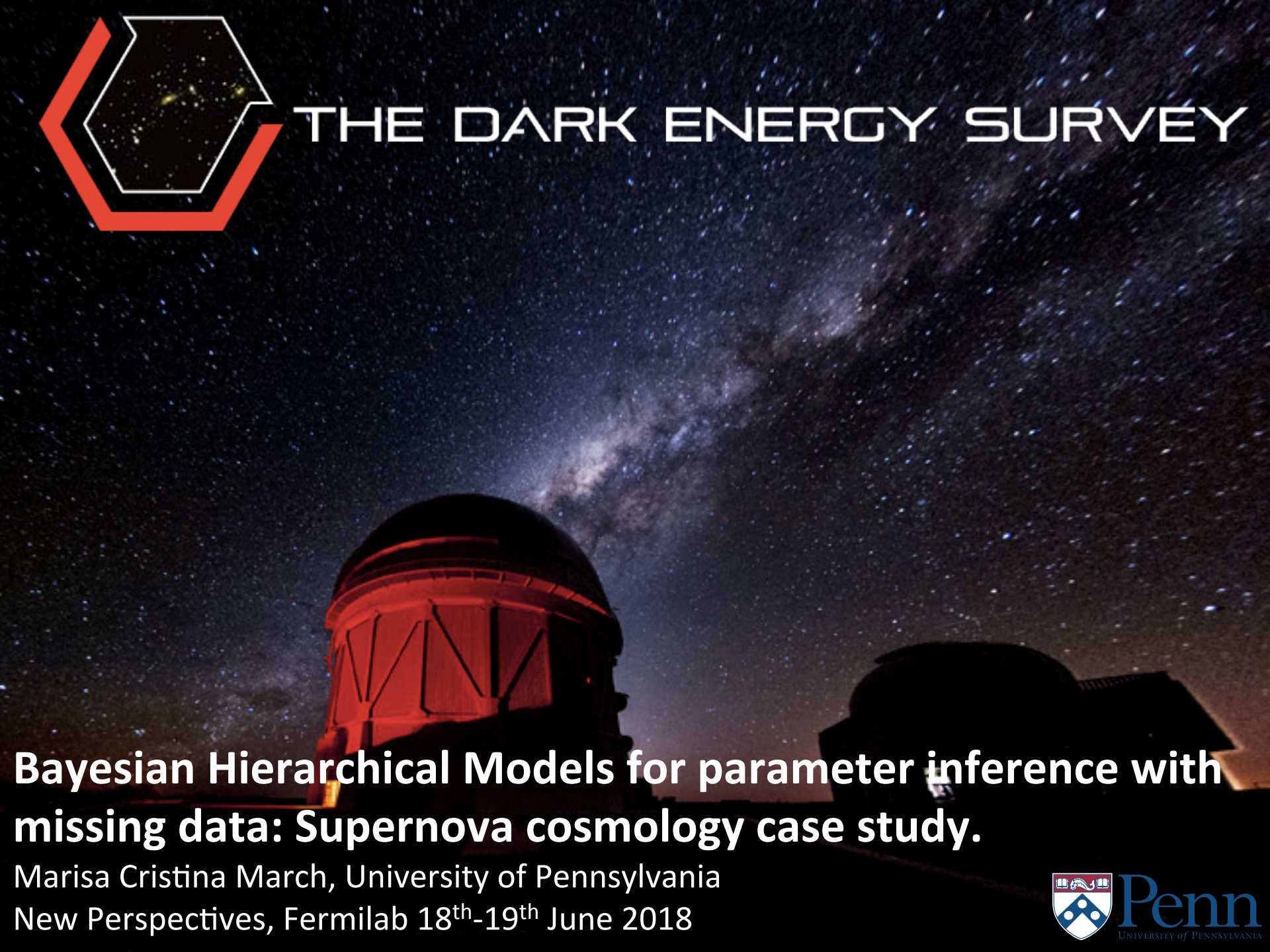


THE DARK ENERGY SURVEY

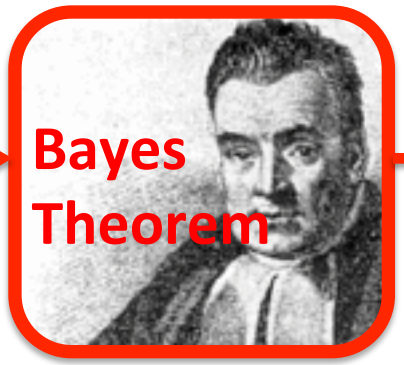
Bayesian Hierarchical Models for parameter inference with missing data: Supernova cosmology case study.

Marisa Cristina March, University of Pennsylvania

New Perspectives, Fermilab 18th-19th June 2018

Penn
University of Pennsylvania

Motivation & Overview



Dark Energy
w=?

Mission: To understand the nature of dark energy

- Using the **Dark Energy Camera** to search for Supernovae Ia
- Using **Bayes Theory** to do the statistical analysis in order to understand the nature of **dark energy**.
- Specific challenge addressed in this talk:
  - How to deal with **missing data** (magnitude limited survey) in a **Bayesian** way, in order to:
    - Use Supernovae Ia to do **Bayesian Model Selection**
    - Understand and reduce **systematics**

Physics concept: Using standard candles to measure dark energy

If you have objects of a standard brightness, you can work out how far away they are based on how bright they appear to be.

