A new approach to calculating dynamic friction for magnetized electron coolers – relevance to future IOTA experiments and to EIC designs

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Motivation – Nuclear Physics

• Electron-ion colliders (EIC)
  – high priority for the worldwide nuclear physics community
• Relativistic, strongly-magnetized electron cooling
  – may be essential for EIC, but never demonstrated

eRHIC concept from BNL

JLEIC concept from Jefferson Lab
Idea for Electron Cooling is 50 Years Old

• Budker developed the concept in 1967

• Many low-energy electron cooling systems:
  – continuous electron beam is generated
  – electrons are nonrelativistic & very cold compared to bunches
  – electrons are magnetized with a strong solenoid field
    • suppresses transverse temperature & increases friction

• Fermilab has shown cooling of relativistic p-bar’s
  – ~5 MeV e-’s (γ ~ 9) from a DC source
  – The electron beam was not magnetized

• Relativistic magnetized cooling not yet demonstrated
  – electron cooling at γ ~ 100 has not been demonstrated
    • a non-magnetized concept was developed for RHIC
    • Fedotov et al., Proc. PAC, THPAS092 (2007).
Risk Reduction is Required for Relativistic Coolers

- eRHIC, JLEIC both need cooling at high energy
  - $100 \text{ GeV/n} \rightarrow \gamma \approx 107 \rightarrow 55 \text{ MeV bunched electrons, } \sim 1 \text{ nC}$

- Electron cooling at $\gamma \sim 100$ requires different thinking
  - friction force scales like $1/\gamma^2$ (Lorentz contraction, time dilation)
    - challenging to achieve the required dynamical friction force
    - not all of the processes that reduce the friction force have been quantified in this regime $\rightarrow$ significant technical risk

- normalized interaction time is reduced to order unity
  - $\tau = t_{\omega_{pe}} \gg 1$ for nonrelativistic coolers
  - $\tau = t_{\omega_{pe}} \sim 1$ (in the beam frame), for $\gamma \sim 100$
    - violates the assumptions of introductory beam & plasma textbooks
    - breaks the intuition developed for non-relativistic coolers
    - as a result, the problem requires careful analysis
Goals

• Simulate magnetized friction force
  – *include all relevant real world effects*
    • e.g. incoming beam distribution
  – *include a wide range of parameters*
  – cannot succeed via brute force
    • improved understanding is required

*from Geller & Weisheit, Phys. Plasmas (1977)*

• Include key aspects of magnetized e- beam transport
  – imperfect magnetization
  – space charge
  – field errors

*from Zhang et al., MEIC design, arXiv (2012)*
Asymptotic model for cold, strongly magnetized electrons

\[ F_\parallel = -\frac{3}{2} \omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \left[ \ln \left( \frac{\rho_{\text{max}}}{\rho_{\text{min}}} \right) \left( \frac{V_\perp}{V_{\text{ion}}} \right)^2 + \frac{2}{3} \right] \frac{V_\parallel}{V_{\text{ion}}} \]

\[ F_\perp = -\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln \left( \frac{\rho_{\text{max}}}{\rho_{\text{min}}} \right) \left( \frac{0.5V_\perp^2 - V_\parallel^2}{V_{\text{ion}}} \right) \frac{V_\perp}{V_{\text{ion}}} \]

\[ r_L = V_{\text{rms},e,\perp}/\Omega_L(B_\parallel) \]
\[ \rho_{\text{min}}^A = \max(r_L, \rho_{\text{min}}) \]
\[ \rho_{\text{max}}^A = \min(r_{\text{beam}}, \rho_{\text{max}}) \]
\[ \rho_{\text{max}} = V_{\text{rel}}/\max(\omega_{pe}, 1/\tau) \]
\[ V_{\text{rel}} = \max(V_{\text{ion}}, V_{e,\text{rms},\parallel}) \]
\[ V_{\text{ion}}^2 = V_\parallel^2 + V_\perp^2 \]


