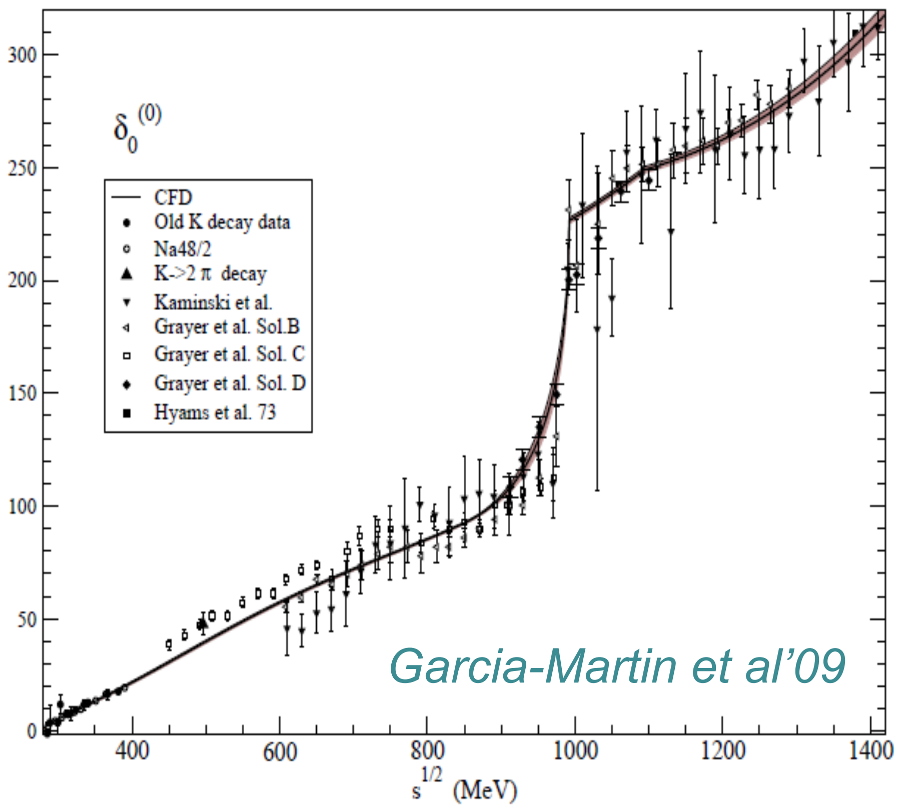


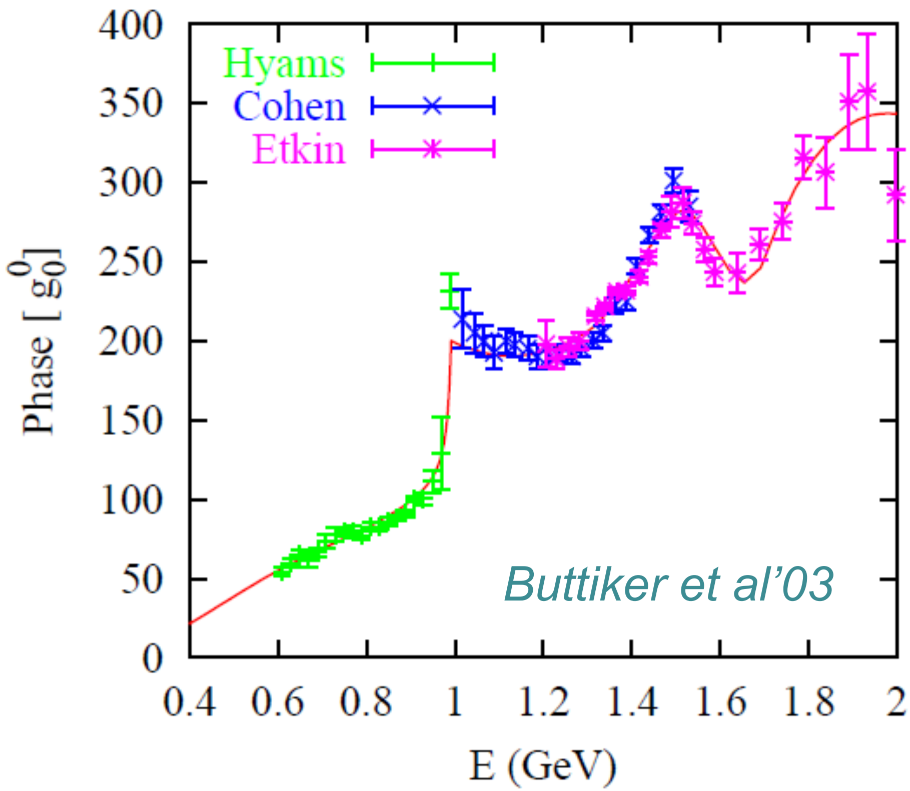
⇨ $$\boxed{\mathrm{Im} F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$ $\quad n = \pi\pi, K\bar{K}$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{ T_{nm}^* \sigma_m(s) X_m^{(N)} \right\} \longrightarrow \mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

  Feynman-Hellmann theorem: $\Rightarrow$

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$\begin{aligned} M^2_{\pi^+} &= (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2) \\ M^2_{K^+} &= (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2) \\ M^2_{K^0} &= (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2) \end{aligned} \Rightarrow$$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$\begin{aligned} M^2_{\pi^+} &= (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2) \\ M^2_{K^+} &= (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2) \\ M^2_{K^0} &= (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2) \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  $\Rightarrow$ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

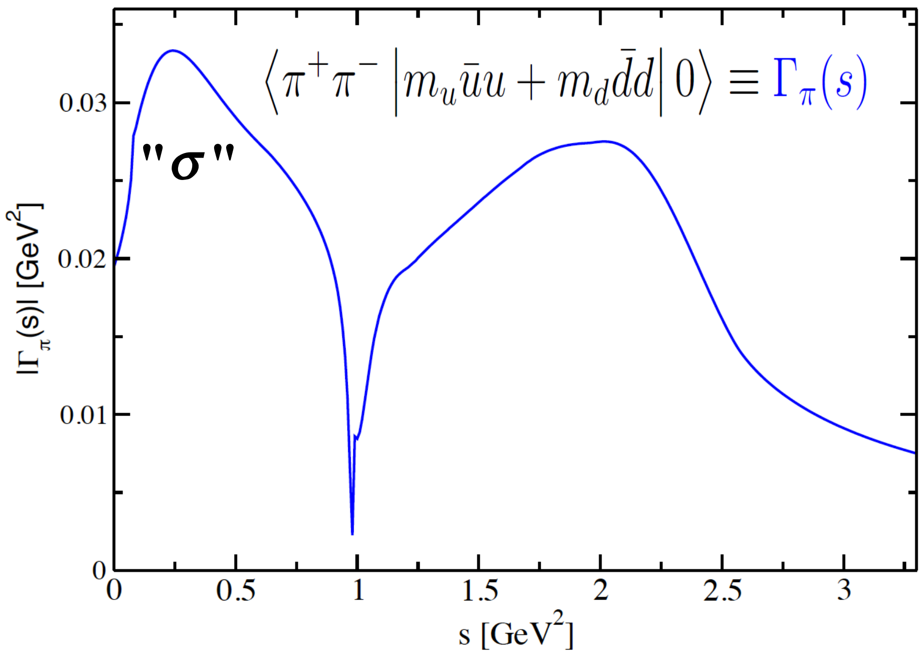
- General solution

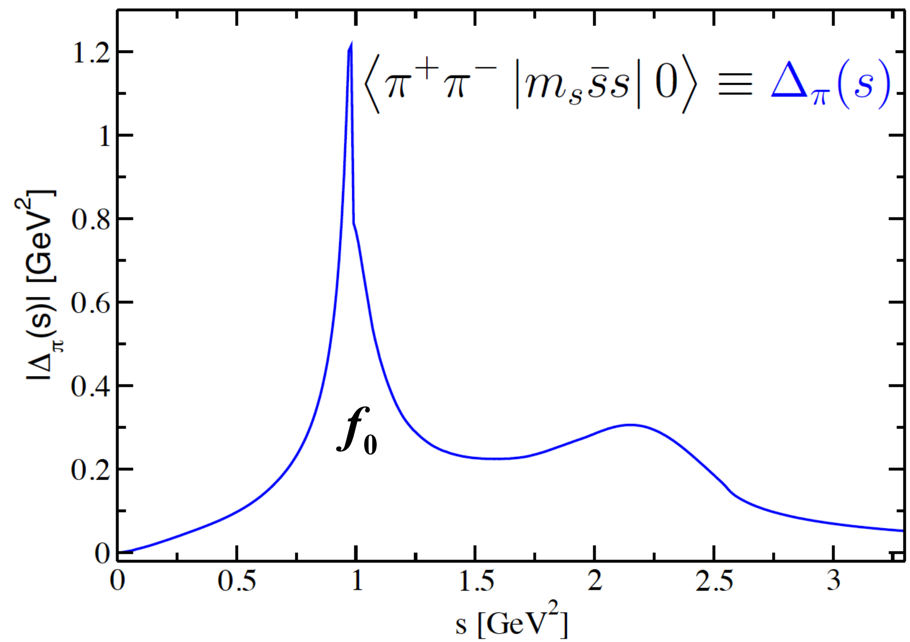
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