Define the 'observed' distance modulus, to be the difference between the apparent (observed) and absolute magnitudes (brightness) of your standard object:

$$\mu^{\text{observed}} = m_B - M_0$$

absolute magnitude

apparent magnitude

The theoretical distance modulus depends on the redshift and the cosmological parameters:

$$\mu^{\text{theory}} = f\{z, \Omega_m, \Omega_\kappa, \Omega_\Lambda, w(z)\}$$

redshift

matter density

curvature density

dark energy density

dark energy equation of state

Recipe:

(1) Measure the apparent magnitude.

(2) Measure the redshift.

(3) Work out what values the cosmological parameters must be to get:

$$\mu^{\text{theory}} = \mu^{\text{observed}}$$

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Evidence for Cosmic Acceleration

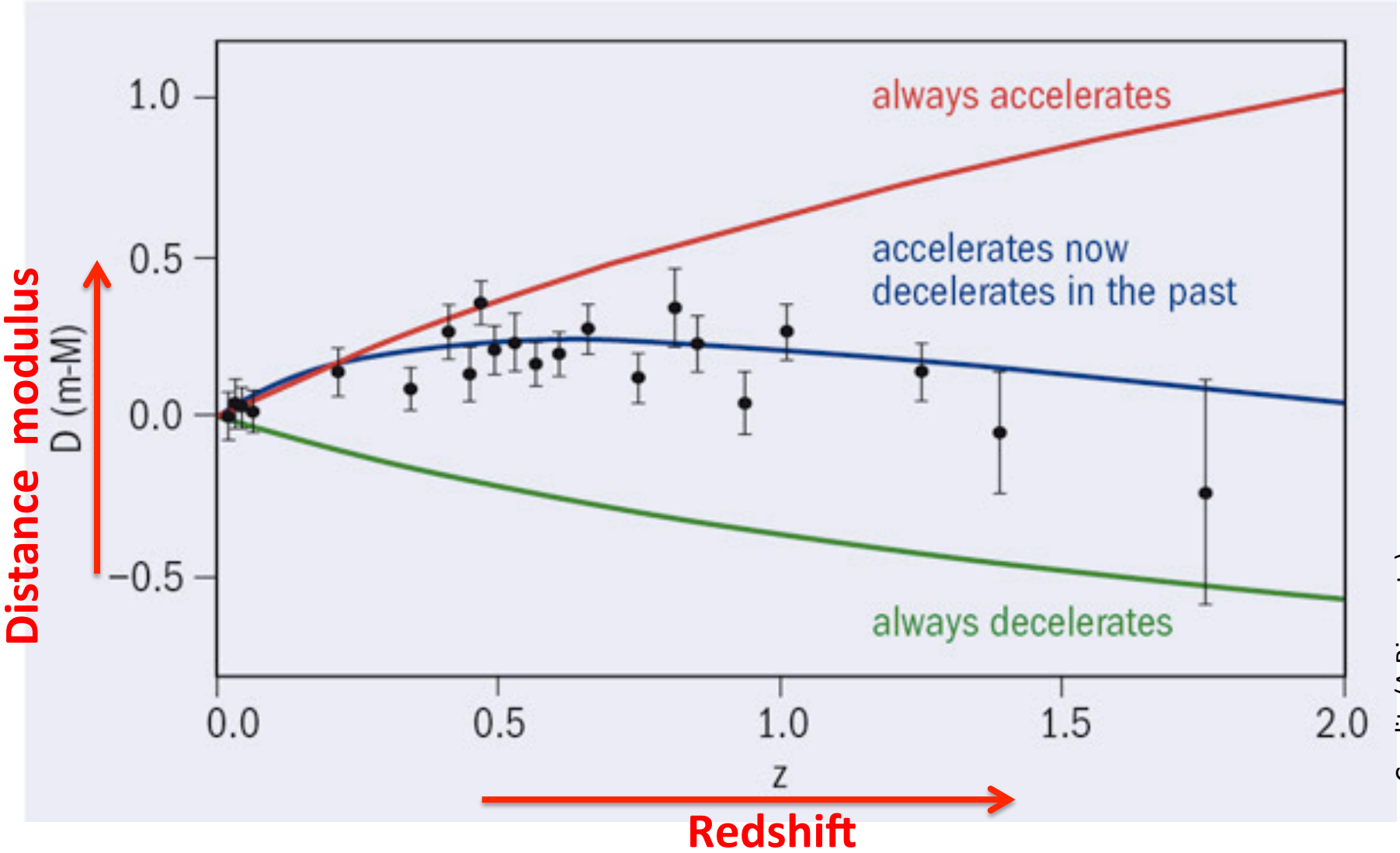


Image Credit: (A Riess et .).
Michael Turner, conference summary; Turner and Huterer 2007.

