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# Lattice Calculation of Parton Distribution Functions

*SPC Summary 2018*

# Introduction

**Working definition** - calculation of x-dependent PDFs and QDAs (quark distribution amplitudes) from Euclidean-space lattice calculations.

- Quasi-PDF (**qPDF**) interpreted in **LaMET** (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \bar{\psi}(z) \gamma^z e^{-ig \int_0^z dz' A^z(z')} \psi(0) | P \rangle + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2)$$

*Quasi distributions* approach light-cone distributions in limit of large  $P^z$

$$q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (**pPDF**) recognizing generalization of PDFs in terms of *loffe Time*.  $\nu = p \cdot z$  **A. Radyushkin, PLB767 (2017)**

$$\mathcal{M}^\alpha(z, p) = \langle p | \bar{\psi}(z) \gamma^\alpha \exp\left(-ig \int_0^z dz' A^z(z')\right) \psi(0) | p \rangle$$

# Introduction - II

- Good “Lattice Cross Sections” (**LCS**) Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle \begin{cases} \mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi) [\bar{\psi}_q \psi](0) \\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi) [\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{cases}$$

whence

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\text{QCD}}^2)$$

KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (**HT**)  $W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq \cdot z} \langle p | J_\mu(z)^\dagger J_\nu(0) | p \rangle$

$$C_4(\vec{p}, \vec{q}, \tau) = \sum_{\vec{x}_f} e^{-i\vec{p} \cdot \vec{x}_f} \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{q} \cdot (\vec{x}_2 - \vec{x}_1)} \langle N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{N}(\vec{0}, t_0) \rangle$$

This is a **four-point** function.

- Moment Methods
  - Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
  - Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)
- Related: calculation of gluon spin operator

# Proposals

**Team:** T. Izubuchi, L. Jin, C. Kallidonis, N. Karthik, S. Mukherjee, P. Petreczky, C. Shugert, and S. Syritsyn

**Project:** Computing Pion Parton Distribution Function on Fine Lattice

**Request:** 0.71M GPU, 80TB Disk, 100TB Tape

**Approach:** qPDF

HISQ sea and Clover valence at very fine lattice spacings of  $a \sim 0.06$  and  $0.04$  fm to get to high momenta.

**Team:** Jian Liang, Keh-Fei Liu, Yi-Bo Yang and Terrence Draper

**Project:** Lattice calculation of nucleon EDM and hadronic tensor

**Request:** 60M JPsi on KNL/SKX, 81TB Disk

**Approach:** HT

Clover on DWF at valence at  $a \sim 0.06$ fm, 370 MeV pion. Expect  $x$  from 0.07 to 0.45.

# Proposals

**Team:** Huey-Wen Lin, Xiangdong Ji, Luchang Jin, Yi-Bo Yang, Yong Zhao, Jiunn-Wei Chen, Yu-Sheng Liu, and Jian-Hui Zhang

**Project:** Three-Dimensional and Flavor Structure of the Nucleon

**Request:** 40.8M JPsi core hours; 400K GPU hours

**Approach:** qPDF

Clover on HISQ at physical quark masses. Extending previous work on PDFs to Generalized Parton Distributions.

**Team:** Bipasha Chakraborty, Robert Edwards, Balint Joo, Kostas Orginos, Jianwei Qiu, Anatoly Radyushkin, David Richards, Raza Sufian, Frank Winter, Colin Egerer, Joe Karpie, Anatoly Radyushkin, Savvas Zafeiropoulos