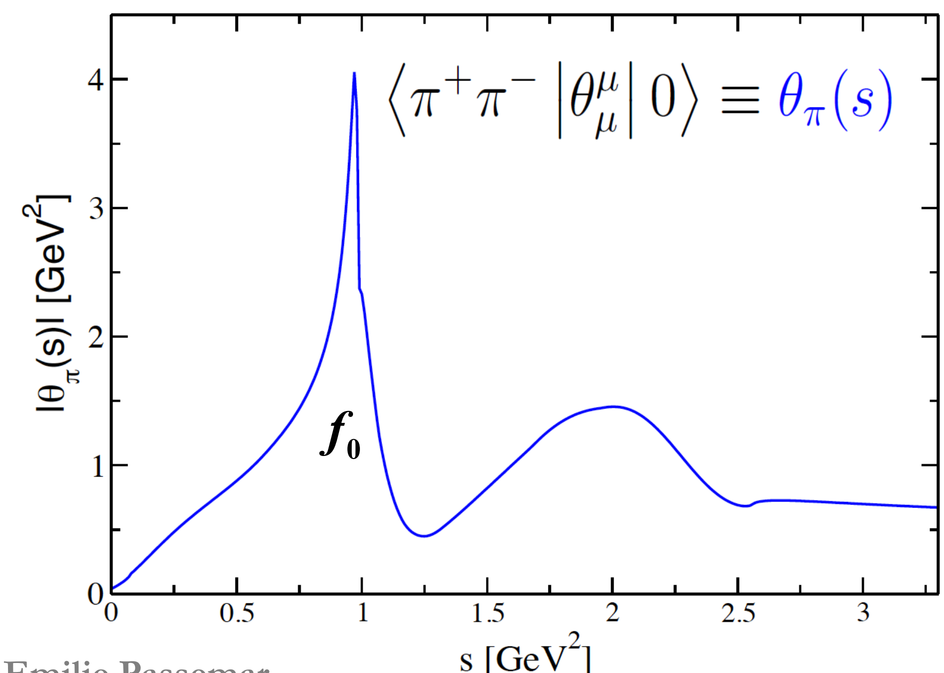
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

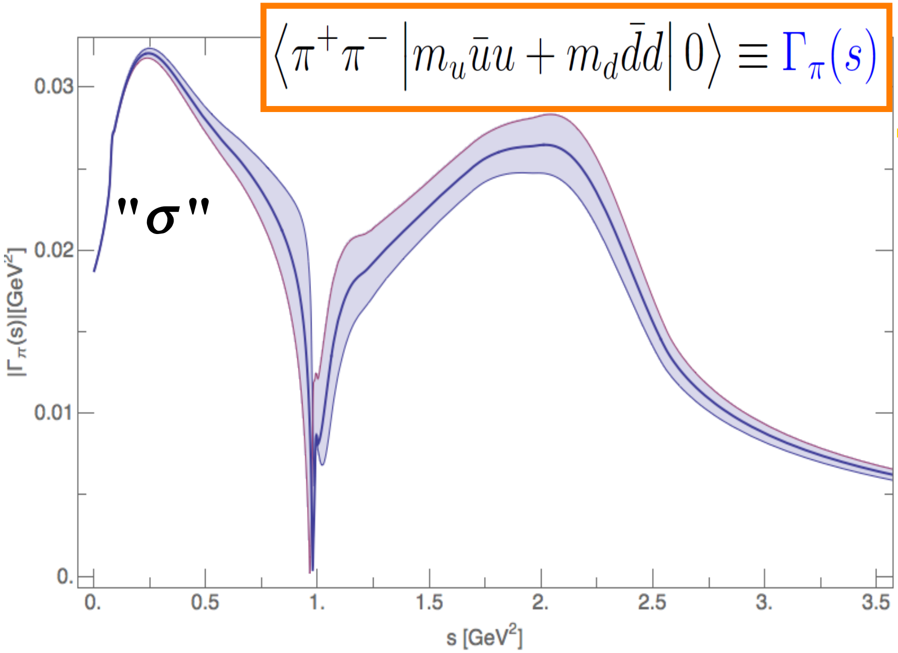
⇒ Relax the constraints and match to ChPT

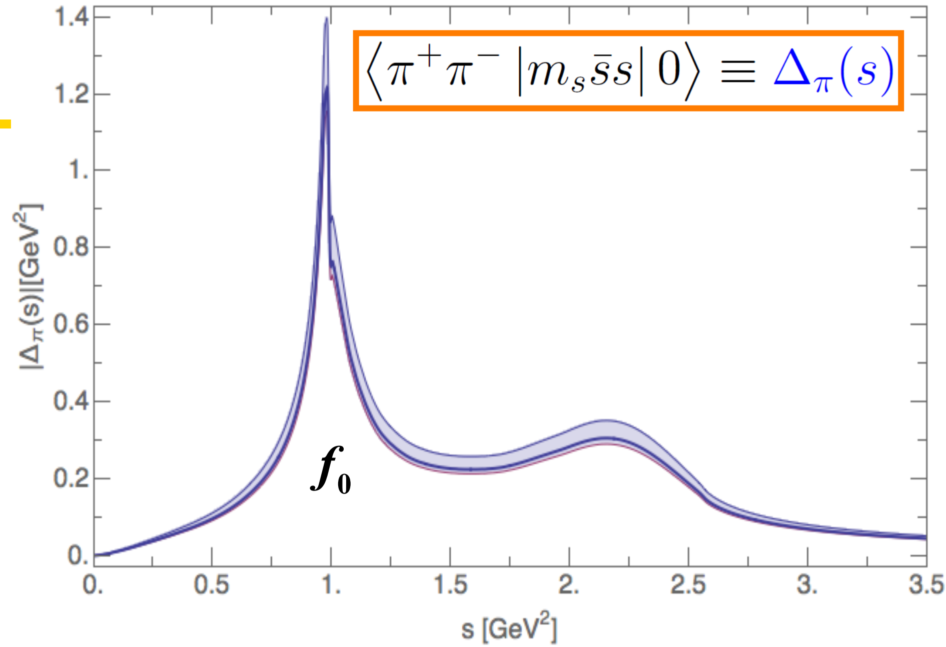
$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$

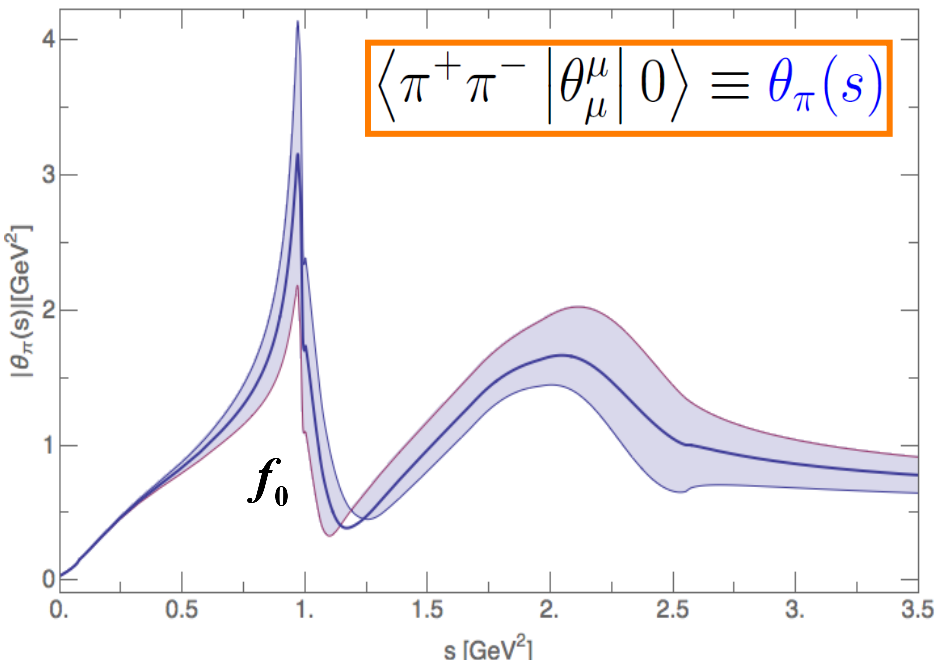

$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$
"σ"
$|\Gamma_\pi(s)|$ [GeV^2]
0
0.01
0.02
0.03
0
0.5
1
1.5
2
2.5
3
s [GeV^2]


$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
f_0
$|\Delta_\pi(s)|$ [GeV^2]
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
s [GeV^2]

