# Spectra from thermal relativistic nuclear field theory

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# Outline

- *Motivation: to build a consistent and predictive approach to describe the entire nuclear chart (ideally, an arbitrary strongly-correlated many-body system), numerically executable and useful for applications, such as r-process, quantum chemistry, fundamental physics etc.*

- *Challenges: the nuclear hierarchy problem, complexity of NN-interaction*

- *Approximate non-perturbative solutions: Relativistic Nuclear Field Theory (RNFT). Emerged as a synthesis of Landau-Migdal Fermi-liquid theory, Copenhagen-Milano NFT and Quantum Hadrodynamics; now put in the context of a systematic equation of motion (EOM) formalism and linked to ab-initio interactions*

- *Technique: Green functions, EOM, time blocking method*

- *Nuclear response to neutral and charge-exchange probes: giant EL, Gamow-Teller, spin dipole etc. (neutron capture, gamma and beta decays, pair transfer, …)*

- *Nuclear response at finite temperature: thermal QFT for transitions between nuclear excited states*

- *Conclusions and perspectives*

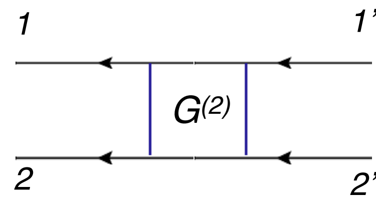
# Definitions and diagrammatic conventions

*One-fermion propagator:*

$$G_{11'}(t-t') = -i\langle T\psi(1)\psi^\dagger(1')\rangle \quad = \quad 1 = \{\xi_1, t\}$$

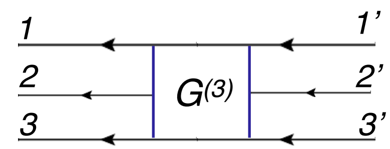

*Two-fermion propagator (two-times):*

$$G(12,1'2') = (-i)^2\langle T\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1')\rangle \quad =$$



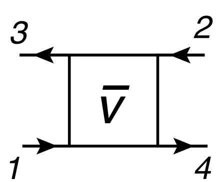

*Three-fermion propagator (two-times):*

$$G(123,1'2'3') = (-i)^3\langle T\psi(1)\psi(2)\psi(3)\psi^\dagger(3')\psi^\dagger(2')\psi^\dagger(1')\rangle \quad =$$



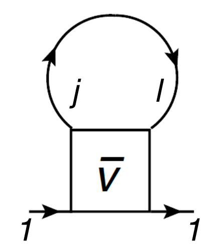

*Two-fermion (antisymmetrized) interaction:*

$$\bar{v}_{1234} \quad =$$




*Static self-energy:*

$$\sum_{jl} \bar{v}_{1j1'l}\rho_{lj} \quad =$$




$$R(12',21') = G(12',21') - G(1,2)G(2',1'),$$ *Response function*

$$\tilde{R}(12',21') = G(12',21') - (G(1,2)G(2',1') - G(1,1')G(2',2))$$ *Fully correlated part*

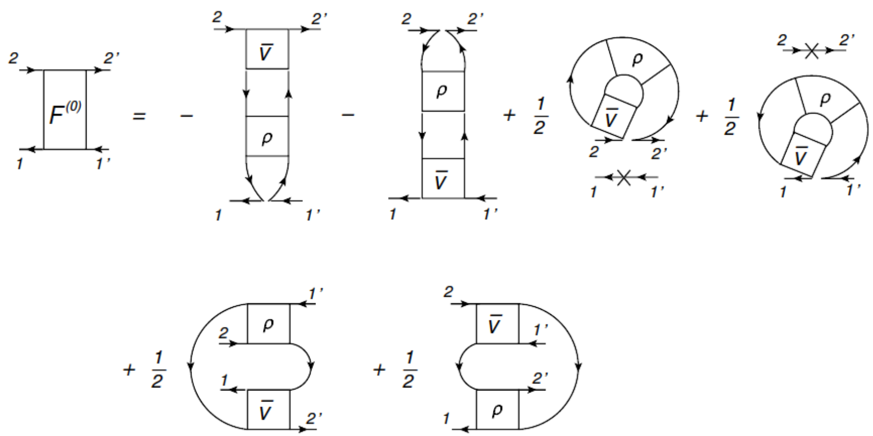
# Exact equations of motion for binary interactions: two-body problem

*particle-hole response:* $R^{(ph)}_{12,1'2'}(t-t') = -i\langle T(\psi_1^\dagger \psi_2)(t)(\psi_{2'}^\dagger \psi_{1'})(t')\rangle$ *Spectra of excitations*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)W(\omega)R(\omega) \qquad (*) \qquad W(t-t') = W^{(0)}\delta(t-t') + W^{(r)}(t-t')$$

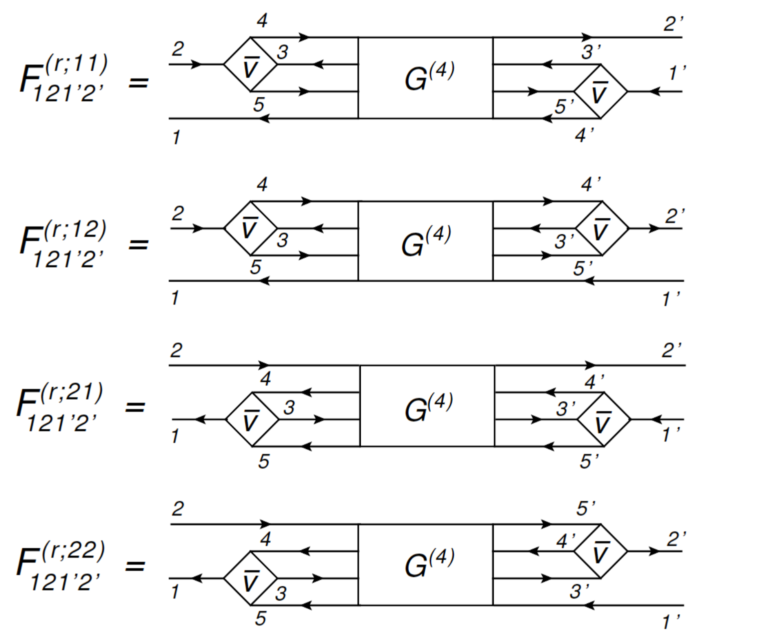
$$R(12',21') = \tilde{R}(12',21') - G(1,1')G(2',2) \qquad W = F^{irr}$$

*instantaneous term ("bosonic" mean field):*




*t-dependent (retarded & advanced) term*

$$F^{(r)}_{12,1'2'}(t-t') = F^{(r;11)}_{12,1'2'}(t-t') + F^{(r;12)}_{12,1'2'}(t-t') + $$
$$+ F^{(r;21)}_{12,1'2'}(t-t') + F^{(r;22)}_{12,1'2'}(t-t')$$




*Mean field $F^{(0)}$, where* $\rho_{12,1'2'} = \lim_{t\to t'-0} R(12,1'2')$

*contains the full solution of (*)* ***including the dynamical term!***

***EOM method:***

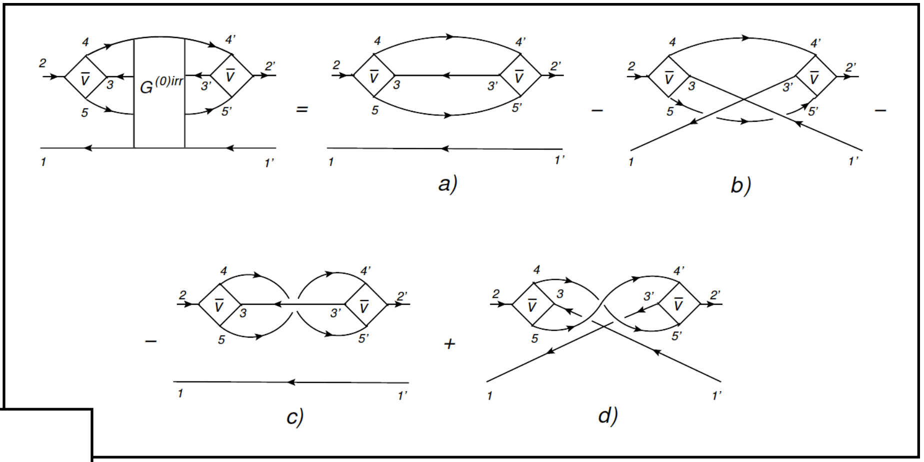

*S. Adachi and P. Schuck, NPA496, 485 (1989).*
*J. Dukelsky, G. Roepke, and P. Schuck, NPA 625, 14 (1995).*
*P. Schuck and M. Tohyama, PRB 93, 165117 (2016).*
*etc.*

# Expansion of the dynamics kernel: $F^{(r;12)irr}$

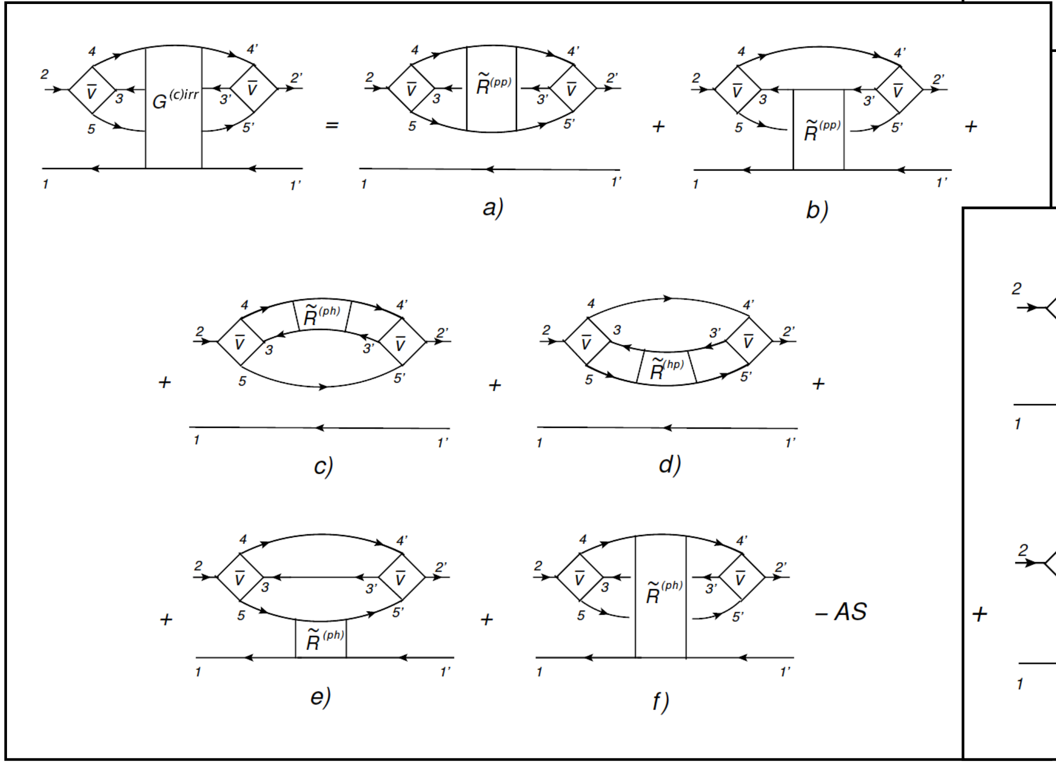
*Irreducible part of $G^{(4)}$ is decomposed into uncorrelated, singly-correlated and doubly-correlated terms:*

$$G^{irr}(543'1',5'4'31) = G^{(0)irr}(543'1',5'4'31) + \\ + G^{(c)irr}(543'1',5'4'31) + G^{(cc)irr}(543'1',5'4'31)$$

