Improving the geometry of a 3D longitudinal network using a stretched wire

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Context

It is difficult to align accurately the components of the particle accelerator in a narrow and straight tunnel using standard 3D measurements, due to the geometry of the network. We propose to use stretched wires to improve the geometry of a 3D longitudinal network. The main advantages of using a stretched wire are:

- At first order, it lies in a vertical plane;
- it is insensitive to horizontal temperature gradients

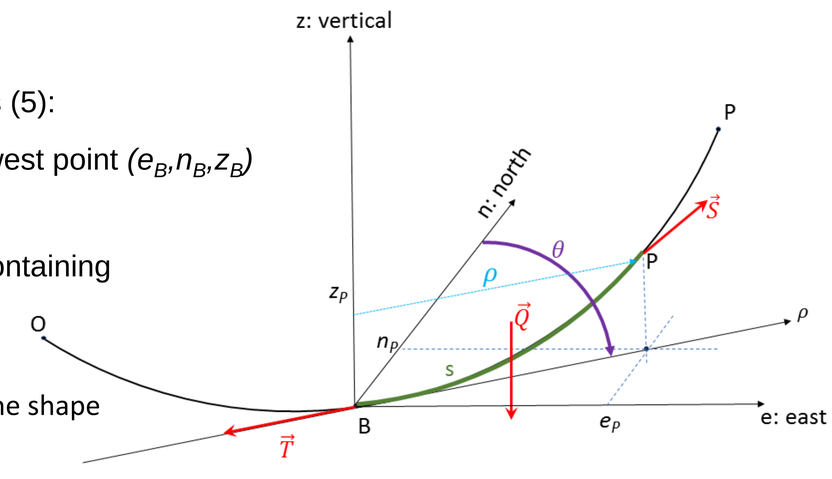
Objectives

- Use a common polypropylene braided rope type, lightly tensed wire
- Perform direct measurements on the wire (in both telescope positions)
- Include these measurements in a network adjustment process
- Study how it can improve the geometry of the network, especially along the radial axis

Stretched wire mechanics

Stretched wire parameters (5):

- Coordinates of the lowest point (e_B, n_B, z_B)
- Azimut of the plane containing the stretched wire: γ
- Coefficient controlling the shape of the wire $\alpha = \frac{T}{qg}$



Catenary equation: $z_P - z_B = \alpha\left(ch\left(\frac{\rho_P}{\alpha}\right) - 1\right)$

T: horizontal tension (Newton)

q: mass per unit length (kg/m)

Wire in use at CERN: $\alpha \approx 65000$; $\emptyset \approx 0{,}20\ mm$; $sag \approx 20\ mm$ (100m-long wire)
Wire in use in this study: $\alpha \approx 650$; $\emptyset \approx 2\ mm$; $sag \approx 1{,}8\ m$ (100m-long wire)

How to include the wire in the adjustment process ?

In a network adjustment process, our aim is to estimate the cartesian coordinates of all the unknown points of the network, in a geocentric cartesian reference system.

(X_P, Y_P, Z_P): cartesian coordinates of point P
(Φ_P, Λ_P): astronomical latitude and longitude of point P
(e_P^v, n_P^v, z_P^v): local coordinates of point *P* in the local geodetic system of point *V*

S: from-station point F: to-station point (on the wire) B: lowest point on the wire

Strategy (1): no wire-information in the adjustment of the data

Strategy (2): Observation equations expressed as function of the arameters of the wire

In the local astronomical coordinate system of the lowest point *B*, for a point F onto the wire, the following relationships exist:

$$e_F^b - e_B^b = \rho_F \sin\gamma \qquad n_F^b - n_B^b = \rho_F \cos\gamma \qquad z_F^b - z_B^b = \alpha\left(ch\left(\frac{\rho_F}{\alpha}\right) - 1\right)$$

