Northern Illinois University

Single particle beam dynamics studies for the IOTA ring with COSY infinity code

Andrei Patapenka
Advisor: proff. Bela Erdelyi

For IOTA/FAST collaboration meeting

May 9, 2018

OUTLINE

- MAPS method in COSY
- EM Field representation in COSY
- IOTA non-linear magnet implementation
- Some results and open issues
- Backup slides

COSY infinity

COSY Environment is Fortran-77 core providing a collection of Differential algebras types and advanced scripting language (FOX).
DA capabilities: Normal form transformations; Tune and resonance strength calculations with NF; Beam matching

FOX addons:

- cosy beam dynamics (contains various types of magnetic and electric elements)
- FMM - 3D Fast Multipole Method (N-body problem and applications for particle accelerator)
- COMFY - space charge effects
- PISCS -electrostatic interactions within a charged particle distribution

Canonical variables in S-Hamiltonian

Longitudinal time-of-flight and energy/momenta deviation variables:

$$\begin{array}{llll} MADX: & MAD8: & SIXTRACK: & COSY: \\ \Delta s = s - \beta ct & c\Delta t = s/\beta_0 - ct & \sigma = s - \beta_0 ct & l = v_0(t - t_0) \quad (1) \\ \delta = \dfrac{P - P_0}{P_0} & p_t = \dfrac{E - E_0}{P_0 c} & p_\sigma = \dfrac{E - E_0}{\beta_0 P_0 c} & \delta_E = \dfrac{E - E_0}{E_0}. \quad (2) \end{array}$$

Re-scaled momenta and vector potential:

$$p_x = P_x/P_0 \qquad p_y = P_y/P_0 \qquad p_s = P_s/P_0 \tag{3}$$

$$a_x = qA_x/P_0 \qquad a_y = qA_y/P_0 \qquad a_s = qA_s/P_0 \tag{4}$$

Horizontal and vertical positions: x, y

COSY: equation of motion

Equation of motion in COSY variables ($E = 0$,and straight reference orbit):

$$\dot{x}(t) = p_x \frac{p_0}{p_z} \qquad \dot{p_x} = (1+\delta_z)\left[\frac{B_y}{\chi_{M_0}} + p_y \frac{p_0}{p_z}\frac{B_z}{\chi_{M_0}}\right] \tag{5}$$

$$\dot{y}(t) = p_y \frac{p_0}{p_z} \qquad \dot{p_y} = (1+\delta_z)\left[\frac{B_x}{\chi_{M_0}} + p_x \frac{p_0}{p_z}\frac{B_z}{\chi_{M_0}}\right] \tag{6}$$

$$\dot{l}(t) = (1+\delta_m)\frac{1+\eta}{1+\eta_0}\frac{p_0}{p_z} \qquad \dot{\delta} = 0 \tag{7}$$

Field representation in COSY

In charge/current free regions filed can be represented with scalar potentials V_E and V_B

$$E = \nabla V_E \qquad B = \nabla V_B \tag{8}$$

$$\Delta V_E = 0 \qquad \Delta V_B = 0 \tag{9}$$

In general V has a form:

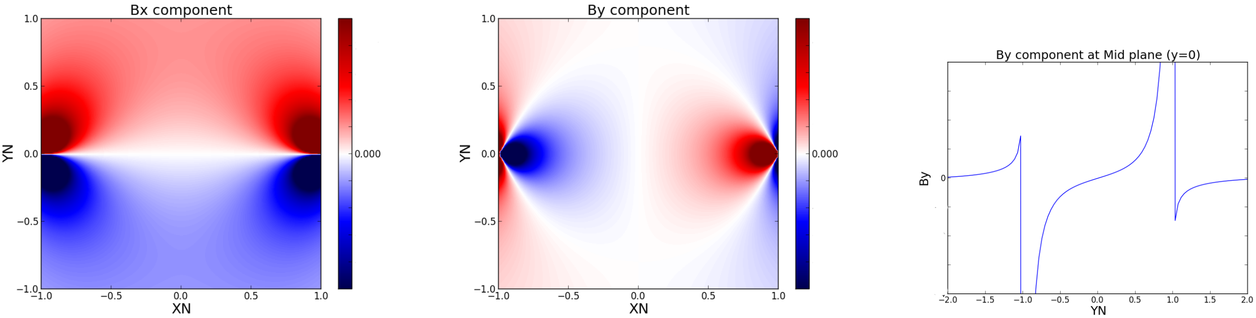
$$V(x, y, s) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{i,j,k} \frac{x^i}{i!} \frac{y^j}{j!} \frac{s^k}{k!} \tag{10}$$

The goal is to find $a_{i,j,k}$ to describe the field.

