Single particle beam dynamics studies for the IOTA ring with COSY infinity code

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• MAPS method in COSY
• EM Field representation in COSY
• IOTA non-linear magnet implementation
• Some results and open issues
• Backup slides
COSY Environment is Fortran-77 core providing a collection of Differential algebras types and advanced scripting language (FOX).
DA capabilities: Normal form transformations; Tune and resonance strength calculations with NF; Beam matching

FOX addons:

- cosy beam dynamics (contains various types of magnetic and electric elements)
- FMM - 3D Fast Multipole Method (N-body problem and applications for particle accelerator)
- COMFY - space charge effects
- PISCS - electrostatic interactions within a charged particle distribution
Canonical variables in S-Hamiltonian

Longitudinal time-of-flight and energy/momenta deviation variables:

\[
\begin{align*}
\Delta s &= s - \beta ct \\
c\Delta t &= s / \beta_0 - ct \\
\delta &= \frac{P - P_0}{P_0} \\
p_t &= \frac{E - E_0}{P_0c} \\
\sigma &= s - \beta_0 ct \\
p_\sigma &= \frac{E - E_0}{\beta_0 P_0 c} \\
l &= v_0(t - t_0) \quad (1) \\
\delta_E &= \frac{E - E_0}{E_0} \quad (2)
\end{align*}
\]

Re-scaled momenta and vector potential:

\[
\begin{align*}
p_x &= P_x / P_0 \\
p_y &= P_y / P_0 \\
p_s &= P_s / P_0 \\
a_x &= qA_x / P_0 \\
a_y &= qA_y / P_0 \\
a_s &= qA_s / P_0
\end{align*}
\]  

Horizontal and vertical positions: \(x, y\)
COSY: equation of motion

Equation of motion in COSY variables ($E = 0$, and straight reference orbit):

\[ \dot{x}(t) = p_x \frac{p_0}{p_z} \]
\[ \dot{y}(t) = p_y \frac{p_0}{p_z} \]
\[ \dot{l}(t) = (1 + \delta_m) \frac{1 + \eta}{1 + \eta_0} \frac{p_0}{p_z} \]

\[ \dot{p}_x = (1 + \delta_z)[\frac{B_y}{\chi} + p_y \frac{p_0}{p_z} \frac{B_z}{\chi_0}] \]  \hspace{1cm} (5)

\[ \dot{p}_y = (1 + \delta_z)[\frac{B_x}{\chi} + p_x \frac{p_0}{p_z} \frac{B_z}{\chi_0}] \]  \hspace{1cm} (6)

\[ \dot{\delta} = 0 \]  \hspace{1cm} (7)
In charge/current free regions field can be represented with scalar potentials $V_E$ and $V_B$

$$E = \nabla V_E$$
$$V_E = 0$$

$$B = \nabla V_B$$
$$V_B = 0$$

In general $V$ has a form:

$$V(x, y, s) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{i,j,k} \frac{x^i y^j s^k}{i! j! k!}$$

The goal is to find $a_{i,j,k}$ to describe the field.
Plane symmetric field

\[ V(x, y, s) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} A_{k,l}(s) \frac{x^k y^l}{k! l!} \]  \hfill (11)

For the case of plane symmetric field to restore the information about the entire field. By component at the midplane is only needed. \( B_y(x, y = 0, s) \) (proved by M. Berz Modern map methods in particle beam physics. Advances in Imaging and Electron Physics, 108:1-318, 1999.)
IOTA nonlinear magnet

By at mid. plane (where $X_n = x/(csqrt\beta)$)

$$B_y(x, y = 0) = \frac{\partial U}{\partial x} = \frac{ct}{\beta^{3/2}} \left[ -\frac{X_n}{1 - X_n^2} \frac{acos(X_n) - \pi/2}{\sqrt{1 - X_n^2}} + \frac{X_n^2(acos(X_n) - \pi/2)}{(1 - X_n^2)^{3/2}} \right]$$

(12)
In COSY fringe field is approximated by Enge functions:

$$B(x, y, s) = F(s)B(x, y, s_0)$$  \hspace{1cm} (13)

where:

$$F(s) = \frac{1}{1 + \exp(a_0 + a_1(s/D) + ... + a_5(s/D)^5)}$$  \hspace{1cm} (14)
Magnet model verification

Bx, By components for IOTA nonlinear magnet compared with ANSYS simulations (ANSYS data provided by F. O’Shea (RadiaBeam). 3D Cartesian grid, distance from the axis: x=1mm, y=1mm
Potential problem

The same data set, but distance to the origin is $x,y=3.6$ mm. Left - truncation order is 10; Right - 16.
To make a better fit to the real field we need higher order.
We used IOTA version with 2 non-linear elements. The lattice was converted to COSY format. Linear optics is the same in MAD/PTC and COSY, but due to the problem with truncation order described above results seem not relevant.

IOTA studies work in progress...
Local integrability in COSY
PISCS module was created by Anthony Gee.

The Poisson Integral Solver with Curved Surfaces (PISCS) is a package written in COSYScript for MSU COSY Infinity v9.2. PISCS is a 3-D Poisson boundary value problem solver accelerated by the fast multipole method (FMM). In this case, the Poisson BVP represents the electrostatic interactions within a charged particle distribution as a supplement to beam physics computations.
Backup slides: PISCs