**Project:** Parton Distribution Functions and Amplitudes of the Pseudoscalar Mesons and Nucleon from Lattice QCD

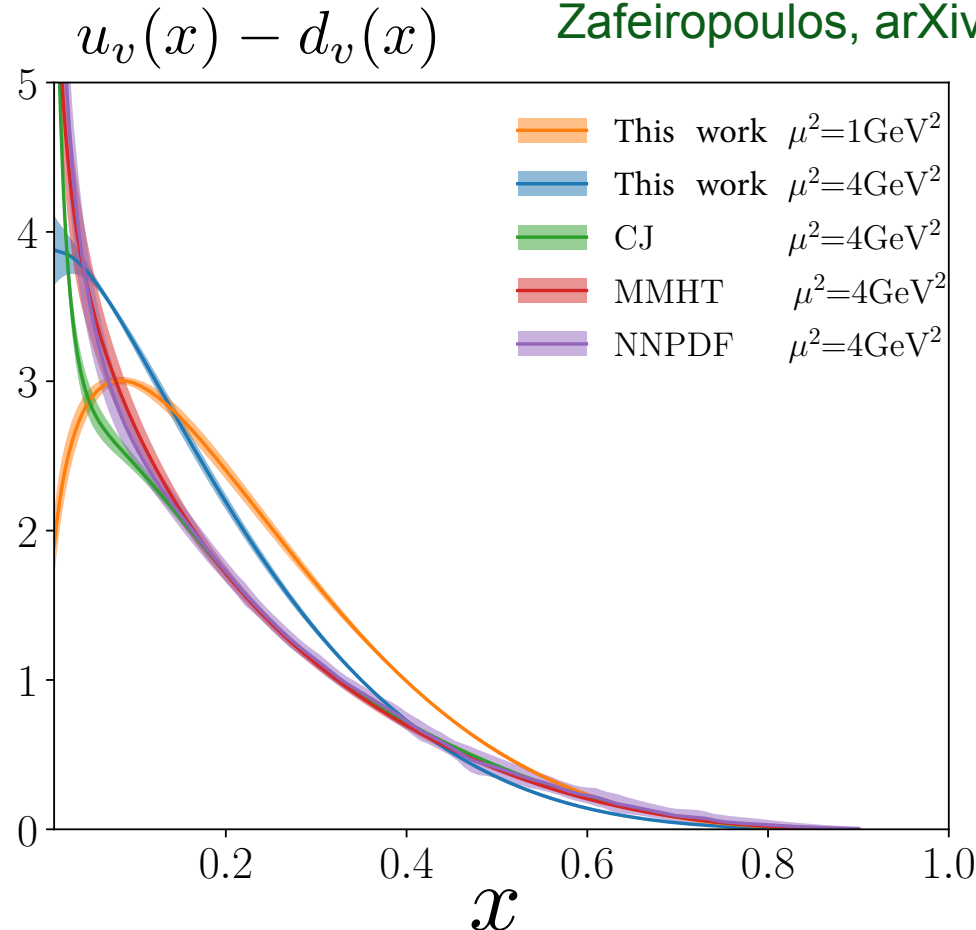
**Request:** 40.8M JPsi core hours; 400K GPU hours

**Approach:** pPDF and LCS

Clover on clover. pPDF for nucleon, and LCS for pion.

# Recent qPDF and pPDF Results

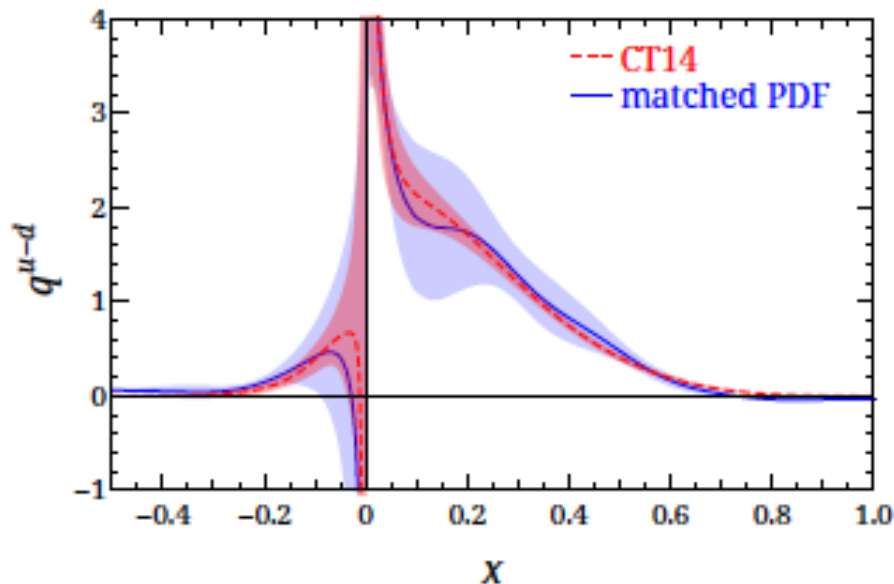
Orginos, Radyushkin, Karpie and  
Zafeiropoulos, arXiv:1706.05373