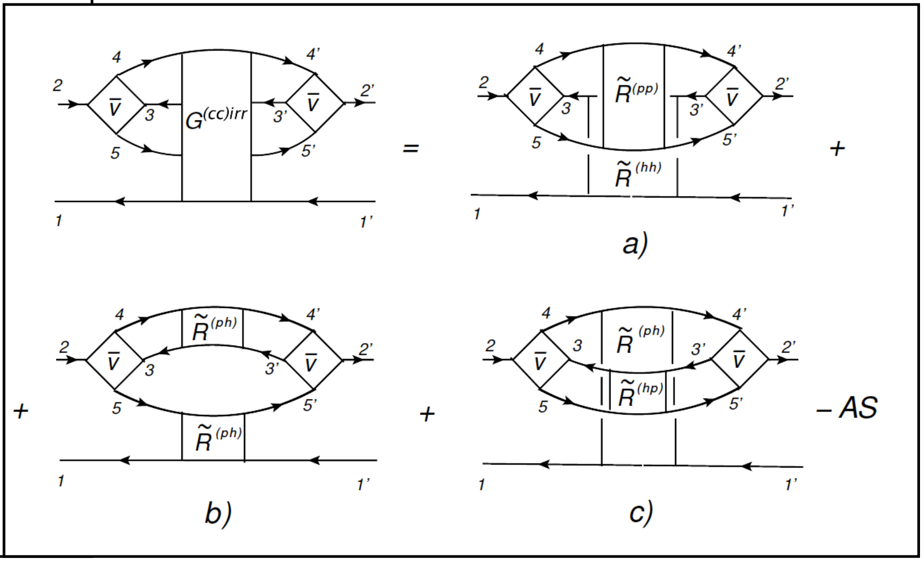
*Uncorrelated terms:*




*Singly-correlated terms (up to phases):*



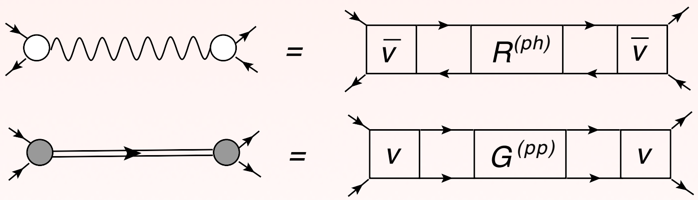

*Doubly-correlated terms (up to phases):*



# Mapping to the (Quasi)particle-Vibration Coupling (QVC, PVC)

*Model-independent mapping to the QVC-TBA:*

$$\sum_{343'4'} \tilde{V}^{*}_{12,34} R_{34,3'4'}(\omega) \tilde{V}_{3'4'1'2'} = \sum_{m} g^{m*}_{12} D_m(\omega) g^{m}_{1'2'}$$

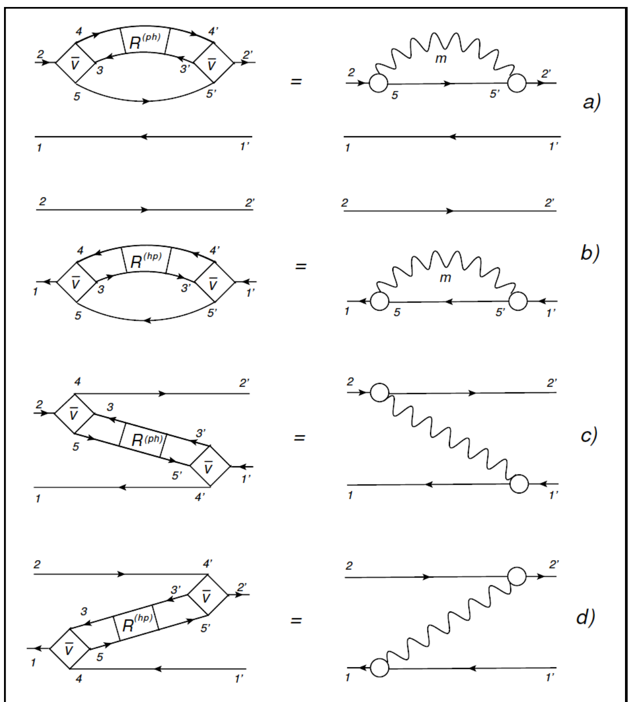



$$R_{12,1'2'}(\omega) = \sum_{m} \Big( \frac{\rho^{m*}_{12}\rho^{m}_{1'2'}}{\omega - \Omega_m + i\delta} - \frac{\rho^{m}_{21}\rho^{m*}_{2'1'}}{\omega + \Omega_m - i\delta} \Big)$$

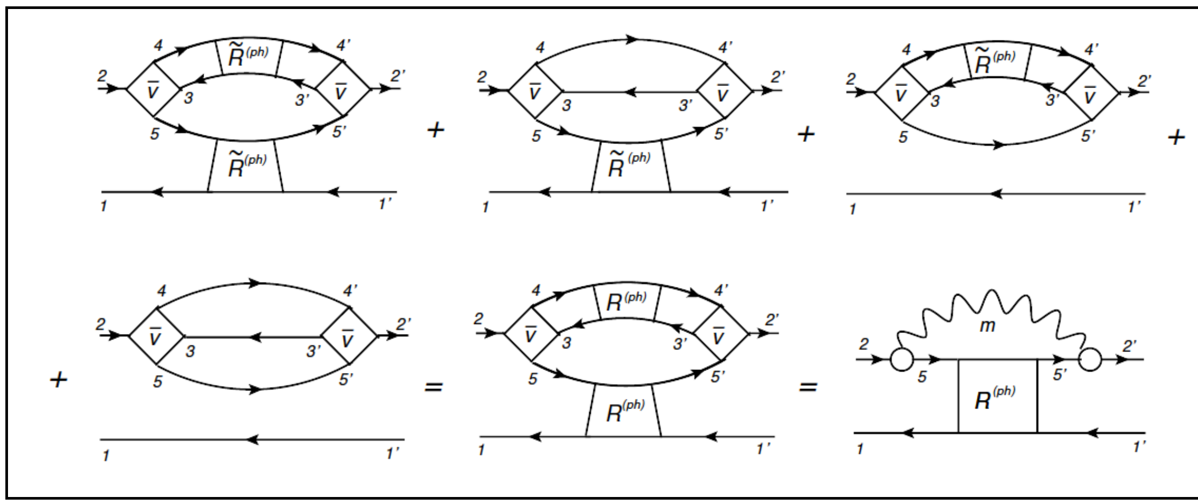
$$g^{m}_{12} = \sum_{34} \tilde{V}_{12,34}\rho^{m}_{34}$$ "phonon" vertex

$$D_m(\omega) = \frac{1}{\omega - \Omega_m + i\delta} - \frac{1}{\omega + \Omega_m - i\delta}$$ "phonon" propagator

*Original QVC: non-correlated and partly singly-correlated terms*




*Generalized QVC meets EOM: ALL correlated terms (preliminary, work in in progress)*



*Self-consistent closed system of equations*

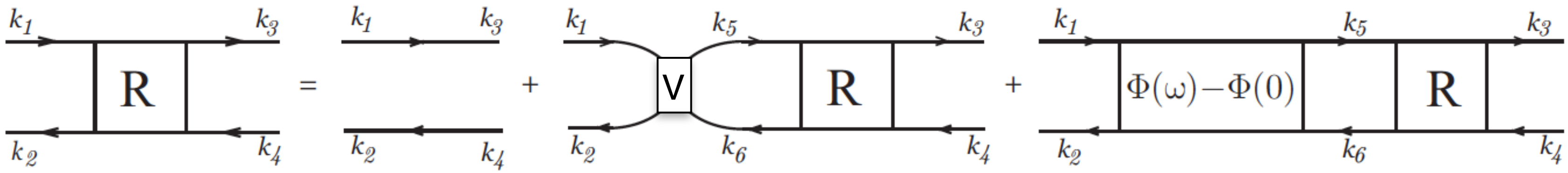
$$\hat{R}(\omega) = \hat{R}^{(0)}(\omega) + \hat{R}^{(0)}(\omega) W[\hat{R}(\omega)] \hat{R}(\omega)$$

***All channels are coupled***

*E.L., P. Schuck, in progress*

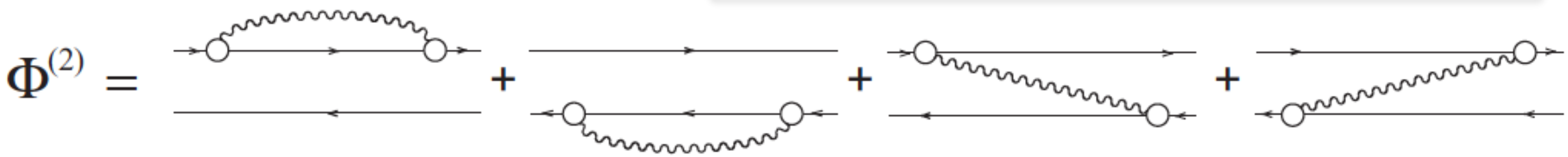
# Nuclear response with QVC in time blocking approximation. Higher orders: toward a complete theory

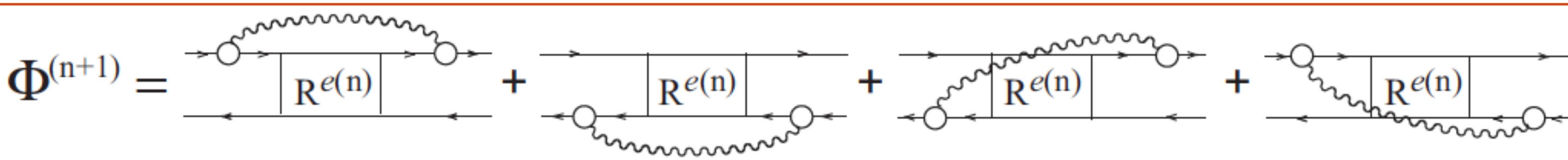
### Bethe-Salpeter Equation:




*Time blocking approximation (TBA):*
*V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)*

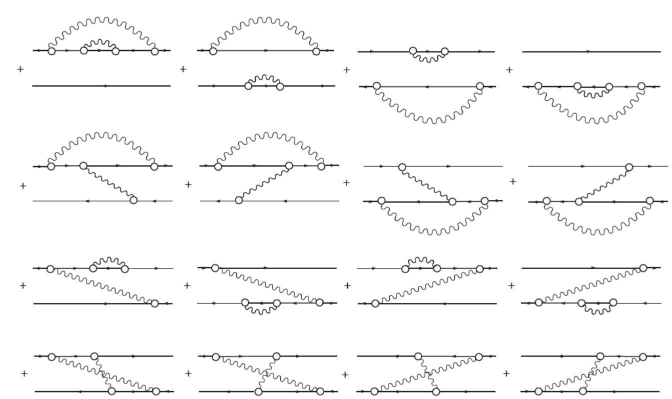
$$R(\omega) = R^0(\omega) + R^0(\omega)\,[V + W(\omega)]\,R(\omega)$$

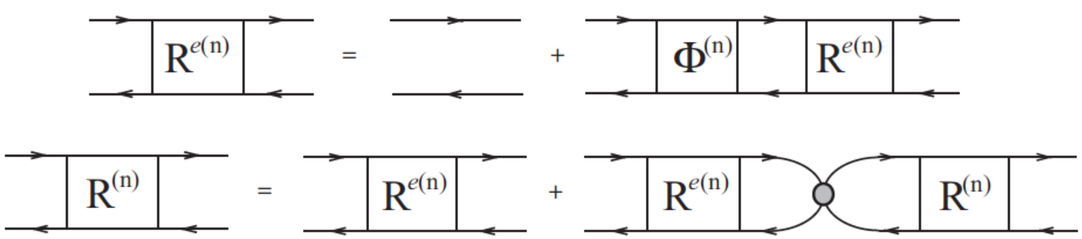



*Generalized TBA for correlated propagator:*
*2-phonon: V. Tselyaev, PRC 75, 024306 (2007)*
*n-th order: E.L. PRC 91, 034332 (2015)*