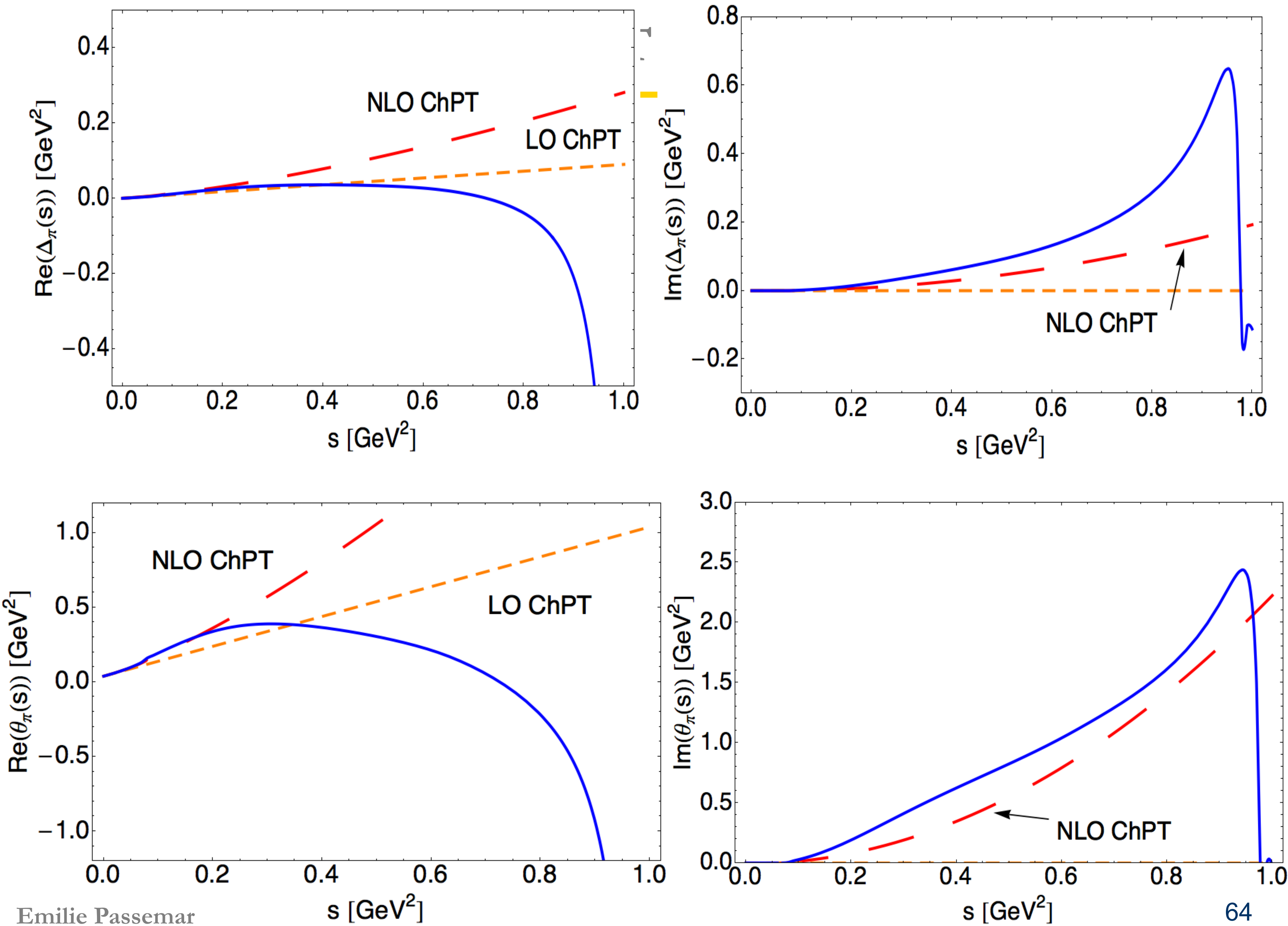

$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
f_0
$|\theta_\pi(s)|$ [GeV^2]
0
1
2
3
4
0
0.5
1
1.5
2
2.5
3
s [GeV^2]







- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs


NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
Im(Δ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV^2]
s [GeV^2]
NLO ChPT
Im(θ_π(s)) [GeV^2]
s [GeV^2]

3.1 Lepton Universality

- What about the *third family*?

A. Pich@KEKFF'15

updated on HFAG'17

$|g_\tau / g_\mu|$

$B_{\tau\to e} \quad \tau_\mu / \tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.9962 ± 0.0027
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.9858 ± 0.0070
$B_{W\to\tau} / B_{W\to\mu}$	1.034 ± 0.013

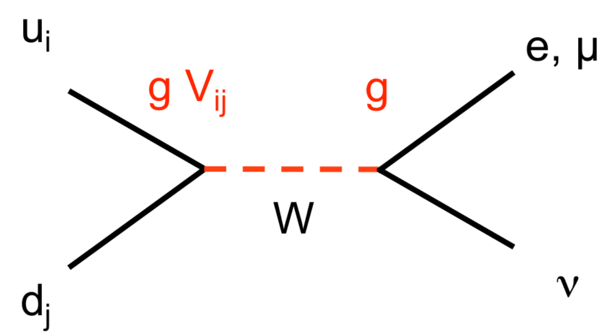
$|g_\tau / g_e|$

$B_{\tau\to\mu} \ \tau_\mu / \tau_\tau$	1.0029 ± 0.0015
$B_{W\to\tau} / B_{W\to e}$	1.031 ± 0.013

- Universality tested at 0.15% level and good agreement except for
  - W decay old anomaly
  - B decays

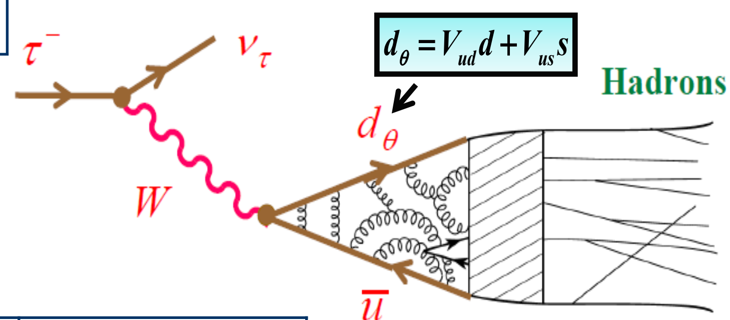
2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays



V_{ud}	$0^+ \to 0^+$ $\pi^{\pm} \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

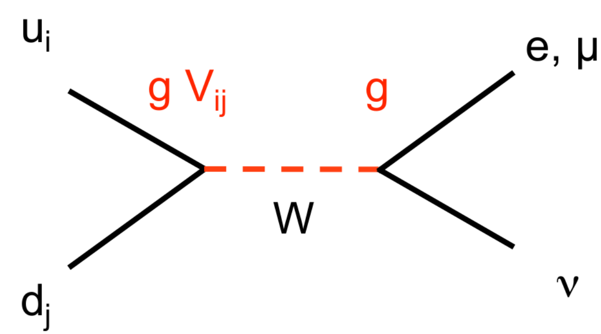
- From τ decays (crossed channel)



V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

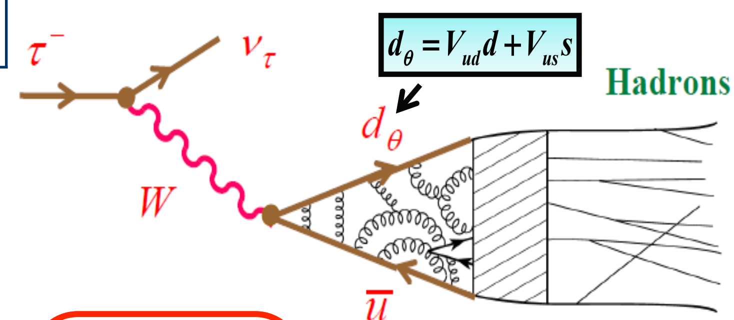
2.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays



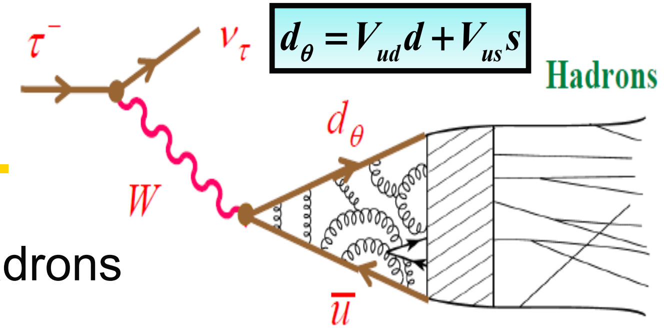
V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

- From τ decays (crossed channel)



V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

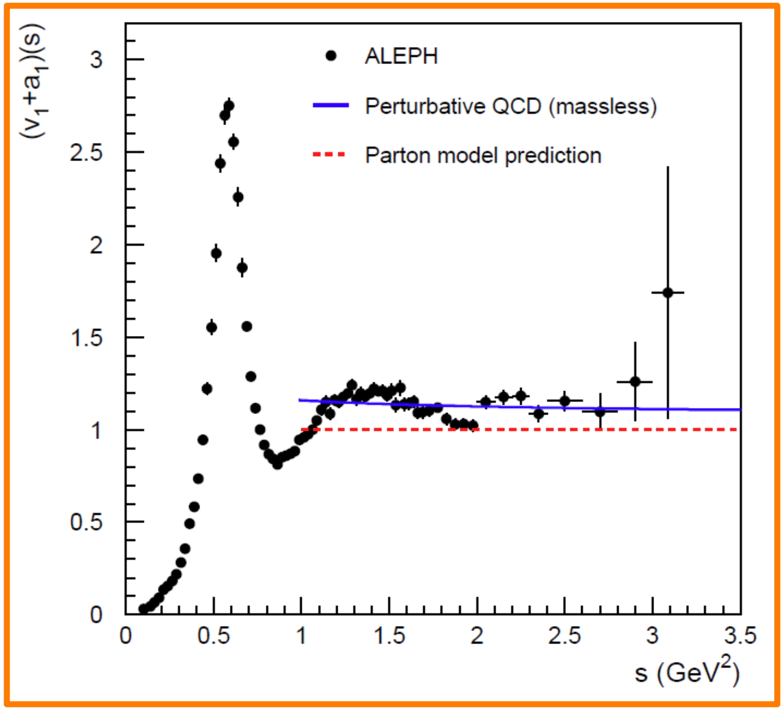
2.3 V_{us} from inclusive measurement



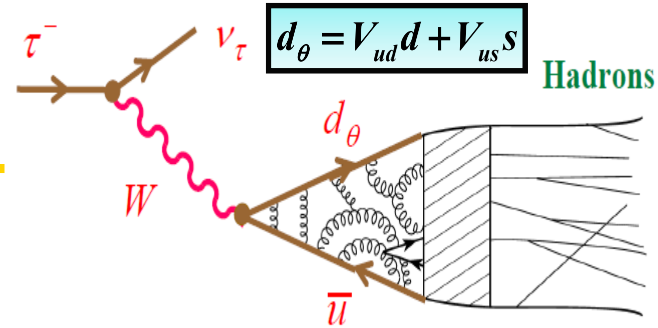
- Tau, the only lepton heavy enough to decay into hadrons
- $\boxed{m_\tau \sim 1.77\,\mathrm{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*
- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $(\alpha_S(m_\tau), |V_{us}|, m_s)$
- We consider $\boxed{\Gamma(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S=0})}$

