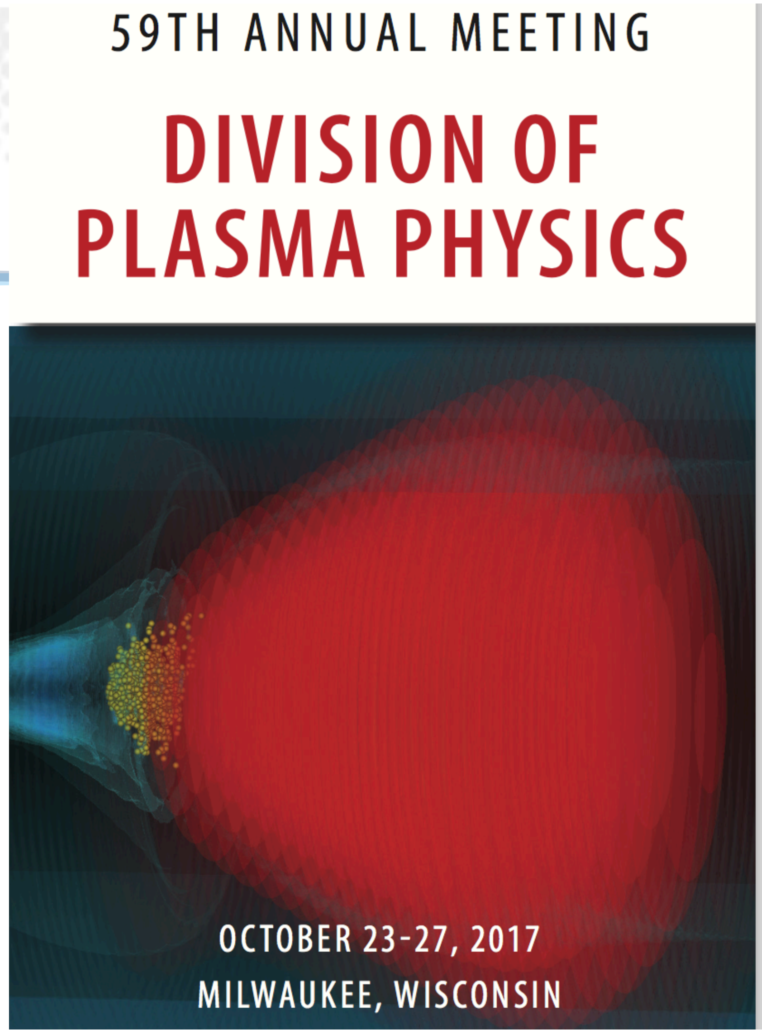
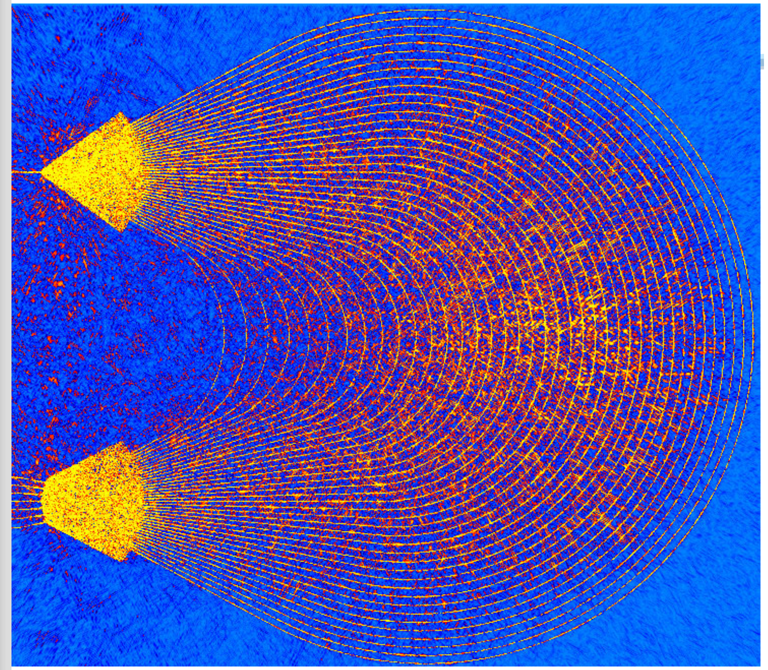
TECH-X

SIMULATIONS EMPOWERING YOUR INNOVATIONS





Optical Society of America's Image of the week, 20180409

When do we move to full ring, self-consistent simulations?