Using Supernovae Type Ia as Standard Candles

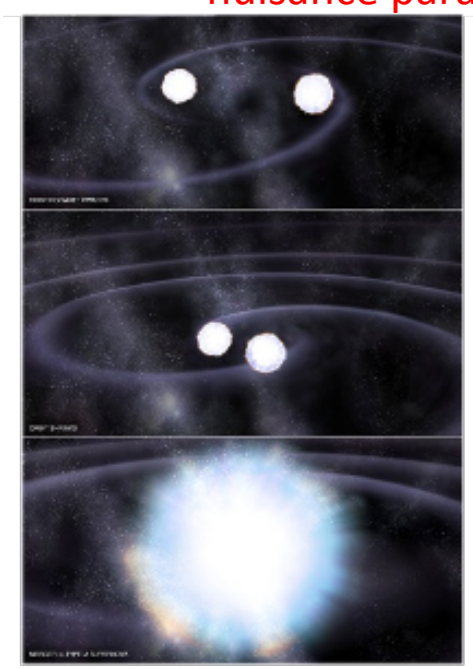
Use the stretch and color of the SNe light curves to apply small corrections to (i.e. to standardize) their brightness.

stretch

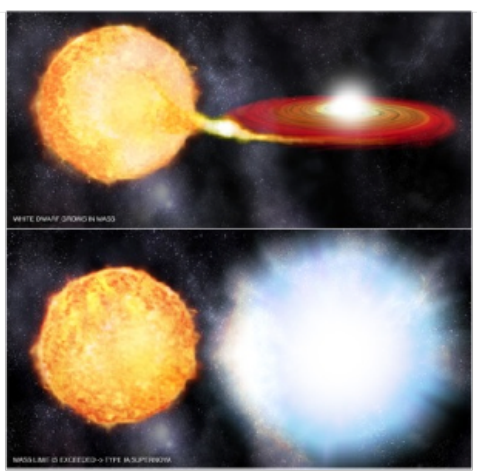
$$\mu^{\text{observed}} = m_B - M_0 + \alpha x_1 - \beta c$$

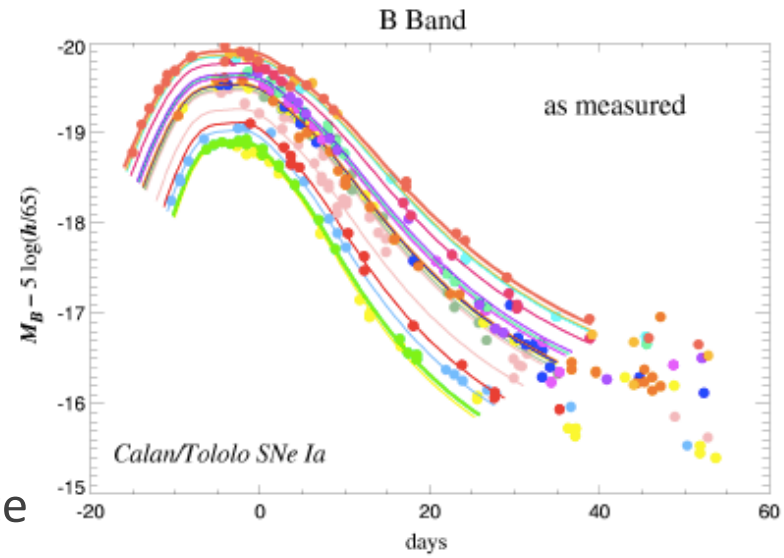
color

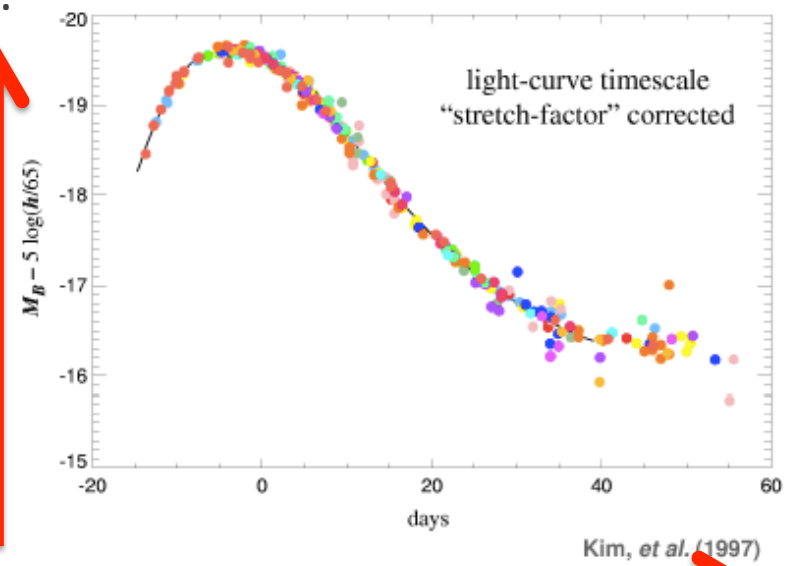
nuisance parameters



SNe Ia thermonuclear explosions come from white dwarf binary mass transfer.