  $\boxed{\Gamma(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S\neq 0})}$
- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!
- Observable studied: $\boxed{R_\tau \equiv \dfrac{\Gamma(\tau^- \to \nu_\tau + \mathrm{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}}$

Davier et al'13



2.4 Theory



- $\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} \approx N_C}$ parton model prediction

- $R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- $\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}}} \Rightarrow |V_{us}|$

QCD switch

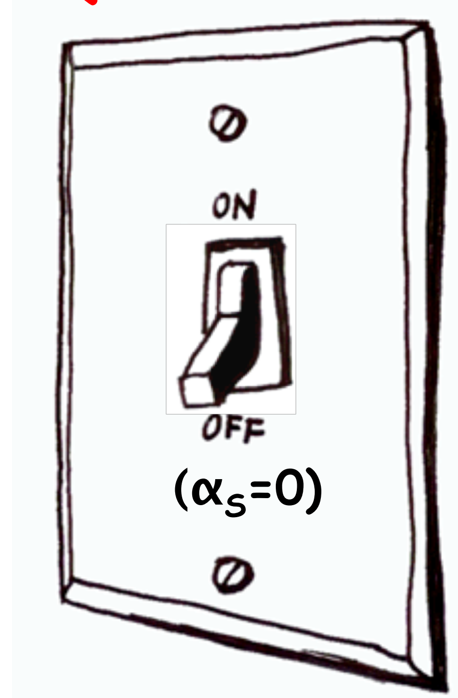
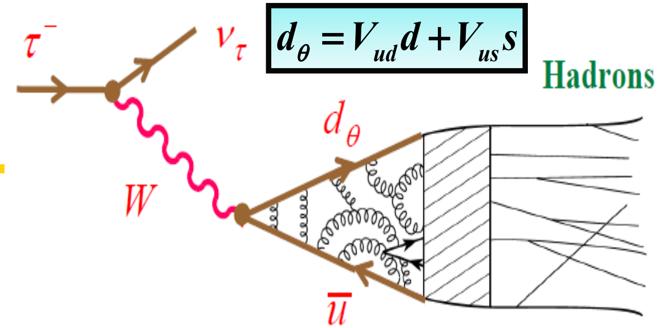


Figure from M. González Alonso'13

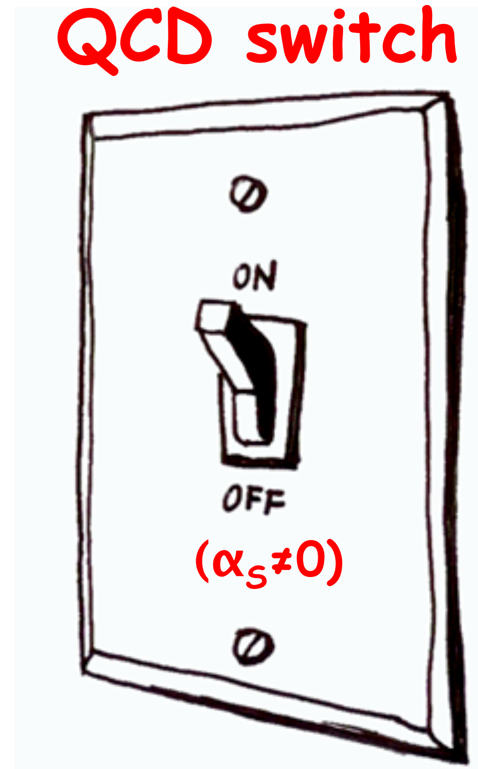
2.4 Theory



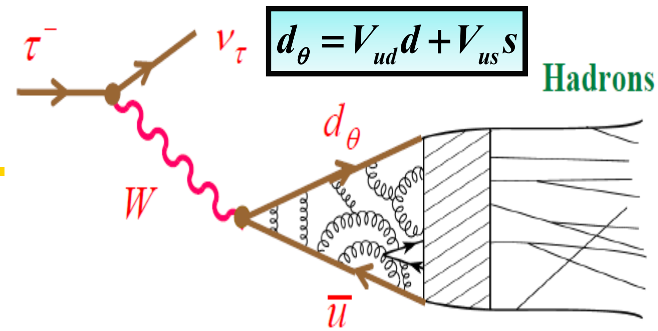
- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$ parton model prediction

- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$



2.4 Theory

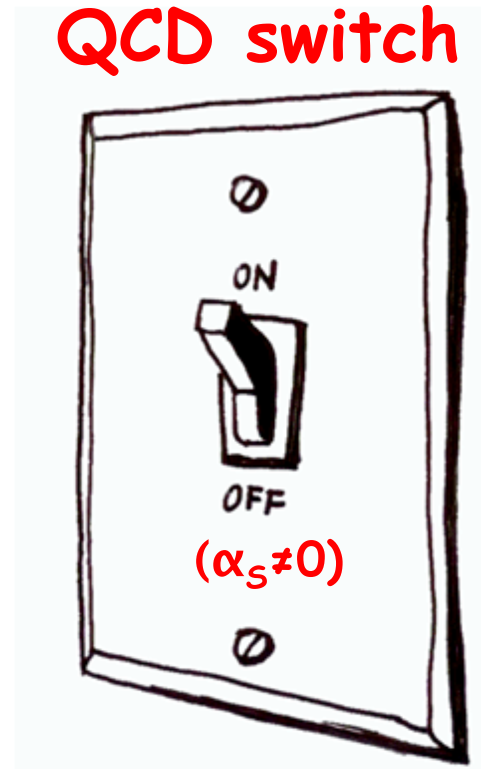


- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C$$ parton model prediction

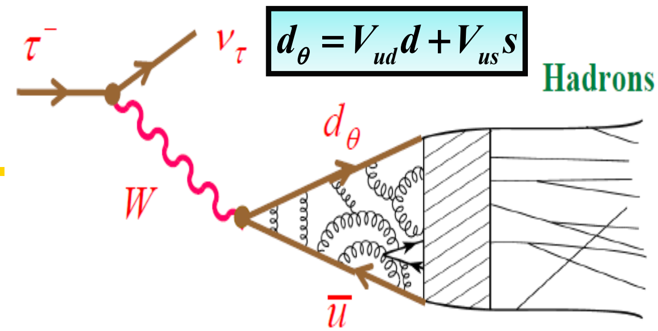
- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$

- Due to *QCD corrections*: $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$

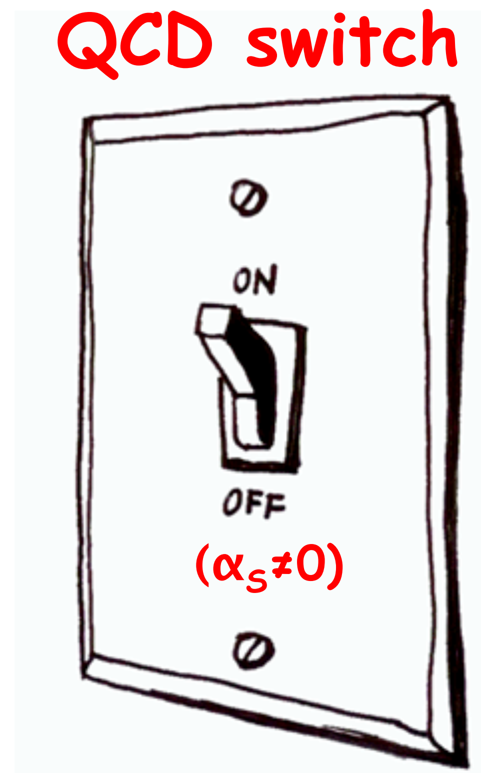


2.4 Theory



- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $\boxed{R_\tau \equiv \dfrac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C}$ naïve QCD prediction



- Extraction of the strong coupling constant :

$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_s) \Rightarrow \alpha_S$$

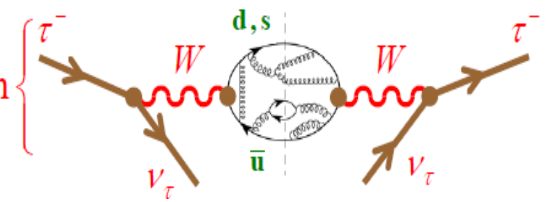
measured calculated

- Determination of V_{us} : $\boxed{\dfrac{|V_{us}|^2}{|V_{ud}|^2} = \dfrac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$

- Main difficulty: compute the QCD corrections with the best accuracy

2.5 Calculation of the QCD corrections

$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \text{Im}$



Braaten, Narison, Pich'92

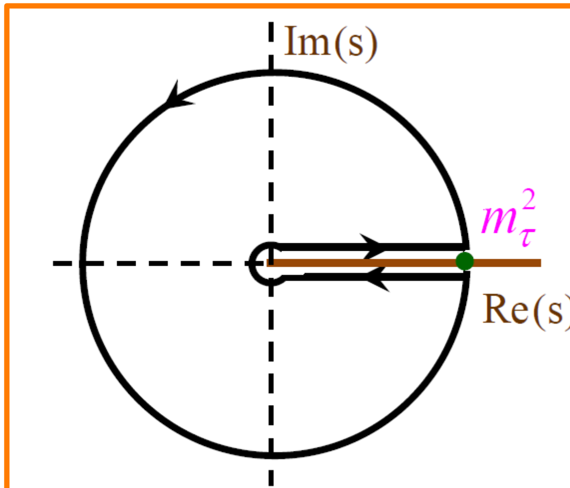
- Calculation of R_τ:

$$\boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$\boxed{R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]}$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O = D} \underbrace{C^{(J)}(s,\mu)}_{\text{Wilson coefficients}} \underbrace{\langle O_D(\mu)\rangle}_{\text{Operators}}$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

