

Lattice Calculation of Hadronic Tensor

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Hadronic tensor and PDF

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z e^{iq \cdot z} \left\langle P, S \left| [J_\mu^\dagger(z), J_\nu(0)] \right| P, S \right\rangle$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2) \quad \hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu \quad x = \frac{Q^2}{2(E_p \nu - \vec{p} \cdot \vec{q})}$$

$$F_1(x, Q^2) \rightarrow q(x, Q^2) \quad F_2(x, Q^2) \rightarrow xq(x, Q^2)$$

On the lattice

$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p} \cdot \vec{x}_f} \sum_{\vec{x}_2 \vec{x}_1} \frac{1}{4\pi} e^{-i\vec{q} \cdot (\vec{x}_2 - \vec{x}_1)} \langle \chi_N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{\chi}_N(\vec{0}, t_0) \rangle$$

finite renormalization for local vector current

$$C_2 = \sum_{\vec{x}_f} e^{-i\vec{p} \cdot \vec{x}_f} \langle \chi_N(\vec{x}_f, t_f) \bar{\chi}_N(\vec{0}, t_0) \rangle$$

$$\tilde{W}_{\mu\nu}(\vec{p}, \vec{q}, \tau) = \frac{E_p}{m_N} \frac{\text{Tr}[\Gamma_e C_4]}{\text{Tr}[\Gamma_e C_2]} \stackrel{t_f \gg t_2, t_1 \gg t_0}{=} \langle p | \sum_{\vec{x}_2 \vec{x}_1} \frac{1}{4\pi} e^{-i\vec{q} \cdot (\vec{x}_2 - \vec{x}_1)} J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) | p \rangle \quad \tau = t_2 - t_1$$

$$\tilde{W}_{\mu\nu}(\vec{p}, \vec{q}, \tau) \sim \sum_n \langle N | J_\mu | N_n \rangle \langle N_n | J_\nu | N \rangle e^{-(E_n - E_p)\tau}$$

$$\tilde{W}_{\mu\nu}(\vec{p}, \vec{q}, \tau) = \int d\nu e^{-\nu\tau} W_{\mu\nu}(\vec{p}, \vec{q}, \nu) \quad \nu = E_n - E_p$$

need to solve the inverse problem!

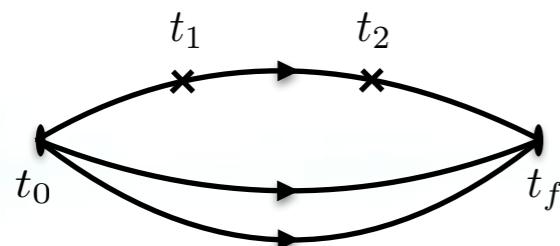
K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.F. Liu, PRD 62, 074501 (2000)

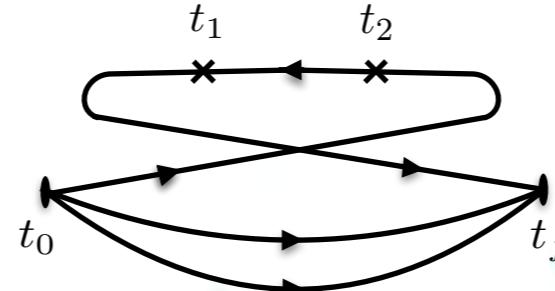
K.F. Liu, PoS, LATTICE2015:115 (2016)

Different contractions

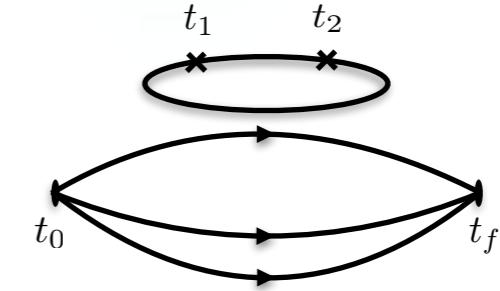
$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2\vec{x}_1} \frac{1}{4\pi} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \langle \chi_N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{\chi}_N(\vec{0}, t_0) \rangle$$



