Lepton Flavor Violation
at the LHC

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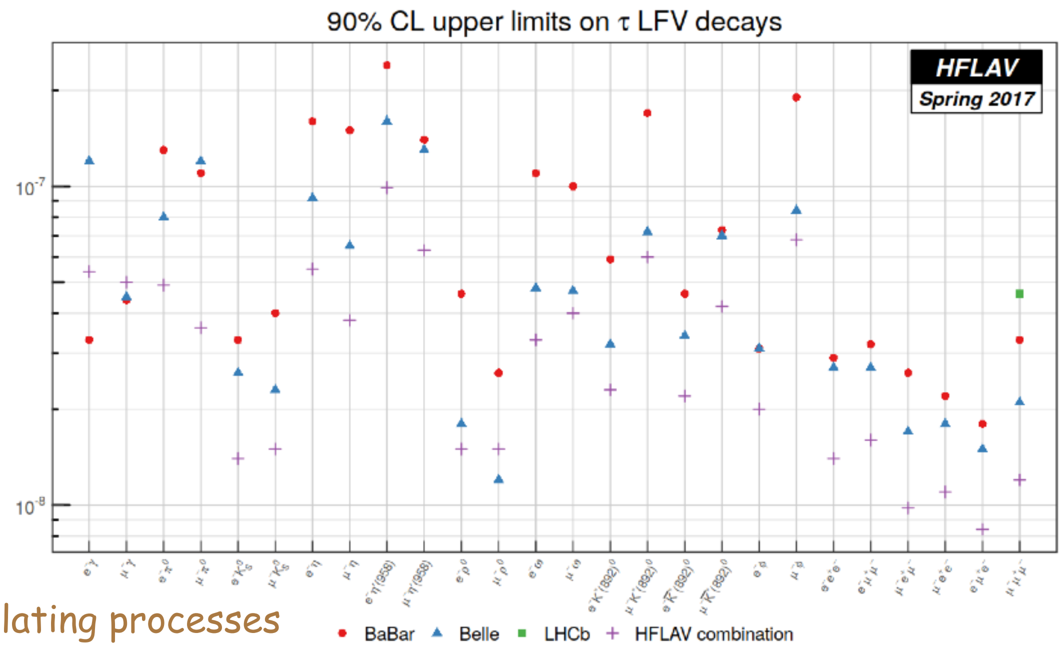
1. Introduction: leptonic FCNC

★ Why study flavor-changing neutral currents (FCNC)?

★ No trivial FCNC vertices in the Standard Model: sensitive NP tests

★ Possible experimental studies in a lepton sector

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+ e^- \to e^- \mu^+$
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - K^+ (B^+, D^+, ...) $\to \pi^+ \mu e$, $\pi^+ \tau e$, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$



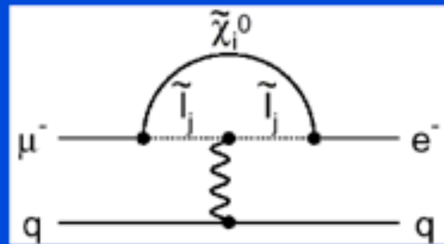
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm2) + e^{\mp}e^{\mp}$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

★ Highly suppressed in the Standard Model, e.g. $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu_i}}{M^2_W}\right|^2 < 10^{-54}$

A multitude of models…

Supersymmetry

Predictions at 10^{-15}

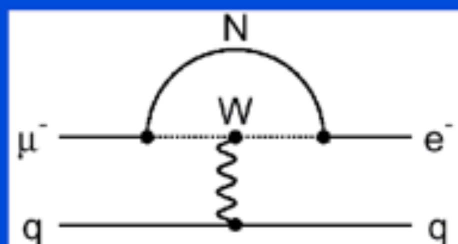


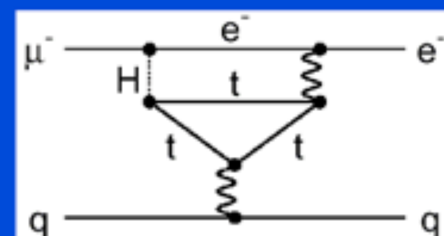
Compositeness

$\Lambda_c = 3000$ TeV

Heavy Neutrinos

$|U_{\mu N}^* U_{eN}|^2 = 8 \times 10^{-13}$





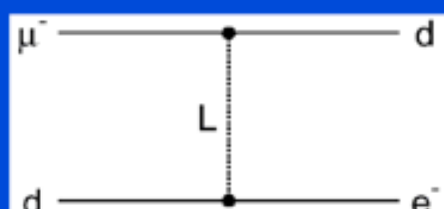
Second Higgs doublet

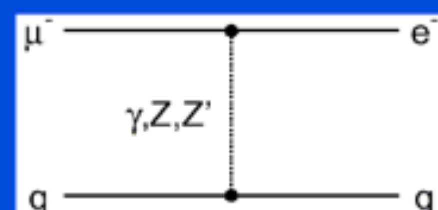
$g_{H_{\mu e}} = 10^{-4} \times g_{H_{\mu\mu}}$

