A new approach to calculating dynamic friction for magnetized electron coolers –

relevance to future IOTA experiments and to EIC designs

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Fermilab Workshop on Megawatt Rings & IOTA/FAST Collaboration Meeting

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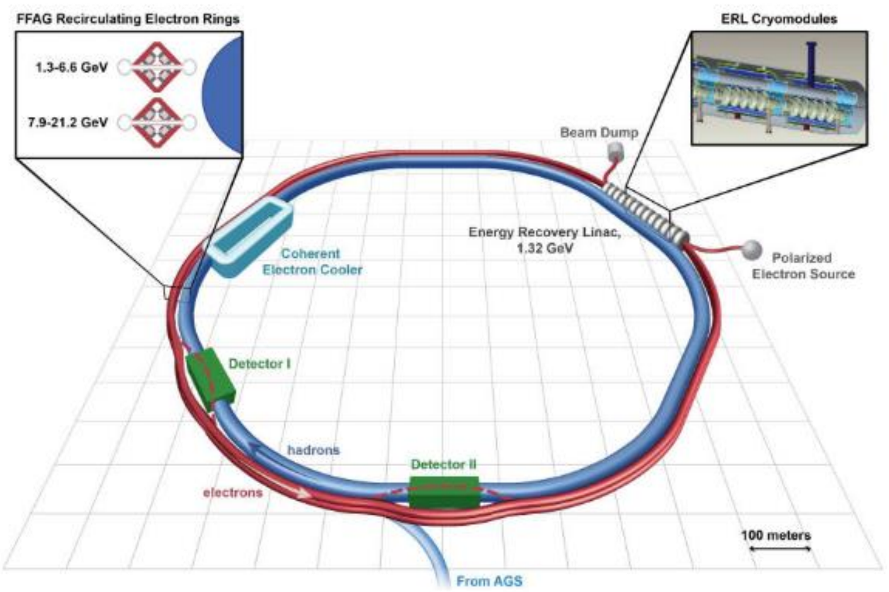
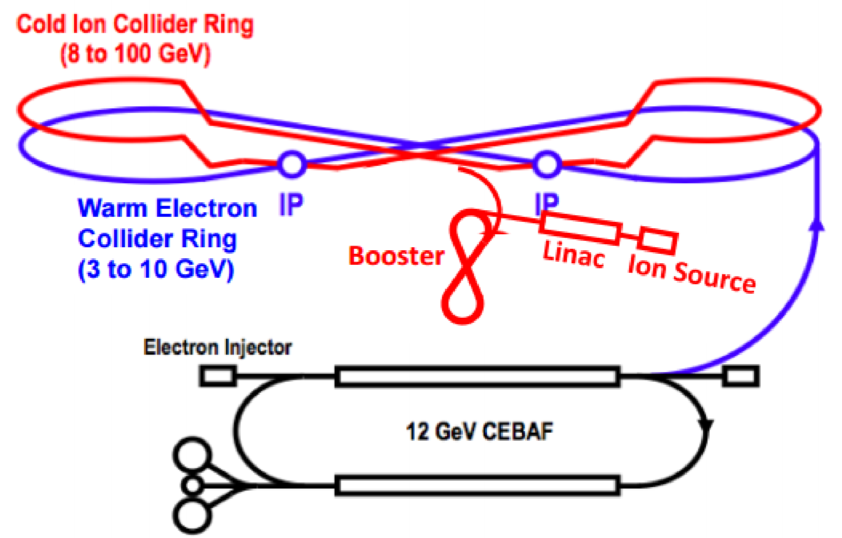
Motivation – Nuclear Physics

- Electron-ion colliders (EIC)
  - *high priority for the worldwide nuclear physics community*
- Relativistic, strongly-magnetized electron cooling
  - *may be essential for EIC, but never demonstrated*

eRHIC concept from BNL



JLEIC concept from Jefferson Lab



Idea for Electron Cooling is 50 Years Old

- Budker developed the concept in 1967
  - *G.I. Budker, At. Energ. 22 (1967), p. 346.*
- Many low-energy electron cooling systems:
  - *continuous electron beam is generated*
  - *electrons are nonrelativistic & very cold compared to bunches*
  - *electrons are magnetized with a strong solenoid field*
    - suppresses transverse temperature & increases friction
- Fermilab has shown cooling of relativistic p-bar's
  - *S. Nagaitsev et al., PRL 96, 044801 (2006).*
  - *~5 MeV e-'s (γ ~ 9) from a DC source*
  - *The electron beam was **not** magnetized*
- Relativistic magnetized cooling not yet demonstrated
  - *electron cooling at γ ~ 100 has not been demonstrated*
    - a non-magnetized concept was developed for RHIC
    - Fedotov *et al.*, Proc. PAC, THPAS092 (2007).

Risk Reduction is Required for Relativistic Coolers

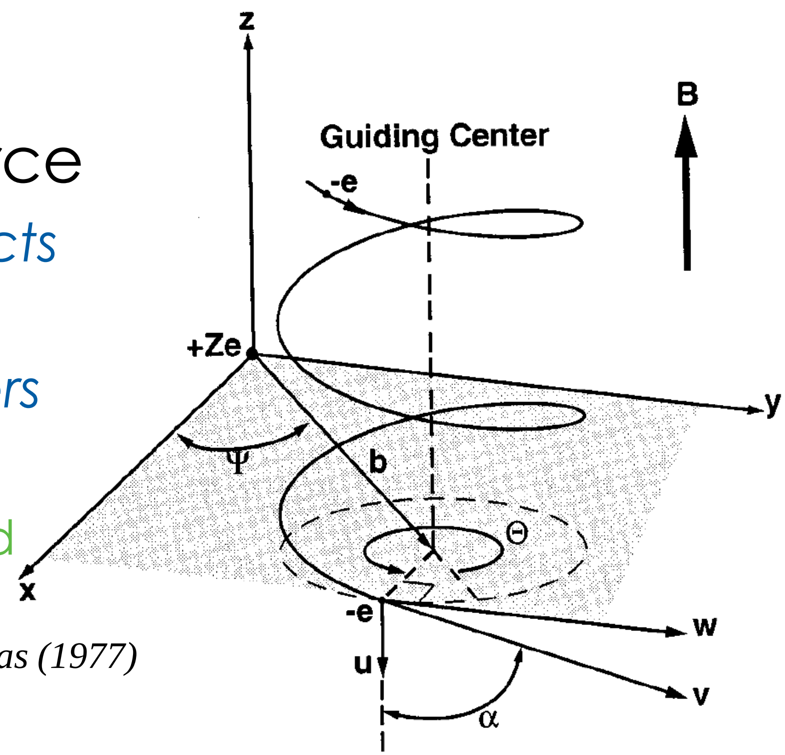
- eRHIC, JLEIC both need cooling at high energy
  - *100 GeV/n → $\gamma \approx 107$ → 55 MeV bunched electrons, ~1 nC*

