EFFECTS OF MODELING METHODS ON ABUNDANCE DETERMINATION UNCERTAINTIES IN R-PROCESS STARS

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ERA OF LARGE SCALE SURVEYS

RAVE (phys.org)

Gaia (esa.int)

SDSS (sdss.org)
ABUNDANCES ARE ONLY AS GOOD AS THEIR MODELS

ABUNDANCES ARE NOT MEASURED, BUT DERIVED!

B. Gustafsson, Astronomical Observatory, Uppsala (2009)
Spectral line formation

To determine relevant properties of a star \((T_{\text{eff}}, \log g, [\text{Fe/H}], \ldots)\) \(\Rightarrow\) Compare models of stellar atmosphere (i.e. emergent flux, SED, ...) to observable quantities.

**Emergent flux calculation**

Radiative transfer equation (plane-parallel):

\[
\mu \frac{dI_{\nu}}{d\tau_{\nu}} = S_{\nu} - I_{\nu} \quad (1)
\]

Optical depth:

\[
\tau_{\nu}(z_0) = \int_{z_0}^{\infty} \kappa_{\nu} \rho \, dz \quad (2)
\]

Source function:

\[
S_{\nu} = \frac{j_{\nu}}{\alpha_{\nu}} \quad (3)
\]

Solution requires adoption of approximations (plane-parallel, monochromatic radiation, static atmosphere, no magnetic fields, ..) and knowledge of source function \(S_{\nu}\).
Abundances are not measured BUT determined using approximations:

- Plane-parallel vs. spherical geometry
- Homogeneity
- Stationarity
- Hydrostatic equilibrium
- 1D vs. 3D atmospheres
- Thermal equilibrium
HOMOGENEITY

\[ X, Y, Z \neq f(r, \theta, \Phi) \] we assume a homogeneous atmosphere as an averaged model. This average model describes the average stellar properties well.
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X, Y, Z ≠ f (r, θ, Φ) we assume a homogeneous atmosphere as an averaged model. This average model describes the average stellar properties well.

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Roughly speaking, the spectra of stars are time-independent on human time scales. We can generally assume \( \frac{d}{dt} = 0 \)
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Abundances are not measured BUT determined using approximations:

- Plane-parallel vs. spherical geometry
- Homogeneity
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- Hydrostatic equilibrium
- 1D vs. 3D atmospheres
- Local thermodynamic equilibrium (LTE)
Spectral line formation

Special case: Local Thermodynamic equilibrium (LTE)

- Matter assumed in equilibrium with the radiation field over a finite volume of gas.
- Properties of gas defined by one $T$ at each depth.

Kirchhoff-Planck's law:

$$S_{\text{LTE}} = B_n = \frac{2\pi n^3}{c^2} \frac{e^{hv/kT}}{1 + e^{hv/kT}}$$

Maxwell velocity distribution:

$$f(v) \, dv = \sqrt{\frac{m^2}{2\pi kT}} \frac{v^3}{4\pi^2} e^{-(mv^2/2kT)} \, dv$$

Boltzmann distribution:

$$n_i/n_j = g_i/g_j e^{c/2kT}$$

The Saha equation:

$$\frac{N_1}{N_0} P_e = \left(\frac{2\pi m_e}{kT}\right)^{3/2} \left(\frac{2\pi m_e}{kT}\right)^{5/2} e^{cT/2kT}$$
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Radiation in LTE

Kirchhoff-Planck’s law:  
$$[S_v]_{LTE} = B_v = \frac{2\nu^3}{c^2} \frac{1}{e^{\frac{\nu kT}{T}} - 1}$$  
(4)
Spectral line formation

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Radiation in LTE

Kirchhoff-Planck’s law:
\[
[S_{\nu}]_{\text{LTE}} = B_{\nu} = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{\hbar \nu / k_B T} - 1}
\] (4)

Matter in LTE

Maxwell velocity distribution:
\[
f(\nu) \, d\nu = \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi \nu^2 e^{-(1/2)m\nu^2 / k_B T} \, d\nu
\] (5)

Boltzmann distribution:
\[
\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-\Delta \chi / k_B T}
\] (6)

The Saha equation:
\[
\left[ \frac{N_1}{N_0} \right]_{P_e} = \frac{(2\pi m_e)^{3/2} (k_B T)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-\chi_0^\infty / k_B T}
\] (7)
Photons carry non-local information: Everything depends on everything, everywhere else!
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Statistical Equilibrium Equation has to be solved simultaneously with the radiative transfer equation:

\[ n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = \sum_{j \neq i} n_j (R_{ji} + C_{ji}) \]
Bulk of atomic data required in NLTE calculations.

Status Quo?

Large uncertainties still associated with collisional rates due to lack of experimental cross-section data, esp. collisions with Hydrogen in cool stars which plays an important role esp. in metal-poor stars.

\[ \frac{n_H}{n_{e^-}} \sim 10^4 \]

Statistical Equilibrium Equation has to be solved simultaneously with the radiative transfer equation:

\[ n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = \sum_{j \neq i} n_j (R_{ji} + C_{ji}) \]
Iron model atom- Fell radiative transitions

Rana Ezzeddine, PhD, 2015
departure coefficient (b) = level population density (NLTE)/level population density (LTE)

Deviations from LTE increase toward lower metallicities
Most r-process stars are old and metal-poor

1694 RAVE r-process stars

Placco et al. 2018

NLTE can be important!
Iron abundance

Its importance

1- Proxy to the total metal content $\sim [\text{Fe/H}]$
2- Wealth of lines in most stellar spectra
3- Opacity contribution
4- Relative elemental abundances $[\text{X}/\text{Fe}]$ used in galactic chemical evolution studies
5- Used to determine spectroscopic $\text{Teff}$, $\log g$, .. iteratively
SPECTROSCOPIC STELLAR PARAMETERS (FE)

Amarsi et al. (2016)

Fe I

Figures 2.