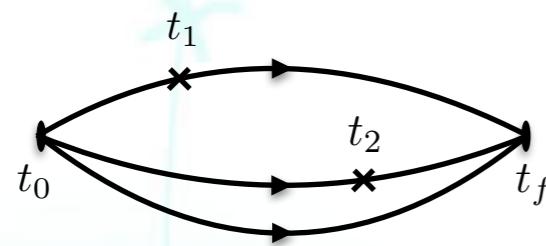
valence and
connected-sea parton



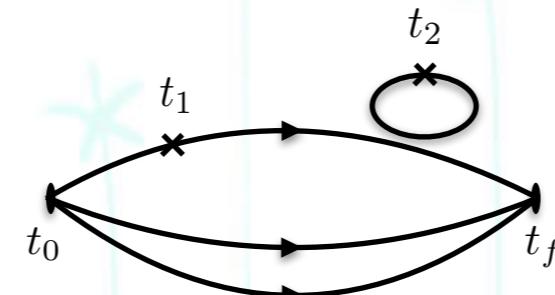
connected-sea
anti-parton



disconnected-sea
parton and anti-parton



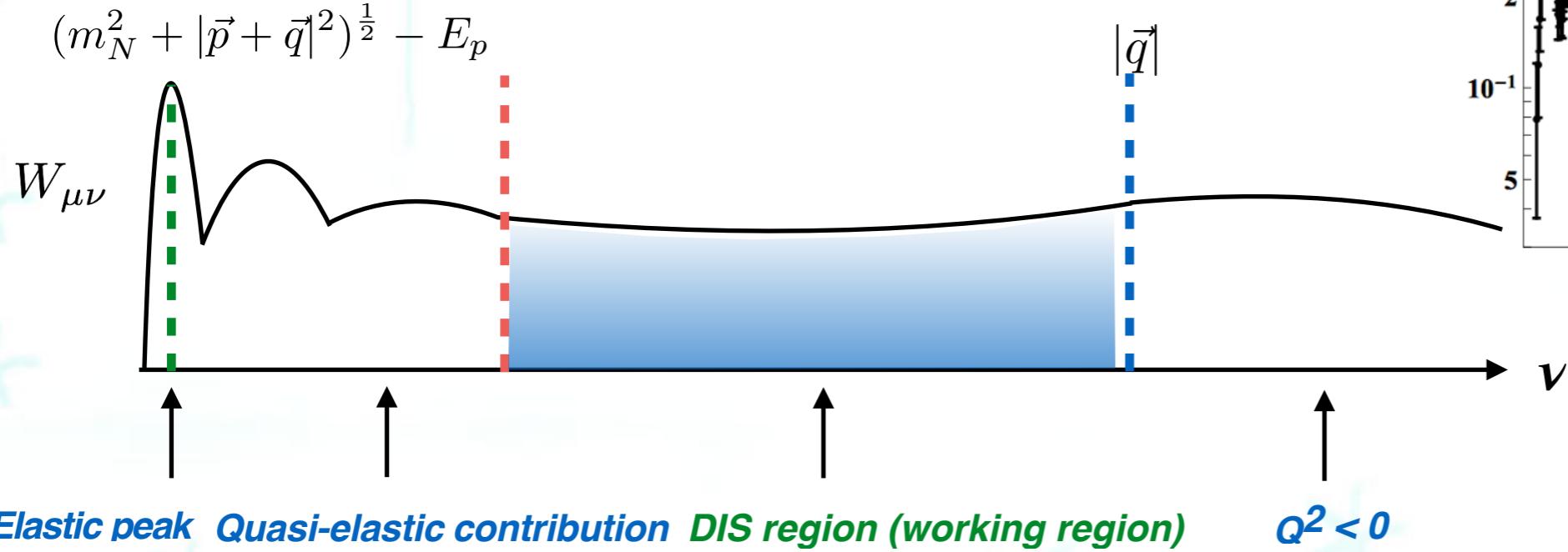
suppressed by $O(1/Q^2)$



The CS u-bar and d-bar contribution are supposed to be responsible for the Gottfried sum rule violation can be revealed explicitly.