- Electron cooling at γ~100 requires different thinking
  - *friction force scales like $1/\gamma^2$ (Lorentz contraction, time dilation)*
    - challenging to achieve the required dynamical friction force
    - not all of the processes that reduce the friction force have been quantified in this regime → significant technical risk

  - *normalized interaction time is reduced to order unity*
    - $\tau = t\omega_{pe} >> 1$ for nonrelativistic coolers
    - $\tau = t\omega_{pe} \sim 1$ (in the beam frame), for γ~100
      - *violates the assumptions of introductory beam & plasma textbooks*
      - *breaks the intuition developed for non-relativistic coolers*
      - *as a result, the problem requires careful analysis*

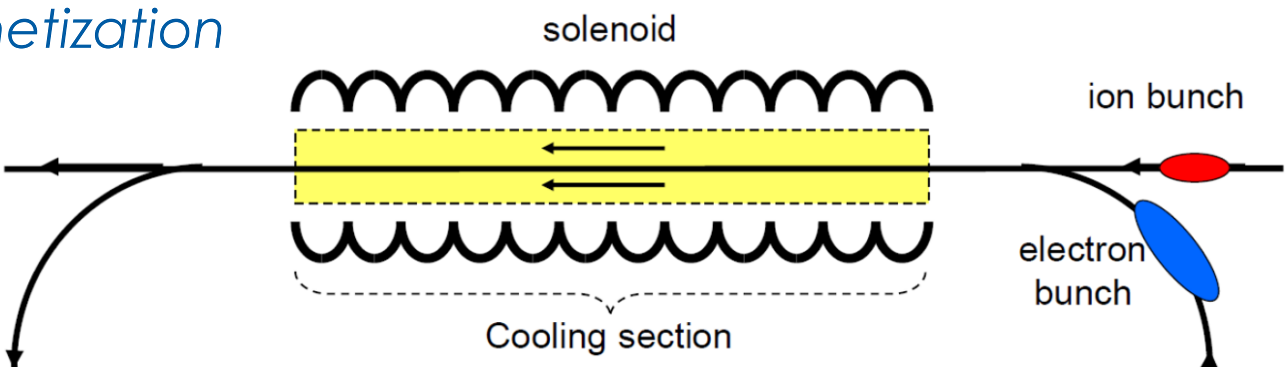
Goals

- Simulate magnetized friction force
  - *include all relevant real world effects*
    - e.g. incoming beam distribution
  - *include a wide range of parameters*
  - *cannot succeed via brute force*
    - improved understanding is required



from Geller & Weisheit, Phys. Plasmas (1977)

- Include key aspects of magnetized e- beam transport
  - *imperfect magnetization*
  - *space charge*
  - *field errors*



from Zhang et al., MEIC design, arXiv (2012)

Asymptotic model for cold, strongly magnetized electrons

$$F_{\parallel} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\left[\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\left(\frac{V_{\perp}}{V_{ion}}\right)^{2}+\frac{2}{3}\right]\frac{V_{\parallel}}{V_{ion}^{3}}$$

$$F_{\perp} = -\omega_{pe}^{2}\frac{(Ze)^{2}}{4\pi\varepsilon_{0}}\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\frac{\left(0.5V_{\perp}^{2}-V_{\parallel}^{2}\right)}{V_{ion}^{2}}\frac{V_{\perp}}{V_{ion}^{3}}$$

$$r_{L} = V_{rms,e,\perp}/\Omega_{L}(B_{\parallel})$$

$$\rho_{\min}^{A} = \max(r_{L},\rho_{\min})$$

$$\rho_{\max}^{A} = \min(r_{beam},\rho_{\max})$$

$$\rho_{\max} = V_{rel}/\max(\omega_{pe},1/\tau)$$

$$V_{rel} = \max(V_{ion},V_{e,rms,\parallel})$$

$$V_{ion}^{2} = V_{\parallel}^{2}+V_{\perp}^{2}$$

Ya. S. Derbenev and A.N. Skrinsky, “The Effect of an Accompanying Magnetic Field on Electron Cooling,” Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, “Magnetization effects in electron cooling,” Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

I. Meshkov, “Electron Cooling; Status and Perspectives,” Phys. Part. Nucl. **25** (1994), 631.

Including thermal effects

$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln\left(\frac{\rho_{\max}+\rho_{\min}+r_L}{\rho_{\min}+r_L}\right)\frac{\mathbf{V}_{ion}}{\left(V_{ion}^2+V_{eff}^2\right)^{3/2}}$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2$$

$$\rho_{\max} = V_{ion}/\max\left(\omega_{pe}, 1/\tau\right)$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\parallel}\right)$$

$$V_{eff}^2 = V_{e,rms,\parallel}^2 + \Delta V_{\perp e}^2$$

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000).