JR Cary

Self-consistent, full-ring simulations

- What do know about self-consistent beam equilibria
- Faster computing
- What can we do in the meantime?

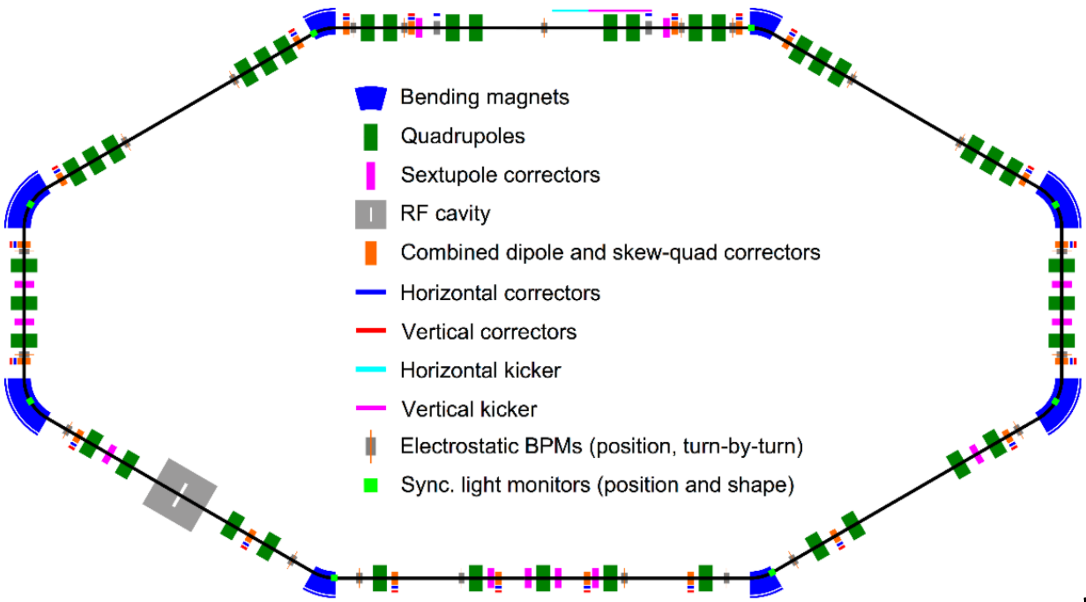


Figure 3. Layout of the Integrable Optics Test Accelerator (IOTA) ring.

BTW

J. R. Cary and I. Doxas, "An Explicit Symplectic Integration Scheme for Plasma Simulations," J. Comp. Phys. **107** (1) 98-104 (1993)

Time-scale hierarchy

- Equilibrium
- Lasts for moderate times (Is it stable?)
- How long will it last? (transport)

PAPER

First use of three-dimensional equilibrium, stability and transport calculations for interpretation of ELM triggering with magnetic perturbations in NSTX

J.M. Canik[1], S.P. Hirshman[1], R. Sanchez[2], R. Maingi[1], J.-W. Ahn[1], R.E. Bell[1], A. Diallo[1], S.P. Gerhardt[3], B.P. LeBlanc[3], J.E. Menard[3] +Show full author list

Published 9 May 2012 • 2012 IAEA, Vienna

Nuclear Fusion, Volume 52, Number 5

Warm-fluid description of intense beam equilibrium and electrostatic stability properties