Leptoquarks

$M_L =$

$3000\sqrt{\lambda_{\mu d}\lambda_{ed}}$ TeV/c^2





Heavy Z', Anomalous Z coupling

$M_{Z'} = 3000$ TeV/c^2

$B(Z \to \mu e) < 10^{-17}$

After W. Marciano

James Miller, 2006

High energy vs low energy

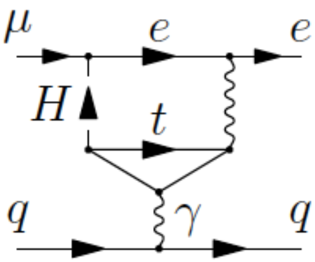
★ Leptonic FCNC could be generated by New Physics

★ E.g. FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$

Harnik, Kopp, Zupan

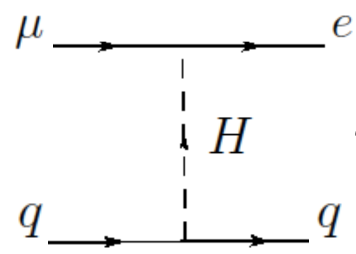
★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level

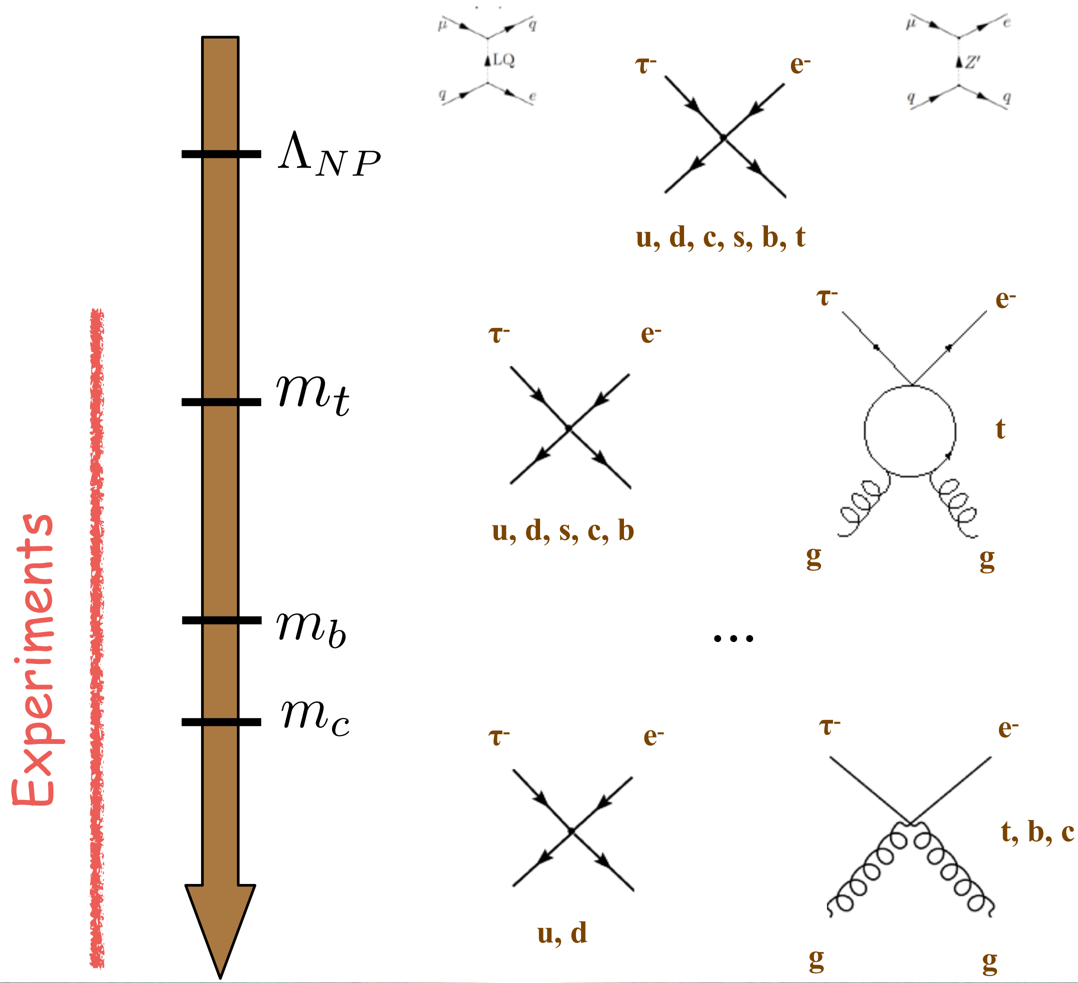
★ … but note: couplings of new physics to light quarks are suppressed

Can we correlate low energy (Belle/BESIII) and high energy (LHC) data?
(will not discuss purely leptonic LFV interactions)

2. Effective Lagrangians for LFV transitions

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy quarks

...

heavy quarks decouple

...