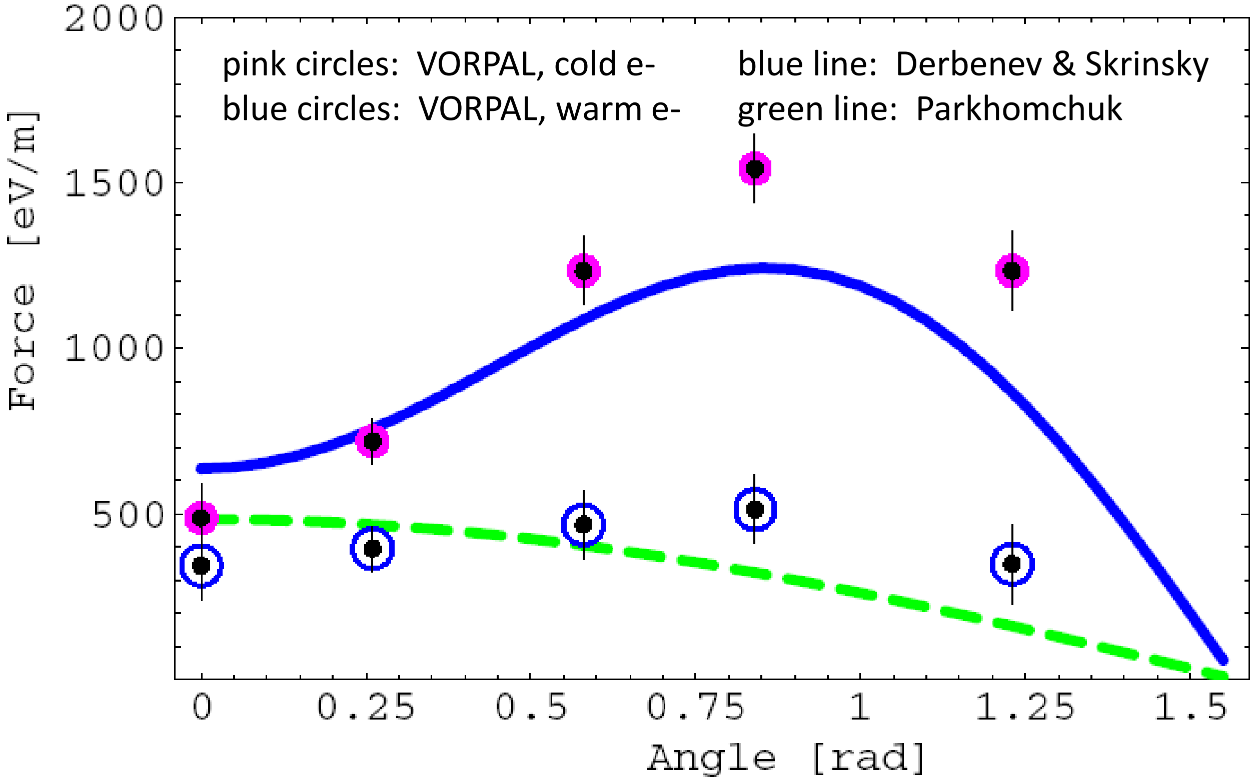
Integrating D&S calculation over thermal electron population:

D.V. Pestrikov, (2002), preprint.

$$F_{\parallel}(0, V_{\parallel}) = -V_{\parallel}\frac{4\pi Z^2 e^4 n_e L_M}{m\Delta_{e,\parallel}^3}\exp\left(-\frac{V_{\parallel}^2}{2\Delta_{e,\parallel}^2}\right)$$

A.V. Fedotov, D.L. Bruhwiler and A.O. Sidorin, "Analysis of the magnetized friction force," Proc. High Brightness (Tsukuba, 2006).

VORPAL modeling of binary collisions clarified differences in formulae for magnetized friction



A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin *et al.*, "Numerical study of the magnetized friction force," Phys. Rev. ST/AB **9**, 074401 (2006).

- D&S asymptotics are accurate for ideal solenoid, cold electrons – not warm
- Parkhomchuk formula often works for typical parameters, but not always
- 3D quad. of D&S with e- dist. works better (modified r_{min}, ideal solenoid)
- In general, direct simulation is required

Detailed simulations of magnetized friction:

WEAY04 Proceedings of HB2006, Tsukuba, Japan

Analysis of the magnetized friction force *

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PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **9**, 074401 (2006)

Numerical study of the magnetized friction force

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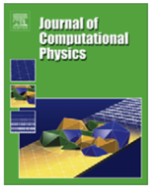

Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam

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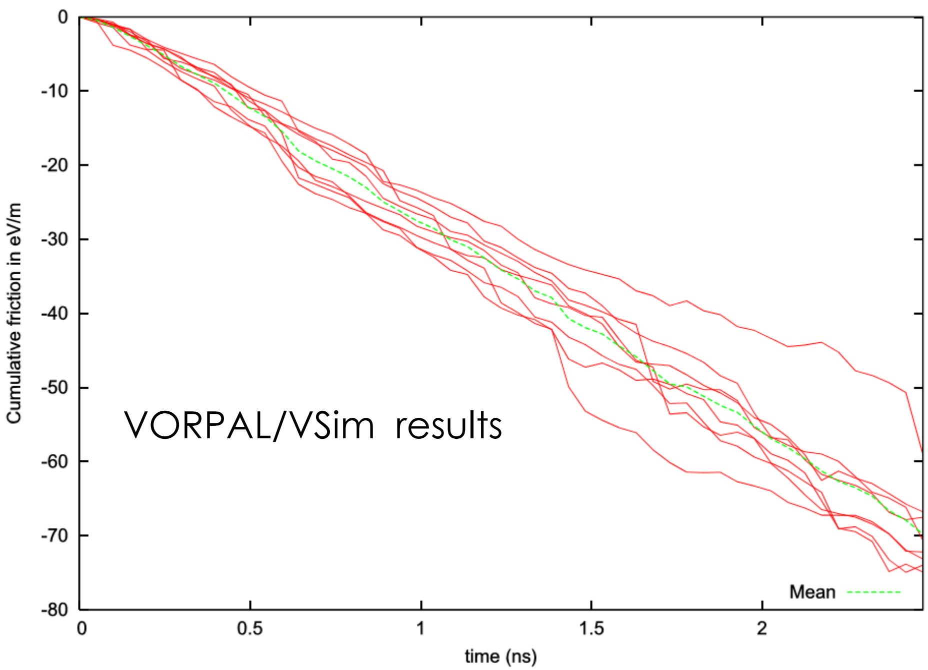
Detailed simulations of magnetized friction:



G.I. Bell, D.L. Bruhwiler, A. Fedotov *et al.*, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam," J. Comp. Phys. **227**, 8714 (2008).

A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin *et al.*, "Analysis of the magnetized friction force," Proc. HB2006, WEAY04 (2006).