2.5 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R$_\tau$:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%$ $\quad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇒ Use of weighted distributions

2.5 Calculation of the QCD corrections

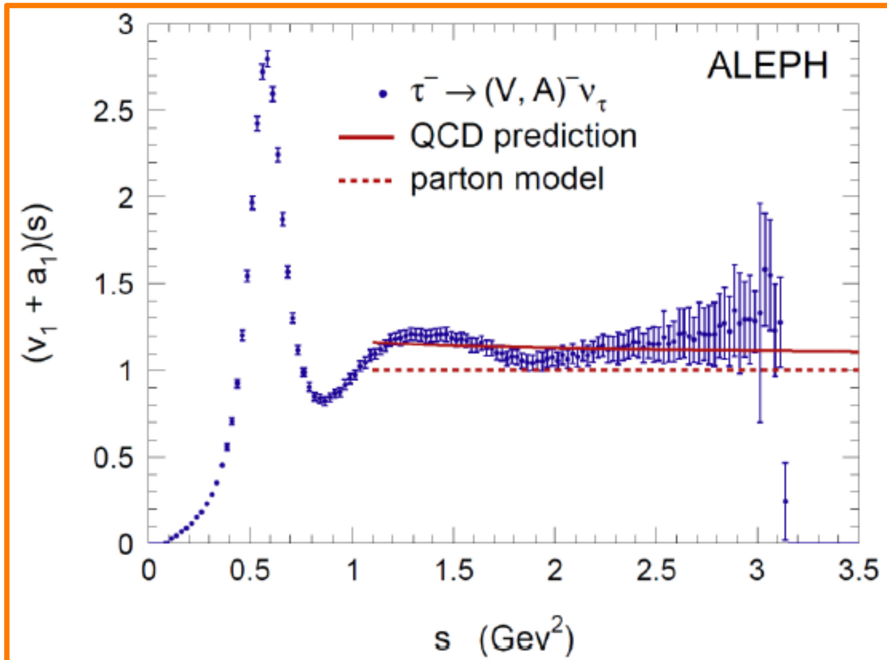
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

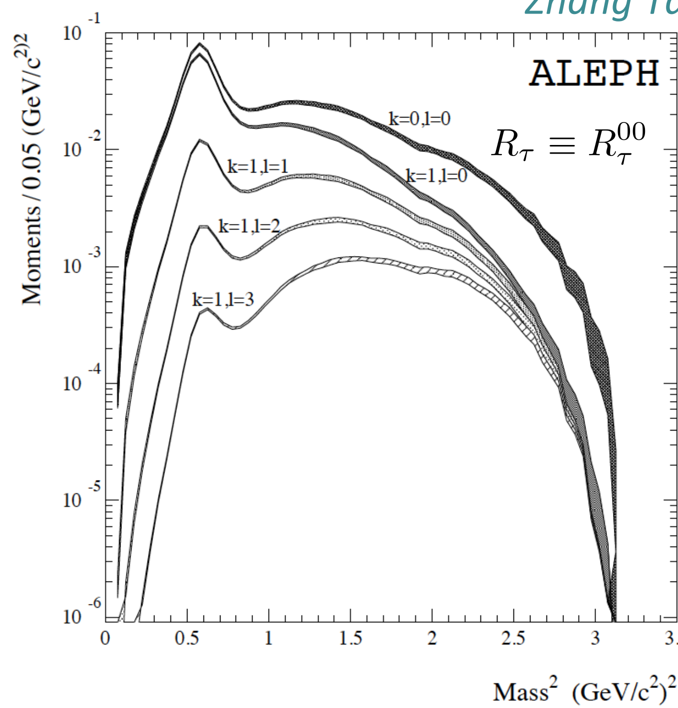
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14

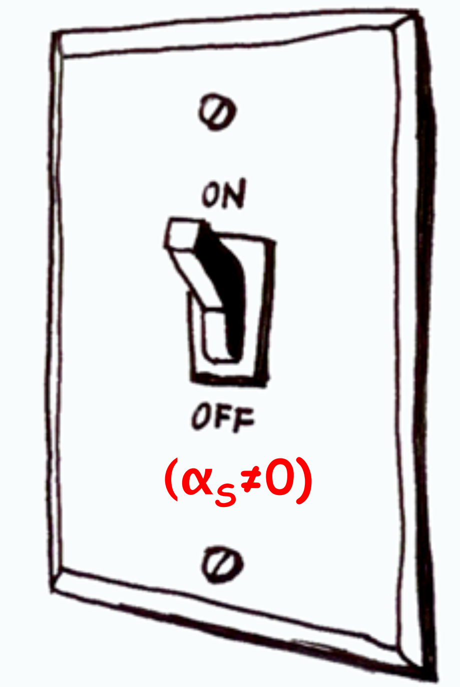


2.5 Inclusive determination of V_{us}

- With QCD on: $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$$

- Use OPE: $$\boxed{R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)}$$

$$\boxed{R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)}$$



- $$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$ *Gamiz et al'07, Maltman'11*

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

HFAG'17

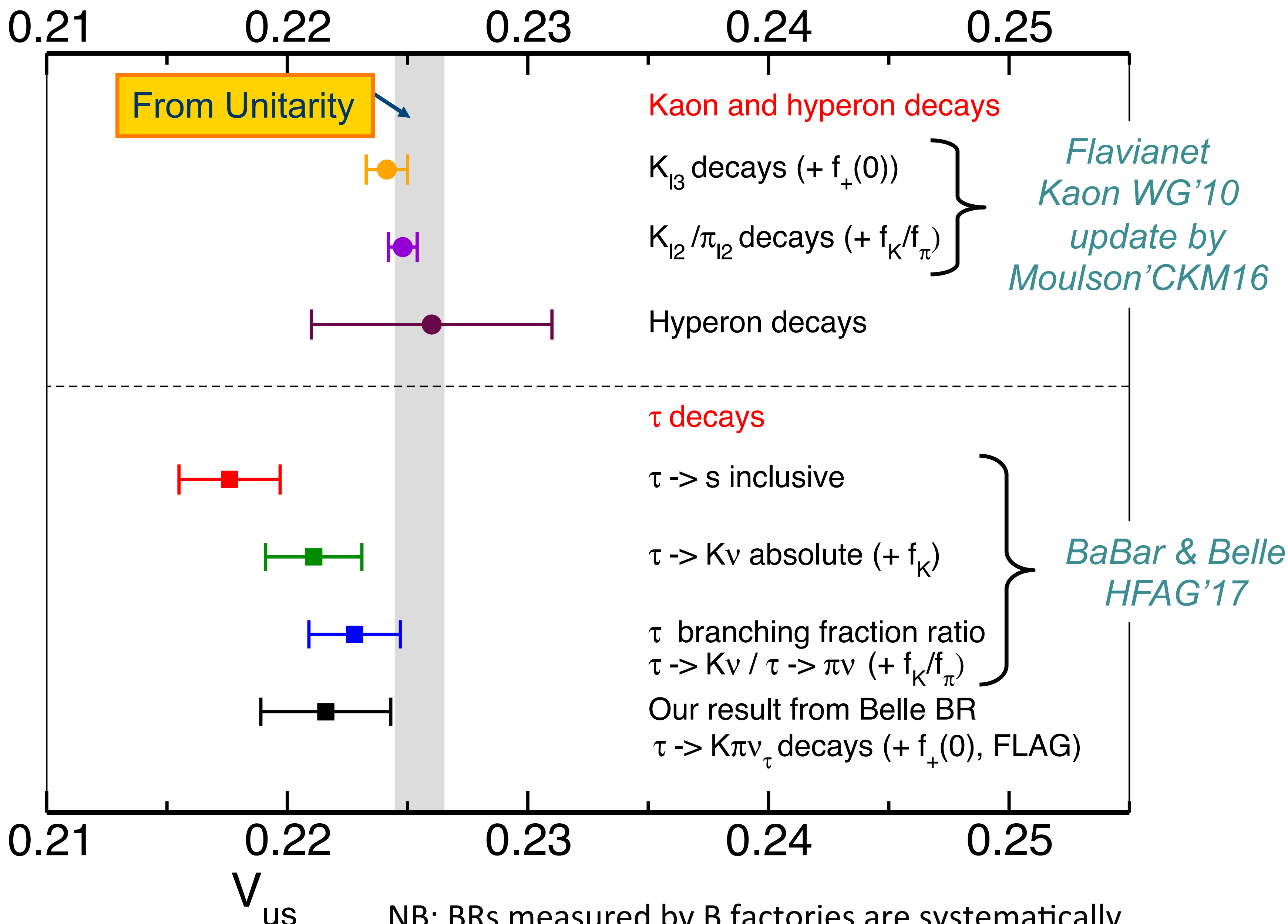
$$R_{\tau,S} = 0.1633(28)$$

$$R_{\tau,NS} = 3.4718(84)$$

$$|V_{ud}| = 0.97417(21)$$

$$\Rightarrow \quad \boxed{|V_{us}| = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

3.1σ away from unitarity!