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

lattice setups



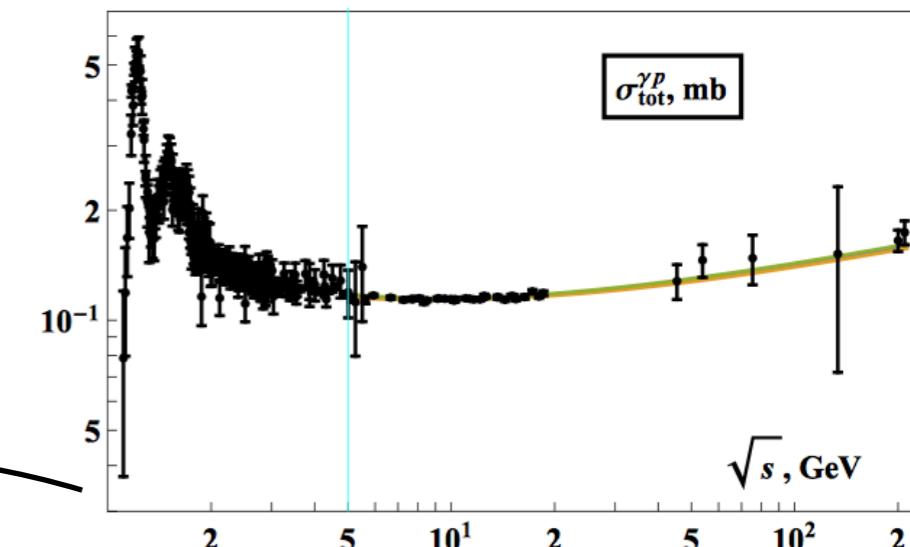
$$Q^2 = |\vec{q}|^2 - \nu^2 > 0 \rightarrow \nu < |\vec{q}| \quad \nu > (E_{n=0} - E_P) + \Delta E \quad (\text{away from the elastic peak})$$

$$\frac{2\pi}{L} = 618 \text{ MeV}$$

	E_p	E_0	$E_0 - E_p$	ν	$ \vec{q} $	Q^2	x
[0. 2. 2.][0. -3. -3.]	2.12	1.48	-0.64	2.21	2.62	2.00	0.108
[0. 2. 2.][0. -3. -3.]	2.12	1.48	-0.64	1.70	2.62	4.00	0.244

$$\mu = \nu = 1 \text{ and } p_1 = q_1 = 0 \quad W_{11}(\nu) = F_1(x, Q^2)$$

clover fermions on domain-wall sea (32lf), $a \sim 0.06$ fm, $m_\pi \sim 371$ MeV
 two sequential sources are used for the 4-point function calculation



Check of elastic case

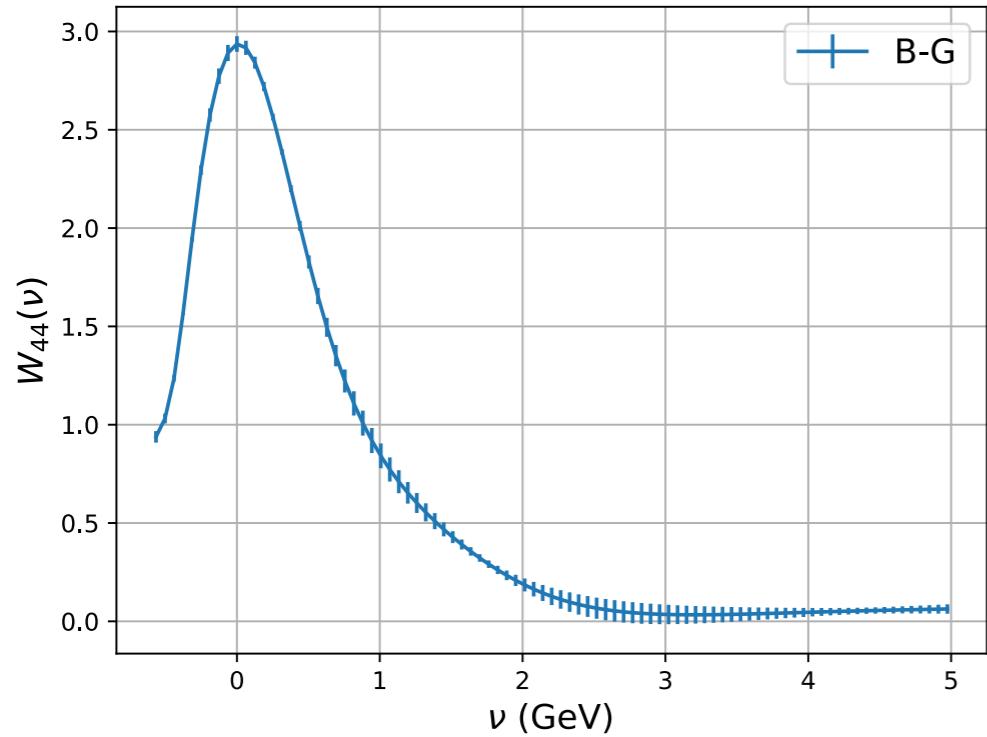
$$\tilde{W}_{\mu\nu}(\vec{p}, \vec{q}, \tau) \sim \sum_n \langle N | J_\mu | N_n \rangle \langle N_n | J_\nu | N \rangle e^{-(E_n - E_p)\tau}$$