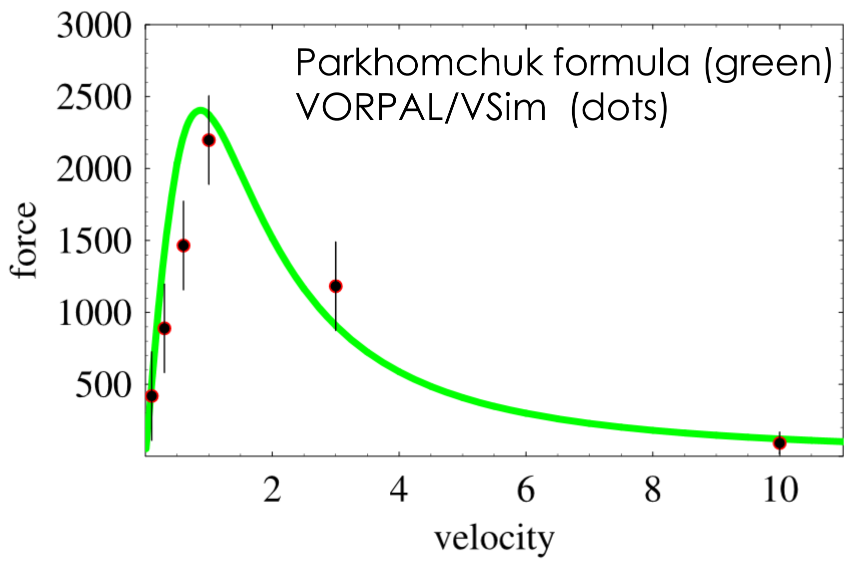
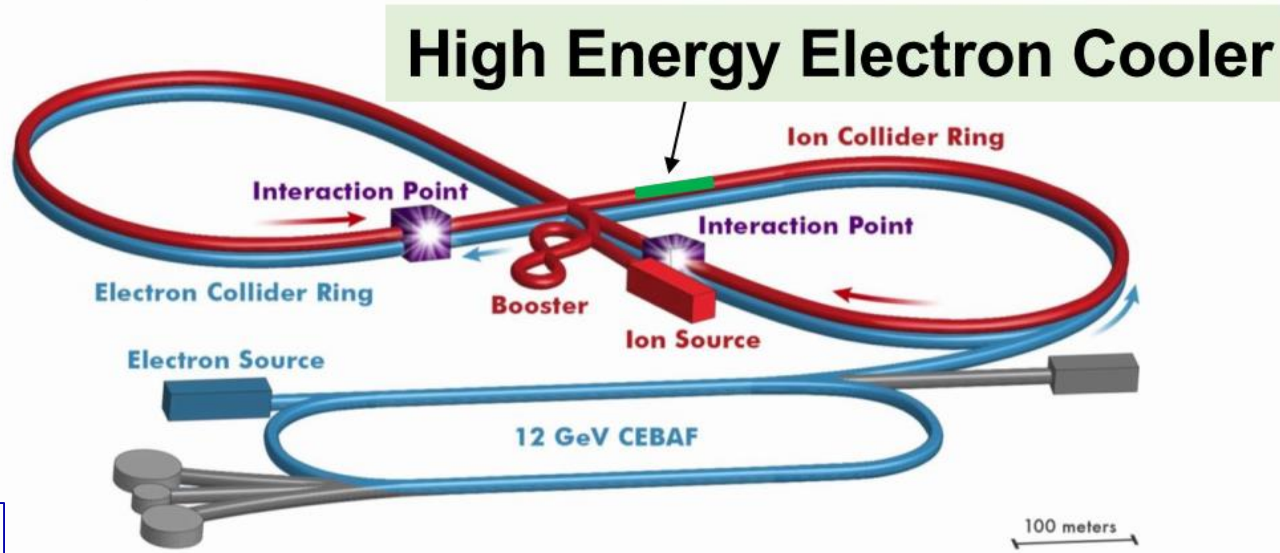
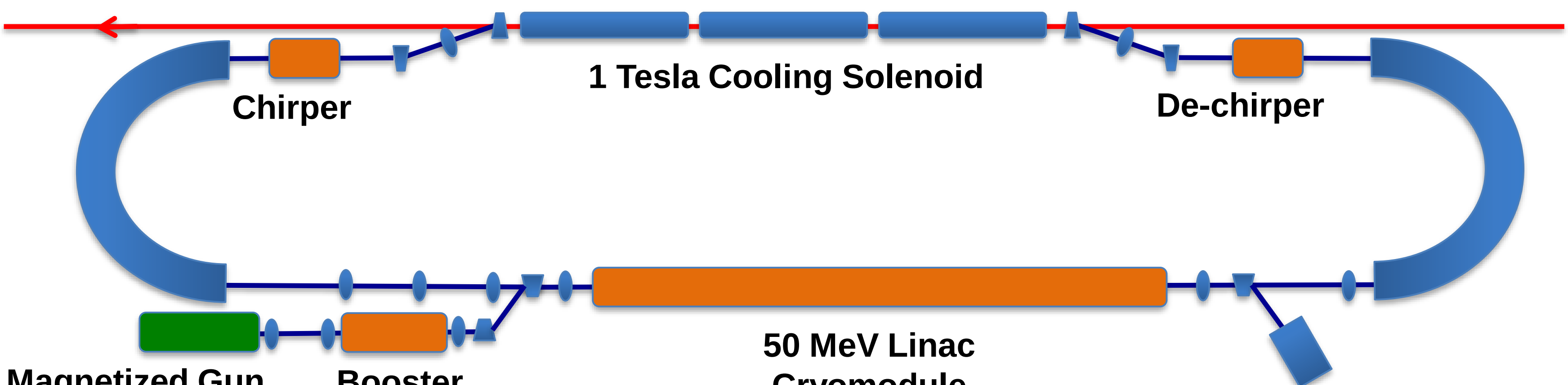


Figure 2: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s] for zero transverse angle $\theta = 0$ with respect to the magnetic field lines. VORPAL results: dots with error bars; Eq. (4) - solid line.

JLab EIC Design:



Images courtesy of Jefferson Lab.



Can we quantify the required solenoidal field quality?

- No, we cannot
  - *Parkhomchuk formula provides a parametric knob*
  - *Derbenev and Skrinsky do not offer quantitative guidance*

- Can we quantify the effects of space charge forces?
  - *No, we cannot*

- Can we quantify the effects of non-Gaussian e- beam phase space distributions?
  - *No, we cannot*

A new dynamical friction calculation is underway…

- We follow the approach described by Y. Derbenev
- However, we begin from a new starting point
  - *analytic momentum transfer between ion and magnetized e-*
  - *proceed step by step with calculation*
- Calculation is defined by the following considerations:

$$\vec{E}(\vec{r},\vec{v},t) = \langle \vec{E}^0 \rangle(\vec{r},t) + \langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t) + \vec{E}^{fl}(\vec{r},\vec{v},t) \quad (1.1)$$

$$\vec{F} = -ze\langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t)\Big|_{\vec{r}=\vec{r}(t),\dot{\vec{r}}(t)=\vec{v}} \quad (1.2)$$

