Neutral Current Weak Form Factors
&
Neutrino Scattering

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USQCD All Hands’ Meeting 2018
Neutrino-Nucleon Neutral Current Elastic Scattering

\[ \nu + p \rightarrow \nu + p \]
\[ \bar{\nu} + p \rightarrow \bar{\nu} + p \]

Matrix element in V-A structure of leptonic current

\[
M = \frac{i}{2\sqrt{2}} G_F \bar{\nu}(q_2) \gamma_\mu (1 - \gamma_5) \nu(q_1) \langle N(p_2)|J^\mu_Z|N(p_1)\rangle.
\]

\[
\langle N(p_2)|J^\mu_Z|N(p_1)\rangle = \bar{u}(p_2)[F^Z_1(Q^2) + F^Z_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} + F^Z_A(Q^2)\gamma^\mu\gamma_5]u(p_1)
\]
(Anti)Neutrino-Nucleon Scattering Differential Cross Section

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 \cdot Q^2}{2\pi \cdot E^2_\nu} (A \pm BW + CW^2)$$

$$W = 4( E_\nu / M_p - \tau)$$

$$A = \frac{1}{4} \{ (G^Z_A)^2 (1 + \tau) - [(F^Z_1)^2 - \tau (F^Z_2)^2] (1 - \tau) + 4\tau F^Z_1 F^Z_2 \}$$

$$B = -\frac{1}{4} G^Z_A (F^Z_1 + F^Z_2)$$

$$C = \frac{1}{64\tau} \{ (G^Z_A)^2 + (F^Z_1)^2 + \tau (F^Z_2)^2 \}$$

Neutral Weak Dirac & Pauli FFs

Weak axial FF
Calculation of $F_1^Z$ and $F_2^Z$

\[
F_{1,2}^{Z,p} = \left(\frac{1}{2} - \sin^2 \theta_W\right) (F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2)) - \sin^2 \theta_W (F_{1,2}^p + F_{1,2}^n) - \frac{F_{1,2}^s}{2}
\]

Nucleon EMFF from Model Independent z-expansion

Strange EMFF from Lattice QCD

PRL 118, 042001 (2017)

RSS, Yang, Alexandru, Draper, Liang, Liu

Physical point
4 lattice spacings
3 volumes

PL B 777 (2018) 8-15

Ye, Arrington, Hill, Lee
Inputs for Previous Neutral Weak EMFFs

Strange EMFF

Nucleon EMFF (total)

PRD 95, 014011(2017)
RSS, de Teramond, Brodsky, Dosch, Deur

PRL 2018
de Teramond, Liu, RSS, Brodsky, Dosch, Deur
Calculation of Neutral Weak EMFFs

\[ G_{E,M}^{Z,p(n)}(Q^2) = \frac{1}{4} \left[ (1 - 4\sin^2 \theta_W)(1 + R_V^{p(n)})G_{E,M}^{\gamma,n(p)}(Q^2) - (1 + R_V^{n(p)})G_{E,M}^{\gamma,n(p)}(Q^2) - G_{E,M}^{s}(Q^2) \right] \]

Radiative corrections for e-p scattering

PRD 96, 093007 (2017) RSS
*Use MiniBooNE data \((0.27 < Q^2 < 0.65 \text{ GeV}^2)\)

**Reason 1:** Uncertainty in \(G^s_{E,M}\) becomes very large and values consistent with zero

**Reason 2:** Nuclear effect can be large for at low \(Q^2\)
Determination of Neutral Current Weak Axial FF

\[ \frac{d\sigma}{dQ^2} \] From MiniBooNE Experiment

\[ F^{p}_{1,2}, F^{n}_{1,2} \} \] From Experiment

\[ F^s_{1,2} \] From Lattice QCD

\[ G_A^Z(0) = -0.751(56) \]

\[ M_{dipole} = 0.95(6) \text{ GeV} \]

In preparation with Keh-Fei Liu & David Richards
Possibility: Since strange quark contribution is small
set \( G_{E,M}^s = 0 \).
Reconstruction of Differential Cross Sections

Nuclear effects, Pauli blocking & nuclear shadowing at $Q^2 < 0.15 \text{ GeV}^2$

BNL E734 data was NOT used in the analysis
Estimate of $G^s_A(0)$

This Calculation

\[ G^Z_A = \frac{1}{2}(-G^{CC}_A + G^s_A) \]

\[ G^Z_A(0) = -0.751(56) \]

\[ G^{CC}_A(0) = 1.2723(23) \]

\[ G^s_A(0) = -0.23(11) \]