NB: BRs measured by B factories are systematically smaller than previous measurements

2.6 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	➡ **(0.713 ± 0.003)%**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	➡ **(0.471 ± 0.018)%**
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	➡ **(0.857 ± 0.030)%**
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	➡ **(2.967 ± 0.060)%**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us}

  ➡ Recent studies

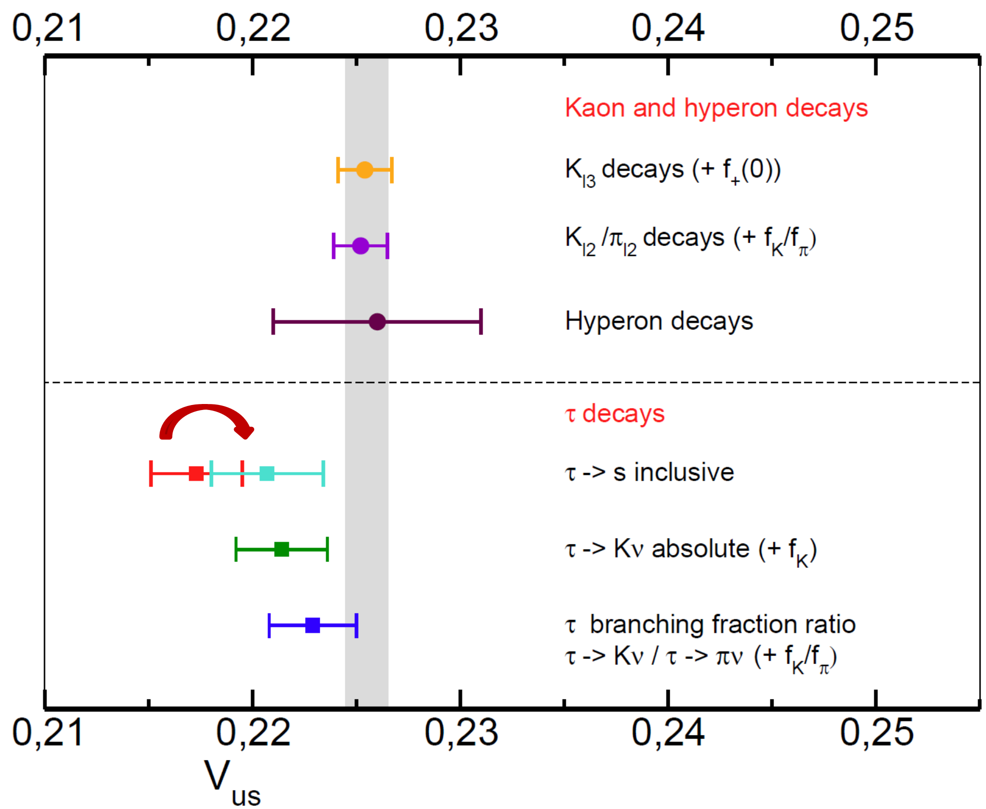
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:

  $\tau \to K\pi\nu_\tau$ Br + spectrum

$$\left|V_{us}\right| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$

➡ need new data



2.6 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	**$(0.713 \pm 0.003)\%$**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	**$(0.471 \pm 0.018)\%$**
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	**$(0.857 \pm 0.030)\%$**
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	**$(2.967 \pm 0.060)\%$**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us}

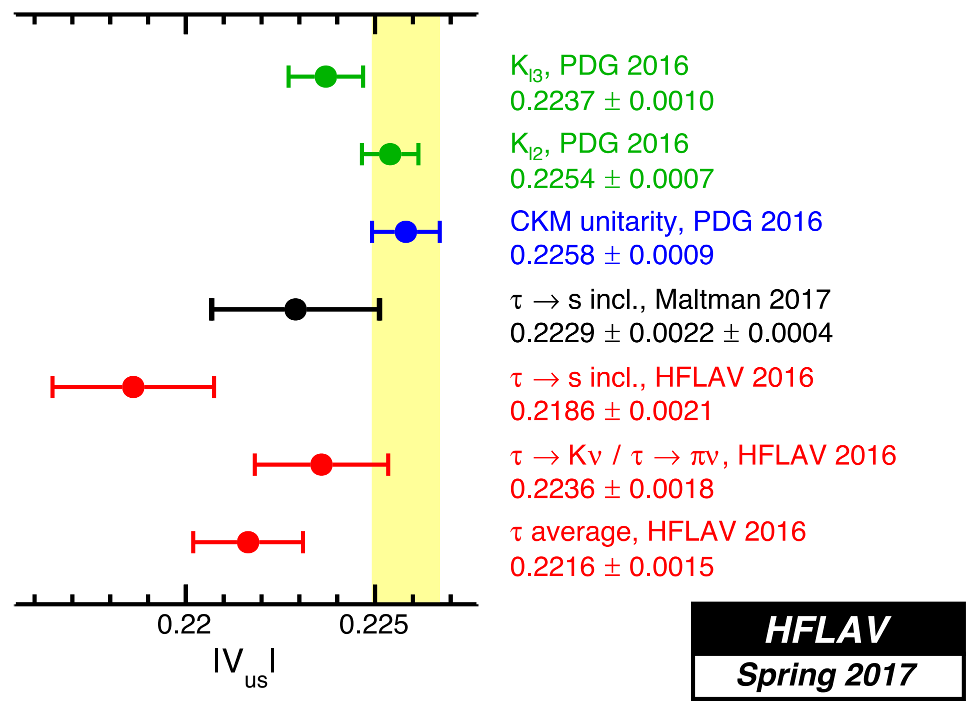
  ⇨ Recent studies

  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:

  $\tau \to K\pi\nu_\tau$ Br + spectrum

$$|V_{us}| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$



⇨ need new data

Very good prospect from Belle II, BES?

4.2 Outlook

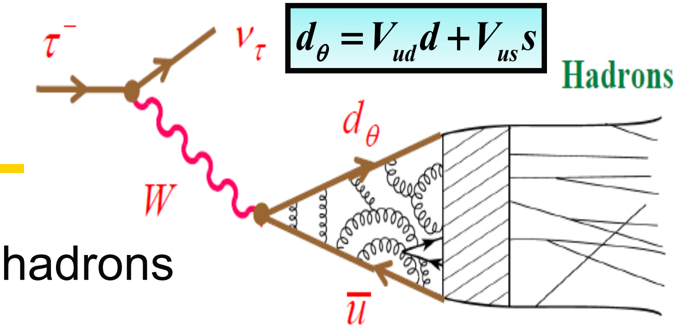
- 45 billion $\tau^+\tau^-$ pairs in full dataset from $\sigma(\tau^+\tau^-)_{E=\Upsilon(4S)}$= 0.9 nb *@Belle II*
- B2TiP initiative: define the first set of measurements to be performed at *Belle II*

  https://confluence.desy.de/display/BI/B2TiP+WebHome

- *Golden/Silver modes* for the Tau, Low Multiplicity and EW working group

Process	Observable	Theory	Sys. limit (Discovery) [ab^{-1}]	vs LHCb/BESIII	vs Belle	Anomaly	NP
$\tau \to \mu\gamma$	$Br.$	$\star\star\star$	-	$\star\star\star$	$\star\star\star$	$\star$	$\star\star\star$
$\tau \to lll$	$Br.$	$\star\star\star$	-	$\star\star\star$	$\star\star\star$	$\star$	$\star\star\star$
$\tau \to K\pi\nu$	A_{CP}	$\star\star\star$	-	$\star\star\star$	$\star\star\star$	$\star\star$	$\star\star$
$e^+e^- \to \gamma A'(\to\text{invisible})$	σ	$\star\star\star$	-	$\star\star\star$	$\star\star\star$	$\star$	$\star\star\star$
$e^+e^- \to \gamma A'(\to \ell^+\ell^-)$	σ	$\star\star\star$	-	$\star\star\star$	$\star\star\star$	$\star$	$\star\star\star$
π form factor	$g-2$	$\star\star$	-	$\star\star\star$	$\star\star$	$\star\star$	$\star\star\star$
ISR $e^+e^- \to \pi\pi$ g-2	$g-2$	$\star\star$	-	$\star\star\star$	$\star\star\star$	$\star\star$	$\star\star\star$

3.1 Introduction



- Tau, the only lepton heavy enough to decay into hadrons

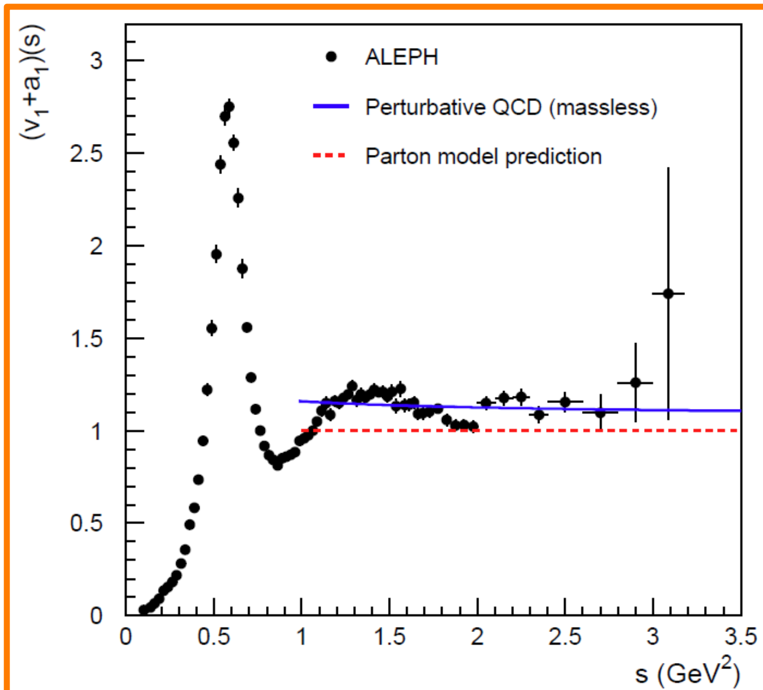
- $\boxed{m_\tau \sim 1.77\,\mathrm{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*

- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $\left(\alpha_S(m_\tau), |V_{us}|, m_s\right)$

- We consider $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S=0}\right)}$

  $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S\neq 0}\right)}$

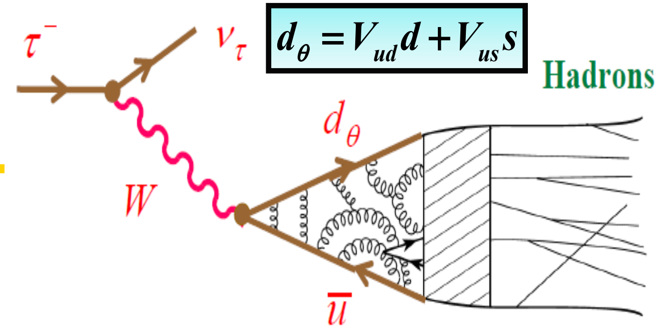
Davier et al'13



- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!