Y. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

THEORY OF ELECTRON COOLING

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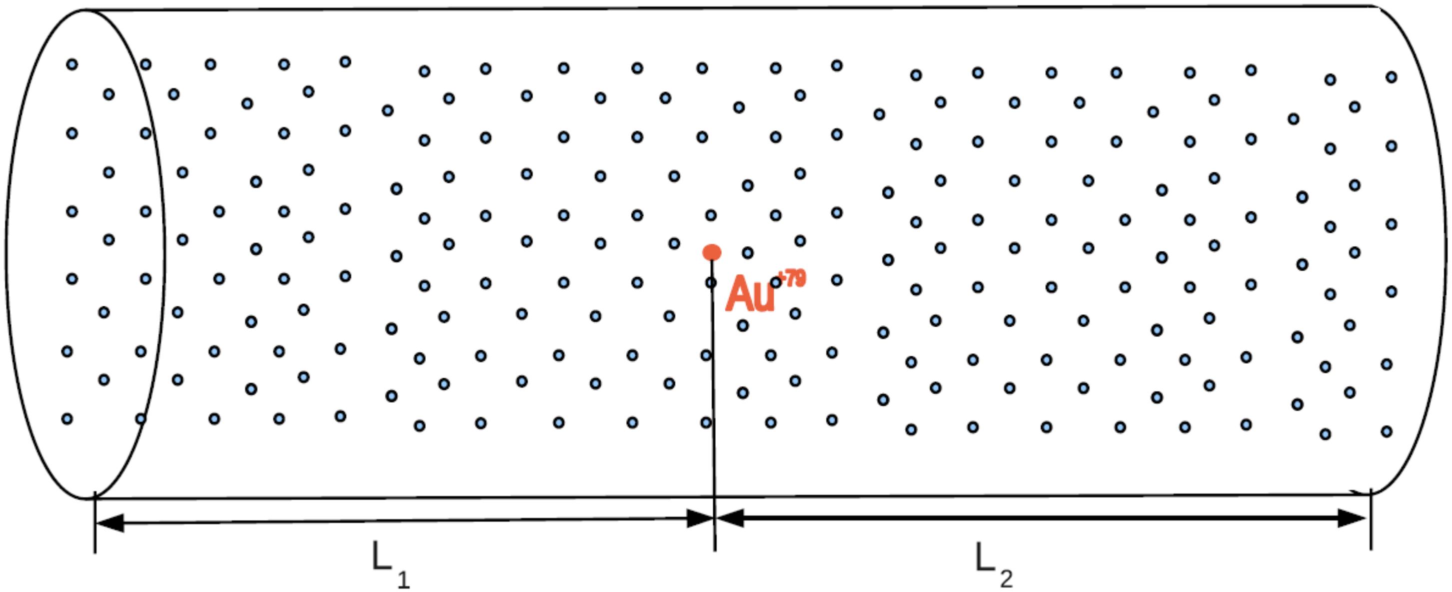
Directly integrate Δp_{ion} to obtain friction force?



$$F = -\frac{n_e m_e}{T} \iiint_{\mathbb{R}^3} d^3 v \iiint_V dr\, dz\, d\varphi\, \Delta v(T, r, \varphi, z, v) r p(v)$$

- Straightforward integration includes space charge, etc.
  - *this approach worked for VORPAL/VSim simulations (w/ effort)*
- Problematic, so we follow Derbenev *et al*.

The required steps are straightforward in principle:

- Calculate the perturbed e- velocities
  - *due to a single ion*
  - *initially, we consider purely longitudinal motion*
- Obtain time-derivative of perturbed E-field
  - *via Poisson and continuity equations*
- Integrate in time to get δE
  - *initially, this is for only a single value of e- velocity*
  - *it is necessary to integrate over thermal e- velocities*
- Integrate δE along ion trajectory to obtain <F>
  - *hence, this is a 2nd-order effect,* $\sim(Ze^2)^2$ *xx*
- Present efforts:
  - *find best way to integrate <F> over e- distribution functions*
  - *consider transverse ion motion*
  - *numerical approaches, testing, etc.*

Hamiltonian for 2-body magnetized collision:

$$H\left(\vec{x}_{ion}, \vec{p}_{ion}, \vec{x}_e, \vec{p}_e\right) = H_0\left(\vec{p}_{ion}, y_e, \vec{p}_e\right) + H_C\left(\vec{x}_{ion}, \vec{x}_e\right)$$

$$\vec{B} = B_0\,\hat{z} \qquad \vec{A} = -B_0 y\,\hat{x} \qquad p_{e,x} = m_e\left(v_{e,x} - \Omega_L y_e\right)$$

$$H_0\left(\vec{p}_{ion}, y_e, \vec{p}_e\right) = \frac{1}{2m_{ion}}\left(p_{ion,x}^2 + p_{ion,y}^2 + p_{ion,z}^2\right) + \frac{1}{2m_e}\left[\left(p_{e,x} + eB_0 y_e\right)^2 + p_{e,y}^2 + p_{e,z}^2\right]$$

$$H_C\left(\vec{x}_{ion}, \vec{x}_e\right) = \frac{-Ze^2}{4\pi\varepsilon_0} \Big/ \sqrt{\left(x_{ion} - x_e\right)^2 + \left(y_{ion} - y_e\right)^2 + \left(z_{ion} - z_e\right)^2}$$

Resulting equations of motion, in the standard drift-kick symplectic form:

$$M\left(\Delta t\right) = M_0\left(\Delta t/2\right) M_C\left(\Delta t\right) M_0\left(\Delta t/2\right)$$

D.L. Bruhwiler and S.D. Webb, "New algorithm for dynamical friction of ions in a magnetized electron beam," in *AIP Conf. Proc.* **1812**, 050006 (2017); http://aip.scitation.org/doi/abs/10.1063/1.4975867

Analytic calculation of $\Delta\mathbf{p}_{ion}$ (1)

$$C_1 = \left(x_{ion} - \frac{p_{gc}}{m_e \Omega_e} \right)^2 + \left(y_{ion} - y_{gc} \right)^2 + \left(z_{ion} - z_e \right)^2 + \frac{2}{m_e \Omega_e} J \tag{14a}$$

$$C_2 = 2\left(x_{ion} - x_{gc} \right) v_{ion,x} + 2\left(y_{ion} - y_{gc} \right) v_{ion,y} + 2\left(z_{ion} - z_e \right)\left(v_{ion,z} - v_{ez} \right) \tag{14b}$$

$$C_3 = v_{ion,x}^2 + v_{ion,y}^2 + \left(v_{ion,x} - v_{ez} \right)^2 \tag{14c}$$

$$b = \left[C_1 + C_2 T + C_3 T^2 \right]^{1/2} \qquad \Delta = 4 C_1 C_3 - C_2^2 \tag{14d}$$

$$D_1 = \left[\frac{2 C_3 T + C_2}{b} - \frac{C_2}{\sqrt{C_1}} \right] \tag{14e}$$

$$D_2 = \left[\frac{2 C_1 + C_2 T}{b} - 2\sqrt{C_1} \right] \tag{14f}$$

Analytic calculation of $\Delta \mathbf{p}_{ion}$ (2)