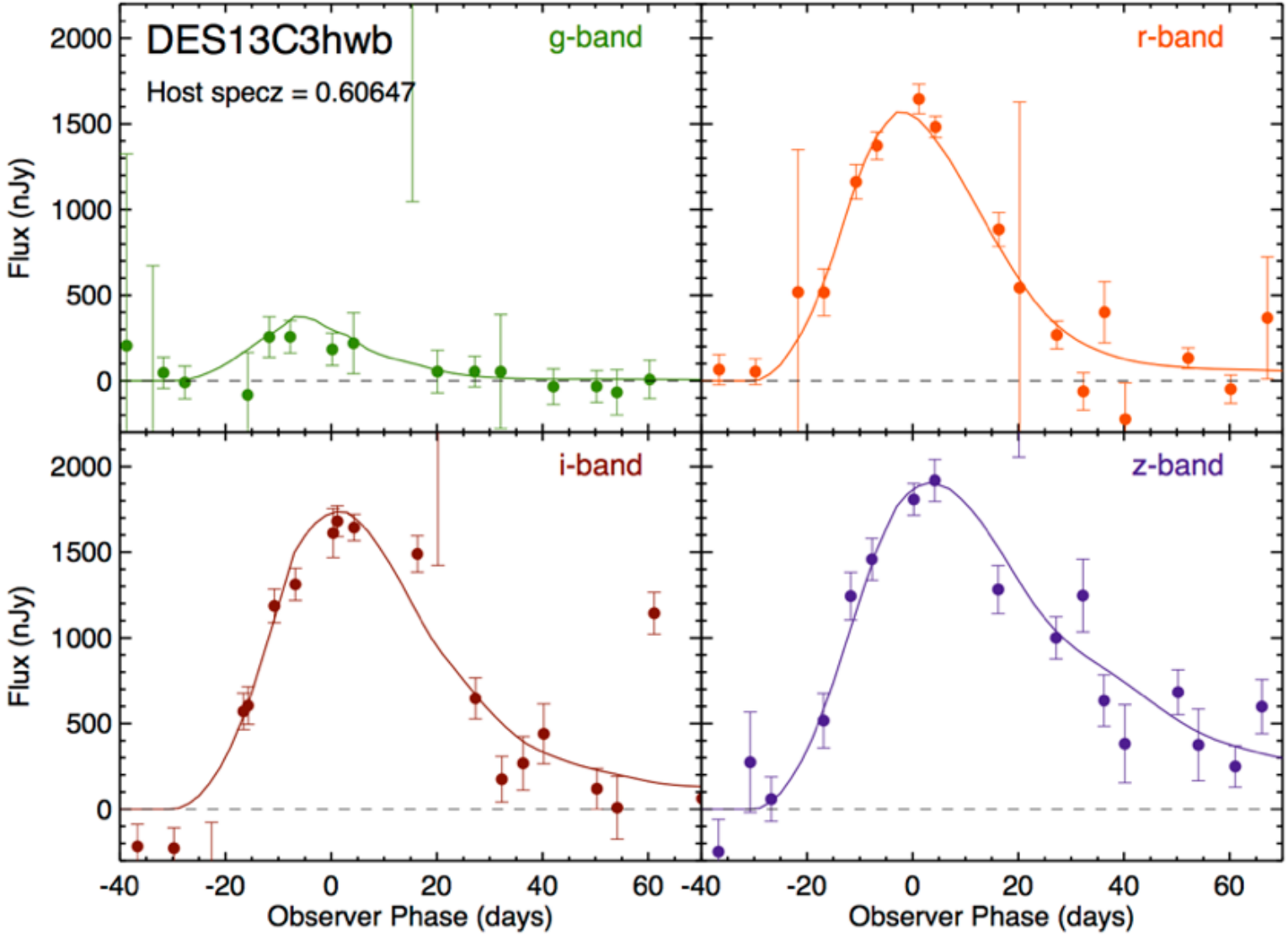


magnitude

days

THE DARK ENERGY SURVEY

Data: Supernova Light Curves



Plot credit: Chris D'Andrea

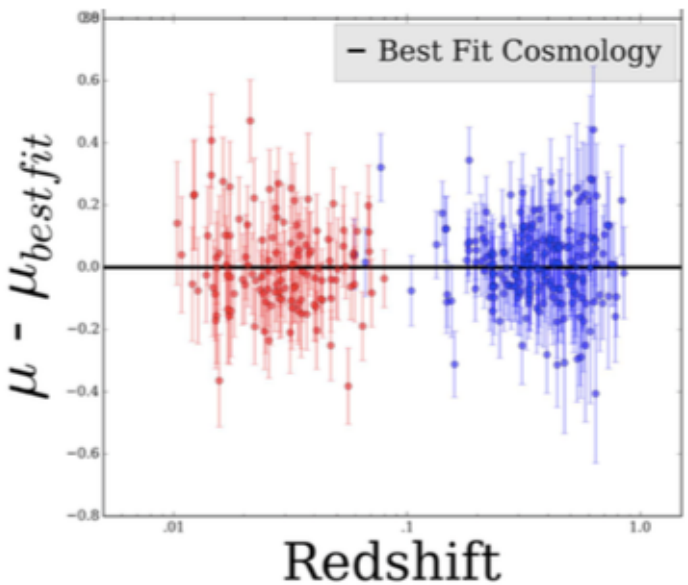
Prelim DES Results! Flat wCDM

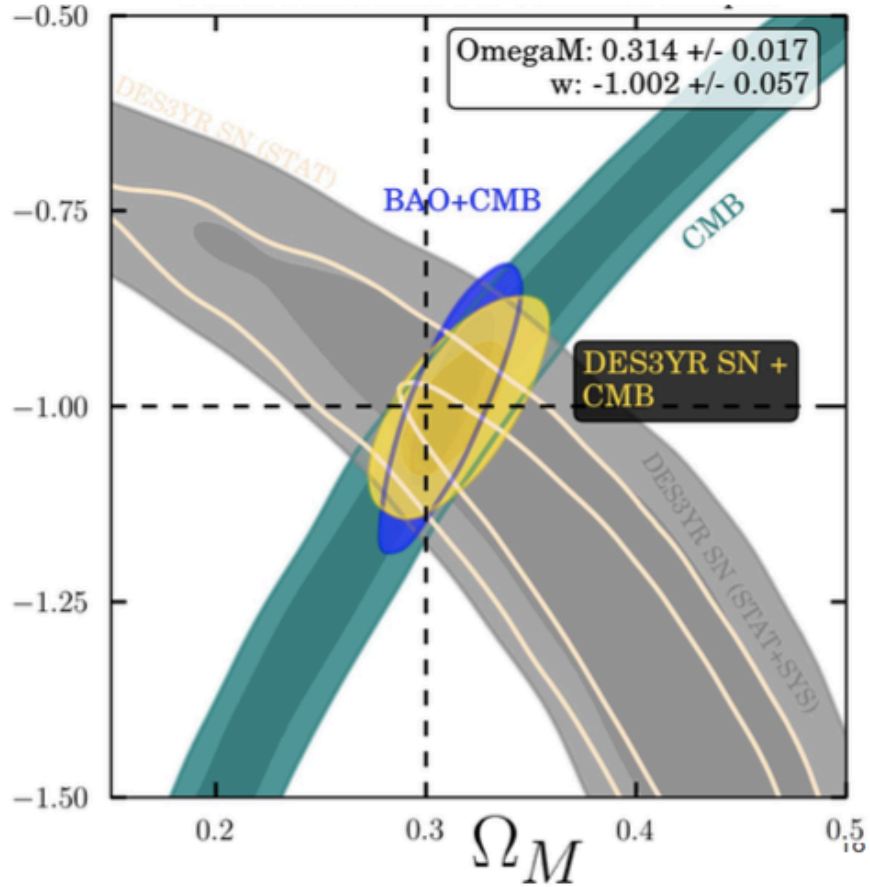
$$w = -1.002 \pm 0.057$$