$$\tilde{W}_{44}(\vec{p} = 0, \vec{q} = 0, \tau) \sim F_1^2(q^2 = 0) = g_V^2$$

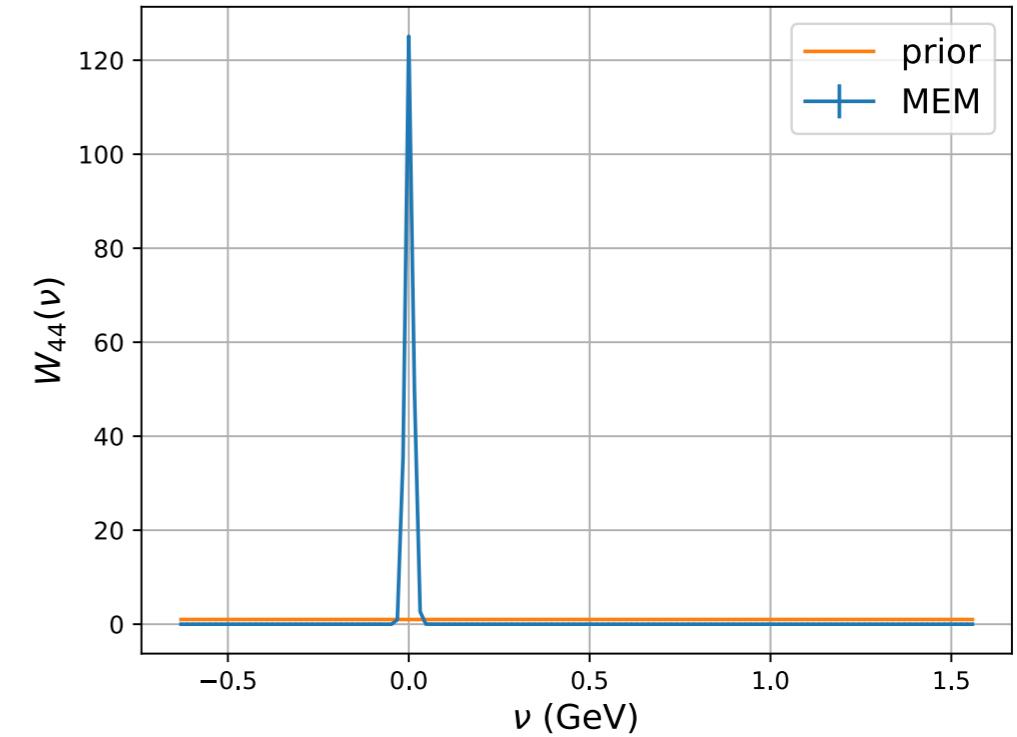
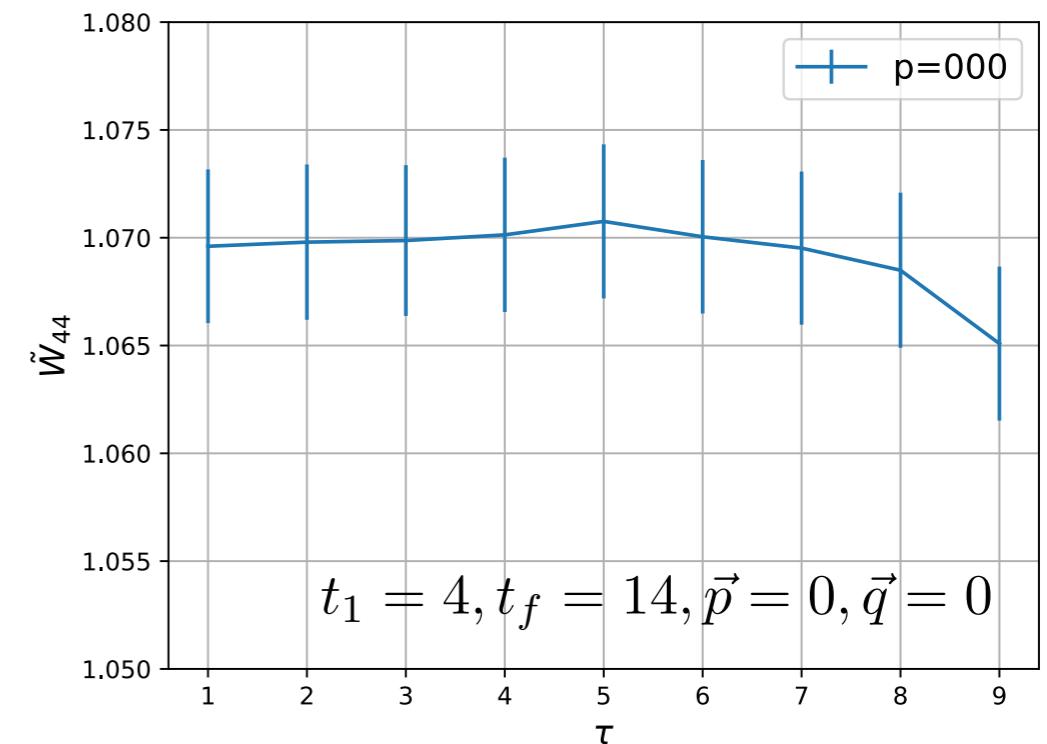
$$g_V = 1.034(2), g_V^2 \sim 1.069(4)$$