SPECTROSCOPIC STELLAR PARAMETERS (FE)

Fe II

Parameters determined iteratively by removing trends between Fe I and Fe II
NLTE EFFECTS : IRON

Departure from LTE can be severe in iron-poor stars! Ezzeddine et al. 2017

\[ \Delta [\text{Fe/H}] = -0.14 \pm 0.04 [\text{Fe/H}]_{\text{LTE}} - 0.15 \pm 0.18 \]
Departure from LTE can be severe in UMP stars!

\[ \Delta [\text{Fe/H}] = -0.14 \pm 0.04 [\text{Fe/H}]_{\text{LTE}} - 0.15 \pm 0.18 \]

Applies well to less metal-poor stars

Fitting slope error bars region ±0.04

\[
\begin{align*}
\square & \quad [\text{Fe/H}]_{\text{3D,NLTE}} - [\text{Fe/H}]_{\text{1D,LTE}} \quad (\text{Nordlander et al. 2016}) \\
\triangle & \quad [\text{Fe/H}]_{\text{1D,NLTE}} - [\text{Fe/H}]_{\text{1D,LTE}} \quad (\text{Nordlander et al. 2016})
\end{align*}
\]
NLTE EFFECTS: Mg & Ca

Sitnova, Ezzeddine et al. (in prep.)
Better agreement between Ca I and Ca II in NLTE vs. LTE
**FE NLTE GRID**

[TEFF, LOGG, [FE/H], MICROTURBULENT VELOCITY (VT)]

SPACE SMALL STEPS OF

- 50K FOR TEMPERATE
- 0.1 DEX FOR GRAVITY
- 0.2 DEX FOR [FE/H] (TO BE REDUCED TO 0.1)
- 0.5 DEX FOR VT

→ TESTED ON BENCHMARK STARS

→ LOGG AGREE WITH GAIA DR2 PARALLAXES

→ CAN BE USED TO DETERMINE NLTE STELLAR PARAMETERS AND FE ABUNDANCES
3D important for CNO elements: large 3D effects

Matthias Steffen
3D COBOLD simulations
In summary, it is particularly important to carry out model atmospheres in the outer layers (compared to observations). This flattens the trend in inferred iron abundance. For the lines indicated with black triangles (non-LTE) and red triangles (low NLTE), weighted by their standard errors (and without systematics), the excitation energies for both species are given relative to the ground state of Fe. The excitation energies for both species are as follows:

- Fe \( \log T_{\text{eff}} = 6435 \text{ K} \)
- Fe \( \log T_{\text{eff}} = 6356 \text{ K} \)
- Fe \( \log g / \text{ cm s}^{-2} = 4.06 \)
- Fe \( \log g / \text{ cm s}^{-2} = 4.26 \)
- Fe \( \log g / \text{ cm s}^{-2} = 3.65 \)
- Fe \( \log g / \text{ cm s}^{-2} = 3.2 \)
- Fe \( \log g / \text{ cm s}^{-2} = 1.88 \text{ km s}^{-1} \)
- Fe \( \log g / \text{ cm s}^{-2} = 1.73 \text{ km s}^{-1} \)
- Fe \( \log g / \text{ cm s}^{-2} = 1.61 \text{ km s}^{-1} \)
- Fe \( \log g / \text{ cm s}^{-2} = 1.60 \text{ km s}^{-1} \)

The 3D model atmospheres used in the figure are consistent with those of Amarsi et al. (2016), listed in Marcs, 2012. These were computed from the 3D model atmospheres of Amarsi et al. (2016). The excitation energies for both species are as follows:

- Fe \( \log T_{\text{eff}} = 4587 \text{ K} \)
- Fe \( \log T_{\text{eff}} = 4987 \text{ K} \)
- Fe \( \log g / \text{ cm s}^{-2} = 4.00 \)
- Fe \( \log g / \text{ cm s}^{-2} = 4.50 \)
- Fe \( \log g / \text{ cm s}^{-2} = 5.00 \)
- Fe \( \log g / \text{ cm s}^{-2} = 5.50 \)
- Fe \( \log g / \text{ cm s}^{-2} = 6.50 \)

The 3D model atmospheres are also used in the figure. The excitation energies for both species are as follows:

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Inferred iron abundance against excitation energy for selected Fe lines is overdrawn; the standard error in the gradient reflects the uncorrelated errors arising from measurement errors in the observed equivalent widths as well as correlated errors arising from errors in the excitation. The 1D stellar atmosphere model is better than the 3D one.
TAKE AWAY POINTS

- Our abundances are only as good as our models
  - Extra care has to be taken when modeling metal-poor stars (i.e., r-process stars)

- Departures from LTE abundances for Fe can be severe
  - Accurate modeling of atmospheres in iron-poor stars (NLTE) is important.
    Ignoring NLTE effects can:
    - underestimate log g ~ 0.2 - 1 dex
    - overestimate Teff ~ 500-600 K
    - underestimate [Fe/H] ~ 0.2 - 1.0 dex
    - underestimate [Mg/H] up to 0.5 dex
    - underestimate [Ca/H] from Ca II lines up to 0.5 dex

- NLTE effects important to include in Spectroscopic Surveys.