Physics of Plasmas **5**, 3028 (1998); https://doi.org/10.1063/1.873027

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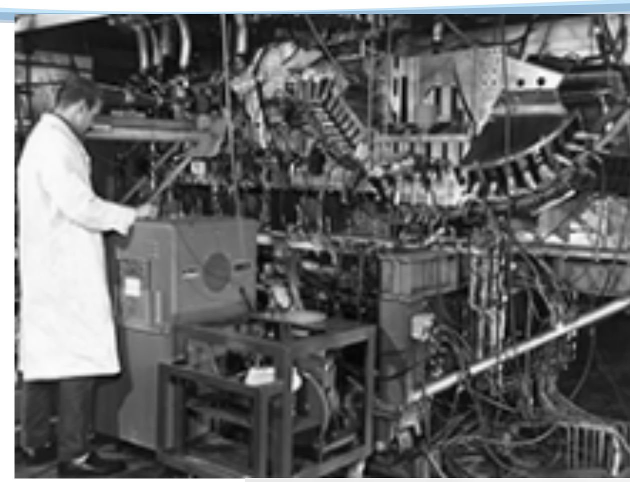
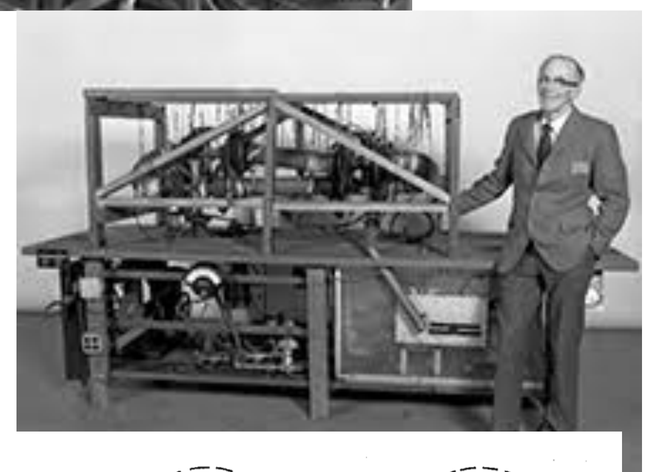
Ronald C. Davidson

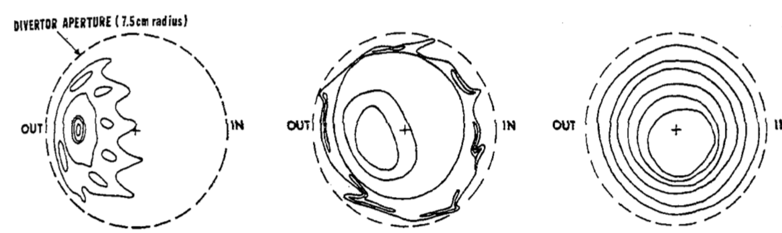
Field has analogies with 3D plasma equilibria of 60's, 70's

- Toroidal magnetic field lines
- Perturbation theory (Lyman Spitzer, 1958) indicated that equilibria existed
- Grad (67) pointed out that equilibria might not exist, even in vacuum
- Model-C stellarator diagnosed by electron beam, horrible surfaces of section
- Stellarator dropped in favor of tokamak
- (Harbinger of chaotic dynamics - rapid loss)



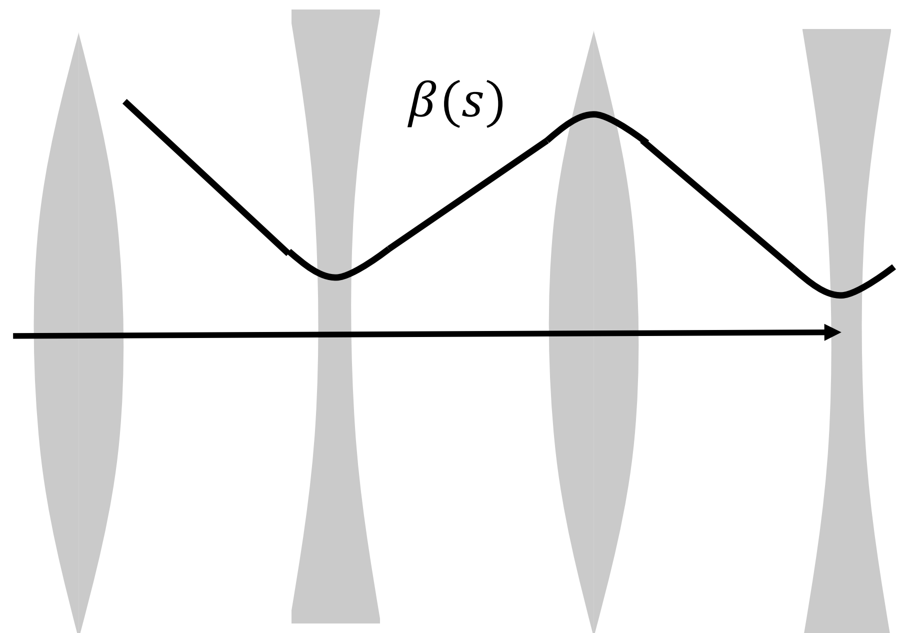


Are we here?



What is an equilibrium?

- Lund: an equilibrium is a solution to the dynamics with distribution having periodicity of the lattice
- Courant-Snyder
- KV
- Nonlinear with space charge?