Plane symmetric field

$$V(x, y, s) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} A_{k,l}(s) \frac{x^k}{k!} \frac{y^l}{l!} \tag{11}$$

For the case of plane symmetric field to restore the information about the entire field. By component at the midplane is only needed. $B_y(x, y = 0, s)$ (proved by M. Berz Modern map methods in particle beam physics. Advances in Imaging and Electron Physics, 108:1-318, 1999.)



By at mid. plane (where $X_n = x/(csqrt\beta)$)

$$B_y(x, y=0) = \frac{\partial U}{\partial x} = \frac{ct}{\beta^{3/2}}[-\frac{X_n}{1-X_n^2} + \frac{acos(X_n)-\pi/2}{\sqrt{1-X_n^2}} + \frac{X_n^2(acos(X_n)-\pi/2)}{(1-X_n^2)^{3/2}}] \tag{12}$$

Fringe field

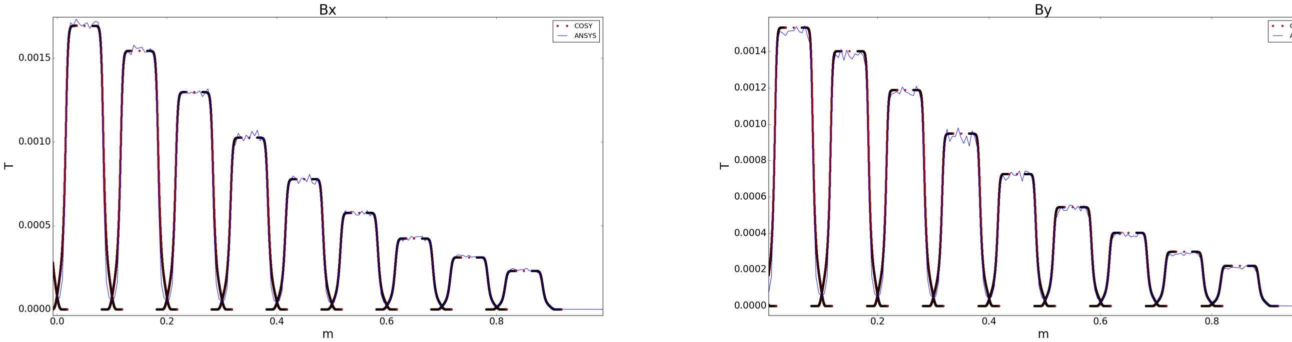
In COSY fringe field is approximated by Enge functions:

$$B(x, y, s) = F(s)B(x, y, s_0) \tag{13}$$

where:

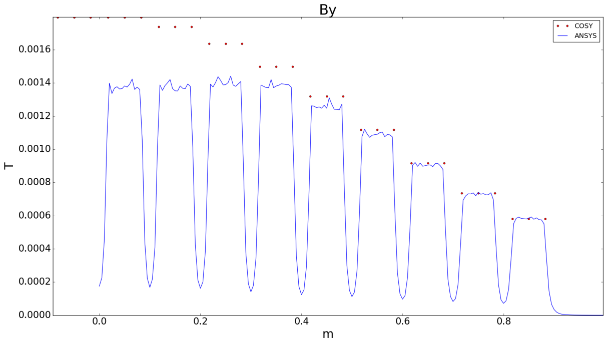
$$F(s) = \frac{1}{1 + exp(a_0 + a_1(s/D) + ... + a_5(s/D)^5)} \tag{14}$$