$$\tilde{W}_{\mu\nu}(\vec{p}, \vec{q}, \tau) = \int d\nu e^{-\nu\tau} W_{\mu\nu}(\vec{p}, \vec{q}, \nu)$$

$$W_{44}(\vec{p} = 0, \vec{q} = 0, \nu) = \delta(q^2 + 2M\nu) 2M F_1^2(q^2 = 0)$$



Backus-Gilbert, area $\sim 1.18(6)$

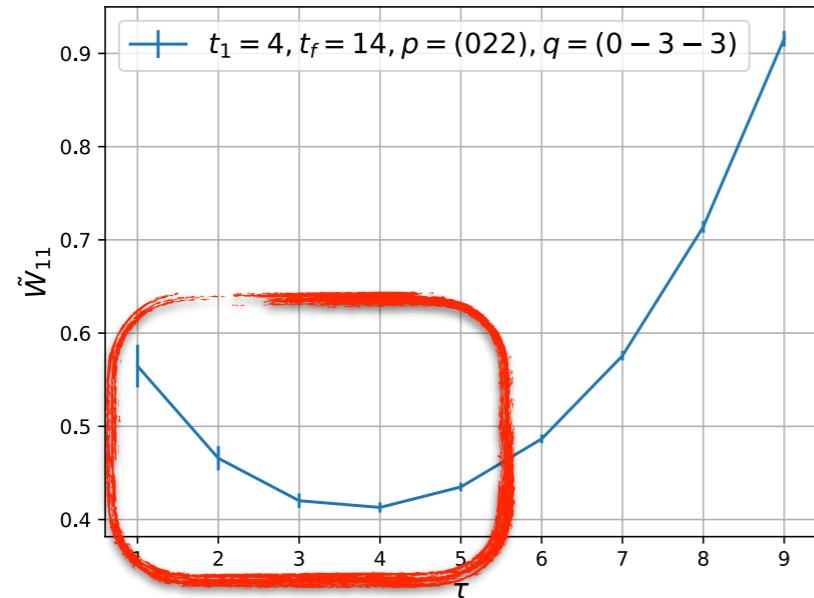


MEM, area $\sim 1.071(3)$

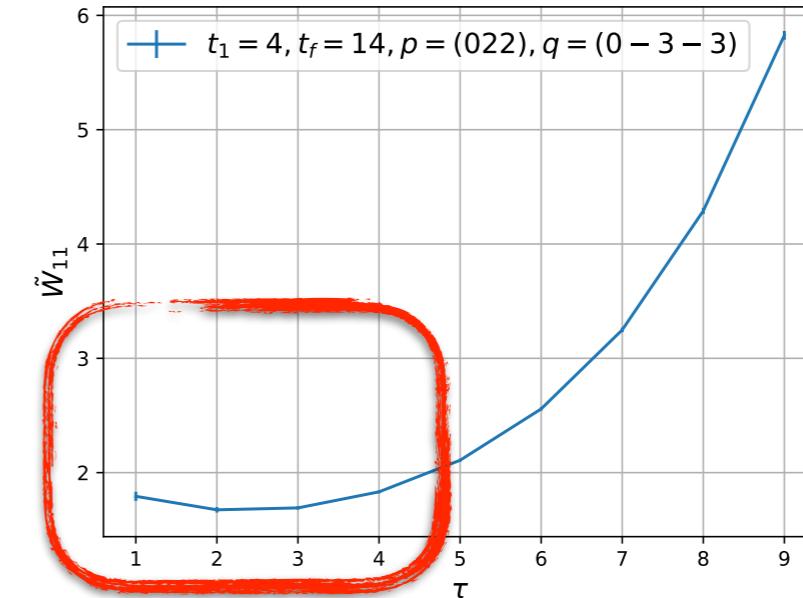
Exploratory results

$$\tilde{W}_{\mu\nu}(q^2, \tau) \propto \sum_n \langle p, s | J_\mu | n \rangle \langle n | J_\nu | p, s \rangle e^{-(E_n - E_p)\tau}$$

$$E_0 = (m_N^2 + |\vec{p} + \vec{q}|^2)^{\frac{1}{2}} \quad \mathbf{p}=(022), \mathbf{q}=(0-3-3)$$