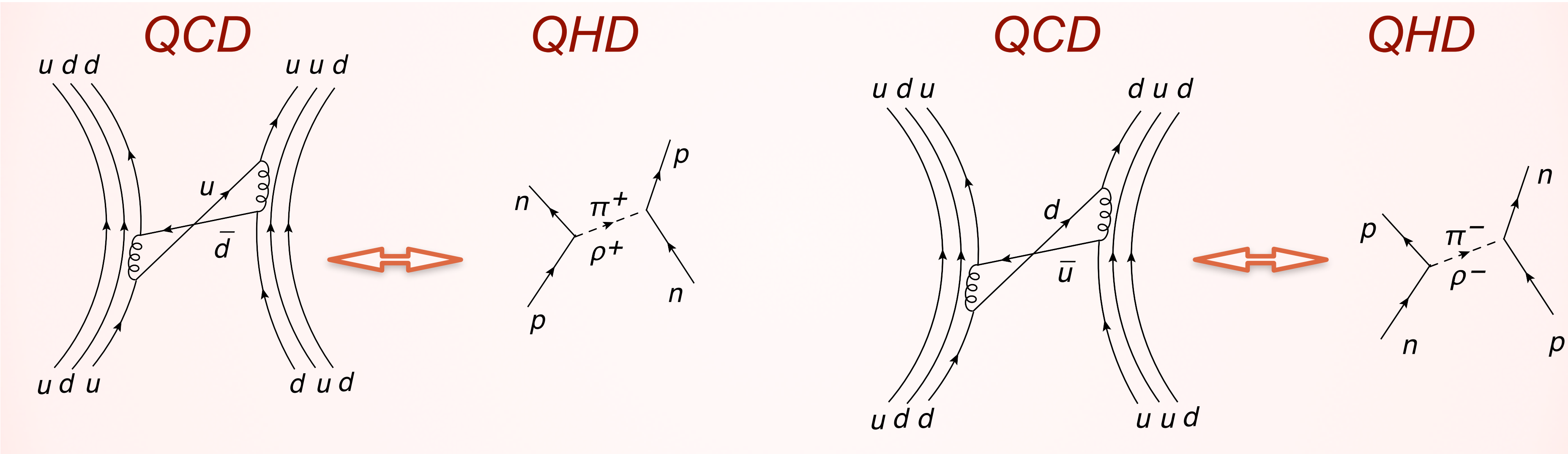
*n = 3: 3p3h correlations*



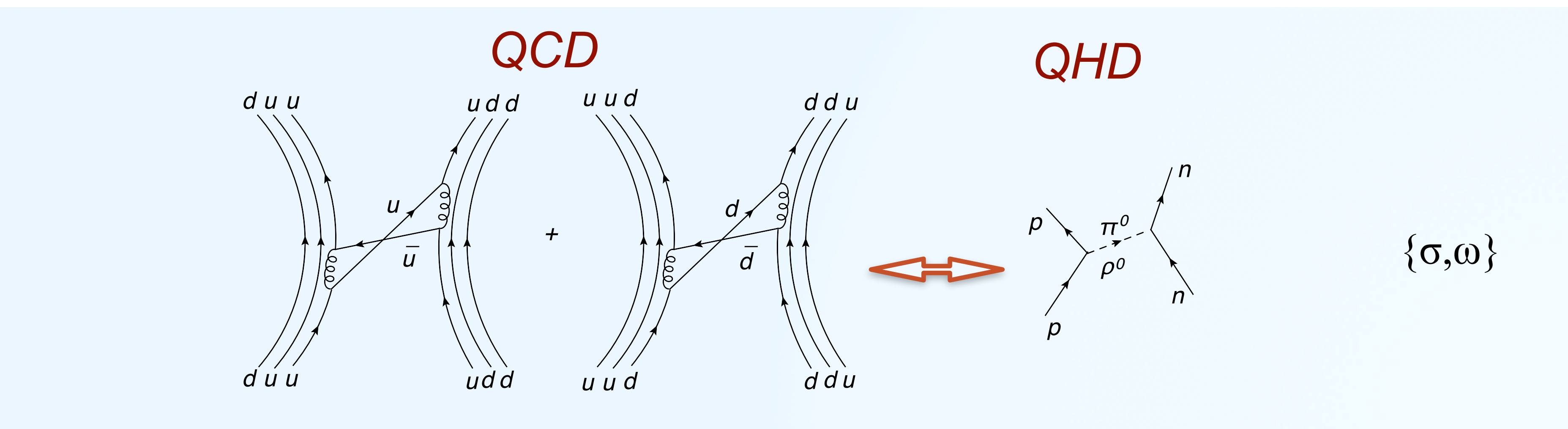


# The underlying mechanism of NN-interaction : meson exchange

Charged mesons:




Neutral mesons:



# Response function in the neutral channel (leading order in QVC): relativistic quasiparticle time blocking approximation (RQTBA)

*Response*

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega) W(\omega) R(\omega)$$

*Interaction*

$$W(\omega) = \underbrace{V_\sigma + V_\omega + V_\rho + V_e}_{} + \Phi(\omega) - \underbrace{\Phi(0)}_{}$$

*Subtraction to avoid double counting (if CDFT-based)*

*Instantaneous meson-exchange: R(Q)RPA*

$$v_\sigma(1,2) = -g_\sigma^2 \gamma_1^0 D_\sigma(1,2) \gamma_2^0$$

$$v_\omega(1,2) = +g_\omega^2 (\gamma^0\gamma_\mu)_1 D_\omega^{\mu\nu}(1,2)(\gamma^0\gamma_\nu)_2$$

$$v_\rho^V(1,2) = +g_\rho^2 (\gamma^0\gamma_\mu\vec{\tau})_1 \vec{\tau}_1 \cdot \vec{\tau}_2 D_\rho^{\mu\nu}(1,2)(\gamma^0\gamma_\nu\vec{\tau})_2$$

*Dynamic (retardation): Quasiparticle-vibration coupling in the (resonant) time blocking approximation*

$$\Phi^\eta_{k_1k_4,k_2k_3}(\omega) =$$

$$= \sum_{\mu\xi} \delta_{\eta\xi} \Big[ \delta_{k_1k_3} \sum_{k_6} \frac{\gamma^{\eta;-\xi}_{\mu;k_6k_2} \gamma^{\eta;-\xi*}_{\mu;k_6k_4}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2k_4} \sum_{k_5} \frac{\gamma^{\eta;\xi}_{\mu;k_1k_5} \gamma^{\eta;\xi*}_{\mu;k_3k_5}}{\eta\omega - E_{k_5} - E_{k_2} - \Omega_\mu}$$

$$- \Big( \frac{\gamma^{\eta;\xi}_{\mu;k_1k_3} \gamma^{\eta;-\xi*}_{\mu;k_2k_4}}{\eta\omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma^{\eta;\xi*}_{\mu;k_3k_1} \gamma^{\eta;-\xi}_{\mu;k_4k_2}}{\eta\omega - E_{k_1} - E_{k_4} - \Omega_\mu} \Big) \Big]$$

E. L., P. Ring, and V. Tselyaev, Phys. Rev. C 78, 014312 (2008)

# Isospin transfer response function: proton-neutron relativistic quasiparticle time blocking approximation (pn-RQTBA)

Response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)\overline{W}(\omega)R(\omega)$$

Interaction

$$\overline{W}(\omega) = V_\rho + V_\pi + V_{\delta\pi} + \Phi(\omega) - \Phi(0)$$

Subtraction to avoid double counting of ρ (if CDFT-based)

Instantaneous meson-exchange: R(Q)RPA

$$V_\rho(1,2) = g_\rho^2 \vec{\tau}_1 \vec{\tau}_2 (\beta\gamma^\mu)_1 (\beta\gamma_\mu)_2 D_\rho(\mathbf{r}_1, \mathbf{r}_2)$$

$$V_\pi(1,2) = -\Big(\frac{f_\pi}{m_\pi}\Big)^2 \vec{\tau}_1 \vec{\tau}_2 (\mathbf{\Sigma}_1 \nabla_1)(\mathbf{\Sigma}_2 \nabla_2) D_\pi(\mathbf{r}_1, \mathbf{r}_2),$$

$$V_{\delta\pi}(1,2) = g' \Big(\frac{f_\pi}{m_\pi}\Big)^2 \vec{\tau}_1 \vec{\tau}_2 \mathbf{\Sigma}_1 \mathbf{\Sigma}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

free-space coupling

fixed strength: (free-space if the Fock term is present)

Dynamic (retardation):

Quasiparticle-vibration coupling

in the (resonant) time blocking approximation

$$\Phi^{\eta_1\eta_4,\eta_2\eta_3}_{k_{1_p}k_{4_n},k_{2_n}k_{3_p}}(\omega) = \delta_{\eta_1,\eta_3}\delta_{\eta_1,-\eta_2}\delta_{\eta_3,-\eta_4} \times \sum_{\mu\xi}\delta_{\xi,\eta_1}$$

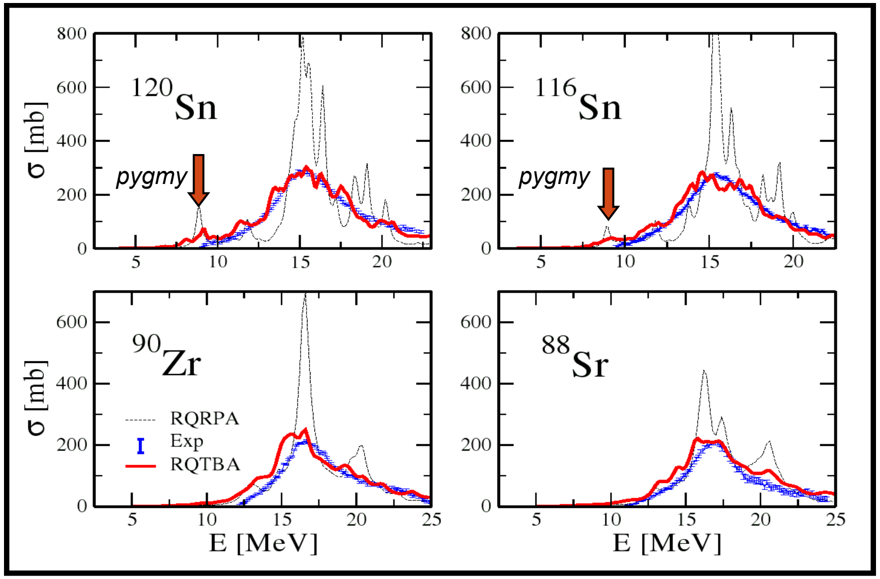
$$\times\Bigg[\delta_{k_{1_p}k_{3_p}} \sum_{k_{6_n}} \frac{\gamma^{\eta_1;-\xi-\xi}_{\mu;k_{6_n}k_{2_n}} \gamma^{\eta_1;-\xi-\xi*}_{\mu;k_{6_n}k_{4_n}}}{\eta_1(\omega-\lambda_{pn}) - E_{k_{1_p}} - E_{k_{6_n}} - \Omega_\mu} + \delta_{k_{2_n}k_{4_n}} \sum_{k_{5_p}} \frac{\gamma^{\eta_1;\xi\xi}_{\mu;k_{1_p}k_{5_p}} \gamma^{\eta_1;\xi\xi*}_{\mu;k_{3_p}k_{5_p}}}{\eta_1(\omega-\lambda_{pn}) - E_{k_{5_p}} - E_{k_{2_n}} - \Omega_\mu}$$

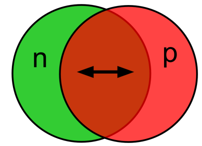
$$- \frac{\gamma^{\eta_1;\xi\xi}_{\mu;k_{1_p}k_{3_p}} \gamma^{\eta_1;-\xi-\xi*}_{\mu;k_{2_n}k_{4_n}}}{\eta_1(\omega-\lambda_{pn}) - E_{k_{3_p}} - E_{k_{2_n}} - \Omega_\mu} - \frac{\gamma^{\eta_1;\xi\xi*}_{\mu;k_{3_p}k_{1_p}} \gamma^{\eta_1;-\xi-\xi}_{\mu;k_{4_n}k_{2_n}}}{\eta_1(\omega-\lambda_{pn}) - E_{k_{1_p}} - E_{k_{4_n}} - \Omega_\mu}\Bigg]$$