$$\sigma_w = 0.041(STAT), 0.040(SYS)$$

The beginning of an era dominated by systematic uncertainties

$$\Omega_M = 0.314 \pm 0.017$$





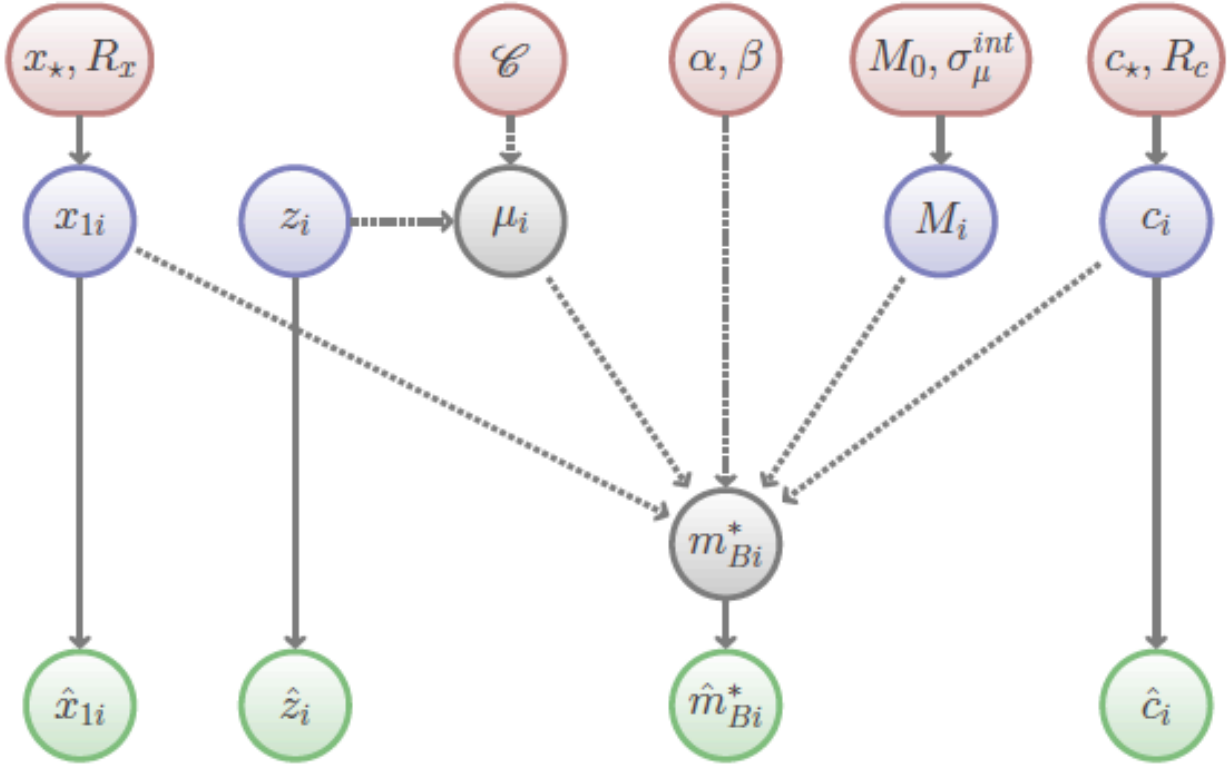
Slide & Plot Credit: Thanks to Dillon Brout!

Beyond the preliminary results:

- Systematics?
- Model Selection?

Use Bayes Theory!