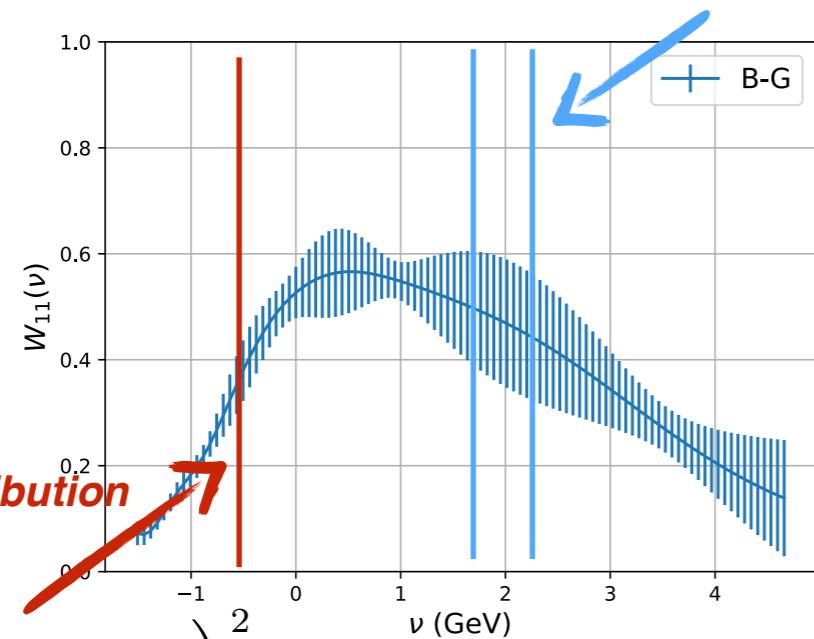


d quark



u quark

working region $\nu \in [1.74, 2.24], x \in [0.24, 0.11], Q^2 \in [4, 2]$

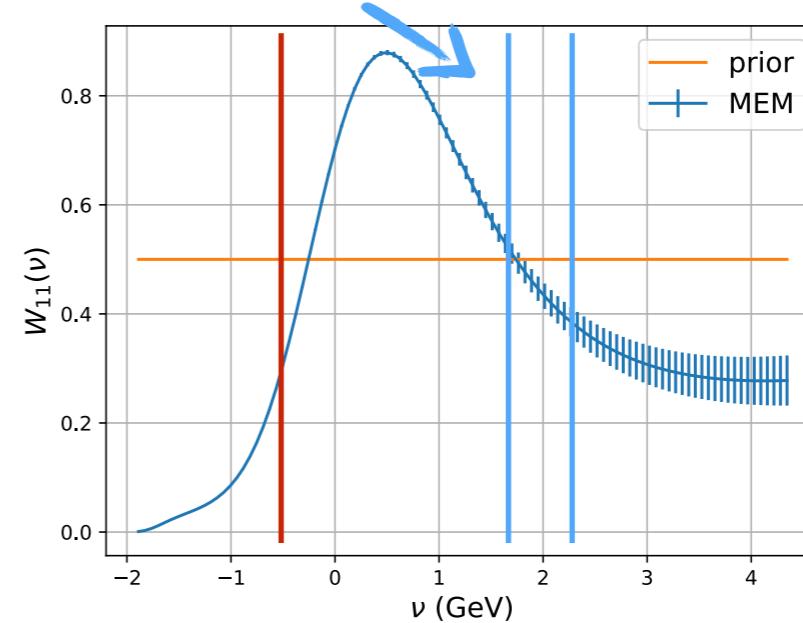


elastic contribution suppressed

B-G, d quark

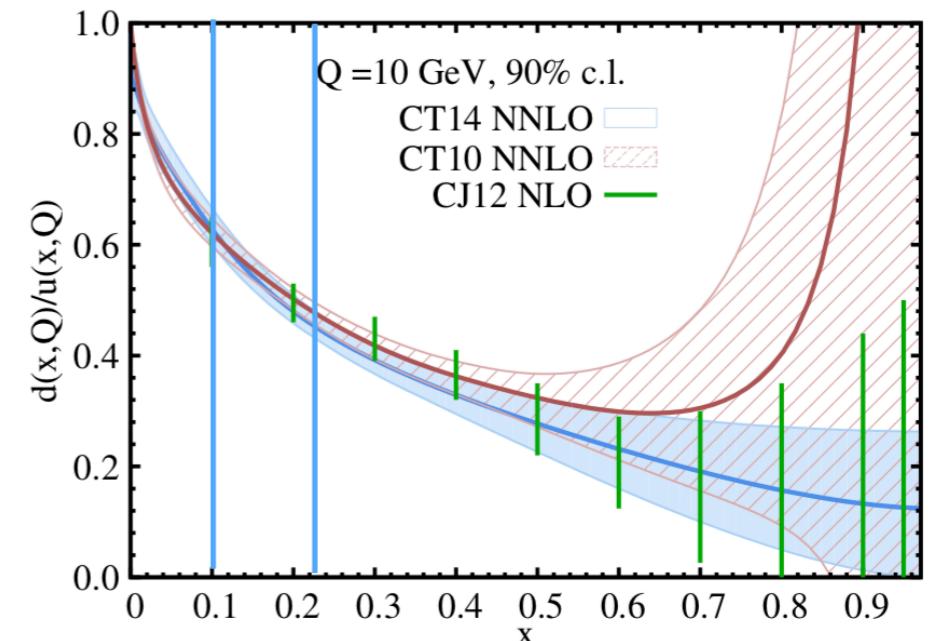
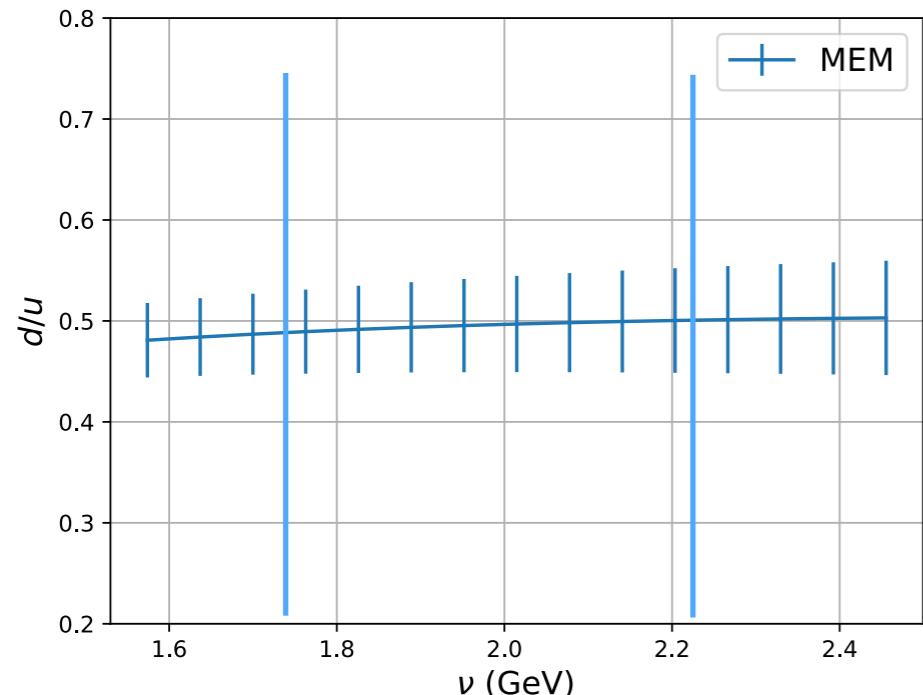
$$G^2(0) \propto \left(\frac{1}{(1 + \frac{Q_{\text{el}}^2}{\Lambda^2})^2} \right)^2$$

**higher intermediate states are seen in this case
still not a perfect case but we get consistent (B-G and MEM) non-zero