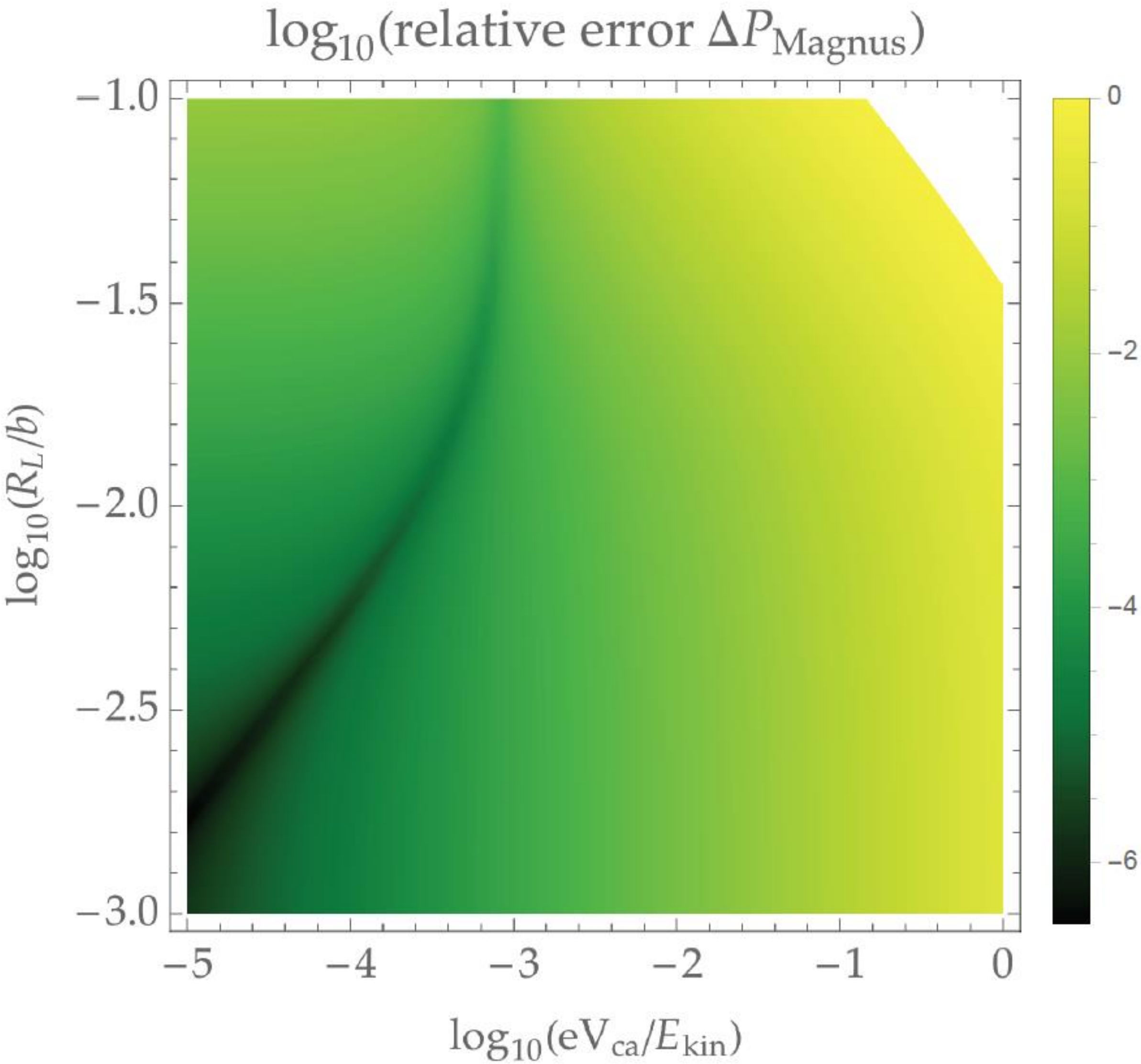
$$\Delta p_{ion,x} = \frac{-2\alpha}{\Delta}\left[\left(x_{ion} - p_{gc}/m_e\Omega_e\right)D_1 - \left(p_{ion,x}/m_{ion}\right)D_2\right] \qquad (15a)$$

$$\Delta p_{ion,y} = \frac{-2\alpha}{\Delta}\left[\left(y_{ion} - y_{gc}\right)D_1 - \left(p_{ion,y}/m_{ion}\right)D_2\right] \qquad (15b)$$

$$\Delta p_{ion,z} = \frac{-2\alpha}{\Delta}\left[\left(z_{ion} - z_e\right)D_1 - \left(\frac{p_{ion,z}}{m_{ion}} - \frac{p_{ez}}{m_e}\right)D_2\right] \qquad (15c)$$

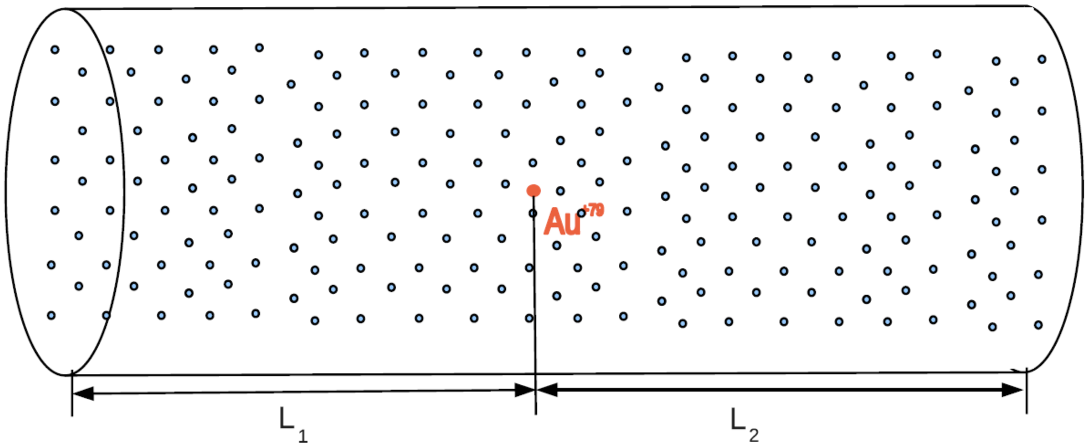
$$\Delta p_{gc} = -\Delta p_{ion,x} \qquad \Delta y_{gc} = -\Delta p_{ion,y}/m_e\Omega_e \qquad (15d)$$