Supernova Bayesian Hierarchical Model



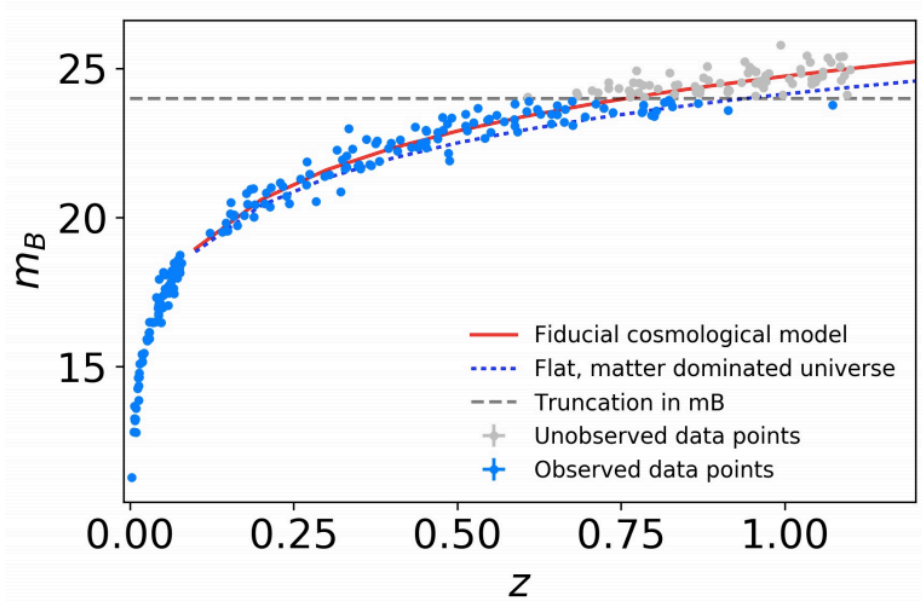
MM. et al. 2011

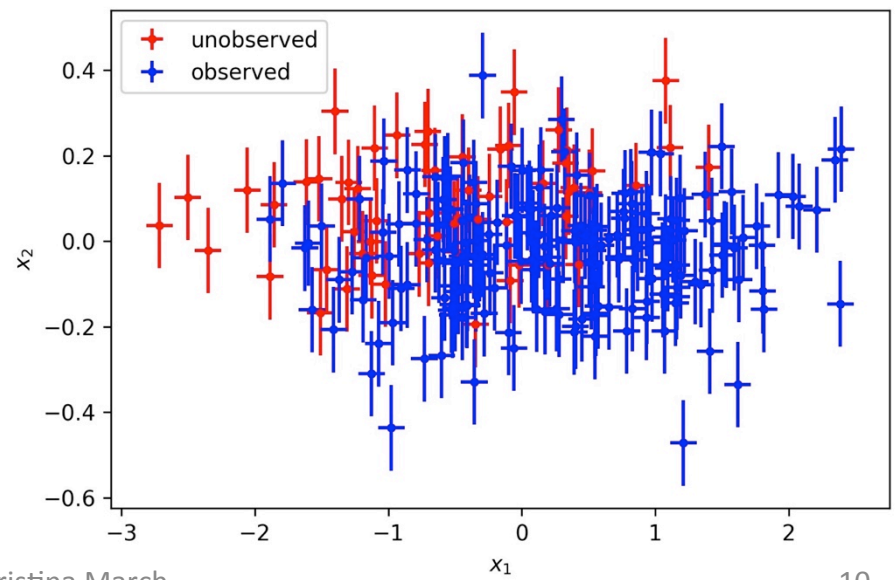
Allows use of Supernova data for Bayesian Model Selection.

- **Which model best explains dark energy?** LCDM, Modified Gravity? Scalar Field? Chameleon Field?
- Uses **latent** or hidden variables and **priors** to model observational data.

Truncated data sets and Malmquist bias in SN cosmology

- Problem is that supernova data sets are **incomplete** in magnitude space. Limit of magnitude is set by instrument and environmental conditions.
- One solution is to **discard data** below a magnitude threshold. Disadvantage is **loss of information.**
- Another solution is to **simulate** surveys and "**correct**" mB data points to recover correct cosmology. Disadvantage is that this cannot be used for Bayesian model selection.
- **Alternative way:** Bayesian Hierarchical Model.





Analytic solution for Malmquist bias (missing data) in Supernova Bayesian Hierarchical Model

$$p(\mathscr{C}, \alpha, \beta | x_1^{\rm obs}, c^{\rm obs} m_B^{\rm obs}, z^{\rm obs}, m_B^{\rm thresh} I, M)$$

$$\propto \int_{N_{\rm obs}}^{\rm inf} dN \iint d\mathbf{R_x}\, d\mathbf{x}_\star \frac{1}{N} \binom{N}{N_{\rm obs}} \prod_i^{N_{\rm obs}} |2\pi\Sigma_{v,i}^{\rm obs}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left((\hat{\mathbf{w}}_{\mathbf{i}}^{\rm obs} - \mathbf{q}_i)^T \Sigma_{v,i}^{\rm obs} (\hat{\mathbf{w}}_{\mathbf{i}}^{\rm obs} - \mathbf{q}_i)\right)\right)$$

$$\times \prod_i^m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{m_B^{\rm thresh}}^{+\infty} \int_0^{\hat{z}^{\rm max}} d\hat{x}_{1i}^{\rm mis}\, d\hat{c}_i^{\rm mis}\, d\hat{m}_{Bi}^{\rm mis}\, d\hat{z}_i^{\rm mis} |2\pi\Sigma_{v,i}^{\rm mis}|^{-\frac{1}{2}}$$

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$$\times p(R_x, x_\star | I, M) p(\mathscr{C}, \alpha, \beta | I, M)$$