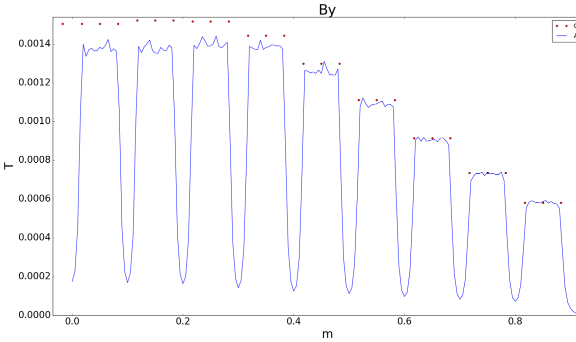
Magnet model verification



Bx, By components for IOTA nonlinear magnet compared with ANSYS simulations (ANSYS data provided by F. O'Shea (RadiaBeam). 3D Cartesian grid, distance from the axis: x=1mm, y=1mm

Potential problem





The same data set, but distance to the origin is x,y=3.6 mm. Left - truncation order is 10; Right - 16.

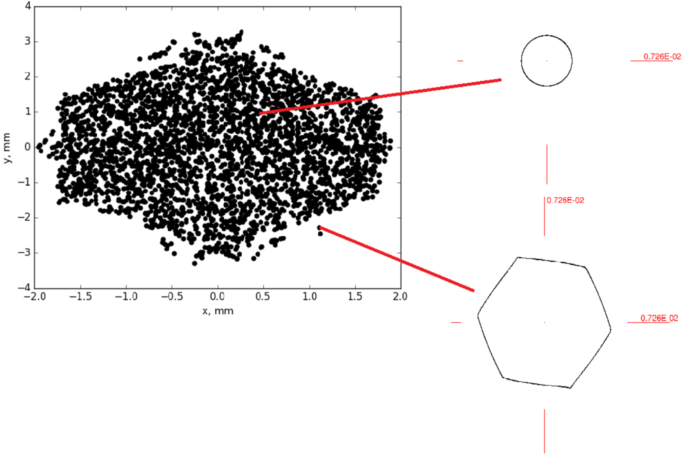
To make a better fit to the real field we need higher order.

IOTA lattice

We used IOTA version with 2 non-linear elements. The lattice was converted to COSY format. Linear optics is the same in MAD/PTC and COSY, but due to the problem with truncation order described above results seems not relevant.

IOTA studies work in progress...

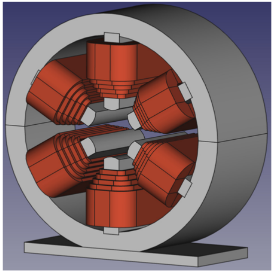
Local integrability in COSY

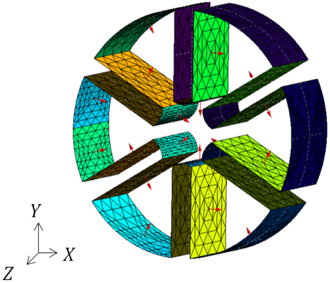


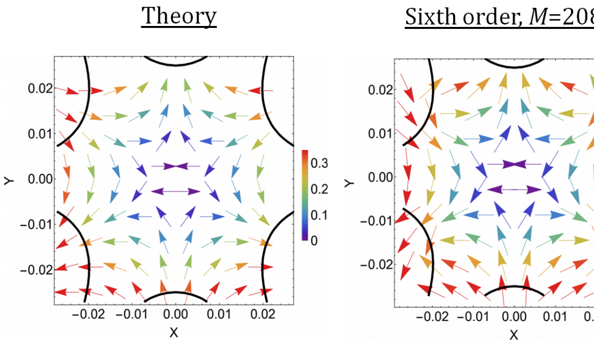
Backup slides: PISCS

PISCS module was created by by Anthony Gee.

The Poisson Integral Solver with Curved Surfaces (PISCS) is a package written in COSYScript for MSU COSY Infinity v9.2. PISCS is a 3-D Poisson boundary value problem solver accelerated by the fast multipole method (FMM). In this case, the Poisson BVP represents the electrostatic interactions within a charged particle distribution as a supplement to beam physics computations.




Y
X
Z


Theory
Sixth order, M=208
0.02
0.01
0.00
−0.01
−0.02
Y
X
0.3
0.2
0.1
0