How to calculate an equilibrium?

- Distribution a function of the invariants (in involution)
- In involution: both are action like, so single-valued functions
- Need not be two invariants if one is confining
- Except: J's cannot generally be found

$$f(J_1, J_2)$$

$$J_1(x, y, p_x, p_y)$$

Without space charge, high confidence of invariants

- Antipov et al, JINST 12

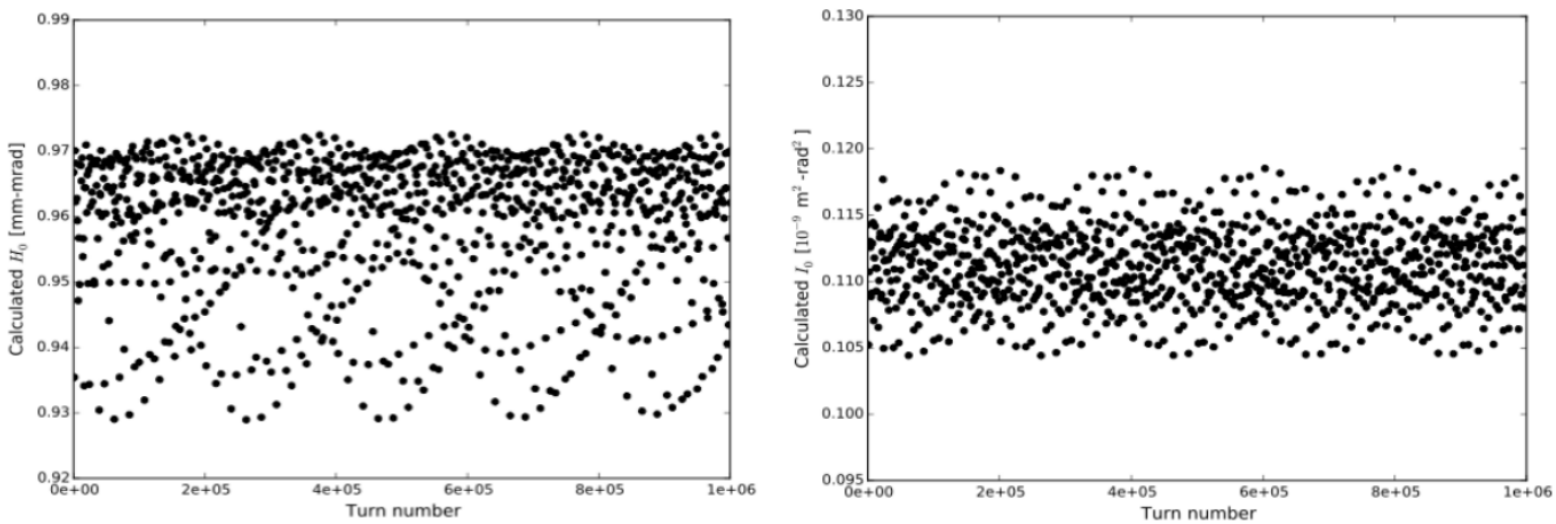


Figure 21. The 1st Danilov and Nagaitsev invariant (i.e. the Hamiltonian) is shown on the left, with the 2nd invariant shown on the right, calculated in the center of the nonlinear insert, showing 1.1% and 2.6% deviation from invariance, respectively, over a million turns. Synergia code was used for the simulations (Plots provided courtesy of N. Cook and S.D. Webb).

To be an equilibrium, collect particles to have constant amplitude

An equilibrium necessary for beam initialization, matching

- Distribution a function of the invariants
- Need not be two invariants
- Self consistency gives an integro-differential equation
- Except: J's cannot generally be found

$$f(J_1, J_2)$$

$$J_1(x, y, p_x, p_y)$$

$$J_i(\phi)$$

$$-\nabla^2 \phi = \int_p d^2 p f(J_1, J_2)$$

Approaches to finding equilibria

- Perturbative: space charge and nonlinearity expansion
- Envelope model (Ryne, earlier)
- Principal orbits (Lund)

$$\frac{d^2X}{ds^2} = -k_x X + \frac{2Q}{X+Y} + \frac{\epsilon_x^2}{X^3},$$
$$\frac{d^2Y}{ds^2} = -k_y Y + \frac{2Q}{X+Y} + \frac{\epsilon_y^2}{Y^3},$$

All of these approaches are asymptotic.
No proofs of existence.
Should we care?