values in the working region**



MEM, d quark

More about the results



S. Dulat et al., PRD 93, 033006 (2016)

$$\nu \in [1.74, 2.24], x \in [0.24, 0.11], Q^2 \in [4, 2]$$

systematics uncertainties

contaminations from quasi-elastic contribution

systematics induced by the inverse methods

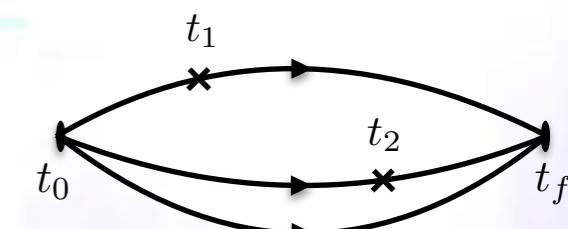
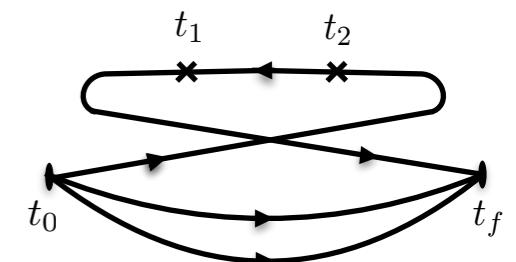
evolution of Q^2 is not considered

excited-states effects

other lattice artifacts (lattice spacing, pion mass, volume...)

Summary and outlook

- We are beginning to have some preliminary results from this hadronic tensor approach
- We need larger **p** and **q** to get into the deep inelastic region
- Other inverse method will be considered
- Finer lattices can help
- We are working to extract the connected-sea anti-parton contribution
- We can calculate the pure higher-twist contribution in the next stage



Thank you!