Scales associated with experiment

Effective Lagrangians

★ Naive power counting: largest contribution from lowest dimensional operators

★ Can write the most general LFV Lagrangian $\mathcal{L}_{LFV} = \mathcal{L}_D + \mathcal{L}_{lq} + \mathcal{L}_G + \ldots$

- dipole operators

$$\mathcal{L}_D = -\frac{m_2}{\Lambda^2}\left[\left(C_{DR}\overline{\ell}_1\sigma^{\mu\nu}P_L\ell_2 + C_{DR}\overline{\ell}_1\sigma^{\mu\nu}P_R\ell_2\right)F_{\mu\nu} + h.c.\right]$$

- four-fermion operators

$$\begin{aligned}\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2}\sum_q \Big[& \left(C_{VR}^{q\ell_1\ell_2}\ \overline{\ell}_1\gamma^\mu P_R\ell_2 + C_{VL}^{q\ell_1\ell_2}\ \overline{\ell}_1\gamma^\mu P_L\ell_2\right)\ \overline{q}\gamma_\mu q \\ & + \left(C_{AR}^{q\ell_1\ell_2}\ \overline{\ell}_1\gamma^\mu P_R\ell_2 + C_{AL}^{q\ell_1\ell_2}\ \overline{\ell}_1\gamma^\mu P_L\ell_2\right)\ \overline{q}\gamma_\mu\gamma_5 q \\ & + m_2 m_q G_F\left(C_{SR}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_L\ell_2 + C_{SL}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_R\ell_2\right)\ \overline{q}q \\ & + m_2 m_q G_F\left(C_{PR}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_L\ell_2 + C_{PL}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_R\ell_2\right)\ \overline{q}\gamma_5 q \\ & + m_2 m_q G_F\left(C_{TR}^{q\ell_1\ell_2}\ \overline{\ell}_1\sigma^{\mu\nu}P_L\ell_2 + C_{TL}^{q\ell_1\ell_2}\ \overline{\ell}_1\sigma^{\mu\nu}P_R\ell_2\right)\ \overline{q}\sigma_{\mu\nu}q\ +\ h.c.\ \Big].\end{aligned}$$

- gluonic operators

$$\begin{aligned}\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[& \left(C_{GR}\overline{\ell}_1 P_R\ell_2 + C_{GL}\overline{\ell}_1 P_L\ell_2\right)G_{\mu\nu}^a G^{a\mu\nu} \\ & + \left(C_{\tilde{G}R}\overline{\ell}_1 P_R\ell_2 + C_{\tilde{G}L}\overline{\ell}_1 P_L\ell_2\right)G_{\mu\nu}^a\widetilde{G}^{a\mu\nu} + h.c.\Big]\end{aligned}$$

Effective Lagrangians: designer states

★ There are many effective operators, so a single operator dominance hypothesis (SODH) is usually applied to get constraints on relevant Wilson coefficients.

$$\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2}\sum_q \Big[\left(C_{VR}^{q\ell_1\ell_2}\, \bar{\ell}_1\gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2}\, \bar{\ell}_1\gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu q$$
$$+ \left(C_{AR}^{q\ell_1\ell_2}\, \bar{\ell}_1\gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2}\, \bar{\ell}_1\gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu\gamma_5 q$$
$$+ m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q$$
$$+ m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q}\gamma_5 q$$
$$+ m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q}\sigma_{\mu\nu} q \ + \ h.c. \Big].$$

- Can (partially) do away with SODH if designer initial/final states are used
- This can be done in case of restricted kinematics (e.g. 2-body decays)

★ Much tighter constraints are obtained from lepton radiative decays: drop from quarkonium decay analyses in what follows

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2}\sum_q \Bigg[& \left(C_{VR}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{VL}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_L\ell_2\right)\ \bar{q}\gamma_\mu q \\
& + \left(C_{AR}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{AL}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_L\ell_2\right)\ \bar{q}\gamma_\mu\gamma_5 q \\
& + m_2 m_q G_F\left(C_{SR}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_L\ell_2 + C_{SL}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_R\ell_2\right)\ \bar{q}q \\
& + m_2 m_q G_F\left(C_{PR}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_L\ell_2 + C_{PL}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_R\ell_2\right)\ \bar{q}\gamma_5 q \\
& + m_2 m_q G_F\left(C_{TR}^{q\ell_1\ell_2}\ \bar{\ell}_1\sigma^{\mu\nu} P_L\ell_2 + C_{TL}^{q\ell_1\ell_2}\ \bar{\ell}_1\sigma^{\mu\nu} P_R\ell_2\right)\ \bar{q}\sigma_{\mu\nu} q + h.c. \Bigg].
\end{aligned}
$$

also dipole operators

Vector meson decays: $\Upsilon(nS)\to\bar{\mu}\tau, \psi(nS)\to\bar{\mu}\tau, \rho\to\bar{\mu}e, \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
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& + \boxed{\left(C_{AR}^{q\ell_1\ell_2}\ \overline{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2}\ \overline{\ell}_1 \gamma^\mu P_L \ell_2 \right)\ \overline{q}\gamma_\mu \gamma_5 q} \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_R \ell_2 \right)\ \overline{q} q \\
& + \boxed{m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2}\ \overline{\ell}_1 P_R \ell_2 \right)\ \overline{q}\gamma_5 q} \\
& + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2}\ \overline{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2}\ \overline{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right)\ \overline{q}\sigma_{\mu\nu} q \ + \ h.c. \Big].
\end{aligned}
$$

also gluonic operators

Pseudoscalar meson decays: $\eta_b \to \overline{\mu} e, \eta_c \to \overline{\mu}\tau, \eta^{(\prime)} \to \overline{\mu} e, \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023
D. Hazard and A.A.P., arXiv:1711.05314

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q \Big[& \left(C_{VR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu q \\
& + \left(C_{AR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu \gamma_5 q \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q \\
& + m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q}\gamma_5 q \\
& + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q}\sigma_{\mu\nu} q \; + \; h.c. \Big].
\end{aligned}
$$

also gluonic operators

Scalar meson decays: $\chi_{b0} \to \bar{\mu}\tau,\ \chi_{c0} \to \bar{\mu}\tau,\ \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023

3a. LFV vector quarkonia decays

★ Most LFV experimental data available V → μe, τe, etc.