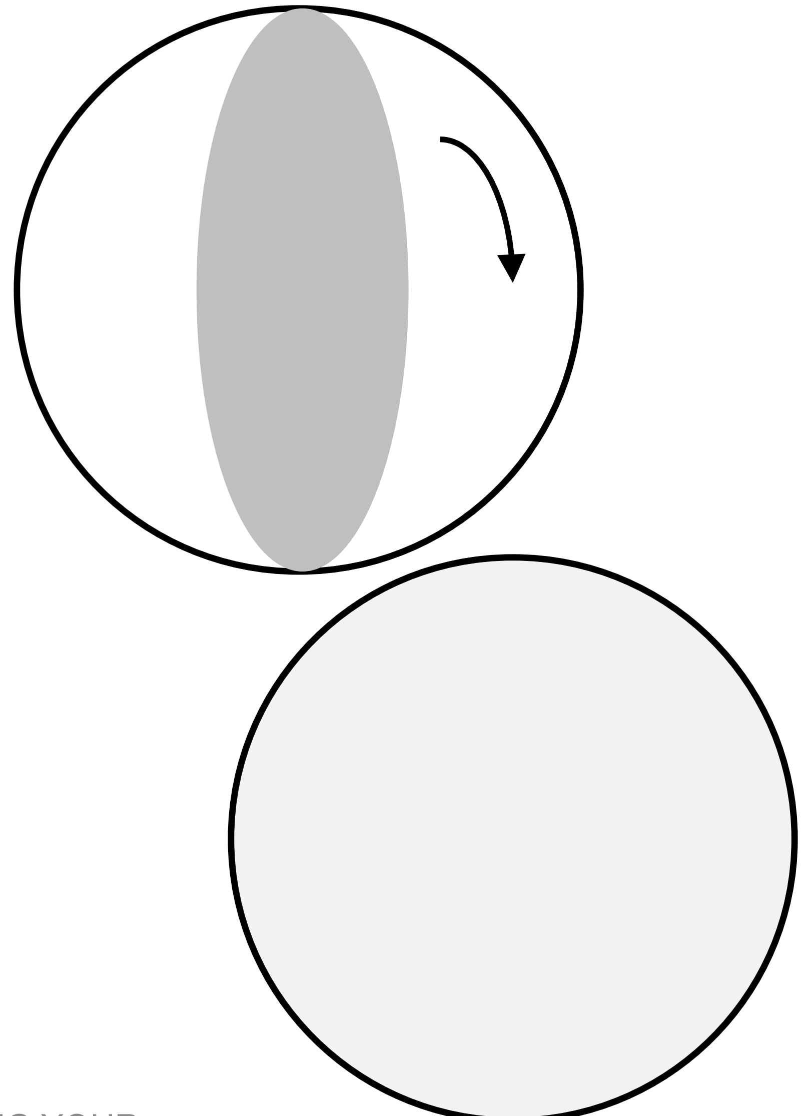
Are there equilibria with space charge? Simulations indicate not, chaotic diffusion

- Amundson: (@CERN) beam expands
- Kesting: PIC noise dominant
- Bruhwiler (2 years ago, 18 IPAC): beam is expanding. Important rate or curiosity?
- If there is a beam equilibrium, then it relies on invariants. To test, the integration method must capture the invariants.

However, simulations cannot distinguish bad initial condition and good with growth

- Courant-Snyder
- Lack of equilibrium
- Phase mixing
- Beam growth until saturation.

- MUST HAVE THEORY OF EQUILIBRIUM



Could it be that there is no equilibrium?

- How would we know?
- Need some sort of algorithm (perturbation theory, principal orbits)
- Continuously refine (higher-order terms, more principal orbits, …)
- If refinements diverge, no equilibrium?
- If attributed to single-particle chaos, should be seeing fractal dynamics
- Claim: we cannot answer the question of chaotic diffusion, beam expansion until we have a theory of equilibria. Any motion can be attributed to relaxation.

Assuming we build the codes to compute equilibria, now what do we do?

- Build IOTA1
- Build IOTA2
- OR
- Simulate

How do we expect loss to occur?

- Expect breakage of KAM surface farther from axis
- 1.5DoF: trajectories are “sticky” (Karney), held up by turnstiles (Meiss), act as if diffusing in space of fractal dimension (Hanson)
- Need work in 2.5 DoF, but similar expected.