- Observable studied: $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$

3.2 Theory



- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^S \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- $$\boxed{\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} = \frac{R_\tau^S}{R_\tau^{NS}}} \Rightarrow \left|V_{us}\right|$$

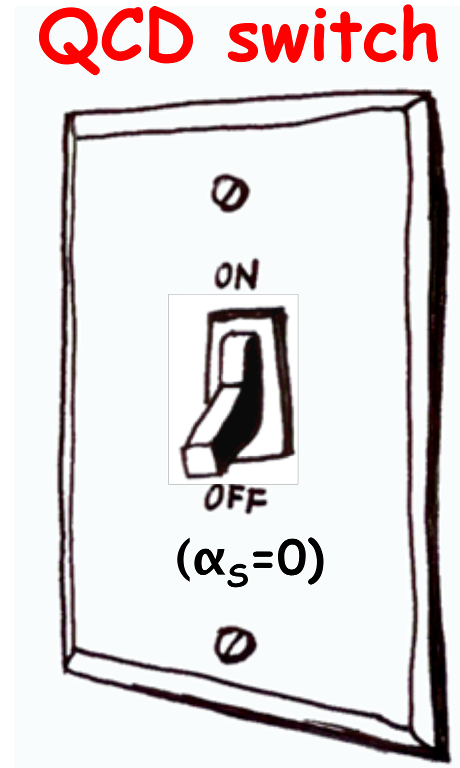
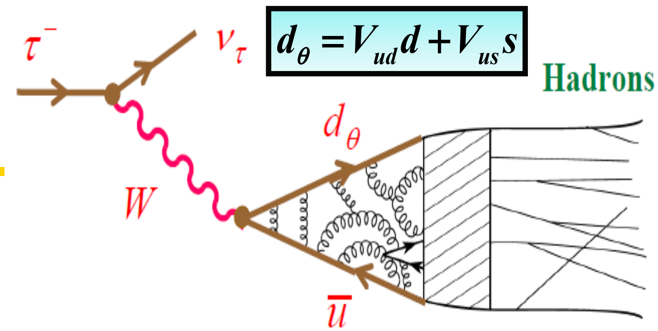


Figure from M. González Alonso'13

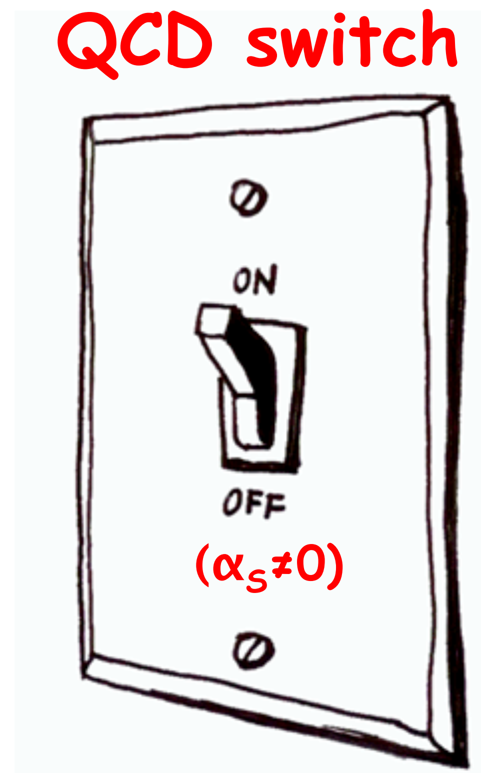
3.2 Theory



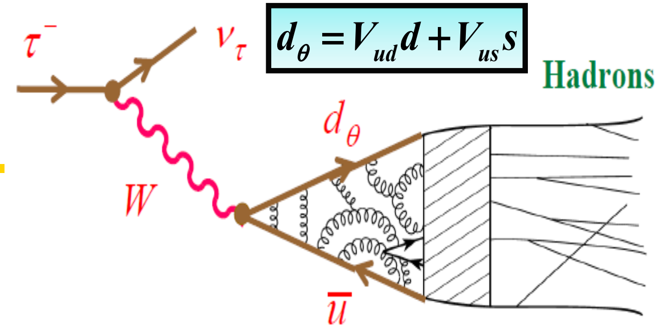
- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$



3.2 Theory

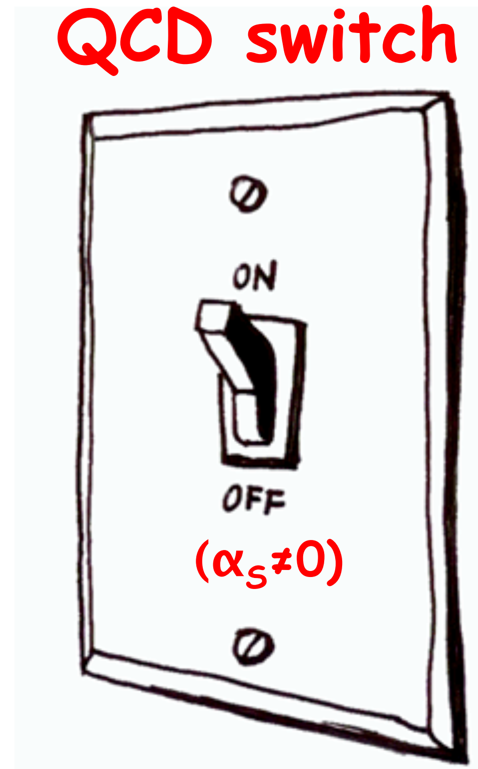


- $\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} \approx N_C}$ parton model prediction

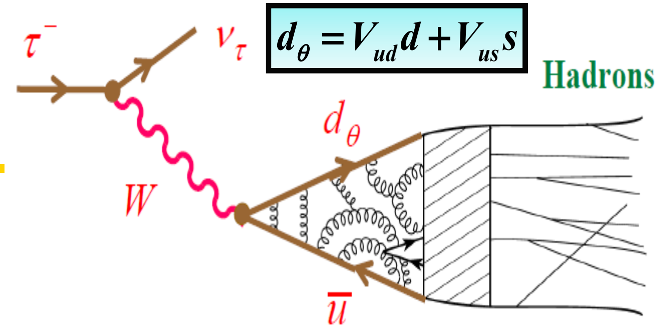
- $R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- Experimentally: $\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$

- Due to *QCD corrections*: $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$

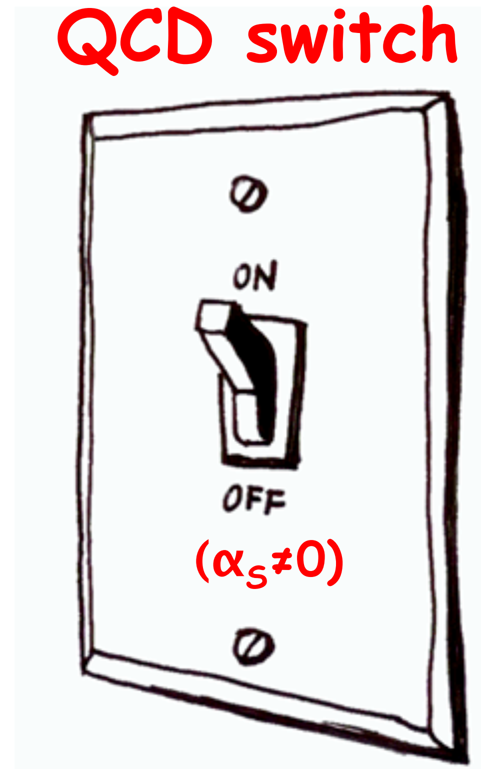


3.2 Theory



- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$ naïve QCD prediction



- Extraction of the strong coupling constant :

$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_s) \Rightarrow \alpha_S$$

measured calculated

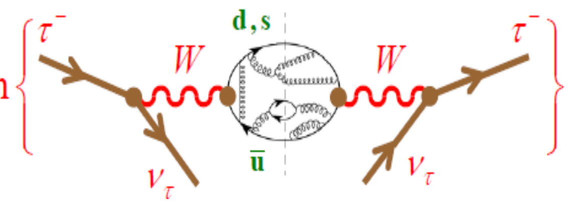
- Determination of V_{us} : $\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$