C. Robin, E.L., Eur. Phys. J. A 52, 205 (2016)

# *Dipole response in medium-mass and heavy nuclei within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)*

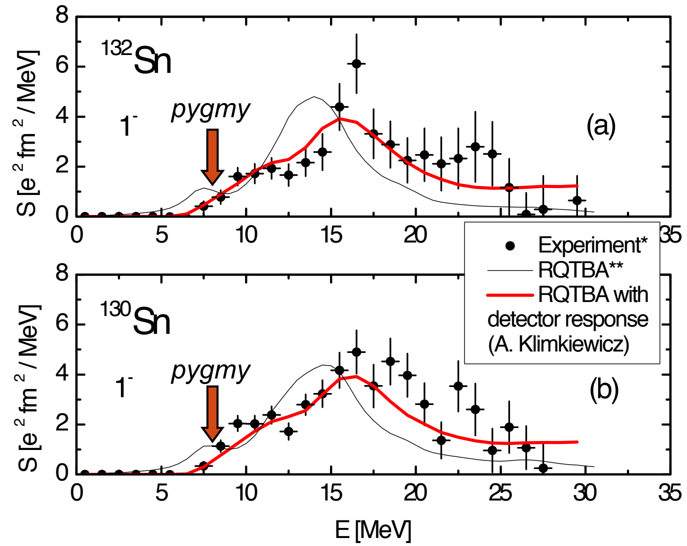
## *Giant dipole resonance (GDR) in stable nuclei*




*Giant & pygmy dipole resonances*

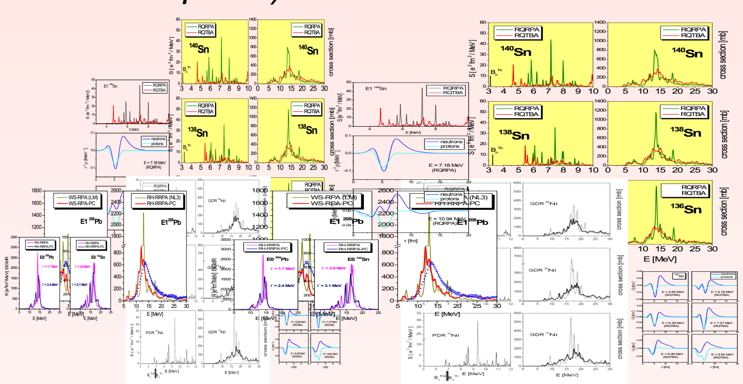
## *Neutron-rich Sn*




* P. Adrich et al., PRL 95, 132501 (2005)

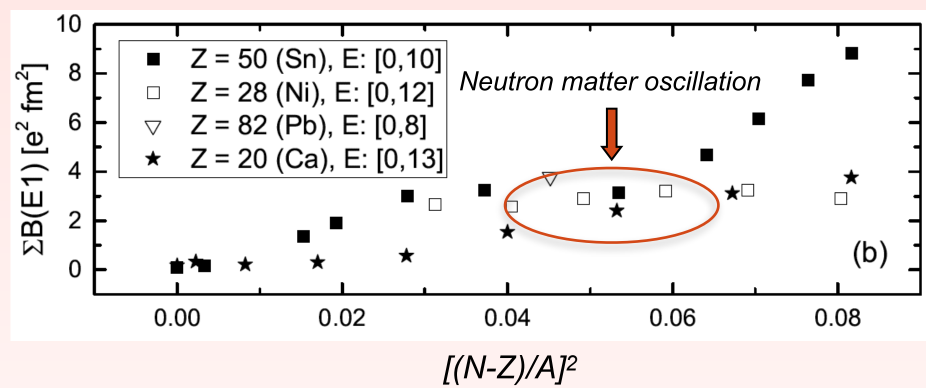
**E. L., P. Ring, and V. Tselyaev, Phys. Rev. C 78, 014312 (2008)

## *2008-2018: Systematic GMR calculations (various multipoles)*



*Used for (n,γ) rates: **see talk of Caroline Robin***

## *Pygmy dipole resonance (PDR) systematics (important for EOS)*

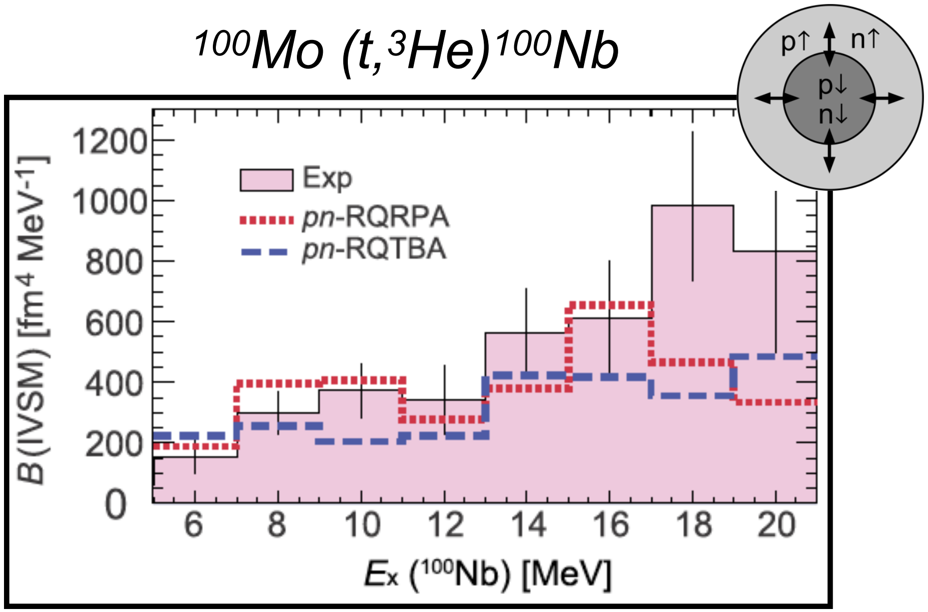



*I.A. Egorova, E. Litvinova, Phys. Rev. C 94, 034322 (2016)*

# *Exotic spin-isospin excitations*

***Recent measurements at MSU***

*$^{100}Mo$ (t,$^{3}$He)$^{100}Nb$*




*Isovector spin monopole resonance*

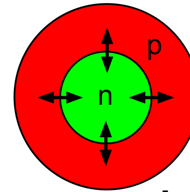
*K. Miki, R.G.T. Zegers,…, E.L., …, C. Robin et al., Phys. Lett. B 769, 339 (2017)*

*Recent developments on spin-isospin response:*

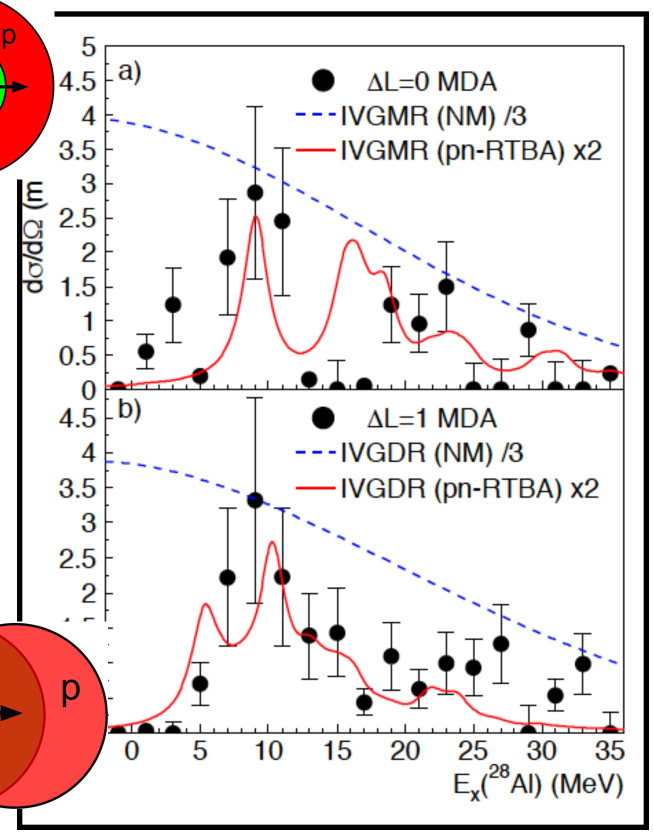
- *Superfluid pairing*
- *Coupling to charge-exchange phonons*
- *Beta decay*
- *QVC-induced ground state correlations (GSC);*
- *Meson-exchange pn-pairing*

***See talk of Caroline Robin***

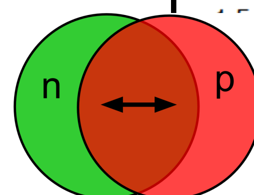
*$^{28}Si$ ($^{10}Be$,$^{10}B$)$^{28}Al$*




*Isovector monopole*




*Isovector dipole*




*M. Scott, R.G.T. Zegers,…, E.L., …, C. Robin et al., Phys. Rev. Lett. 118, 172501 (2017)*

# *Nuclear systems at finite temperature: Experimental data*

- *J.J. Gaardhøje, C. Ellegaard, B. Herskind, S.G. Steadman, Phys. Rev. Lett. 53, 148 (1984).*
- *J.J. Gaardhøje, C. Ellegaard, B. Herskind, et al., Phys. Rev. Lett. 56, 1783 (1986).*
- *D.R. Chakrabarty, S. Sen, M. Thoennessen et al., Phys. Rev. C 36, 1886 (1987).*
- *A. Bracco, J.J. Gaardhøje, A.M. Bruce et al., Phys. Rev. Lett. 62, 2080 (1989).*
- *G. Enders, F.D. Berg, K. Hagel, et al., Phys. Rev. Lett. 69, 249 (1992).*
- *H.J. Hofmann, J.C. Bacelar, M.N. Harakeh, et al., Nucl. Phys. A 571, 301 (1994).*
- *E. Ramakrishnan, T. Baumann, A. Azhari et al., Phys. Rev. Lett. 76, 2025 (1996).*
- *P. Heckman, D. Bazin, J.R. Beene, Y. Blumenfeld, et al., Phys. Lett. B 555, 43 (2003).*
- *F. Camera, A. Bracco, V. Nanal, et al., Phys. Lett. B 560, 155 (2003).*
- *M. Thoennessen, Nucl. Phys. A 731, 131 (2004).*

***A (relatively) recent survey:***

*D. Santonocito and Y. Blumenfeld, Eur. Phys. J. A 30, 183 (2006).*

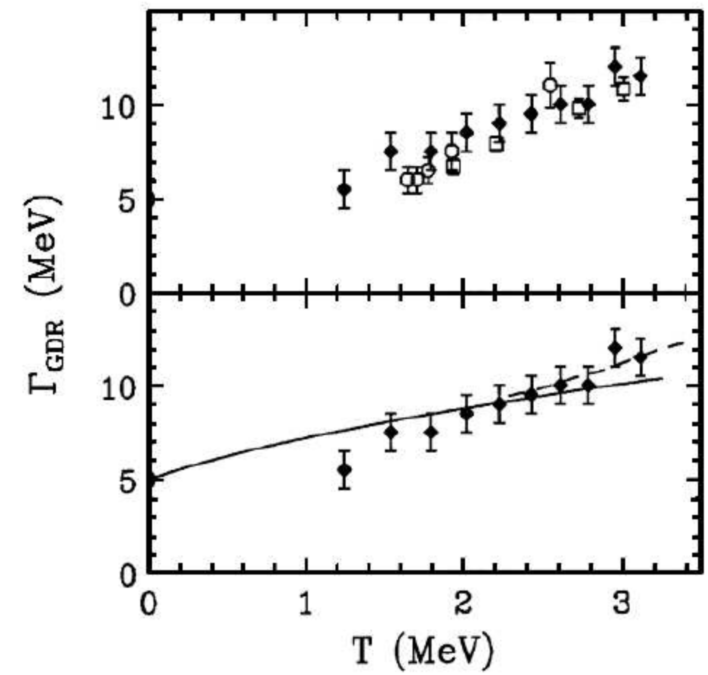



**Fig. 4.** Comparison of the GDR width extracted from $50A$ MeV $\alpha$-particle inelastic-scattering experiment (full symbols) on $^{120}$Sn [29] and from fusion reaction data (open symbols) on $^{108-112}$Sn nuclei [23–25]. The lower part shows the comparison of the $\alpha$ inelastic-scattering experiment results with adiabatic coupling calculations [32] shown as a full line. The dashed line includes the contribution to the width due to particle evaporation width [35].