Including thermal effects

\[ F = -\frac{1}{\pi} \omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln \left( \frac{\rho_{\text{max}} + \rho_{\text{min}} + r_L}{\rho_{\text{min}} + r_L} \right) \frac{V_{\text{ion}}}{(V_{\text{ion}}^2 + V_{\text{eff}}^2)^{3/2}} \]

\[ \rho_{\text{min}} = \frac{(Ze^2/4\pi\varepsilon_0)/m_{\text{ion}}V_{\text{ion}}^2}{\rho_{\text{max}} = V_{\text{ion}}/\max(\omega_{pe}, 1/\tau)} \]

\[ r_L = \frac{V_{\text{rms,e,\perp}}}{\Omega_L(B_\parallel)} \]

\[ V_{\text{eff}}^2 = V_{e,\text{rms,\parallel}}^2 + \Delta V_{\perp e}^2 \]


Integrating D&S calculation over thermal electron population:

\[ F_{\|}(0, V_{\|}) = -V_{\|} \frac{4\pi Z^2 e^4 n_e L_M}{m \Delta_{e,\|}^3} \exp \left( -\frac{V_{\|}^2}{2 \Delta_{e,\|}^2} \right) \]


VORPAL modeling of binary collisions clarified differences in formulae for magnetized friction

![Graph showing force vs. angle]

- D&S asymptotics are accurate for ideal solenoid, cold electrons – not warm
- Parkhomchuk formula often works for typical parameters, but not always
- 3D quad. of D&S with e- dist. works better (modified $r_{\text{min}}$, ideal solenoid)
- In general, direct simulation is required

Detailed simulations of magnetized friction:

Analysis of the magnetized friction force

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Numerical study of the magnetized friction force


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Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam

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Detailed simulations of magnetized friction:


Figure 2: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s] for zero transverse angle $\theta = 0$ with respect to the magnetic field lines. VORPAL results: dots with error bars; Eq. (4) - solid line.
JLab EIC Design:

High Energy Electron Cooler

Images courtesy of Jefferson Lab.
Can we quantify the required solenoidal field quality?

• No, we cannot
  – Parkhomchuk formula provides a parametric knob
  – Derbenev and Skrinsky do not offer quantitative guidance

• Can we quantify the effects of space charge forces?
  – No, we cannot

• Can we quantify the effects of non-Gaussian e-beam phase space distributions?
  – No, we cannot
A new dynamical friction calculation is underway...

- We follow the approach described by Y. Derbenev
- However, we begin from a new starting point
  - analytic momentum transfer between ion and magnetized e-
  - proceed step by step with calculation
- Calculation is defined by the following considerations:

\[
\begin{align*}
\vec{E}(\vec{r}, \vec{u}, t) &= \langle \vec{E}^0 \rangle(\vec{r}, t) + \langle \Delta \vec{E} \rangle(\vec{r}, \vec{u}, t) + \vec{E}^{fl}(\vec{r}, \vec{u}, t) \\
\vec{F} &= -ze\langle \Delta \vec{E} \rangle(\vec{r}, \vec{u}, t) \bigg|_{\vec{r}=\vec{r}(t), \vec{r}(t)=\vec{u}}
\end{align*}
\] (1.1) (1.2)

Y. Derbenev, “Theory of Electron Cooling,” arXiv (2017);
https://arxiv.org/abs/1703.09735

THEORY OF ELECTRON COOLING

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Translation supported by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, worldwide license to publish or reproduce this manuscript for U.S. Government purposes.
Directly integrate $\Delta p_{\text{ion}}$ to obtain friction force?

$$F = -\frac{n_e m_e}{T} \iiint_{\mathbb{R}^3} d^3v \int_{V} \int dr dz d\varphi \Delta v(T, r, \varphi, z, v) r p(v)$$

- Straightforward integration includes space charge, etc.
  - *this approach worked for VORPAL/VSim simulations (w/ effort)*
- Problematic, so we follow Derbenev et al.
The required steps are straightforward in principle:

- Calculate the perturbed e- velocities
  - due to a single ion
  - initially, we consider purely longitudinal motion
- Obtain time-derivative of perturbed E-field
  - via Poisson and continuity equations
- Integrate in time to get $\delta E$
  - initially, this is for only a single value of e- velocity
  - it is necessary to integrate over thermal e- velocities
- Integrate $\delta E$ along ion trajectory to obtain $<F>$
  - hence, this is a 2nd-order effect, $\sim(Ze^2)^2 xx$
- Present efforts:
  - find best way to integrate $<F>$ over e- distribution functions
  - consider transverse ion motion
  - numerical approaches, testing, etc.
Hamiltonian for 2-body magnetized collision:

\[
H(\vec{x}_{\text{ion}}, \vec{p}_{\text{ion}}, \vec{x}_e, \vec{p}_e) = H_0(\vec{p}_{\text{ion}}, y_e, \vec{p}_e) + H_C(\vec{x}_{\text{ion}}, \vec{x}_e)
\]

\[
\vec{B} = B_0 \hat{z}, \quad \vec{A} = -B_0 y \hat{x}, \quad p_{e,x} = m_e(v_{e,x} - \Omega_L y_e)
\]

\[
H_0(\vec{p}_{\text{ion}}, y_e, \vec{p}_e) = \frac{1}{2m_{\text{ion}}} \left( p_{\text{ion},x}^2 + p_{\text{ion},y}^2 + p_{\text{ion},z}^2 \right) + \frac{1}{2m_e} \left[ (p_{e,x} + eB_0 y_e)^2 + p_{e,y}^2 + p_{e,z}^2 \right]
\]

\[
H_C(\vec{x}_{\text{ion}}, \vec{x}_e) = -\frac{Ze^2}{4\pi\varepsilon_0} \sqrt{(x_{\text{ion}} - x_e)^2 + (y_{\text{ion}} - y_e)^2 + (z_{\text{ion}} - z_e)^2}
\]

Resulting equations of motion, in the standard drift-kick symplectic form:

\[
M(\Delta t) = M_0(\Delta t/2) M_C(\Delta t) M_0(\Delta t/2)
\]

Analytic calculation of $\Delta p_{\text{ion}}$  (1)

\[
C_1 = \left( x_{\text{ion}} - \frac{p_{\text{gc}}}{m_e \Omega_e} \right)^2 + \left( y_{\text{ion}} - y_{\text{gc}} \right)^2 + \left( z_{\text{ion}} - z_{e} \right)^2 + \frac{2}{m_e \Omega_e} J
\]  (14a)

\[
C_2 = 2(x_{\text{ion}} - x_{\text{gc}})v_{\text{ion},x} + 2(y_{\text{ion}} - y_{\text{gc}})v_{\text{ion},y} + 2(z_{\text{ion}} - z_{e})(v_{\text{ion},z} - v_{e,z})
\]  (14b)

\[
C_3 = v_{\text{ion},x}^2 + v_{\text{ion},y}^2 + (v_{\text{ion},x} - v_{e,z})^2
\]  (14c)

\[
b = \left[ C_1 + C_2 T + C_3 T^2 \right]^{1/2} \quad \Delta = 4C_1 C_3 - C_2^2
\]  (14d)

\[
D_1 = \left[ \frac{2C_3 T + C_2}{b} - \frac{C_2}{\sqrt{C_1}} \right]
\]  (14e)

\[
D_2 = \left[ \frac{2C_1 + C_2 T}{b} - 2\sqrt{C_1} \right]
\]  (14f)
Analytic calculation of $\Delta p_{\text{ion}}$ (2)

\[ \Delta p_{\text{ion},x} = -\frac{2\alpha}{\Delta} \left[ \left( x_{\text{ion}} - p_{gc} / m_e \Omega_e \right) D_1 - \left( p_{\text{ion},x} / m_{\text{ion}} \right) D_2 \right] \] (15a)

\[ \Delta p_{\text{ion},y} = -\frac{2\alpha}{\Delta} \left[ \left( y_{\text{ion}} - y_{gc} \right) D_1 - \left( p_{\text{ion},y} / m_{\text{ion}} \right) D_2 \right] \] (15b)

\[ \Delta p_{\text{ion},z} = -\frac{2\alpha}{\Delta} \left[ \left( z_{\text{ion}} - z_e \right) D_1 - \left( \frac{p_{\text{ion},z}}{m_{\text{ion}}} - \frac{p_{ez}}{m_e} \right) D_2 \right] \] (15c)

\[ \Delta p_{gc} = -\Delta p_{\text{ion},x} \quad \Delta y_{gc} = -\Delta p_{\text{ion},y} / m_e \Omega_e \] (15d)
Time-explicit vs analytic shows agreement:

- Two small parameters are required:
  - Larmor radius must be small compared to impact param.
    - averaging
  - Kinetic energy must be large compared to max potential energy
    - perturbative
Choice of coordinate system is important:

\[
\Delta v_{e,z} = \frac{Ze^2}{m_e} \left\{ \left[ x_{gc}^2 + y_{gc}^2 + \rho_{gc}^2 + z_e^2 \right]^{-1/2} - \left[ x_{gc}^2 + y_{gc}^2 + \rho_{gc}^2 + (z_e - v_{rel}T)^2 \right]^{-1/2} \right\}
\]

\[v_{rel} = v_{i,z} - v_{e,z}\]

\[
\frac{\partial E}{\partial T} = -\vec{j} \\
\quad r^2 = x_{gc}^2 + y_{gc}^2
\]

\[
\frac{\partial E_z}{\partial T} \bigg|_{r=0} = \frac{Ze^2}{m_e} \left\{ \left[ \rho_{gc}^2 + z_e^2 \right]^{-1/2} - \left[ \rho_{gc}^2 + (z_e - v_{rel}T)^2 \right]^{-1/2} \right\}
\]
Integrate twice to obtain friction force:

\[ E_z(r = 0) = (n_0 e) \frac{Z e^2}{m_e v_{rel}} \left\{ T \left[ \rho_{gc}^2 + z_e^2 \right]^{-1/2} + \frac{1}{v_{rel}} \ln \left[ \frac{\left[ \rho_{gc}^2 + z_e^2 \right]^{1/2} + z_e}{\left[ \rho_{gc}^2 + (z_e - v_{rel} T)^2 \right]^{1/2} + (z_e - v_{rel} T)} \right] \right\} \]

Let \( z_e = v_{i,z} T \) and then integrate over \( T \) to obtain:

\[
\langle F \rangle = (n_0 e) \frac{n_0 (Z e^2)^2}{m_e v_{rel} T} \left\{ \frac{T}{v_{rel}} \ln \left[ \left[ \rho_{gc}^2 + (v_{i,z} T)^2 \right]^{1/2} + v_{i,z} T \right] - \frac{v_{e,z}}{v_{i,z} v_{rel}} \left[ \rho_{gc}^2 + (v_{i,z} T)^2 \right]^{1/2} \right\} - \frac{T}{v_{rel}} \ln \left[ \left[ \rho_{gc}^2 + (v_{e,z} T)^2 \right]^{1/2} + v_{e,z} T \right] - \frac{1}{v_{e,z} v_{rel}} \left[ \rho_{gc}^2 + (v_{e,z} T)^2 \right]^{1/2} \]

There is an integrable singularity for cold electrons.
The challenge now is to integrate over thermal velocities
Long-term goals:
Include other effects in Magnus Expansion

\[ H(\vec{x}_{\text{ion}}, \vec{p}_{\text{ion}}, \vec{x}_e, \vec{p}_e) = H_0(\vec{p}_{\text{ion}}, y_e, \vec{p}_e) + H_C(\vec{x}_{\text{ion}}, \vec{x}_e) + H_{\text{space-charge}}(\vec{x}_{\text{ion}}, \vec{x}_e) + H_{\text{solenoid-field-errors}}(??) \]

• Quantitative treatment of space charge & field errors?
  – space charge should work
  – field errors are more challenging

• Requires generalization of Magnus expansion
  – we are optimistic this can be done
Future Electron cooling experiments in IOTA

- Could be used to test friction force equations...?
  - RadiaSoft is interested to collaborate
JSPEC – new software for IBS and e-cooling

JSPEC on GitHub, https://github.com/zhanghe9704/electroncooling
Primary developer is He Zhang of JLab

A new GUI for JSPEC is under development, as part of the open source cloud computing initiative, Sirepo http://sirepo.com
Thank You!

Questions?