Integrating SLRS results with Geodetic Network Adjustment.

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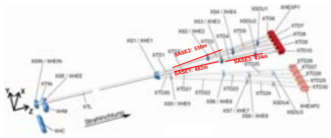
Straight Line Reference

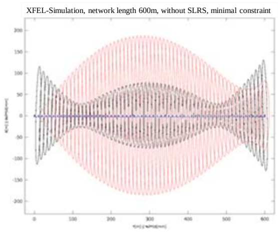


Accuracy demands for XFEL are as of 0.5mm/500m. This is not reliably achievable with normal angular measurements as with laser trackers or theodolites.
A Straight Line Reference System (SLRS) was developed to overcome these restrictions and to guarantee the required accuracy.
This SLRS is not mechanically connected to accelerator components, it is a separate system installed "somewhere" in the tunnel. It can be imagined as a giant ruler; during the normal measurement of the tunnel network additional observations of the "ruler" have to be made to stabilize the network.

The SLRS does not produce "normal" geodetic observations, like angles, distances or height differences.
Instead it produces a different type of observations, called "straightness".

To be able to adjust the complete tunnel network with a geodetic Gauss-Markov model, a new type of observations has to be included in the model.
A new software that can include this type of observations has to be developed.

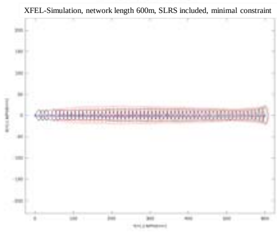
Network accuracy w/o SLRS (simulation)



BLACK: Absolute error ellipses of network points, M=50:1
RED: Relative error ellipses of network points related to P0 at 0m, 50:1

The simulation of a linear tunnel network, measured with laser tracker, shows clearly that with an à priori accuracy of $\sigma_h = \sigma_z = 0.15$mgon and $\sigma_d = 0.05$mm we get a far worse à posteriori accuracy in lateral direction and height as in longitudinal direction.
While the accuracy for height can be greatly improved by introducing gravity related measurement – either by levelling or by using the vertical axis information of the laser tracker – there is no common geodetic type of measurement that can improve the lateral accuracy of this network.
The simulation shows the effect of random errors only, systematic effects, mostly because of refraction in the tunnel, are generally unknown, but could sum up to a multitude of the simulated accuracy.

Network accuracy with SLRS (simulation)



BLACK: Absolute error ellipses of network points, M=50:1
RED: Relative error ellipses of network points related to P0 at 0m, 50:1

Including SLRS straightness information into the same network shows that the single point accuracy as well as the relative accuracy are greatly improved.

The simulation shows that the accuracy requirement of 0.5mm/600m in horizontal and height is fulfilled, the relative error (P=99%) is up to 0.4mm in horizontal.

The systematic effect of refraction is also greatly reduced due to the vacuum system (~1mbar).

Network adjustment, SLRS data included

The network adjustment software used at DESY (PANDA) is not able to integrate constraints in a convenient way. Because of time restrictions, it was not possible to have a dedicated adjustment software ready in time.

The actual approach for solving a network with SLRS data is the following:

1. Estimate a free network solution (laser tracker and levelling data) **without** using SLRS information, however, this solution **does** include SLRS monuments measured by laser tracker
2. **Compare** actual solution of SLRS monuments of free network solution (1.) with SLRS measurement and **calculate corrections**
3. **Apply corrections** from SLRS (2.) to actual point coordinates estimated by free network solution (1.)
4. Introduce corrected point coordinates (3.) of SLRS monuments into new network adjustment as **fixed points**, thus making it an over-constrained network solution.
5. Since the network was estimated as free network first, this step (4.) does not introduce any constraints but those coming from SLRS

The actual solution will be close to the correct solution, concerning the coordinates of all reference monuments.
However, the stochastic model is wrong. This is not important for actual alignment work, but matters for possible analysis in the future, like deformation analysis.

Adjustment models

There are three different models of Least Squares Adjustment, that all provide Best Linear Unbiased Estimators (BLUE).

Least Squares Model

$$\varphi(X) = L$$

A simple model where each single observation from the total vector of observations (L) has to be expressed by a function of the unknowns (X). This model is used in PANDA and lots of other geodetic network adjustment packages. When using only geodetic observations (horizontal and vertical angles, distances, height differences) this is the most appropriate model because of its relative simplicity and smaller matrices compared to the generalized model.