***General observations:***

- ***Broadening of the GDR with temperature***
- ***"Disappearance" of the GDR at T~5 MeV***

# History and current status of finite-temperature QFT approaches

- *Finite-Temperature Green function formalism*

> *T. Matsubara, Prog. Theor. Phys. 14, 351 (1955).*
> *A.A. Abrikosov, L.P. Gor'kov, and I.E. Dzyaloshinski, Methods of Quantum Field Theory in Statistical Physics*

- *Finite-Temperature Hartree-Fock, Hartree-Fock-Bogolyubov and random phase approximations*

> *A.L. Goodman, Nucl. Phys. A352, 30 (1981).*
> *P. Ring et al., Nucl. Phys. A419, 261 (1983).*
> *H.M. Sommermann, Ann. Phys. 151, 163 (1983).*
> *Y.F. Niu et al., Phys. Lett. B 681, 315 (2009).*

- *Continuum RPA and QRPA at finite temperature*

> *J. Bar-Touv, Phys. Rev. C 32, 1369 (1985).*
> *V.A. Rodin and M.G. Urin, PEPAN 31, 975 (2000).*
> *E.V. Litvinova, S.P. Kamerdzhiev, and V.I. Tselyaev, Phys. At. Nucl. 66, 558 (2003).*
> *E. Khan, N. Van Giai, M. Grasso, Nucl. Phys. A731, 311 (2004).*
> *E. Litvinova and N. Belov, Phys. Rev. C88, 031302(R) (2013).*

- *Finite-Temperature approaches* ***beyond RPA***

> *P.F. Bortignon et al., Nuc. Phys. A460, 149 (1985).*
> *D. Lacroix et al., PRC 58, 2154 (1998).*

- ***FT-RPA, FT-CRPA and FT-QRPA seem to be understood, however, microscopic calculations beyond one-loop approximations are still very limited, sometimes contradicting, and their results are not assessed systematically.***
- ***Open questions: What are the microscopic mechanisms of the GMR's broadening with temperature? What happens to the soft modes and to the low-lying strength at T>0?***

# Nucleus in the thermal equilibrium: a compound state

$$\Omega(\lambda, T) = E - \lambda N - TS$$

*Grand thermodynamical potential to be minimized with the Covariant Energy Density Functional (NL3, P. Ring et al.)*

$$E[\mathcal{R},\phi] = Tr[(\vec{\alpha}\vec{p} + \beta m)\mathcal{R}] + \sum_m \left\{ Tr[(\beta\Gamma_m\phi_m)\mathcal{R}] \mp \int d^3r \left[\frac{1}{2}(\vec{\nabla}\phi_m)^2 + U(\phi_m)\right] \right\}$$

$$S = -k\mathrm{Tr}(\mathcal{R}\ln\mathcal{R})$$

*Entropy (maximized)*

$$N = \mathrm{Tr}(\mathcal{R}\mathcal{N})$$

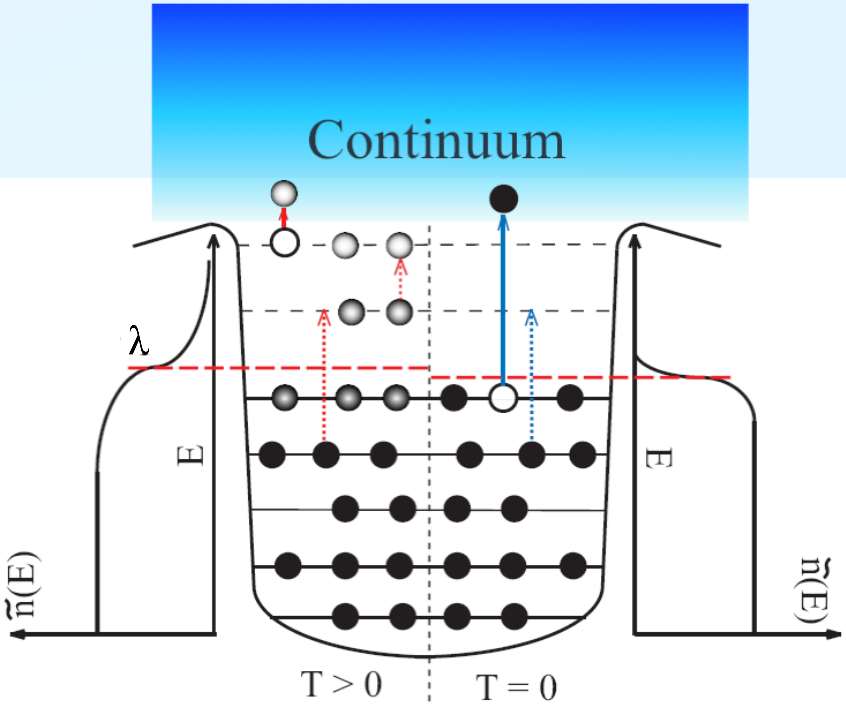
*Particle number*

$$\mathcal{R} = \frac{e^{-(\mathcal{H}-\lambda\mathcal{N})/kT}}{\mathrm{Tr}\left[e^{-(\mathcal{H}-\lambda\mathcal{N})/kT}\right]}$$

*Density matrix*

$$\mathcal{H} = \frac{\delta E[\mathcal{R}]}{\delta\mathcal{R}}$$

*Single-particle Hamiltonian*



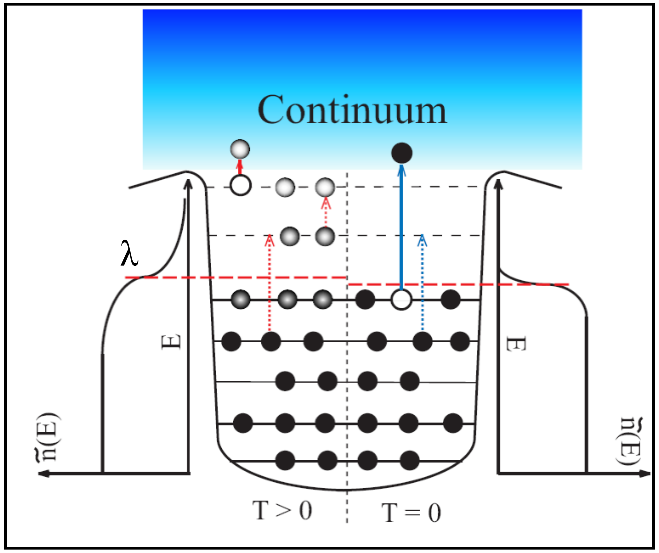
# Nucleus in the thermal equilibrium: a compound state

***Fractional occupancies and thermal unblocking:***

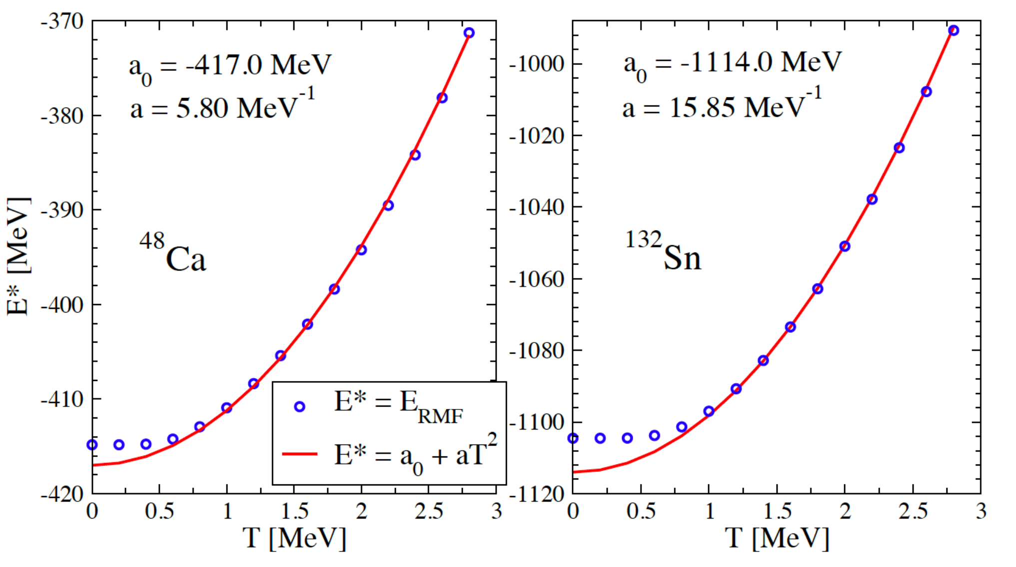
$$n_i(T) = n(\varepsilon_i, T) = \frac{1}{1 + exp\{\varepsilon_i/T\}}$$ **Fermions**

$$\varepsilon_i = \tilde{\varepsilon}_i - \lambda$$

$$N(\Omega_\mu, T) = \frac{1}{exp\{\Omega_\mu/T\} - 1}$$ **Bosons**




***RMF excitation energies vs temperature***
***Calculations of H. Wibowo (WMU):***




*Parabolic fit of the RMF E*(T) gives the level density parameters $a_{RMF}$ close to those of the empirical Fermi gas model*

# *Matsubara imaginary-time Green function formalism for T>0*

*Free single-fermion propagator in t-representation (imaginary time):*

$$\tilde{\mathcal{G}}(1,2) = -\delta_{12}\big(\theta(\tau)(1-n_1) - \theta(-\tau)n_1\big)e^{-\varepsilon_1\tau}, \qquad \tau = t_1 - t_2$$

$$1 = \{\xi_1, t_1\}$$

*or*

$$\tilde{\mathcal{G}}(1,2) = \tilde{\mathcal{G}}_{12}(\tau) = -\sigma\delta_{12}\theta(\sigma\tau)n(-\sigma\varepsilon_1)e^{-\varepsilon_1\tau}, \qquad \sigma = sign(\tau)$$

*To be compared to T=0 case:*

$$\tilde{G}(1,2) = -i\sigma_1\delta_{12}\theta(\sigma_1\tau)e^{-i\varepsilon_1\tau}, \qquad \sigma_1 = sign(\varepsilon_1)$$

*Fourier transform to the imaginary* ***discrete energy variable****:*

$$\tilde{\mathcal{G}}_{12}(i\xi_l) = \frac{1}{2}\int_{-1/T}^{1/T} d\tau\, \tilde{\mathcal{G}}_{12}(\tau)e^{i\xi_l\tau} = \frac{\delta_{12}}{i\xi_l - \varepsilon_1}, \qquad \xi_l = (2l+1)\pi T$$

*Dyson equation for the single-fermion propagator:*

$$\mathcal{G}(1,2) = \tilde{\mathcal{G}}(1,2) + \sum_{1'2'}\tilde{\mathcal{G}}(1,1')\Sigma^e(1'2')\mathcal{G}(2',2)$$

# Bethe-Saltpeter equation for the nuclear particle-hole response

*Bethe-Saltpeter equation (BSE) for the response function:*

$$\mathcal{R}(14,23) = \mathcal{G}(1,3)\mathcal{G}(4,2) + \sum_{5678} \mathcal{G}(1,5)\mathcal{G}(6,2)V(58,67)\mathcal{R}(74,83)$$

*BSE in terms of the uncorrelated one-fermion propagator:*

$$\mathcal{R}(14,23) = \tilde{\mathcal{R}}(14,23) + \sum_{5678} \tilde{\mathcal{R}}(16,25)\mathcal{W}(58,67)\mathcal{R}(74,83)$$

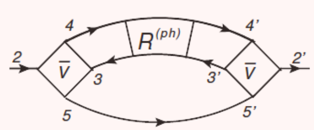
*Free (uncorrelated) response:* $\tilde{\mathcal{R}}^{(0)}(14,23) = \tilde{\mathcal{G}}(1,3)\tilde{\mathcal{G}}(4,2)$