$\ell_1\ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(\Upsilon(1S)\to\ell_1\ell_2)$	6.0×10^{-6}	$\cdots$	$\cdots$
$\mathcal{B}(\Upsilon(2S)\to\ell_1\ell_2)$	3.3×10^{-6}	3.2×10^{-6}	$\cdots$
$\mathcal{B}(\Upsilon(3S)\to\ell_1\ell_2)$	3.1×10^{-6}	4.2×10^{-6}	$\cdots$
$\mathcal{B}(J/\psi\to\ell_1\ell_2)$	2.0×10^{-6}	8.3×10^{-6}	1.6×10^{-7}
$\mathcal{B}(\phi\to\ell_1\ell_2)$	FPS	FPS	4.1×10^{-6}
$\mathcal{B}(\ell_2\to\ell_1\gamma)$	4.4×10^{-8}	3.3×10^{-8}	5.7×10^{-13}

★ Decay amplitude:

$$\mathcal{A}(V\to\ell_1\bar{\ell}_2)=\bar{u}(p_1,s_1)\left[A_V^{\ell_1\ell_2}\gamma_\mu+B_V^{\ell_1\ell_2}\gamma_\mu\gamma_5+\frac{C_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu\right.$$
$$\left.+\frac{iD_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu\gamma_5\right]v(p_2,s_2)\ \epsilon^\mu(p).$$

★ Decay rate:

$$\frac{\mathcal{B}(V\to\ell_1\bar{\ell}_2)}{\mathcal{B}(V\to e^+e^-)}=\left(\frac{m_V(1-y^2)}{4\pi\alpha f_V Q_q}\right)^2[(|A_V^{\ell_1\ell_2}|^2+|B_V^{\ell_1\ell_2}|^2)$$
$$+\frac{1}{2}(1-2y^2)(|C_V^{\ell_1\ell_2}|^2+|D_V^{\ell_1\ell_2}|^2)$$
$$+y\mathrm{Re}(A_V^{\ell_1\ell_2}C_V^{\ell_1\ell_2*}+iB_V^{\ell_1\ell_2}D_V^{\ell_1\ell_2*})].$$

Form-factors depend on vector, tensor, and dipole Wilson coefficients

LFV vector quarkonia decays

★ Most general decay rate for V → μe, τe, etc. ($V = \Upsilon(nS), \psi(nS), \rho, \phi, \ldots$):

$$\frac{\mathcal{B}(V \to \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \to e^+ e^-)} = \left(\frac{m_V(1-y^2)}{4\pi\alpha f_V Q_q}\right)^2 [(|A_V^{\ell_1\ell_2}|^2 + |B_V^{\ell_1\ell_2}|^2) + \frac{1}{2}(1-2y^2)(|C_V^{\ell_1\ell_2}|^2 + |D_V^{\ell_1\ell_2}|^2) + y\mathrm{Re}(A_V^{\ell_1\ell_2} C_V^{\ell_1\ell_2 *} + i B_V^{\ell_1\ell_2} D_V^{\ell_1\ell_2 *})].$$

D. Hazard and A.A.P., PRD94 (2016), 074023

… and the decay rate is

$$A_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2}[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}) + \kappa_V (C_{VL}^{q\ell_1\ell_2} + C_{VR}^{q\ell_1\ell_2}) + 2y^2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} + C_{TR}^{q\ell_1\ell_2})],$$

$$B_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2}[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}) - \kappa_V (C_{VL}^{q\ell_1\ell_2} - C_{VR}^{q\ell_1\ell_2}) - 2y^2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} - C_{TR}^{q\ell_1\ell_2})],$$

$$C_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2} y[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1\ell_2} + C_{DR}^{\ell_1\ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} + C_{TR}^{q\ell_1\ell_2})],$$

$$D_V^{\ell_1\ell_2} = i\frac{f_V m_V}{\Lambda^2} y[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1\ell_2} - C_{DR}^{\ell_1\ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1\ell_2} - C_{TR}^{q\ell_1\ell_2})].$$

LFV vector quarkonia decays: 4f operators

★ Constraints on Wilson coefficients of four-fermion low energy operators

Wilson coefficient [GeV^{-2}]	Leptons $\ell_1\ell_2$	Initial state (quark) $\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$
$\lvert C_{VL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	$\cdots$	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.0×10^{-5}	2×10^{-3}
$\lvert C_{VR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	$\cdots$	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.0×10^{-5}	2×10^{-3}
$\lvert C_{TL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	$\cdots$	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	4.8	1×10^{4}
$\lvert C_{TR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	$\cdots$	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	4.8	1×10^{4}

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LFV (pseudo)scalar quarkonia decays

★ Most general decay rate for P/S → μe, τe, etc ($P = \eta_b, \eta_c, \eta^{(\prime)}, \ldots$):
$S = \chi_{b0}, \chi_{c0}, \ldots$