Time-explicit vs analytic shows agreement:



- Two small parameters are required:
  - *Larmor radius must be small compared to impact param.*
    - **averaging**
  - *Kinetic energy must be large compared to max potential energy*
    - **perturbative**

Choice of coordinate system is important:



$$\Delta v_{e,z} = \frac{Ze^2}{m_e}\left\{\left[x_{gc}^2 + y_{gc}^2 + \rho_{gc}^2 + z_e^2\right]^{-1/2} - \left[x_{gc}^2 + y_{gc}^2 + \rho_{gc}^2 + (z_e - v_{rel}T)^2\right]^{-1/2}\right\}$$

$$v_{rel} = v_{i,z} - v_{e,z}$$

$$\frac{\partial \vec{E}}{\partial T} = -\vec{J} \qquad r^2 = x_{gc}^2 + y_{gc}^2$$

$$\left.\frac{\partial E_z}{\partial T}\right|_{r=0} = \frac{Ze^2}{m_e}\left\{\left[\rho_{gc}^2 + z_e^2\right]^{-1/2} - \left[\rho_{gc}^2 + (z_e - v_{rel}T)^2\right]^{-1/2}\right\}$$

Integrate twice to obtain friction force:

$$E_z(r=0) = (n_0 e)\frac{Ze^2}{m_e v_{rel}}\left\{T\left[\rho_{gc}^2 + z_e^2\right]^{-1/2} + \frac{1}{v_{rel}} ln\left[\frac{\left[\rho_{gc}^2 + z_e^2\right]^{1/2} + z_e}{\left[\rho_{gc}^2 + (z_e - v_{rel}T)^2\right]^{1/2} + (z_e - v_{rel}T)}\right]\right\}$$

Let $z_e = v_{i,z}T$ *and then integrate over* T *to obtain*:

$$< F > = (n_0 e)\frac{n_0 (Ze^2)^2}{m_e v_{rel} T}\left\{\frac{T}{v_{rel}} ln\left(\left[\rho_{gc}^2 + \left(v_{i,z}T\right)^2\right]^{1/2} + v_{i,z}T\right) - \frac{v_{e,z}}{v_{i,z}^2 v_{rel}}\left(\left[\rho_{gc}^2 + \left(v_{i,z}T\right)^2\right]^{\frac{1}{2}} - \rho_{gc}\right) - \frac{T}{v_{rel}} ln\left(\left[\rho_{gc}^2 + \left(v_{e,z}T\right)^2\right]^{1/2} + v_{e,z}T\right) - \frac{1}{v_{e,z} v_{rel}}\left(\left[\rho_{gc}^2 + \left(v_{e,z}T\right)^2\right]^{\frac{1}{2}} - \rho_{gc}\right)\right\}$$

There is an integrable singularity for cold electrons.
The challenge now is to integrate over thermal velocities

Long-term goals:
Include other effects in Magnus Expansion

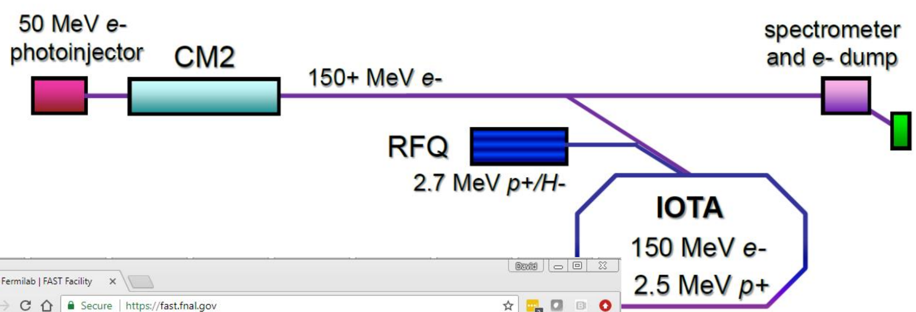
$$H\left(\vec{x}_{ion},\vec{p}_{ion},\vec{x}_e,\vec{p}_e\right)=H_0\left(\vec{p}_{ion},y_e,\vec{p}_e\right)+H_C\left(\vec{x}_{ion},\vec{x}_e\right)$$
$$+H_{space-charge}\left(\vec{x}_{ion},\vec{x}_e\right)+H_{solenoid-field-errors}\left(??\right)$$

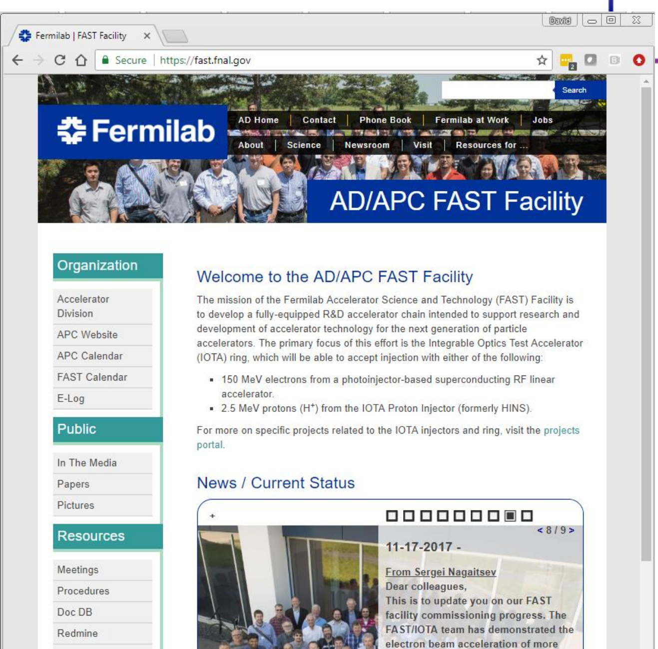
- Quantitative treatment of space charge & field errors?
  - *space charge should work*
  - *field errors are more challenging*