- Main difficulty: compute the QCD corrections with the best accuracy

3.3 Calculation of the QCD corrections

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \right.$$



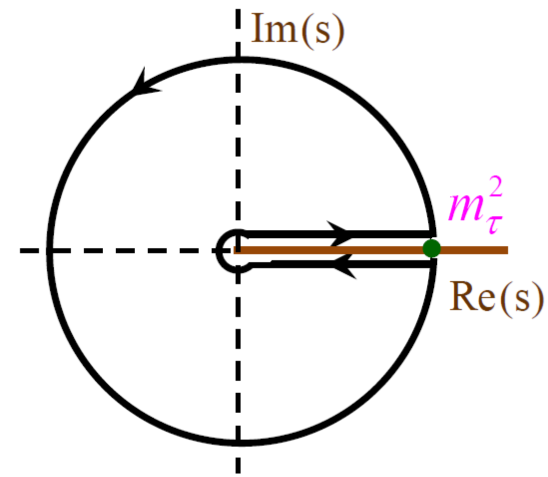
Braaten, Narison, Pich'92

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- Analyticity: Π is analytic in the entire complex plane except for s real positive

  ⇒ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \text{C}^{(J)}(s,\mu)\, \langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

3.3 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%$ $\quad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

3.3 Calculation of the QCD corrections

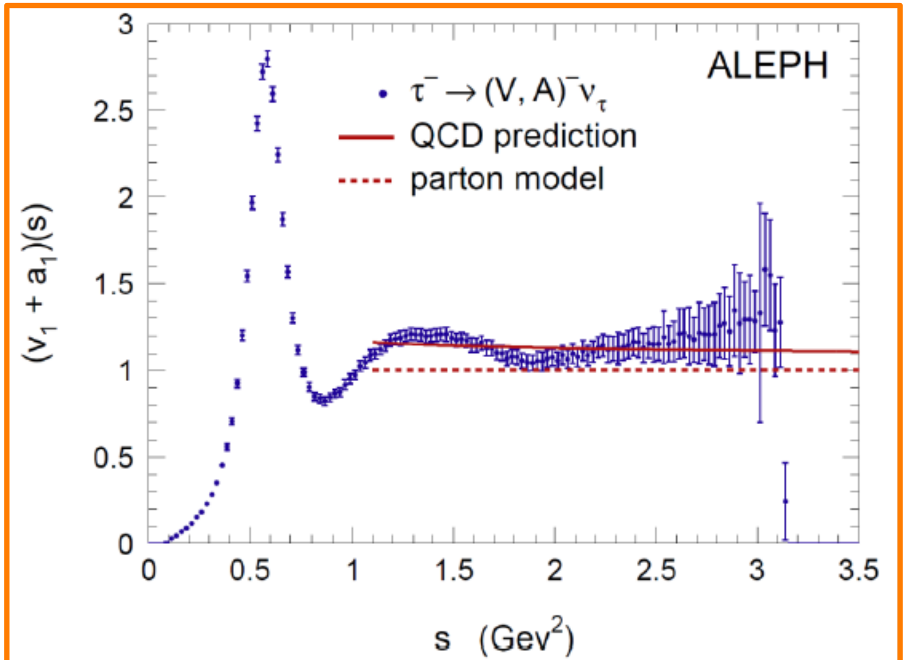
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

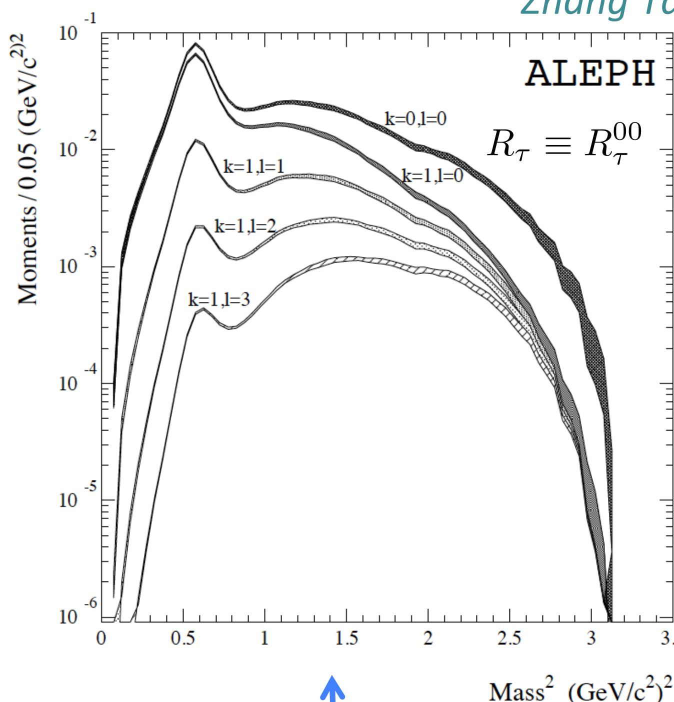
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R^{k\ell}_{\tau,U}(s_0) = \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

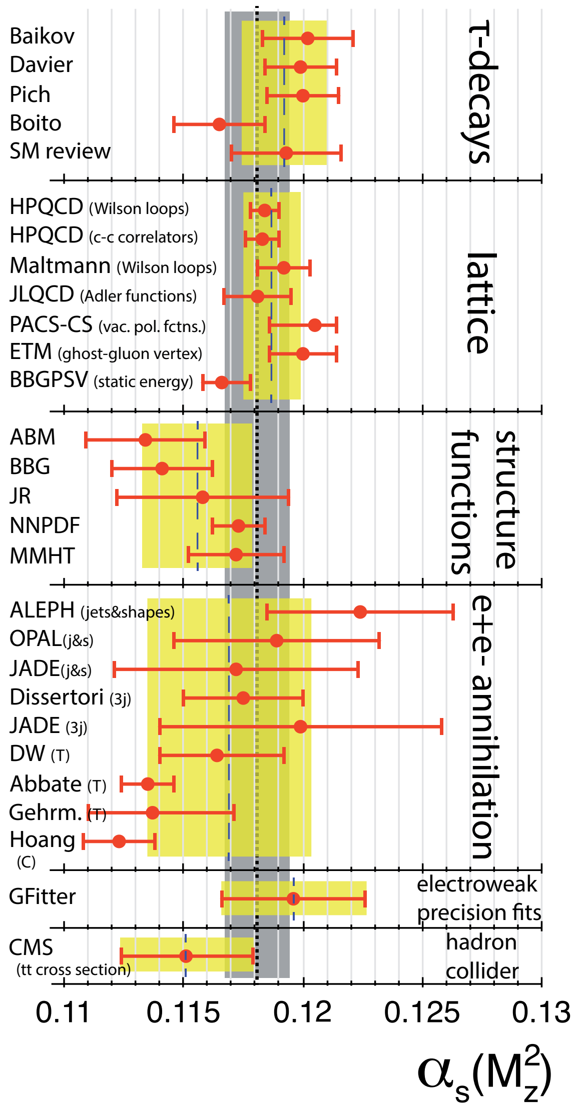


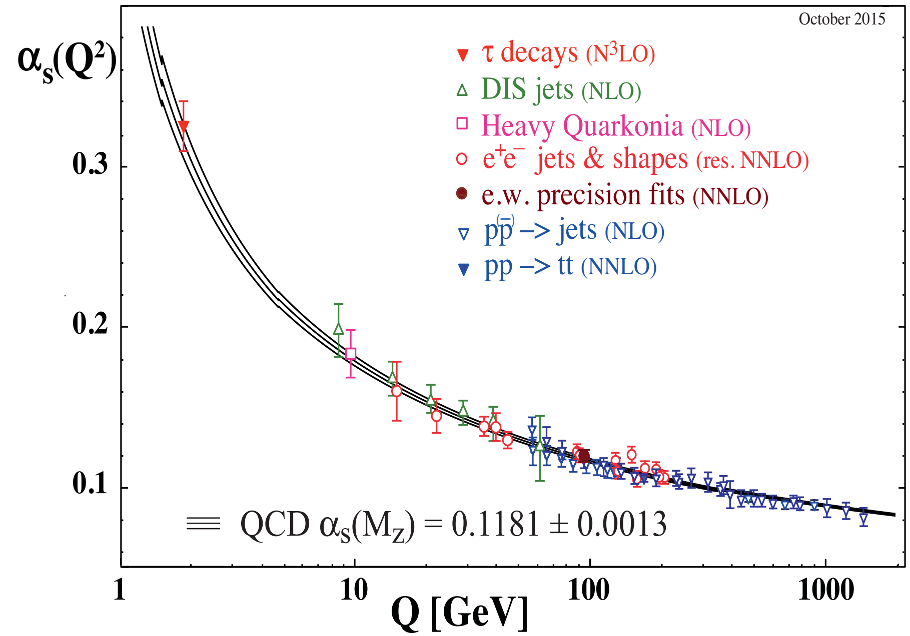
Zhang'Tau14



3.4 Extraction of α_s

Bethke, Dissertori, Salam, PDG'15





- *Extraction of α_s* from hadronic τ very interesting : Moderate precision at the τ mass ⇨ very good precision at the Z mass

- Beautiful test of the QCD running

3.4 Extraction of α_s

- Several delicate points:
  - How to compute the perturbative part: CIPT vs. FOPT?
  - How to estimate the non perturbative contribution? Where do we truncate the expansion, what is the role of higher order condensates?
  - Which weights should we use?
  - What about duality violations?

⇨ A MITP topical workshop in Mainz: March 7-12, 2016

Determination of the fundamental parameters of QCD

A session on Tuesday afternoon

- New data on spectral functions needed to help to answer some of these questions