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$$\mathcal{B}(M \to \ell_1 \bar{\ell}_2) = \frac{m_M}{8\pi\Gamma_M} \left(1 - y^2\right)^2 \left[\left|E_M^{\ell_1\ell_2}\right|^2 + \left|F_M^{\ell_1\ell_2}\right|^2\right]$$

... for pseudoscalar operators

$$E_P^{\ell_1\ell_2} = y\frac{m_P}{4\Lambda^2}\left[-i f_P \left[2\left(C_{AL}^{cc\ell_1\ell_2} + C_{AR}^{cc\ell_1\ell_2}\right) - m_P^2 G_F \left(C_{PL}^{cc\ell_1\ell_2} + C_{PR}^{cc\ell_1\ell_2}\right)\right]\right]$$

$$F_P^{\ell_1\ell_2} = -y\frac{m_P}{4\Lambda^2}\left[f_P \left[2\left(C_{AL}^{cc\ell_1\ell_2} - C_{AR}^{cc\ell_1\ell_2}\right) - m_P^2 G_F \left(C_{PL}^{cc\ell_1\ell_2} - C_{PR}^{cc\ell_1\ell_2}\right)\right]\right]$$

... and for scalar operators

$$E_S^{\ell_1\ell_2} = i y f_S m_c \frac{m_S^2 G_F}{2\Lambda^2}\left(C_{SL}^{ccl_1l_2} + C_{SR}^{ccl_1l_2}\right)$$

$$F_S^{\ell_1\ell_2} = y f_S m_c \frac{m_S^2 G_F}{2\Lambda^2}\left(C_{SL}^{ccl_1l_2} - C_{SR}^{ccl_1l_2}\right)$$

Gluonic operators?

LFV pseudoscalar/scalar quarkonia decays

★ Constraints on Wilson coefficients of low energy operators

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Wilson coefficient	Leptons $\ell_1\ell_2$	Initial state η_b	η_c	$\eta(u/d)$	$\eta(s)$	$\eta'(u/d)$	$\eta'(s)$
$\|C_{AL}^{q\ell_1\ell_2}/\Lambda^2\|$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$\|C_{AR}^{q\ell_1\ell_2}/\Lambda^2\|$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$\|C_{PL}^{q\ell_1\ell_2}/\Lambda^2\|$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	2×10^{3}	1×10^{3}	3.9×10^{4}	3.6×10^{4}
$\|C_{PR}^{q\ell_1\ell_2}/\Lambda^2\|$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	2×10^{3}	1×10^{3}	3.9×10^{4}	3.6×10^{4}

★ More data is needed: use radiative decays: $\mathcal{B}(V\to\gamma\ell_1\overline{\ell}_2)=\mathcal{B}(V\to\gamma M)\mathcal{B}(M\to\ell_1\overline{\ell}_2)$

$$\mathcal{B}(\psi(2S)\to\gamma\chi_{c0}(1P))=9.99\pm0.27\%,$$
$$\mathcal{B}(\psi(3770)\to\gamma\chi_{c0}(1P))=0.73\pm0.09\%.$$
$$\mathcal{B}(J/\psi\to\gamma\eta_c)=1.7\pm0.4\%,$$
$$\mathcal{B}(\psi(2S)\to\gamma\eta_c)=0.34\pm0.05\%.$$

$$\mathcal{B}(\Upsilon(2S)\to\gamma\chi_{b0}(1P))=3.8\pm0.4\%,$$
$$\mathcal{B}(\Upsilon(3S)\to\gamma\chi_{b0}(1P))=0.27\pm0.04\%,$$
$$\mathcal{B}(\Upsilon(3S)\to\gamma\chi_{b0}(2P))=5.9\pm0.6\%.$$

LFV pseudoscalar/scalar quarkonia decays

★ Very scarce LFV experimental data available P/S → μe, τe, etc.

- no data for pseudoscalar heavy-flavored meson decays
- no data for any scalar meson decays
- maybe use B-decays?

$\ell_1\ell_2$	$e\mu$
$\mathcal{B}(\eta \to \ell_1\ell_2)$	6×10^{-6}
$\mathcal{B}(\eta' \to \ell_1\ell_2)$	4.7×10^{-4}
$\mathcal{B}(\pi^0 \to \ell_1\ell_2)$	3.6×10^{-10}

$$P = \eta_b, \eta_c, \eta^{(\prime)}, \ldots$$
$$S = \chi_{b0}, \chi_{c0}, \ldots$$

★ Constraints are available for quark off-diagonal currents from B/D → μe, τe, etc.

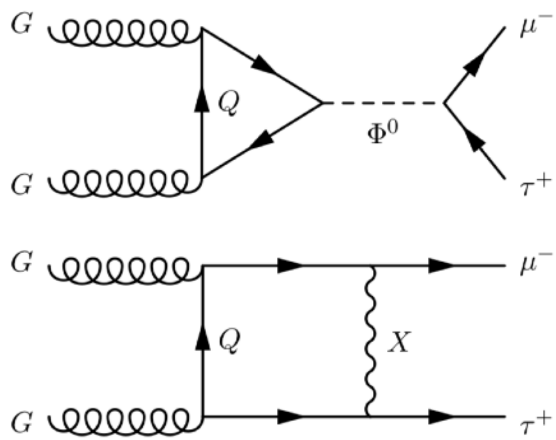
$\ell_1\ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(B_d^0 \to \ell_1\ell_2)$	2.2×10^{-5}	2.8×10^{-5}	1.0×10^{-9}
$\mathcal{B}(B_s^0 \to \ell_1\ell_2)$	$\cdots$	$\cdots$	5.4×10^{-9}
$\mathcal{B}(\bar{D}^0 \to \ell_1\ell_2)$	FPS	$\cdots$	1.3×10^{-8}
$\mathcal{B}(K_L^0 \to \ell_1\ell_2)$	FPS	FPS	4.7×10^{-12}