- Pseudo-PDF
- Quenched
- pion mass 600 MeV

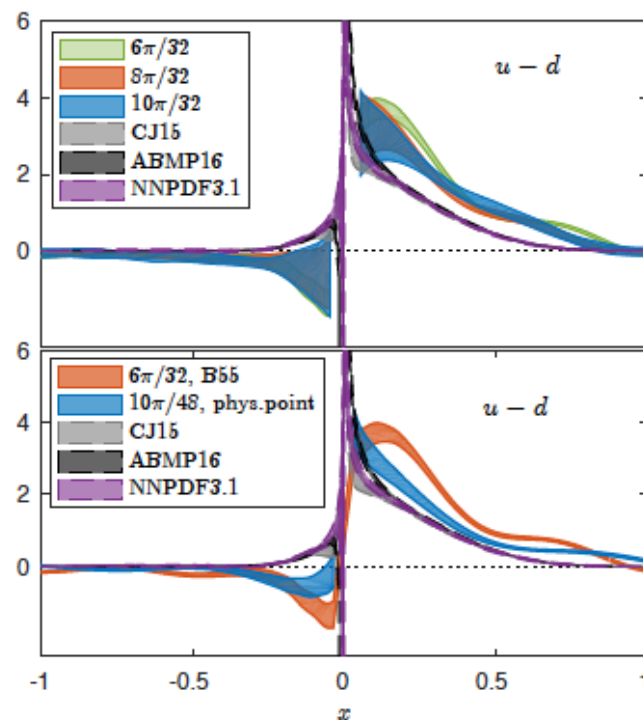
# Recent qPDF and pPDF Results

Pioneers LP3 Collaboration (JW Chen et al), arXiv:1803.04393



- Quasi-PDF approach
- Physical Pion Mass

Alexandrou et al, arXiv:1803.02685



# Challenges/Questions

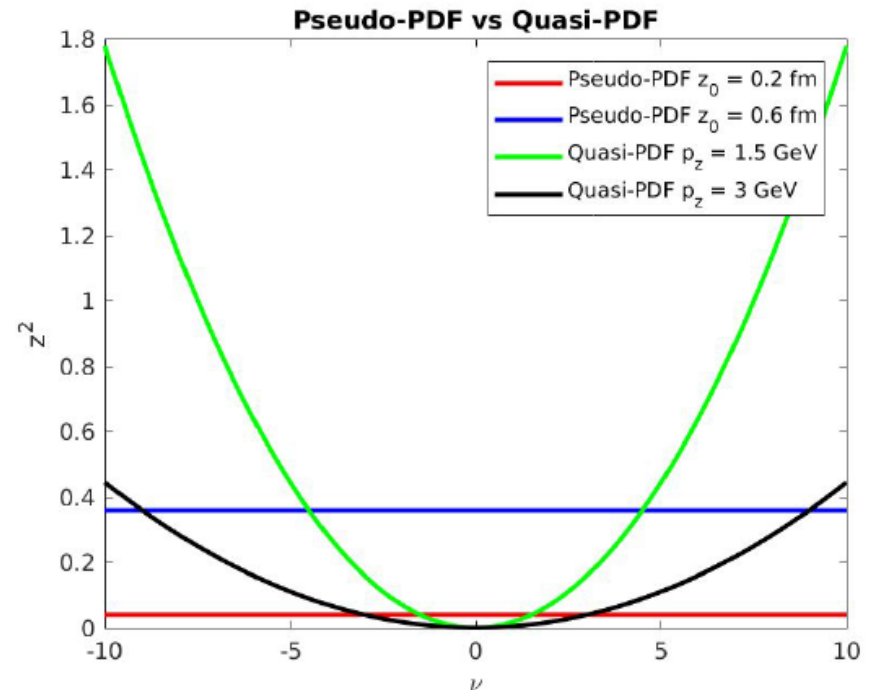
Joe Karpie

Relation between qPDF and pPDF approaches

- Both integrals of Ioffe-Time Distribution Function
- Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$





# Challenges/Questions - II

## Renormalization and Matching

- LaMET/**qPDF** - see talk by **Yi-Bo Yang**

- **pPDF**

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)}{M(0, z^2)}$$

Form *reduced matrix element*

Free of UV divergences; well-defined continuum limit

- **HT** and **LCS**

Local current operators, continuum perturbation theory.

# Challenges/Questions - III

## High spatial momentum and lattice systematics

- Momentum smearing to reach high momenta
- Finite-volume effects
- Range of Bjorken-x
- Relation to moment calculations

## Inverse Problem

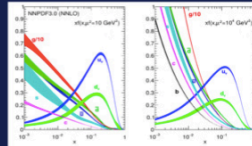
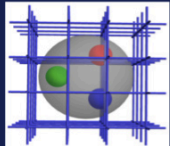
$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\text{QCD}}^2)$$

Calculate on Lattice

Extract PDF?

Calculate in PQCD

→ Hadronic-tensor Talk Jian Liang



## Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

## Whitepaper

[Home](#)

[Registration](#)

[Participants](#)

[Venue Details](#)

[Visa Information](#)

[Travel Information](#)

[Timetable](#)

[Final Bulletin](#)

### Administrative Responsible

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The workshop will be hosted at [Balliol College](#), sessions will start at 2pm on 22/03/2017 and will end at 4pm on 24/03/2017