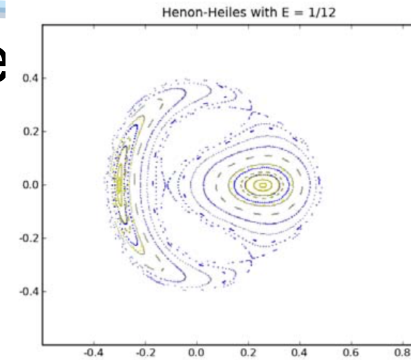
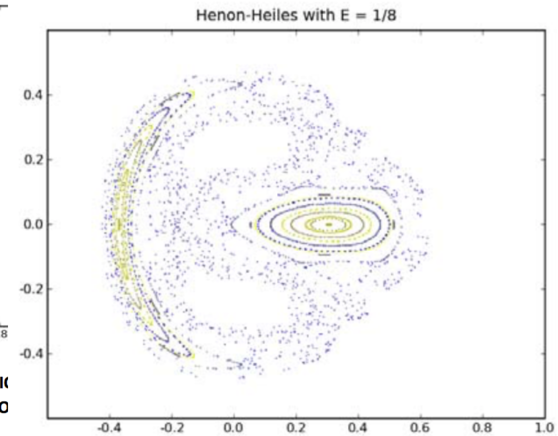


FIGURE 1: POINCARÉ PLOT OF 15 INITIAL CONDITI
DIMENTIONLESS ENERGY, E=1/12, NON-CHAOTIC O



Power law decay

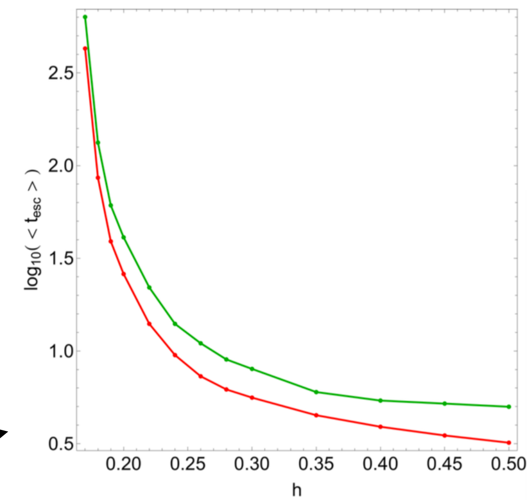
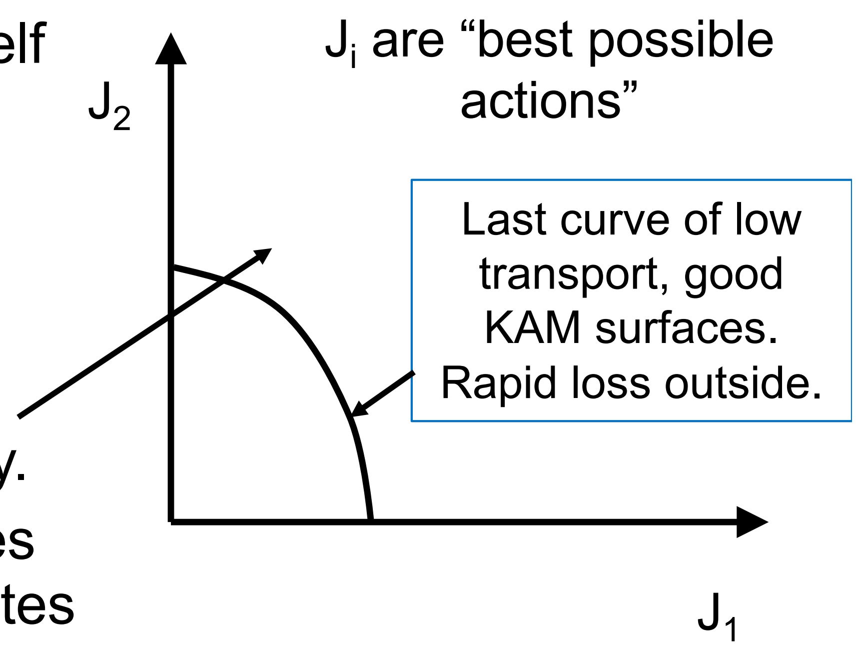


Fig. 6 Evolution of the logarithm of the average escape time of the orbits, $\log_{10}(< t_{esc} >)$, as a function of the total orbital energy h, for both the configuration (x, y) space (green) and the phase $(y\dot{y})$ space (red).