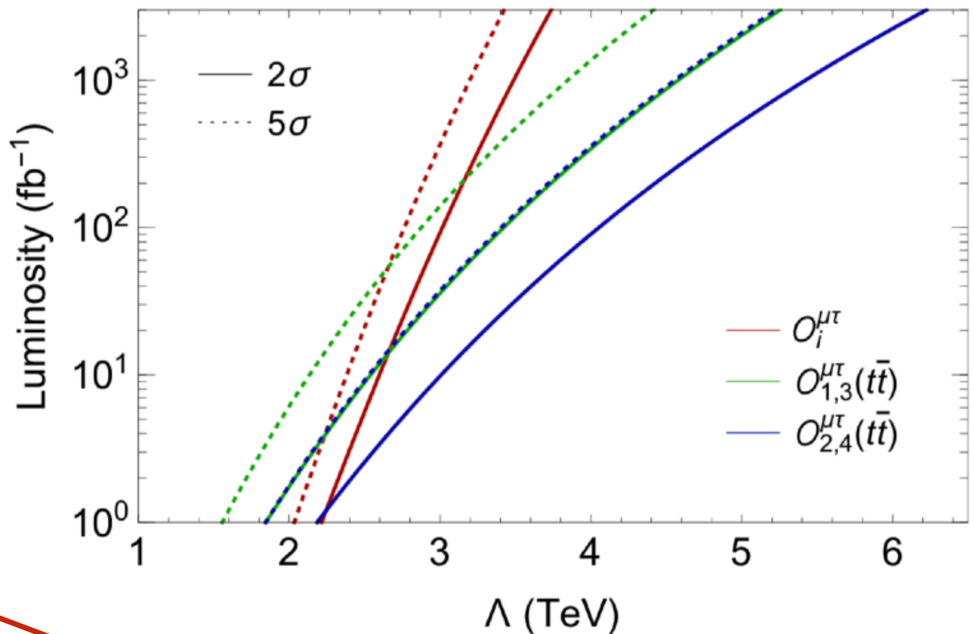
D. Hazard and A.A.P., PRD94 (2016), 074023
D. Hazard and A.A.P., arXiv:1711.05314

3b. Probing LFV gluonic operators with LHC

★ Large gluon luminosity of the LHC makes it possible to probe gluonic operators at LHC directly

Bhattacharya, Morgan, Osborne, AAP: arXiv: 1802.06082





NP "Q"

SM "Q"

$$\mathcal{L}_{\rm eff} = \sum_{i=1}^{4} \mathcal{C}_i^{\mu\tau} \mathcal{O}_i^{\mu\tau} + {\rm h.c.}\,,$$

$$\begin{aligned}
\mathcal{O}_1^{\mu\tau} &= (\overline{\mu}_L \tau_R)\, G\cdot G\,, \qquad C_i^{\mu\tau} = 4\pi v\, g_s^2/\sqrt{2}\Lambda^4 \\
\mathcal{O}_2^{\mu\tau} &= (\overline{\mu}_L \tau_R)\, G\cdot \widetilde{G}\,, \\
\mathcal{O}_3^{\mu\tau} &= (\overline{\mu}_R \tau_L)\, G\cdot G\,, \\
\mathcal{O}_4^{\mu\tau} &= (\overline{\mu}_R \tau_L)\, G\cdot \widetilde{G}\,.
\end{aligned}$$

$$\boxed{\begin{aligned} \Lambda_{2\sigma} &\approx 3000 \text{ GeV} \\ \Lambda_{5\sigma} &\approx 2800 \text{ GeV} \end{aligned}}$$

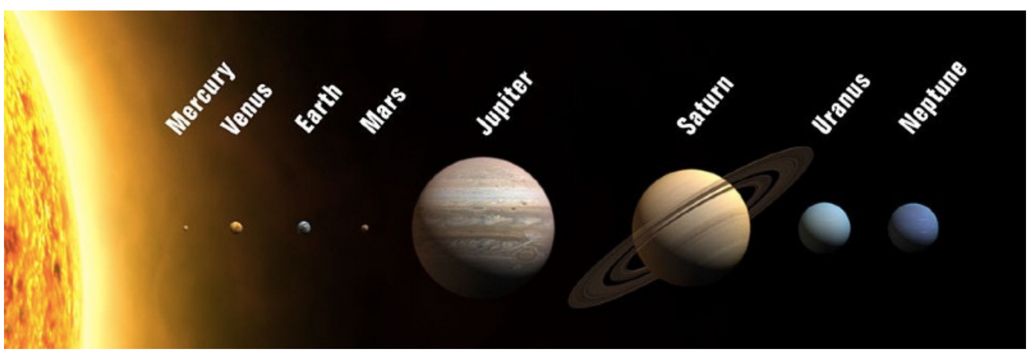
$$\mathcal{L}_{\mu\tau}^{(6)} \supset \frac{1}{\Lambda^2} \sum_{i=1}^{4} C_i^{q\mu\tau} \mathcal{O}_i^{q\mu\tau} + {\rm h.c.}\,,$$