Using astronomical latitude and longitude of points *S* and *B*, it is possible to consider the rotation matrix R such as:

$$\begin{bmatrix} e_F^s - e_B^s \\ n_F^s - n_B^s \\ z_F^s - z_B^s \end{bmatrix} = [R_{ij}] \begin{bmatrix} e_F^b - e_B^b \\ n_F^b - n_B^b \\ z_F^b - z_B^b \end{bmatrix} \qquad \Delta_i = R_{i1} h_F \sin\gamma + R_{i2} h_F \cos\gamma + R_{i3}\alpha\left(ch\left(\frac{h_F}{\alpha}\right) - 1\right)$$

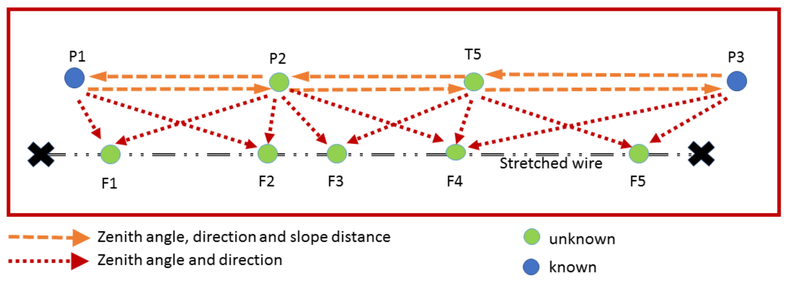
Then, the observation equations could be rewritten as follows:

$$L_{SF} = arctan\left(\frac{\Delta_1 + e_B^s - e_S^s}{\Delta_2 + n_B^s - n_S^s}\right) - G_{0,S}$$

$$A_{SF} = arctan\left(\frac{\sqrt{(\Delta_1 + e_B^s - e_S^s)^2 + (\Delta_2 + n_B^s - n_S^s)^2}}{\Delta_3 + z_B^s - z_S^s}\right)$$

The observation equations are written as function of the parameters of the wire, the longitudinal coordinate of the point onto the wire and the cartesian coordinates of the points outside the wire.

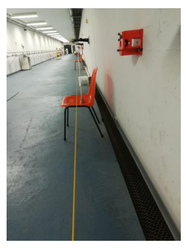
Test campaign



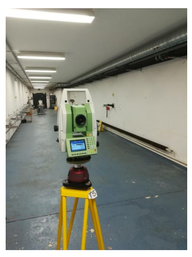
Information	Strategy 1	Strategy 2
Nb. of observations	40	40
Nb. of unknowns (total count)	25	20
Nb. of unknowns (coordinates)	21	6
Nb. of unknowns (due to the wire)	0	10
Degree of freedom	15	20
Estimated variance factor	0.97	0.93

- Polypropylene braided rope type wire stretched over 50 m (azimuth ≈ 43°)
- 5 marks (with a pen) on the wire: F1 to F5
- Points P1 and P3 with fixed coordinates

- Experiment was conducted in the ESGT laboratory
- All measurements were taken in both telescope faces
- Atmospheric EDM errors were corrected using meteorological sensors
- Zero and cyclic errors of the EDM were determined using the 50m-long interferometric bench of the laboratory
- Standard deviation for EDM measurements: $\pm 0{,}5mm \pm 0{,}5ppm$
- Standard deviation for angle measurements: $0{,}15\ mgon$
- Centring error of 0,5 mm for points located onto the wire and of 0,3mm for the others



Results: points coordinates accuracies

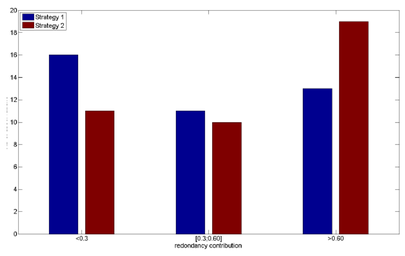


Point	Strategy 1			Strategy 2		
	Radial	Long.	Vert.	Radial	Long.	Vert.
P2	0.9	0.7	0.4	0.7 (17)	0.7 (3)	0.4 (4)
T5	0.9	0.7	0.4	0.8 (18)	0.7 (3)	0.4 (4)
F1	1.1	2.8	0.8	1.0 (8)	2.6 (5)	0.7 (12)
F2	1.4	1.9	0.9	0.8 (42)	1.7 (11)	0.6 (32)
F3	1.5	2.9	0.9	0.8 (47)	2.7 (8)	0.6 (38)
F4	1.4	1.8	0.7	0.8 (44)	1.6 (8)	0.6 (12)
F5	1.0	3.9	0.8	1.0 (3)	3.8 (3)	0.8 (1)