Generalized Least Squares Model

$$\varphi(L, X) = 0$$

The Least Squares Model is a special case of this Generalized LSM. When using non-standard observations, or having unknowns other than coordinates of network points, it is sometimes difficult or even impossible to separate observations from unknowns in the functional equations. In this case the Generalized LSM provides a solution, but at the cost of a higher computational effort and a more complicated model.

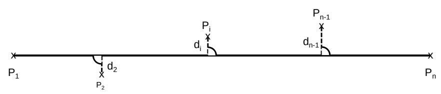
Least Squares Model with constraints

$$\varphi_1(X) = L\ ;\ \varphi_2(X) = 0$$

This model introduces additional constraints between unknowns. It is possible to constrain certain network monuments to form a straight line, a well-defined curve, ellipse, circle, etc. This model generally has similar computational cost as the standard LSM, however it is still necessary to express each single observation as a function of the unknowns.

Constrained Model with SLRS observations

SLRS observations constrain certain points to form a straight line. With n being the number of SLRS measurements, we get n-2 constraints.



Functional Model (SLRS only)

$$\varphi_2(X) = \begin{bmatrix} d_2 \\ \vdots \\ d_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{|(\vec{P}_2 - \vec{P}_1) \times (\vec{P}_n - \vec{P}_1)|}{|\vec{P}_n - \vec{P}_1|} \\ \vdots \\ \frac{|(\vec{P}_{n-1} - \vec{P}_1) \times (\vec{P}_n - \vec{P}_1)|}{|\vec{P}_n - \vec{P}_1|} \end{bmatrix} = 0$$

$$\varphi_2(X) = \begin{bmatrix} |(\vec{P}_2 - \vec{P}_1) \times (\vec{P}_n - \vec{P}_1)| \\ \vdots \\ |(\vec{P}_{n-1} - \vec{P}_1) \times (\vec{P}_n - \vec{P}_1)| \end{bmatrix} = 0$$

or, equivalent but numerically more stable

$$\varphi_2(X) = \begin{bmatrix} |(\vec{P}_3 - \vec{P}_2) \times (\vec{P}_2 - \vec{P}_1)| \\ \vdots \\ |(\vec{P}_n - \vec{P}_{n-1}) \times (\vec{P}_{n-1} - \vec{P}_{n-2})| \end{bmatrix} = 0$$

Derivatives (straightness observations)

$$\frac{\partial \varphi_i}{\partial X_1} = 2(P_1 - P_i)(P_1 - P_n)(2P_1 - P_i - P_n)$$

$$\frac{\partial \varphi_i}{\partial X_i} = 2(P_i - P_1)(P_1 - P_n)^2$$

$$\frac{\partial \varphi_i}{\partial X_n} = 2(P_n - P_1)(P_1 - P_i)^2$$

all other derivatives are 0

Design Matrix (SLRS)

$$A_2 = \begin{bmatrix} \frac{\partial \varphi_2}{\partial X_1} & \cdots & \frac{\partial \varphi_2}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \varphi_{n-2}}{\partial X_1} & \cdots & \frac{\partial \varphi_{n-2}}{\partial X_n} \end{bmatrix}$$

Normal Equation

$$\begin{bmatrix} x \\ -k \end{bmatrix} = \begin{bmatrix} A_1' P A_1 & A_2' \\ A_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} A_1' P l \\ -w \end{bmatrix}$$

Enhanced adjustment Model

Integrating SLRS data involves several coordinate systems

- $\boldsymbol{X}_{(x,y,z)}$: overall tunnel coordinate system
- $\boldsymbol{S}_{(x,y,z)}$: SLRS camera CS, Origin is principal point, X-Axis is parallel to laser beam
- $\boldsymbol{Z}_{i,(x,y,z)}$: SLRS target CS, each target *i* has its own CS

The following approximations apply
(for reference only, no restriction for the adjustment):

- $\boldsymbol{Z}_{(0,0,0)} = \boldsymbol{S}_{i,(l,0,0)}$
- $\boldsymbol{S}_x \parallel \boldsymbol{Z}_x$
- $\boldsymbol{S}_y \parallel \boldsymbol{Z}_y$
- $\boldsymbol{S}_z \parallel \boldsymbol{Z}_z$

The following restrictions are introduced into the model

- $\boldsymbol{S}_{(y,z)} \perp \boldsymbol{laser\ beam}$
- $\boldsymbol{Z}_{(y,z)} \parallel \boldsymbol{S}_{(y,z)}$
- $\boldsymbol{X}$ *and* $\boldsymbol{Z}_i$ all share the same scale

These restrictions are **alignment requirements** of the SLRS. It has to be made sure that they are met before putting the system in operation.