The goal of this workshop is to bring together the global PDF analysis and lattice-QCD communities to explore ways to improve current PDF determinations. In particular, we plan to set precision goals for lattice-QCD calculations so that these calculations, together with experimental input, can achieve more reliable determinations of PDFs. In addition we will discuss what impact such improved determinations of PDFs will have on future new-physics searches.

Workshop participation is by invitation only

There will be introductory-level talks on the first day to bring state of the other. The remainder of the workshop will be primarily by topic. Short presentations will be scheduled as the require questions from both communities, and set 5-year precision goals for impact quantities. The findings of the workshop will be compiled into a report.

Organized by:

- Huey-Wen Lin (Michigan State University)
- Emanuele Nocera (Oxford University)
- Fred Olness (Southern Methodist University)
- Kostas Orginos (College of William and Mary)
- Juan Rojo (Oxford University)

With support from [the European Research Council](#) through

## Lattice PDF Workshop, 6–8 April 2018



### Overview

[Scientific Programme](#)

[Timetable](#)

[Contribution List](#)

[Registration](#)

[Participant List](#)

[Location](#)

[Lodging](#)

MCFP Workshop on Lattice Parton Physics  
Maryland Center for Fundamental Physics (MCFP)  
University of Maryland  
College Park, MD

April 6-8, 2018

Organizer:

Taku Izubuchi (BNL)  
Karl Jansen (DESY)  
Xiangdong Ji (University of Maryland/TD Lee Institute, Shanghai)  
Jianwei Qiu (JLab)  
Feng Yuan (LBNL)