Table: Estimated standard deviation at a 95%-confidence level, along the radial, longitudinal and vertical components, in mm, and percentage of improvement in parenthesis

- Improvement between 11% and 17% for points outside the wire (radial)
- Improvement up to 47% for point F3 (radial)
- Improvement up to 38% for points on the wire (vertical)
- Improvement up to 10% for all points (longitudinal)

Results: reliability of the network



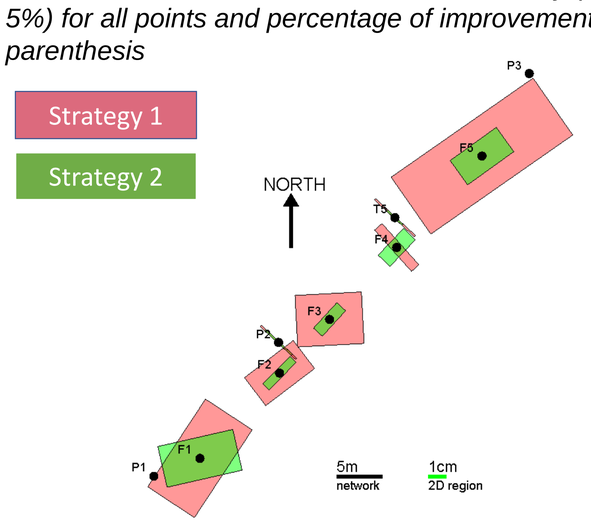
Distribution of redundancy contribution values for the observations in intervals [0, 0.3[, [0.3, 0.6] and]0.6, 1]

Improvements mainly due to:

- A greater degree of freedom
- A change in the geometry of the network (functional models

Point	Strategy	1/2 Length (mm)	1/2 Width (mm)	Azim (deg)	Vertical (mm)
P2	1	13.2	0.5	133	0.5
	2	9.9 (25)	0.5 (0)	133	0.5 (2)
T5	1	14.9	0.5	133	0.5
	2	5.6 (62)	0.5 (0)	133	0.5 (4)
F1	1	28.7	15.3	33	1.6
	2	21.0 (27)	11.4 (25)	77	1.4 (15)
F2	1	16.8	9.6	53	1.9
	2	10.6 (37)	2.4 (75)	45	0.5 (72)
F3	1	18.2	14.1	87	1.7
	2	9.4 (48)	3.3 (77)	42	0.5 (70)
F4	1	15.1	2.7	138	1.1
	2	10.1 (33)	4.5 (-65)	44	0.5 (50)
F5	1	47.3	19.0	55	1.7
	2	15.5 (67)	8.2 (57)	54	1.4 (14)

2D horizontal and 1D vertical external reliability (95%, 5%) for all points and percentage of improvement in parenthesis



For points outside the wire (P2 and T5):

- 25 to 62% improvement along the radial axis (1/2 length)
- A change in the geometry of the network (functional models and degree of freedom)

For other points:

- 25 to 77% improvement along the radial and longitudinal axes
- Up to 70% improvement along the vertical axis

Conclusion

- Using a stretched wire in a 3D longitudinal network can improve both accuracy and reliability of the network, especially along the radial axis.
- The main limitation is currently the pointing accuracy on the wire

Next steps

- Propose a more complex geometry for the network
  - Using a stretched wire to connect several sub networks
  - Using several stretched wires with different lengths
- Perform distance and automatic measurements on the wire (using spherical pearls)