Posterior probability of parameters in a truncated data set

$$\mathbf{x}_\star = [x_{1,\star} c_\star] \in \mathbb{R}^2$$

$$\mathbf{R_x} = \begin{bmatrix} R_{x1} & 0 \\ 0 & R_c \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\hat{\mathbf{w}}_{\mathbf{i}} = \begin{bmatrix} \hat{m}_{B,i} \\ \hat{x}_{1,i} \\ \hat{c}_i \end{bmatrix} \in \mathbb{R}^3$$

$$\Sigma_{c,i} = \begin{bmatrix} \sigma_{mi}^2 & \sigma_{mi,x1i} & \sigma_{mi,ci} \\ \sigma_{mi,x1i} & \sigma_{x1i}^2 & \sigma_{c,x1i} \\ \sigma_{mi,ci} & \sigma_{x1i,ci} & \sigma_{ci}^2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

arXiv:1804.02474

Parameters of interest

$$p(\boxed{\mathscr{C}, \alpha, \beta} | x_1^{\rm obs}, c^{\rm obs} m_B^{\rm obs}, z^{\rm obs}, m_B^{\rm thresh} I, M)$$

$$\propto \int_{N_{\rm obs}}^{\rm inf} dN \iint d\mathbf{R_x}\, d\mathbf{x}_\star\, \frac{1}{N} \binom{N}{N_{\rm obs}} \prod_i^{N_{\rm obs}} |2\pi\Sigma_{v,i}^{\rm obs}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left((\hat{\mathbf{w}}_\mathbf{i}^{\rm obs} - \mathbf{q}_i)^T \Sigma_{v,i}^{\rm obs} (\hat{\mathbf{w}}_\mathbf{i}^{\rm obs} - \mathbf{q}_i)\right)\right)$$

$$\times \prod_i^m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{m_B^{\rm thresh}}^{+\infty} \int_0^{\hat{z}^{\rm max}} d\hat{x}_{1i}^{\rm mis}\, d\hat{c}_i^{\rm mis}\, d\hat{m}_{Bi}^{\rm mis}\, d\hat{z}_i^{\rm mis} |2\pi\Sigma_{v,i}^{\rm mis}|^{-\frac{1}{2}}$$

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$$\times\, p(R_x, x_\star | I, M) p(\mathscr{C}, \alpha, \beta | I, M) \qquad (45)$$

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Parameters of interest

data

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Parameters of interest

data

Likelihood of observed data

$$p(\mathscr{C},\alpha,\beta|x_1^{\rm obs},c^{\rm obs}m_B^{\rm obs},z^{\rm obs},m_B^{\rm thresh}I,M$$

$$\propto \int_{N_{\rm obs}}^{\rm inf} dN \iint d\mathbf{R_x}\, d\mathbf{x}_\star\, \frac{1}{N}\binom{N}{N_{\rm obs}} \prod_i^{N_{\rm obs}} |2\pi\Sigma_{v,i}^{\rm obs}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left((\hat{\mathbf{w}}_{\mathbf{i}}^{\rm obs}-\mathbf{q}_i)^T\Sigma_{v,i}^{\rm obs}(\hat{\mathbf{w}}_{\mathbf{i}}^{\rm obs}-\mathbf{q}_i)\right)\right)$$

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Parameters of interest

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missing data

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missing data

$$\times p(R_x, x_\star | I, M) p(\mathscr{C}, \alpha, \beta | I, M) \qquad (45)$$