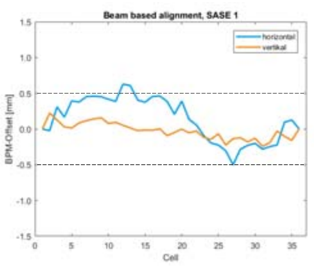
CS transformations

- $\boldsymbol{Z} \rightarrow \boldsymbol{S}$ 3 translations, 1 rotation (around $\boldsymbol{Z}_x$), 1 scale, per $\boldsymbol{Z}$
- $\boldsymbol{S} \rightarrow \boldsymbol{X}$ 3 translations, 3 rotations

Observations and Unknowns

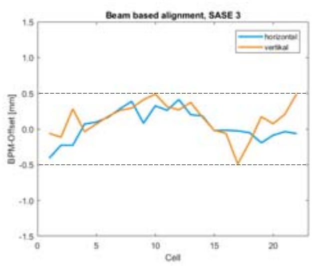
observations: image coordinates of poisson spots, measured in pixel, two pairs (y,z) per target
constants: fiducialization data of targets
unknowns: parameters of coordinate transformations, (laser tracker targets on the outside of the measurement boxes)

Beam Based Alignment (BBA) results



SASE1

SASE1 was the first undulator and optical beamline section aligned using the information from SLRS in combination with laser tracker observations.
After the initial alignment done by the alignment department, a beam based alignment (BBA) was performed by the machine operators. The deviation from the ideal straight line was recorded by BPM. The BBA is, however, not sensitive for absolute position and orientation of the beam inside the tunnel, so the zero-position and the orientation of the beam axis can be chosen arbitrarily.
The beam based alignment showed horizontal deviations from the straight line mostly inside the expected error band of ±0.5mm. For the vertical direction the error is even smaller, due to the integration of levelling observations.



SASE3

SASE3 was the second undulator that was aligned using SLRS data.
This SLRS also performed very well, horizontal alignment stayed well inside the expected error band of ±0.5mm. The vertical alignment, while also staying inside the required error band, was performing slightly worse than for SASE1. The reason for this is unclear, but some vertical tunnel movements could be assumed in this area due to heavy ground work above the tunnel in the second half part of the undulator.

SASE2

SASE2 was the last undulator put in operation and thus also the last one that was aligned using SLRS information.
As it can be clearly seen from the graph above, this SLRS did not operate inside its specifications. While the vertical alignment was well inside the specs, the horizontal alignment was exceeding the allowed tolerance by almost a factor of three. Due to time constraints it was not possible to successfully re-measure this section with geodetic techniques. Instead the section was re-aligned using the BBA-data.
There are investigations ongoing why this SLRS did not produce the same quality of results as the other two SLRS, but at the moment the reason is still unclear.

Summary

Adjustment model

While the constrained model seems to provide a nice and clean solution for the combined adjustment, this approach has drawbacks. It is necessary to perform a lot of pre-calculations with the SLRS data to reduce the real observations to a single straight line information that can be inserted into the constrained model.
Whenever using derived observations, information is dropped from the model – the opportunity to detect additional flaws in the data might be missed, the stochastic model is disturbed.
For the SLRS adjustment there are a couple of geometric parameters that should be clearly part of the overall adjustment process and not eliminated before by pre-estimation:

1. Image scale, distance dependent
2. Fiducialization parameters of targets
3. One roll angle per target around beam
4. Overall orientation of SLRS in tunnel network

To estimate these additional parameters, it will be necessary to use a Generalized LSM. The complete model is quite complex and not yet complete.

Preliminary model

Evaluation of existing data sets with a preliminary model has proven to work well (SASE1 and SASE3), the remaining errors are within the expected range, according to the BBA results.
Why the similar system (same hardware, same software) of SASE2 did fail to produce proper results, is currently under investigation. It is unlikely that the fail arises from a flaw in the preliminary adjustment model, which has proven to work for SASE1 and SASE3.