*Interaction kernel:*

$$\mathcal{W}(14,23) = \tilde{V}(14,23) + V^e(14,23) + i\tilde{\mathcal{G}}^{-1}(1,3)\Sigma^e(4,2) + i\Sigma^e(1,3)\tilde{\mathcal{G}}^{-1}(4,2) - i\Sigma^e(1,3)\Sigma^e(4,2)$$

*Leading-order self-energy and induced interaction:*

$\Sigma^e_{22'}$ =

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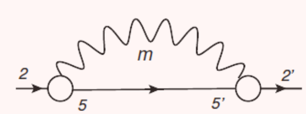
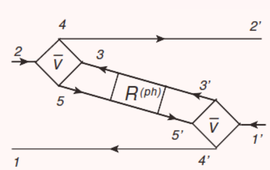



$V^e_{22'}$ =

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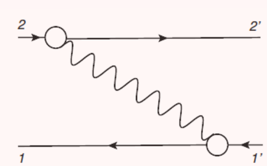
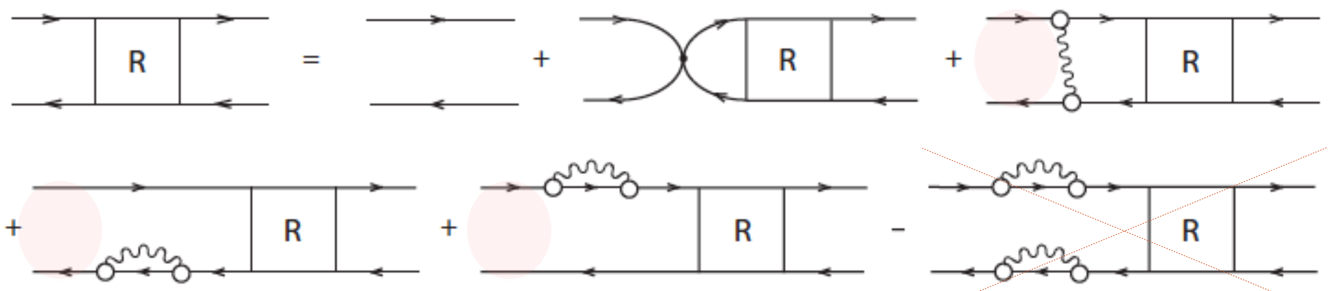


# Time blocking method at T=0




*Formally, the same BSE at T=0 and T>0*

next-order, GSC/PVC, ***(see talk of Caroline Robin)***

$$R = \tilde{R}^0 - i\tilde{R}^0 W R$$

$$\tilde{R}^0(14,23) = \tilde{G}(1,3)\tilde{G}(4,2) \rightarrow \tilde{D}^0(14,23) = \Theta(14,23)\tilde{G}(1,3)\tilde{G}(4,2)$$

*Time- projection operator:*

$$\Theta(14,23) = \delta_{\sigma_1,-\sigma_2}\theta(\sigma_1 t_{14})\theta(\sigma_1 t_{23})$$

*V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)*

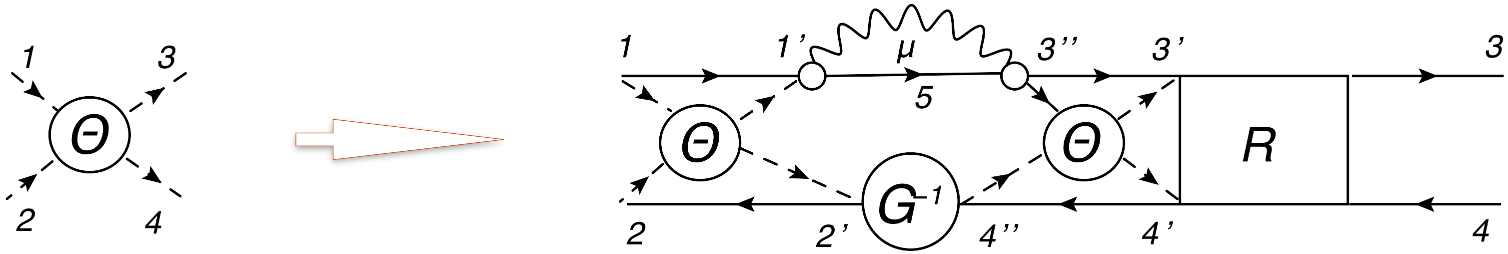
$$\tilde{R}^0_{14,23}(\omega,\varepsilon,\varepsilon') = 2\pi\delta_{13}\delta_{24}\delta(\varepsilon-\varepsilon')\tilde{G}_1(\varepsilon+\omega)\tilde{G}_2(\varepsilon)$$

*Non-separable*

$$\tilde{D}^0_{14,23}(\omega,\varepsilon,\varepsilon') = i\delta_{\sigma_1,-\sigma_2}\delta_{13}\delta_{24}\sigma_1(\omega-\varepsilon_{12}+i\sigma_1\delta)\tilde{G}_1(\varepsilon+\omega)\tilde{G}_2(\varepsilon)\tilde{G}_3(\varepsilon'+\omega)\tilde{G}_4(\varepsilon')$$

*Separable*

# Time blocking method at T=0



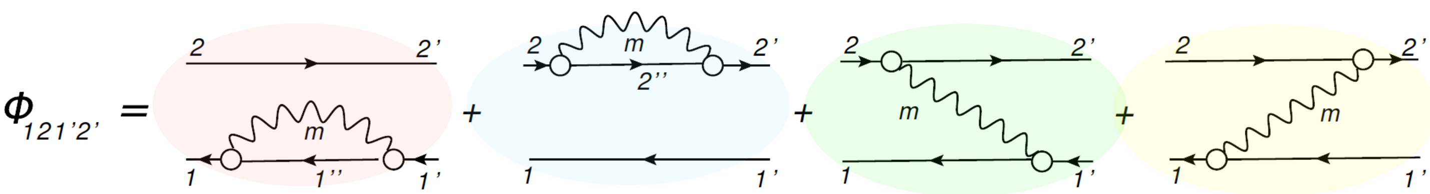

*Single-frequency variable equation for the response function:*

$$R_{14,23}(\omega) = \tilde{R}^0_{14,23}(\omega) + \sum_{1'2'3'4'} \tilde{R}^0_{12',21'}(\omega)\left[\tilde{V}_{1'4',2'3'} + \delta\Phi_{1'4',2'3'}(\omega)\right] R_{3'4,4'3}(\omega)$$

$$\delta\Phi_{1'4',2'3'}(\omega) = \Phi_{1'4',2'3'}(\omega) - \Phi_{1'4',2'3'}(0)$$

Correction of the double counting

*Dynamical kernel (time-ordered), the resonant part without the GSC/PVC:*




$$\Phi^{(ph,ph)}_{12,1'2'}(\omega) = \sum_m \left[\delta_{22'} \sum_{1''} \frac{g^m_{11''} g^{m*}_{1'1''}}{\omega - \varepsilon_{1''} + \varepsilon_2 - \Omega_m} + \delta_{11'} \sum_{2''} \frac{g^m_{2''2} g^{m*}_{2''2'}}{\omega - \varepsilon_1 + \varepsilon_{2''} - \Omega_m} - \frac{g^{m*}_{1'1} g^m_{2'2}}{\omega - \varepsilon_1 + \varepsilon_{2'} - \Omega_m} - \frac{g^m_{11'} g^{m*}_{22'}}{\omega - \varepsilon_{1'} + \varepsilon_2 - \Omega_m}\right]$$

# Time blocking method at T>0

**How to transform the BSE at T>0?**

*Free two-fermion propagator:*

$$\tilde{\mathcal{R}}^0(14,23) = \tilde{\mathcal{G}}(1,3)\tilde{\mathcal{G}}(4,2)$$

*Fourier transform to the imaginary discrete energy variables:*

$$\tilde{\mathcal{R}}^0_{14,23}(i\omega_n, i\xi_l, i\xi_{l'}) = \frac{\delta_{13}\delta_{24}\delta_{ll'}}{T(i\xi_l - \varepsilon_2)(i\omega_n + i\xi_l - \varepsilon_1)} = \frac{\delta_{13}\delta_{24}\delta_{ll'}}{T}\mathcal{G}_1(i\omega_n + i\xi_l)\mathcal{G}_2(i\xi_l)$$

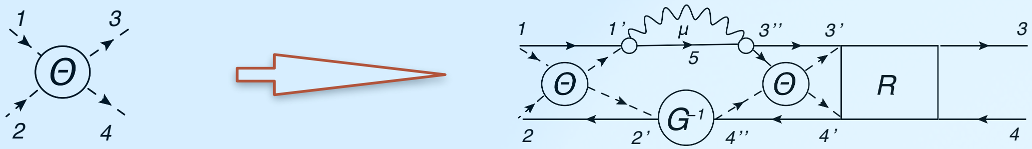
- ***Which projection operator can bring*** $\tilde{\mathcal{R}}^0_{14,23}(i\omega_n, i\xi_l, i\xi_{l'})$ ***to a symmetric form at T>0 ?***
- ***The operator*** $\Theta(14,23) = \delta_{\sigma_1,-\sigma_2}\theta(\sigma_1 t_{14})\theta(\sigma_1 t_{23})$ ***used at T=0 can not…***
- ***We have found that the operator***

$$\Theta(14,23;T) = \delta_{\sigma_1,-\sigma_2}\Big[n(\sigma_1\varepsilon_2,T)\theta(\sigma_1 t_{12}) + n(-\sigma_1\varepsilon_1,T)\theta(-\sigma_1 t_{12})\Big]\theta(\sigma_1 t_{14})\theta(\sigma_1 t_{23})$$

$$\lim_{T\to 0}\theta(12,T) = \lim_{T\to 0}\Big[n(\sigma_1\varepsilon_2,T)\theta(\sigma_1 t_{12}) + n(-\sigma_1\varepsilon_1,T)\theta(-\sigma_1 t_{12})\Big] = 1$$

***can do this***

# Time blocking at T>0




"Soft" time blocking at T>0 leads to a single-frequency variable equation for the response function

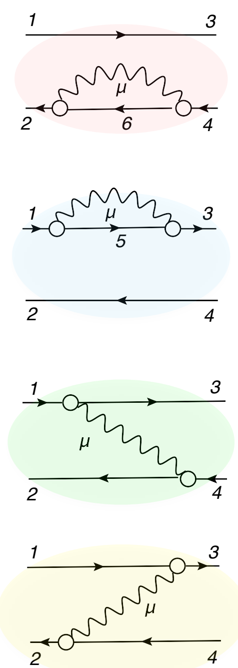
$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + \\ + \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\left[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\right]\mathcal{R}_{3'4,4'3}(\omega,T)$$

$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

*T > 0:*

$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)} \sum_{\mu;\eta_\mu=\pm1} \eta_\mu \Big[\delta_{13}\sum_6 \gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_6(T)\big)\big(n(\varepsilon_6-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_6-\eta_\mu\Omega_\mu} + \\ + \delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_5(T)\big)}{\omega-\varepsilon_5+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_2(T)\big)\big(n(\varepsilon_2-\eta_\mu\Omega_\mu,T)-n_3(T)\big)}{\omega-\varepsilon_3+\varepsilon_2-\eta_\mu\Omega_\mu} - \\ - \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42} \times \\ \times \frac{\big(N(\eta_\mu\Omega_\mu)+n_4(T)\big)\big(n(\varepsilon_4-\eta_\mu\Omega_\mu,T)-n_1(T)\big)}{\omega-\varepsilon_1+\varepsilon_4-\eta_\mu\Omega_\mu}\Big],$$