Other Calculations

MiniBooNE, PRD 82 (2010) \[ G^s_A(0) = 0.08(26) \]

BNL E734, PRC 48 (1993) \[ G^s_A(0) = -0.21(10) \]

From Jeremy Green’s Talk
Summary

- Precise estimate of NC weak axial form factor $G_{ZA}^Z$
- Strange quark contribution cannot be ignored
- Reconstruction of (anti)neutrino- nucleon diff. cross sections with correct prediction of $G_{ZA}^Z$ and lattice input of $G_{E,M}^s$
- Lattice QCD calculation of $G_{sA}^s$ in the continuum and infinite volume limit with controlled systematic uncertainties required
An Example: LQCD Constraint on Models

Many models of meson-baryon fluctuations to study $s(x) - \bar{s}(x)$ asymmetry

\[ \Lambda(uds) \]

\[ K^+(u\bar{s}) \]
TABLE II. Two solutions for the strange form factors at $Q^2 = 0.5$ GeV$^2$ produced from the E734 and HAPPEX data.

<table>
<thead>
<tr>
<th></th>
<th>Solution 1</th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_E^s$</td>
<td>0.02 ± 0.09</td>
<td>0.37 ± 0.04</td>
</tr>
<tr>
<td>$G_M^s$</td>
<td>0.00 ± 0.21</td>
<td>−0.87 ± 0.11</td>
</tr>
<tr>
<td>$G_A^s$</td>
<td>−0.09 ± 0.05</td>
<td>0.28 ± 0.10</td>
</tr>
</tbody>
</table>

$Q^2 = 0.5$ GeV$^2$
\[ A_{PV}^p = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{1}{[\epsilon(G^p_E)^2 + \tau(G^p_M)^2]} \times \{ (\epsilon(G^p_E)^2 + \tau(G^p_M)^2)(1 - 4\sin^2\theta_W)(1 + R_V^p) \\
- (\epsilon G^p_E G^m_E + \tau G^p_M G^m_M)(1 + R_V^n) \\
- (\epsilon G^p_E G^s_E + \tau G^p_M G^s_M)(1 + R_V^{(0)}) \\
- \epsilon'(1 - 4\sin^2\theta_W)G^p_M G^e_A \}, \]

with

\[
\tau = \frac{Q^2}{4M^2_p}, \quad \epsilon = \left(1 + 2(1 + \tau)\tan^2\frac{\theta}{2}\right)^{-1}, \\
\epsilon' = \sqrt{\tau(1 + \tau)(1 - \epsilon^2)},
\]

\[
\begin{array}{c|ccc}
 & R_A^{T=1} & R_A^{T=0} & R_A^{(0)} \\
\hline
\text{one-quark} & -0.172 & -0.253 & -0.551 \\
\text{many-quark} & -0.086(0.34) & 0.014(0.19) & \text{N/A} \\
\text{total} & -0.258(0.34) & -0.239(0.20) & -0.55(0.55)
\end{array}
\]
\[ \nu_{\mu} + n \rightarrow \mu^- + p \quad , \quad \bar{\nu}_{\mu} + p \rightarrow \mu^+ + n , \]
\[ \nu_e + n \rightarrow e^- + p \quad , \quad \bar{\nu}_e + p \rightarrow e^+ + n . \]

<table>
<thead>
<tr>
<th>Particle</th>
<th>Lifetime (ns)</th>
<th>Decay mode</th>
<th>Branching ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ )</td>
<td>26.03</td>
<td>( \mu^+ + \nu_{\mu} ) ( e^+ + \nu_e )</td>
<td>99.9877 0.0123</td>
</tr>
<tr>
<td>( K^+ )</td>
<td>12.385</td>
<td>( \mu^+ + \nu_{\mu} ) ( \pi^0 + e^+ + \nu_e ) ( \pi^0 + \mu^+ + \nu_{\mu} )</td>
<td>63.44 4.98 3.32</td>
</tr>
<tr>
<td>( K_L^0 )</td>
<td>51.6</td>
<td>( \pi^- + e^+ + \nu_e ) ( \pi^+ + e^- + \bar{\nu}<em>e ) ( \pi^- + \mu^+ + \nu</em>{\mu} ) ( \pi^+ + \mu^- + \bar{\nu}_\mu )</td>
<td>20.333 20.197 13.551 13.469</td>
</tr>
<tr>
<td>( \mu^+ )</td>
<td>2197.03</td>
<td>( e^+ + \nu_e + \bar{\nu}_\mu )</td>
<td>100.0</td>
</tr>
</tbody>
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