$$\begin{aligned}
\mathcal{O}_1^{q\mu\tau} &= (\overline{\mu}_L \tau_R)\,(\overline{q}_L q_R) \\
\mathcal{O}_2^{q\mu\tau} &= (\overline{\mu}_L \tau_R)\,(\overline{q}_R q_L) \\
\mathcal{O}_3^{q\mu\tau} &= (\overline{\mu}_R \tau_L)\,(\overline{q}_L q_R) \\
\mathcal{O}_4^{q\mu\tau} &= (\overline{\mu}_R \tau_L)\,(\overline{q}_R q_L)
\end{aligned}$$

Can probe LFV couplings of top quarks!

4. Takeaway

- Flavor-changing neutral current transitions provide great opportunities for studies of lepton flavor in the SM and BSM
  - charge lepton transitions offer practically SM-background-free playground
    - large contributions from New Physics are possible, but not seen
    - EFT approach can be useful in studies of quarkonium/tau FCNC decays
    - ... as current methods do not constrain NP-heavy fermion couplings very well
    - LFV decays might shed some light on LFUV transitions in B-decays
- Need more data from Belle-II (or LHCb) on LFV quarkonia decays!
  - there is NO DATA for LFV radiative decays, e.g. $\psi(nS) \to \gamma \overline{\mu} e, \gamma \overline{\mu} \tau, \ldots$
- More data from ATLAS/CMS/(LHCb?) on pp $\to \tau\mu$ + X
  - studies of gluonic operators from gg $\to \tau\mu$ due to large gluon luminosity of LHC



- Goal: global analysis of LFV.
- Is there a flavor problem?

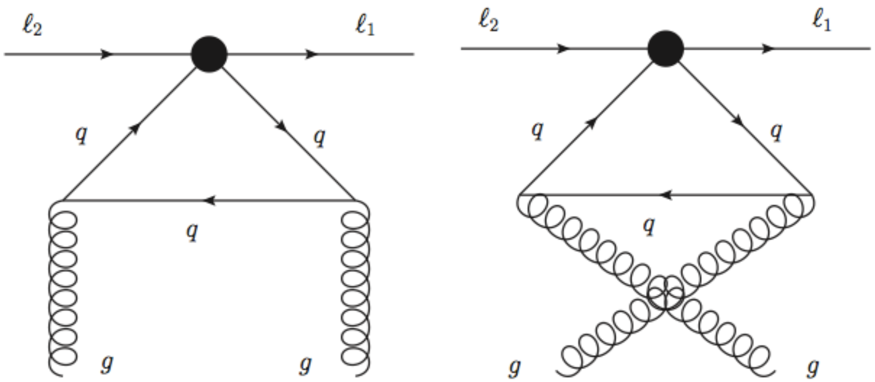
Why is $m_t \gg m_u$?

Why is $M_{Jupiter} \gg M_{Mercury}$?


EXTRA!

Effective Lagrangians: gluonic operators

★ Coefficients of gluonic operators depend on the number of active flavors



$$\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[\Big(C_{GR}\overline{\ell}_1 P_R \ell_2 + C_{GL}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu} G^{a\mu\nu} + \Big(C_{\tilde{G}R}\overline{\ell}_1 P_R \ell_2 + C_{\tilde{G}L}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} + h.c.\Big]$$

★ we can calculate their contribution to meson or tau decay rates!
★ also relevant for muon conversion experiments
★ c_i probe couplings of heavy quarks to New Physics

AAP and D. Zhuridov
PRD89 (2014) 3, 033005

Effective Lagrangians: gluonic operators

★ ... get an effective Lagrangian

$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \text{H.c.},$$

AAP and D. Zhuridov
PRD89 (2014) 3, 033005

...where we defined operators

$$O_1^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_2^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

$$O_3^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_4^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

...and Wilson coefficients

$$c_1^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} + C_2^{q\ell_1\ell_2}\right),$$

$$c_2^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} - C_2^{q\ell_1\ell_2}\right),$$

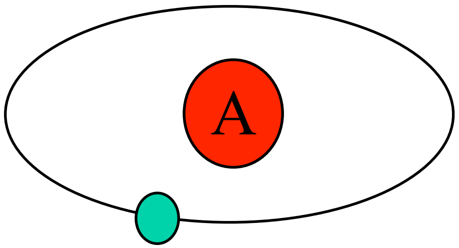
$$c_3^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} + C_4^{q\ell_1\ell_2}\right),$$

$$c_4^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} - C_4^{q\ell_1\ell_2}\right),$$

$$I_1 = \frac{1}{3}, \qquad I_2 = \frac{1}{2}.$$

Probing LFV gluonic operators with muon conversion

★ Basic idea for the muon conversion experiment



★ take low energy muons (~ 30 MeV) to be stopped in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^{\ell} Z^{3/2}$
overlap probability: $p \sim r^{2\ell} Z^3$