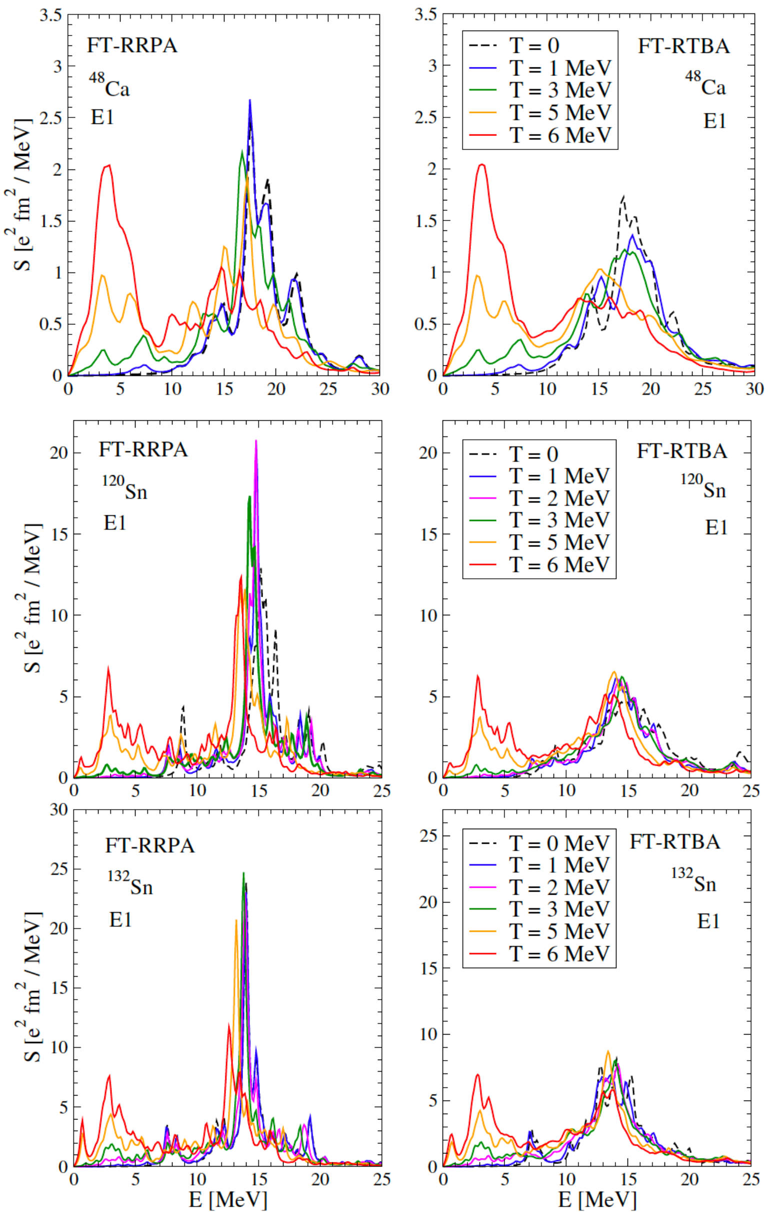
*Dynamical kernel:*



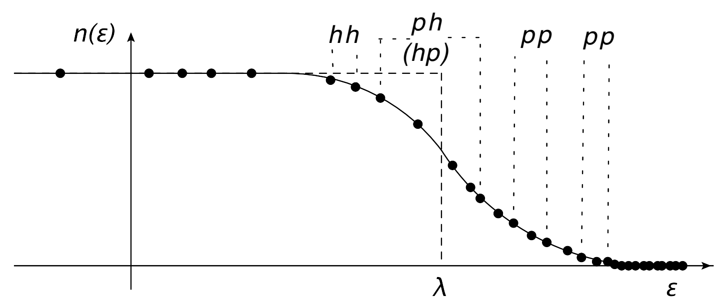

*T = 0:*

$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times \\ \times\Big[\delta_{13}\sum_6 \frac{\gamma^{\mu}_{62}\gamma^{\mu *}_{64}}{\omega-\varepsilon_1+\varepsilon_6-\Omega_\mu} + \\ +\delta_{24}\sum_5 \frac{\gamma^{\mu}_{15}\gamma^{\mu *}_{35}}{\omega-\varepsilon_5+\varepsilon_2-\Omega_\mu} - \\ - \frac{\gamma^{\mu}_{13}\gamma^{\mu *}_{24}}{\omega-\varepsilon_3+\varepsilon_2-\Omega_\mu} - \\ - \frac{\gamma^{\mu *}_{31}\gamma^{\mu}_{42}}{\omega-\varepsilon_1+\varepsilon_4-\Omega_\mu}\Big]$$

# Giant Dipole Resonance in $^{48}$Ca and $^{120,132}$Sn




Thermal unblocking:




Uncorrelated propagator:

$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

- *All states are additionally fragmented due to the thermal effects*
- *More phonon modes to be included in the PVC self-energy*
- *Broadening of the resulting GDR spectrum*
- *Development of the low-energy part => a feedback to GDR*

- *The spurious translation mode is properly decoupled as the mean field is modified consistently*
- *The role of the new terms in the Φ amplitude increases with temperature*
- *A very little fragmentation of the low-energy peak (possibly due to the absence of GSC/PVC)*

# The role of the exponential factor

$$S(E,T) = -\frac{1}{\pi} \lim_{\Delta \to +0} Im \langle V^{0\dagger} \mathcal{R}(E + i\Delta, T) V^{0} \rangle$$

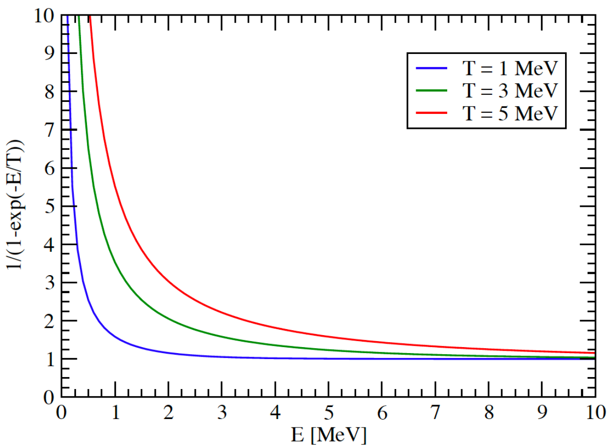
*The final strength function at T>0:*

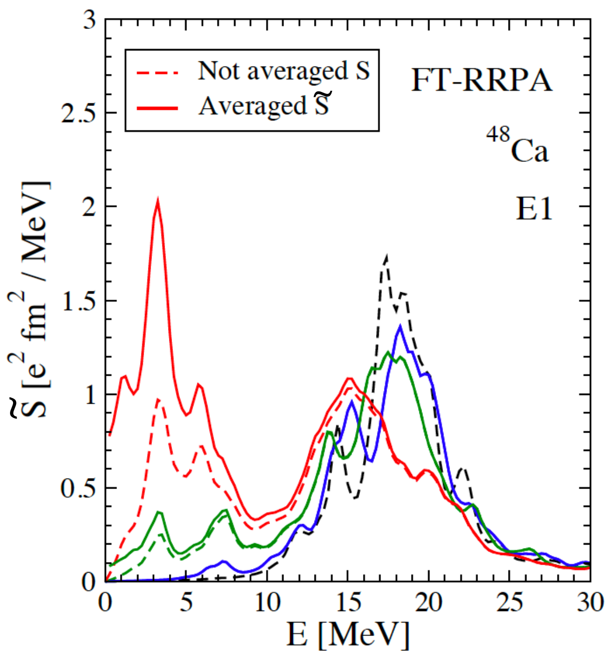
$$\tilde{S}(E,T) = \frac{S(E,T)}{1 - e^{-E/T}}$$

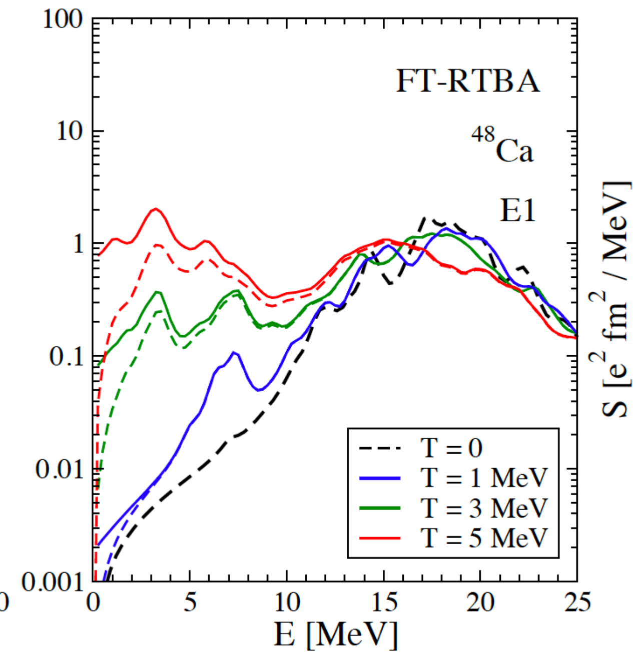
*Averaging over the initial state energies, Detailed balance at T>0*

$$\lim_{E \to 0} S(E,T) = 0$$

*The exponential factor:*



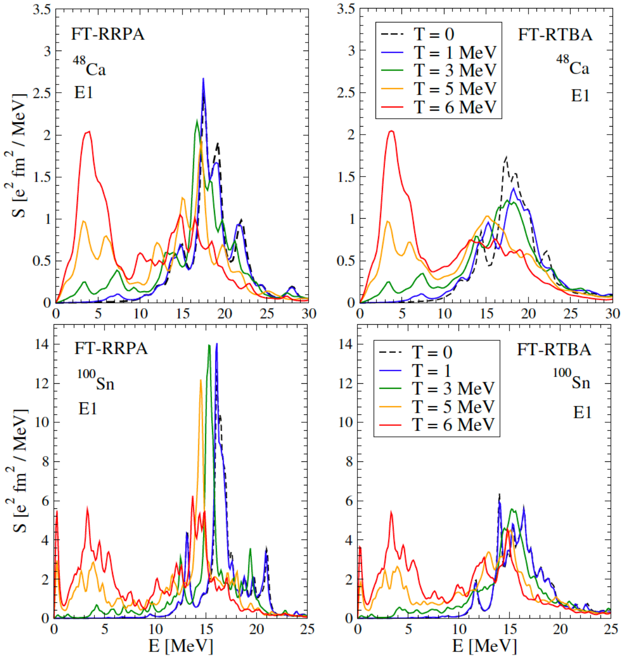



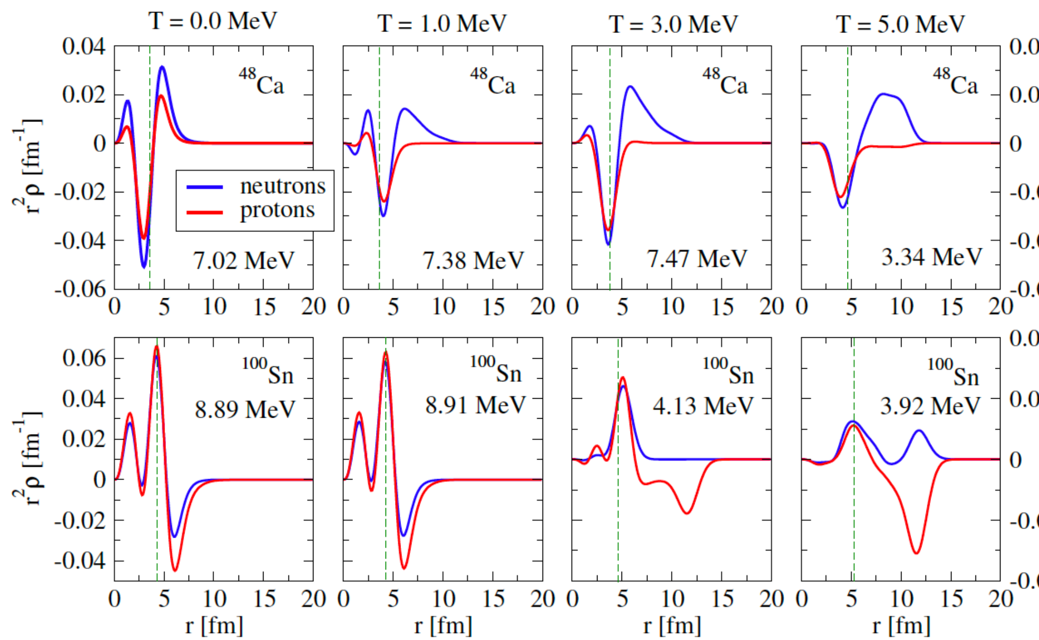

- ***The exponential factor brings an additional enhancement at E<T energy region and provides the finite zero-energy limit of the strength function (regardless its spin-parity)***