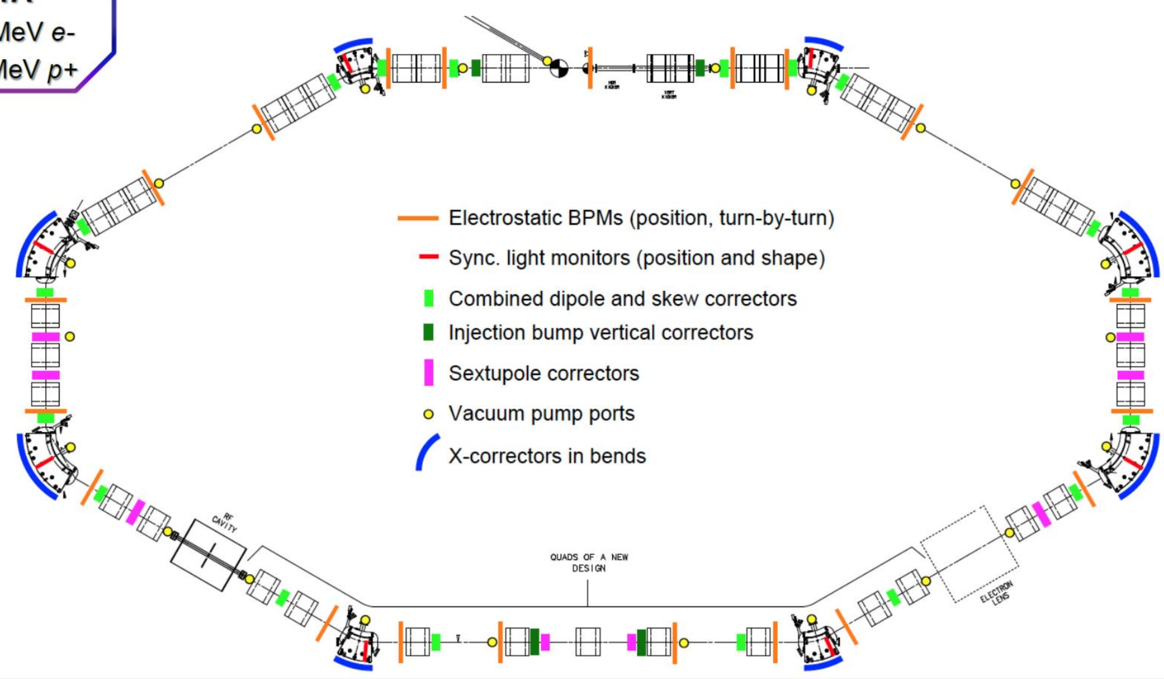
- Requires generalization of Magnus expansion
  - *we are optimistic this can be done*

Future Electron cooling experiments in IOTA

- Could be used to test friction force equations…?
  - *RadiaSoft is interested to collaborate*



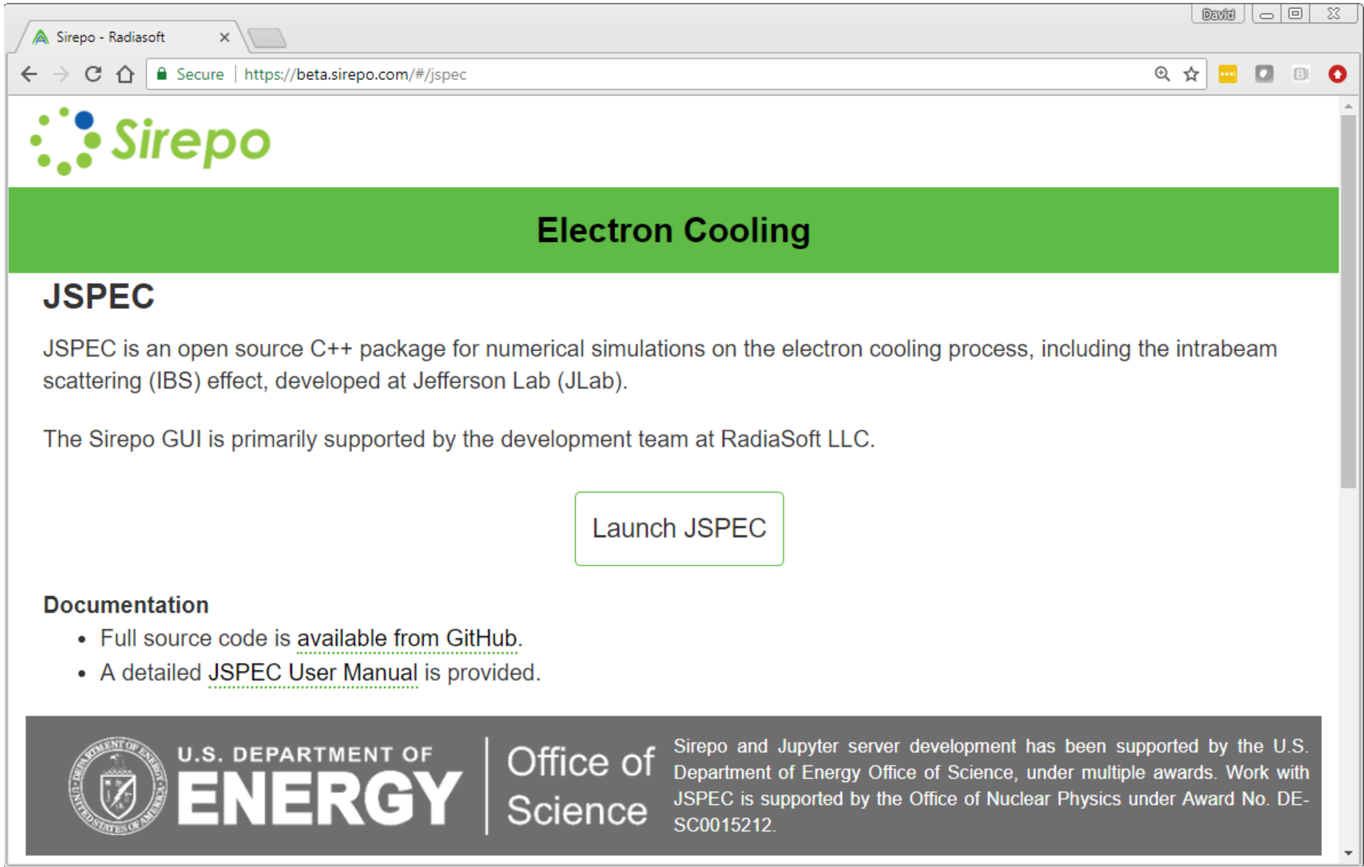




JSPEC – new software for IBS and e- cooling

JSPEC on GitHub, https://github.com/zhanghe9704/electroncooling
Primary developer is He Zhang of JLab

A new GUI for JSPEC is under development, as part of the open source cloud computing initiative, Sirepo http://sirepo.com



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Thank You!

Questions?