- Meiss - Thirty Years of Turnstiles and Transport, arXiv:1501.04364 (2015)
- Hanson et al, Algebraic Decay in Self-Similar Markov Chains, 1985
- Zotos, “An overview of the escape dynamics in the Henon-Heiles Hamiltonian system”, 2017

Conjecture: contraction, not expansion?

- Assume matched, so no halo particles created by core oscillations, good surfaces without self fields.
- Self-fields increasingly create chaos moving outwards, with some regions of denser KAM surface.
- Trajectories at edge in highly chaotic region connected to the wall, so particles lost rapidly.
- Particles fill region out to where KAM surfaces are dense (period of expansion, but asymptotes to given size, not diffusive).
- Particles in edge chaotic region slowly cross last fairly good surface, then are rapidly lost to wall.
- Particle loss in edge chaotic region causes decrease in RMS beam size.

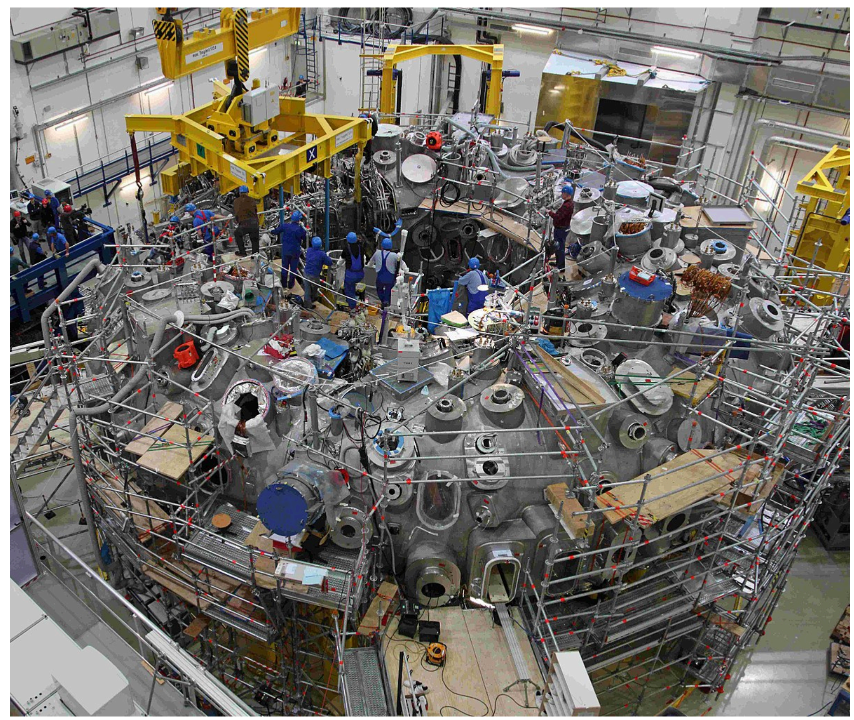


Stellarator equilibrium existence never answered mathematically, but well enough

- Shown how to eliminate chaotic regions
- Targets identified for equilibria
  - Quasihelical
  - Omnigenous
- Codes developed for finding equilibria assuming magnetic surfaces
- Wendelstein 7-x built using omnigenity
- First plasma Dec 2015 (40 years after PPPL gave up on Stellarator)

Quasi-helically symmetric toroidal stellarators
J **Nührenberg**, R **Zille** - Physics Letters A, 1988 - adsabs.harvard.edu
AA(Max-Planck-Institut für Plasmaphysik, IPP-EURATOM Association, D-8046 Garching near München, FRG; The work presented here only became possible with the advent of 3D MHD equilib codes exploiting the assumption of nested toroidal surfaces. We are thus indebted to F …
☆ 99 Cited by 362 Related articles All 5 versions

Helical Plasma Confinement Devices with Good Confinement Properties
John R. Cary and Svetlana G. Shasharina
Phys. Rev. Lett. **78**, 674 – Published 27 January 1997



If self-consistent beam dynamics follows similar path…

- ✓ Find vacuum fields with invariants (MacMillen, Cary, Danilov).
- Find general conditions (analogous to quasihelicity, omnigenity) for non-chaotic trajectories.
- Develop methods (even asymptotic) for computing equilibria with above properties.
- Initialize with matched (equilibrium) beams.
- Measure transport to understand quality of solution.

Questions?