# Evolution of the pygmy dipole resonance (PDR) at T>0

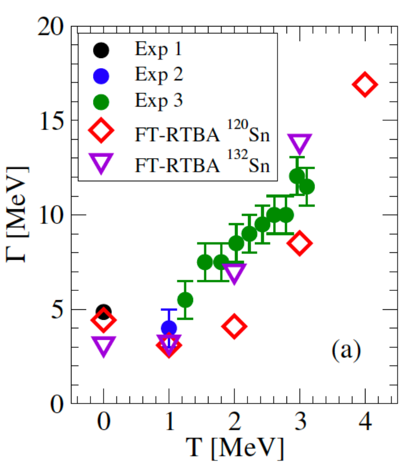
## Strength distribution



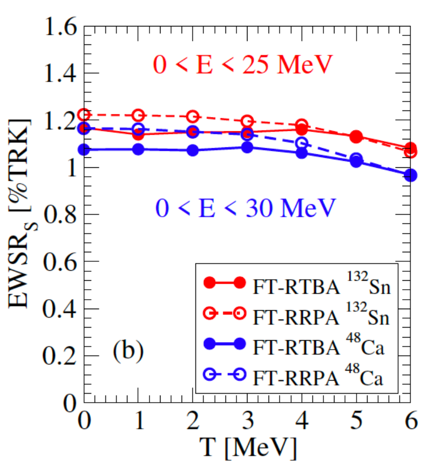

## Transition density for the low-energy peak




## GDR's width



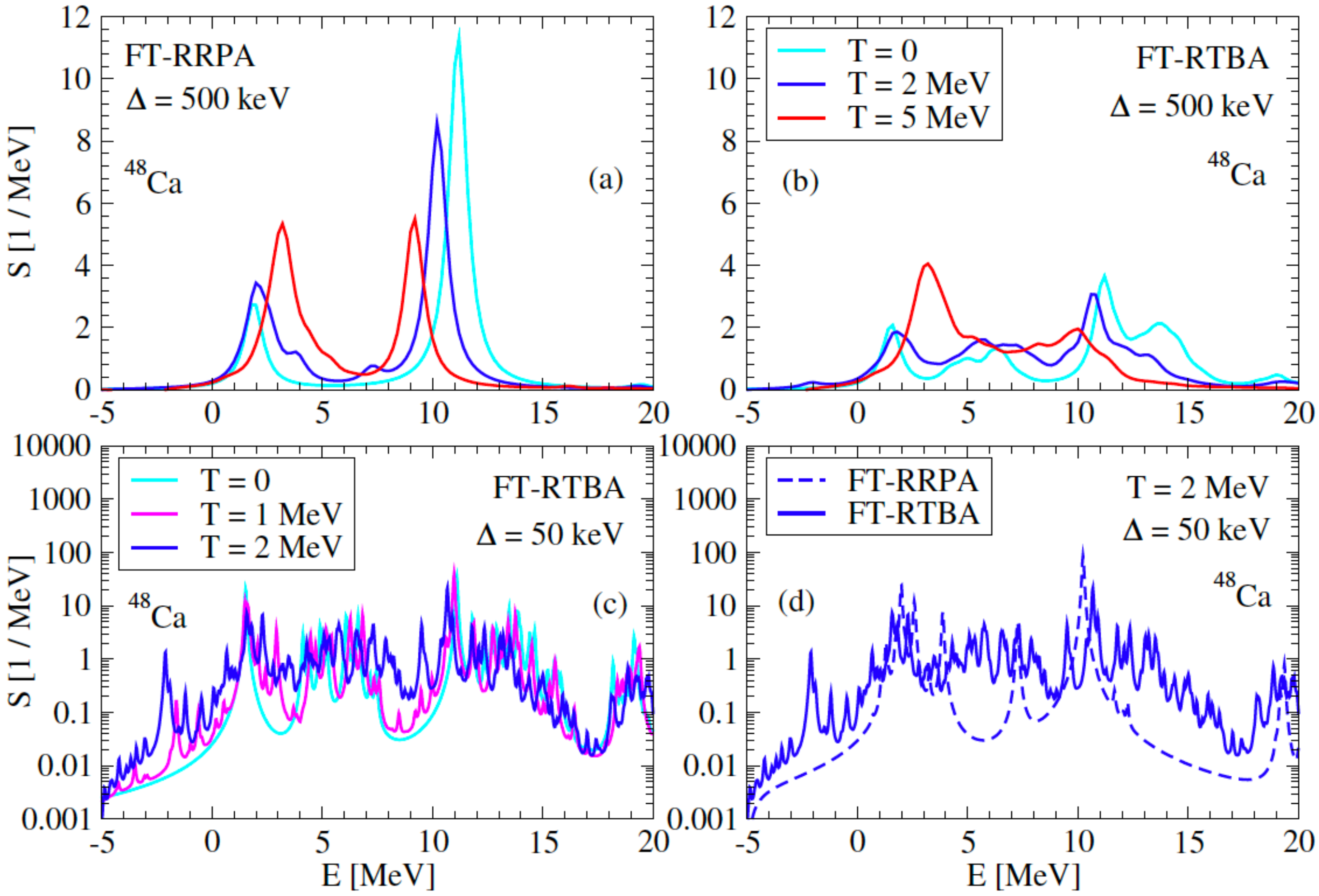

## Energy-weighted sum rule




- *The low-energy peak (PDR) gains the strength from the GDR with the temperature growth: EWSR ~ const*
- *The total width Γ ~ T² (as in the Landau theory)*
- *The PDR develops a new type of collectivity originated from the thermal unblocking*
- *The same happens with other low-lying modes => strong PVC => "destruction" of the GDR at high temperatures*

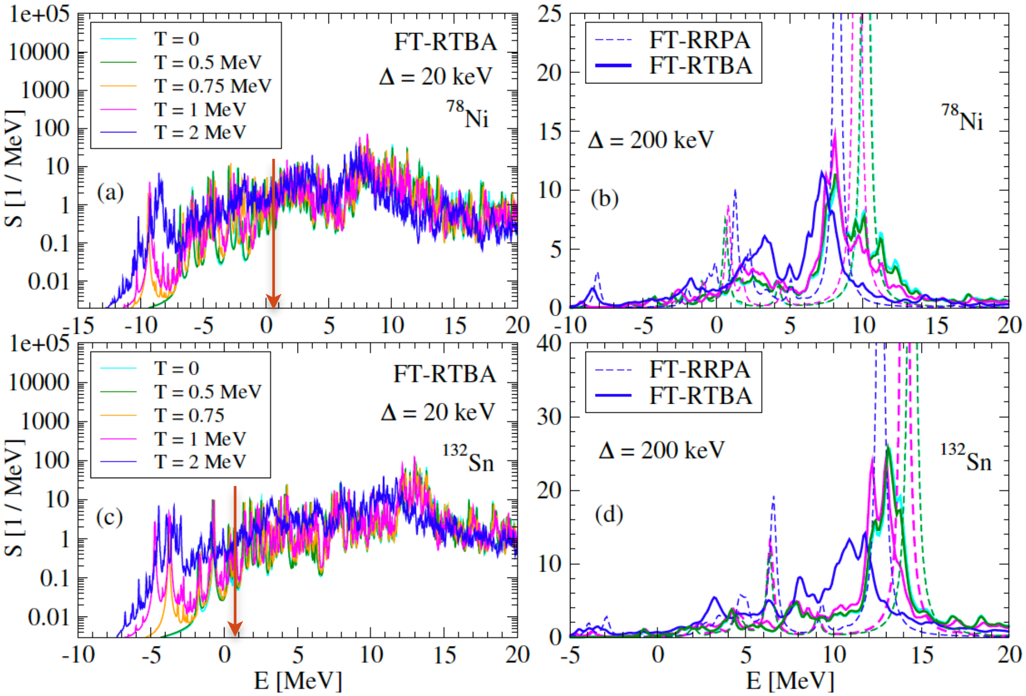
*E.L., H. Wibowo, arXiv:1804.10228, PRL (2018), accepted.*

# Temperature dependence of the Gamow-Teller Resonance (GTR): $^{48}Ca$ case




- The GTR shows a stronger sensitivity to temperature than the neutral GDR.
- The strength gets "pumped" into the low-energy peak with the temperature increase.
- New states appear in the lowest-energy sector due to the thermal unblocking.
- PVC fragmentation effects remain strong.

# Gamow-Teller Resonance: $^{78}$Ni and $^{132}$Sn



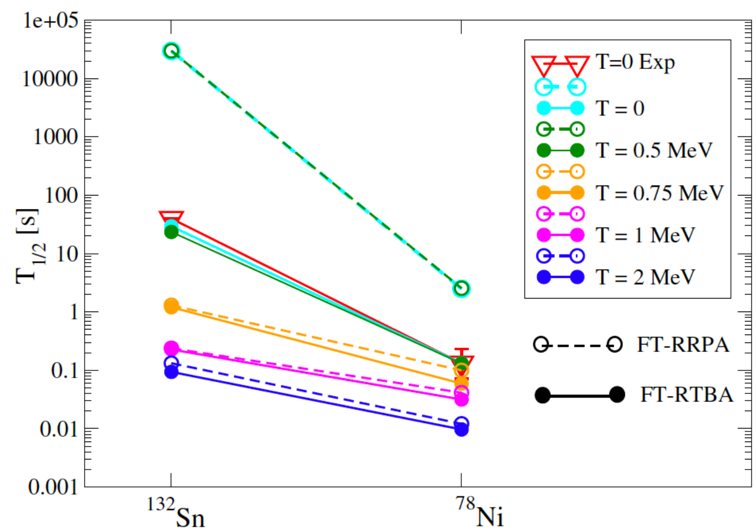

### Beta decay half-life $T_{1/2}$

$$T_{1/2}^{-1} = \frac{g_A^2}{D}\int_{\Delta B}^{\Delta_{nH}} f(Z,\Delta_{np}-E)S_{GT^-}(E)dE$$

*$\Delta_{nH}$ = 0.78 MeV; $g_A$ = 1.27 (unquenched)*

*E.L., C. Robin, H. Wibowo, in preparation*

- *The thermally unblocked transitions increase the GTR strength within the $Q_\beta$ window. This causes the decrease of the $T_{1/2}$ with temperature.*
- *At the typical r-process temperatures T~0.2-0.3 MeV the thermal unblocking is still suppressed by the large shell gaps, however, the situation should change in the open-shell nuclei.*



# *Outlook*

### ***Summary:***

- *Relativistic NFT offers a powerful framework for a high-precision solution of the nuclear many-body problem at zero and finite temperature;*
- *The self-consistent Green function formalism and the non-perturbative response theory based on QHD and including high-order correlations are available for a large class of nuclear excited states in even-even and odd-odd nuclei; now generalized to finite temperature*
- *The first application to the dipole response has explained the the dependence of the GDR's width on temperature and "disappearance" of the GDR at T~6 MeV in medium-heavy nuclei. A temperature evolution of the GTR was studied within a proton-neutron version of the FT-RTBA.*

### ***Current and future developments:***

- *An approach to nuclear response including both continuum and PVC at finite temperature, for both neutral and charge-exchange excitations (in progress);*
- *Inclusion of the superfluid pairing to extend the application range (r-process);*
- *Extension of FT-RTBA to pairing channels and applications to neutron stars;*
- *Toward an "ab initio" description: realization of the approach based on the bare relativistic meson-exchange potential*

# *Many thanks for collaboration and support:*

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*Peter Schuck (IPN Orsay)*
*Peter Ring (TU München)*
*Irina Egorova (WMU, JINR Dubna)*
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*Dario Vretenar (U Zagreb)*
*Remco Zegers (NSCL)*
*And others…*

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