large overlap for an s-wave and high-Z nucleus

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

Experimental ideas

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

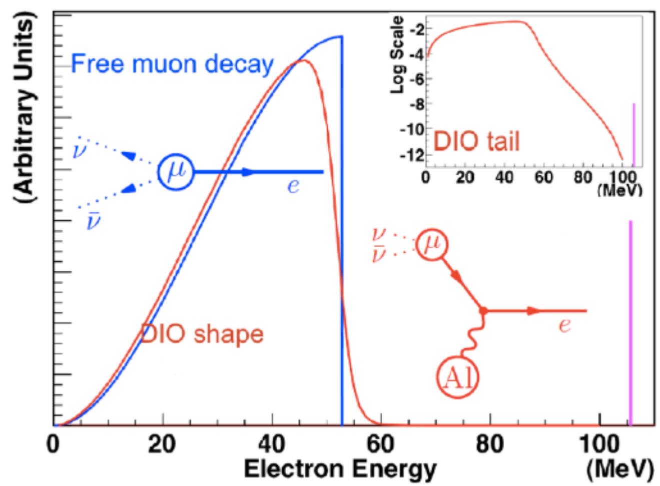
★ The experiment is tricky

✓ Muon conversion gives monoenergetic electrons...
✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

$$\text{SINDRUM II (PSI), 2006}: \quad R_{\mu e} < 7 \times 10^{-13}$$

$$\text{M2e goal}: \quad R_{\mu e} < \text{a few} \times 10^{-17}$$



Czarnecki, Marciano, Tormo

Probing LFV gluonic operators with muon conversion

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Nuclear averages are often done as an approximation. For a general operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z\rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle\right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \qquad \int d^3\rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) \ll m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

Probing LFV gluonic operators with muon conversion

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Lepton wave functions are taken as solutions of Dirac equation
- with usual substitutions $u_1(r) = r\, g(r)$ and $u_2(r) = r\, f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi^\mu_\kappa = \begin{pmatrix} g(r)\chi^\mu_\kappa(\theta,\phi) \\ i f(r)\chi^\mu_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

$$\text{SINDRUM II (PSI), 2006 :} \qquad R_{\mu e} < 7 \times 10^{-13}$$

$$\text{M2e goal :} \qquad R_{\mu e} < \text{a few} \times 10^{-17}$$

Probing gluonic operators with muons

★ Let's calculate the conversion amplitude

$$M_{NN'}^{\mu e} = \frac{1}{\Lambda^2 M_Q}\int d^3x \left[\left(c_1\bar{\psi}_{\kappa,W}^{\mu(e)}P_L\psi_{1s}^{(\mu)} + c_3\bar{\psi}_{\kappa,W}^{\mu(e)}P_R\psi_{1s}^{(\mu)}\right)\langle N'|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a G^{a\mu\nu}|N\rangle + \left(c_2\bar{\psi}_{\kappa,W}^{\mu(e)}P_L\psi_{1s}^{(\mu)} + c_4\bar{\psi}_{\kappa,W}^{\mu(e)}P_R\psi_{1s}^{(\mu)}\right)\langle N'|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a \widetilde{G}^{a\mu\nu}|N\rangle\right]$$

★ Relate nucleon and nuclear matrix elements...

$$\left\langle N\left|\frac{\beta_L}{4\alpha_s}G_{\mu\nu}^a G^{a\mu\nu}\right|N\right\rangle = -\frac{9}{2}\left[ZG^{(g,p)}\rho^{(p)} + (A-Z)G^{(g,n)}\rho^{(n)}\right]$$

★ ... and calculate (relevant) parity-conserving nucleon matrix element

$$G^{(g,\mathcal{N})} = \left\langle \mathcal{N}\left|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a G^{a\mu\nu}\right|\mathcal{N}\right\rangle = \text{-189 MeV}$$

Numerical estimates

★ Conversion probability (factoring out lightest heavy quark mass)

$$\Gamma_{\text{conv}}(\mu N \to eN) = \frac{4}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right) a_N^2$$

★ ... where we defined $a_N = G^{(g,p)} S^{(p)} + G^{(g,n)} S^{(n)}$

★ ... and also

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A-Z) \rho^{(n)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right)$$

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}$$

Nucleus	Model	c, fm	z, fm	$S^{(p)}$	$S^{(n)}$
${}^{48}_{22}\mathrm{Ti}$	FB	–	–	0.0368	0.0435
${}^{197}_{79}\mathrm{Au}$	2pF	6.38	0.535	0.0614	0.0918

TABLE I. Nucleon densities model parameters and the overlap integrals in the unit of $m_\mu^{5/2}$ for several nuclei.

Numerical estimates

★ Conversion probability

$$B^N_{\mu e} \equiv \Gamma_{\rm conv}(\mu^- N \to e^- N_{\rm g.s.})/\Gamma_{\rm capture}(\mu^- N)$$

★ ... results in constraints on scale/Wilson coefficient

Coefficient	Bound on $\lvert c_i^{e\mu}\rvert/\Lambda^2$, GeV^{-3}	
	conversion on $^{48}_{22}$Ti	conversion on $^{197}_{79}$Au
c_1	2.5×10^{-11}	1.2×10^{-11}
c_2	—	—
c_3	2.5×10^{-11}	1.2×10^{-11}
c_4	—	—

AAP and D. Zhuridov
PRD89 (2014) 3, 033005

★ Important: can only probe parity-conserving operators!!!