priors

$$\mathbf{x}_\star = [x_{1,\star} c_\star] \in \mathbb{R}^2$$

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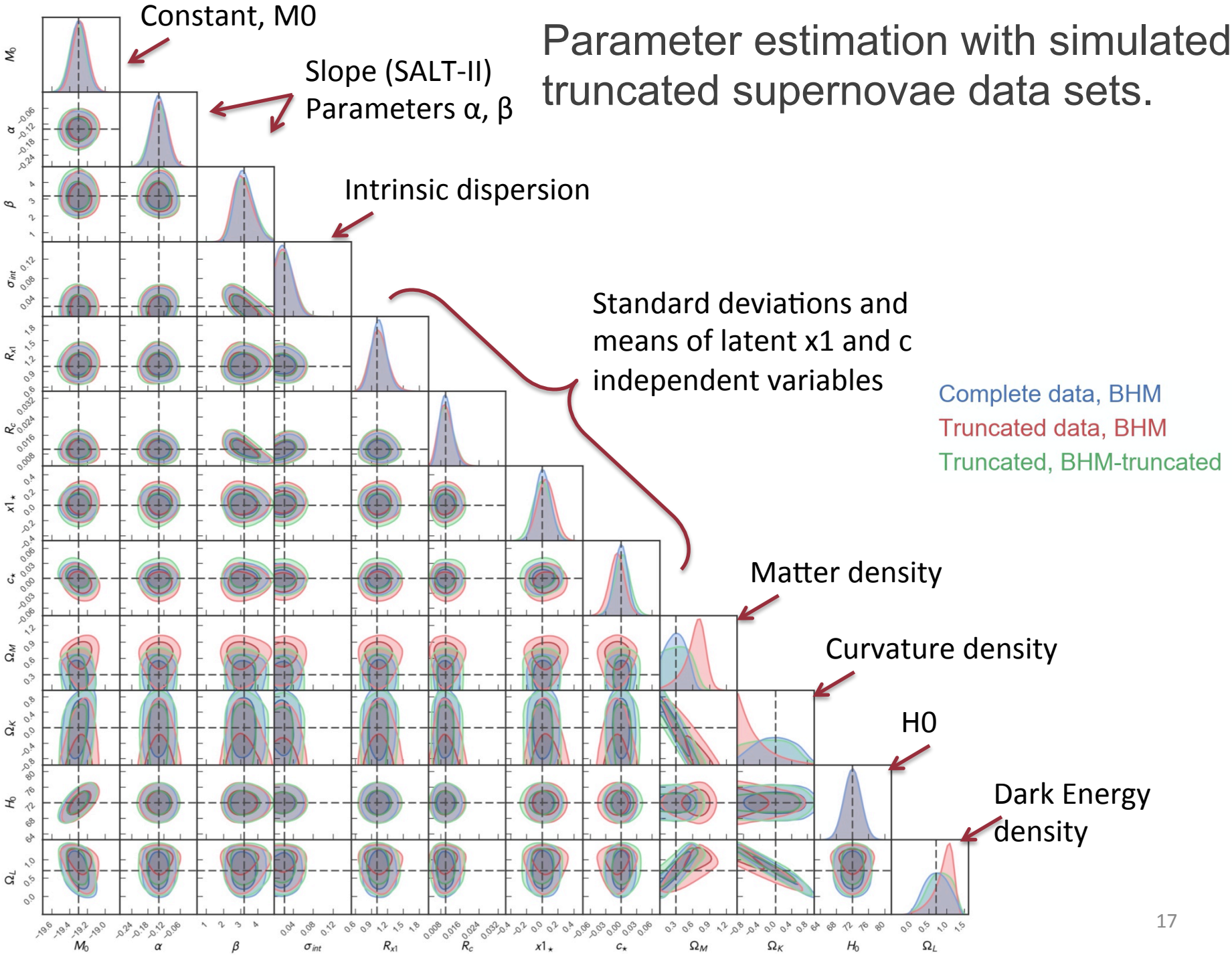
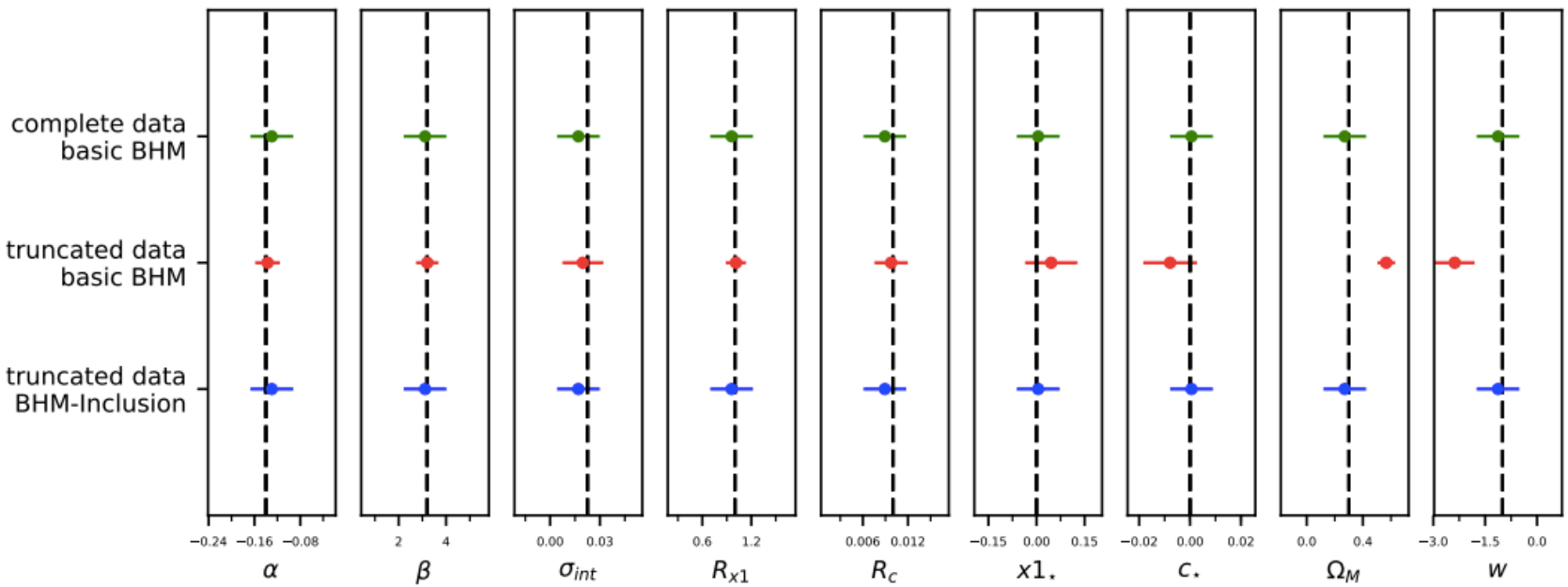
Parameter estimation with simulated truncated supernovae data sets.



Summary & Next Steps



Summary:

- If you want to do Bayesian **Model selection**, you need to have the correct Bayesian **Posterior.**
- How do you account for **missing data** in a Bayesian way?
- See arXive: **1804.02474**

Done:

Analytic solution for missing data.
Tested on basic simulations.

Next Steps:

- Include refined selection function, test on SNANA DES like simulations.
- Account for uncertainty in **typing.**