Renormalization for LaMET

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L a t t i c e
P a r t o n
P h y s i c s
P r o j e c t

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Large momentum effective theory

LaMET

- Proton physics corresponds to taking $P \to \infty$ before $\Lambda \to \infty$.
- Light-cone object

- If $\Lambda \to \infty$ is taken prior to $P \to \infty$
- Not light-cone object, but calculable on the lattice.
- Result will depend on P

- The IR physics are the same; the UV difference between there two can be calculated perturbatively.

- Can also be used to calculate the PDF, glue spin, meson DA, GPD, and so on.

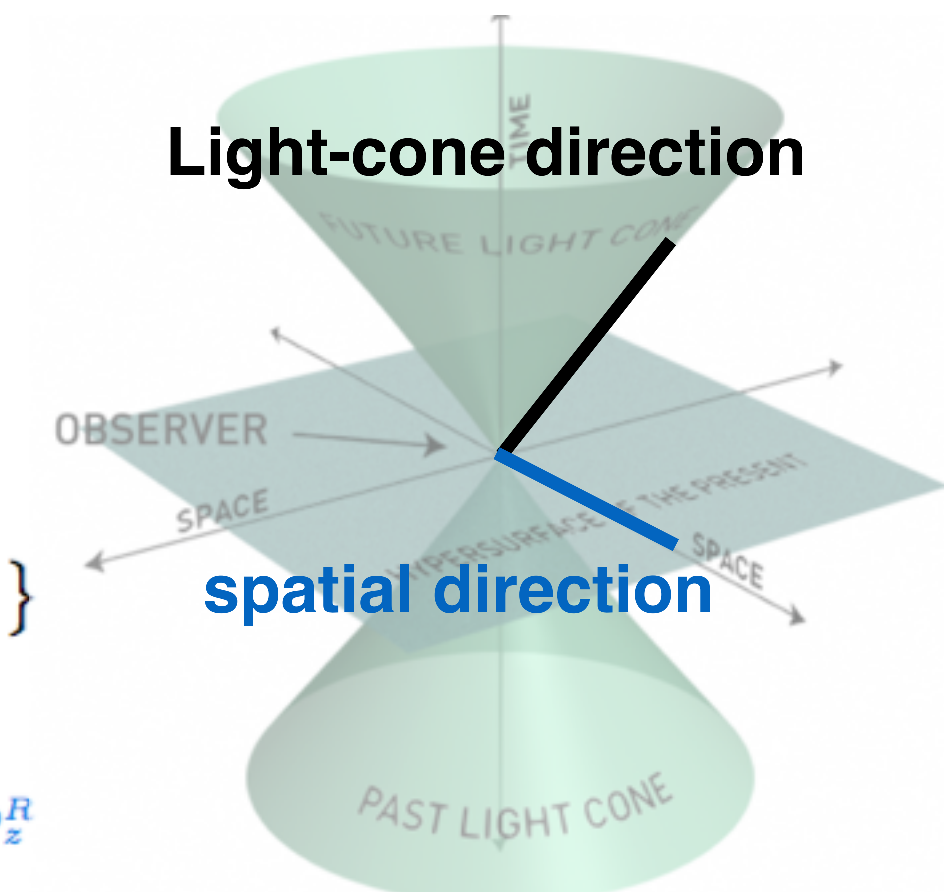
X. Ji, PRL 110 (2013) 262002, 1305.1539
X. Ji. SCPMA 57 (2014) 1407, 1404.6680

Large momentum effective theory

The light-cone PDF is defined by

quasi-PDF

$$\boxed{\begin{aligned} q(x,\mu^2) &= \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\bar{\psi}(\xi^-)\gamma^+ \\ &\quad \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle \end{aligned}}$$



and can be accessed by,

$$\begin{aligned} q(x,\mu) &= \int_{-\infty}^{\infty} \frac{dy}{|y|} \left\{\delta(1-\frac{x}{y}) - \frac{\alpha_s C_F}{2\pi} C_1\left(\frac{x}{y}, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}, \frac{\mu_R^2}{p_z^{R\,2}}\right)\right\} \\ &\quad \int_{-\infty}^{\infty} \frac{e^{iyP_z z}}{4\pi} \langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R_{p^2=\mu_R^2, p_z=p_z^R} \\ &\quad + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \alpha_s^2\right), \end{aligned}$$

with the lattice calculation of the ***RI/MOM renormalized quasi-PDF****,*

$$\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R_{p^2=\mu_R^2, p_z=p_z^R}.$$

C. Alexandrou et. al., NPB923 (2017) 394, 1706.00265
I. Stewart, Y. Zhao, PRD97 (2018) 054512
LP³, 1803.04393

Multiplicative renormalization

of the non-local operator

- The quark quasi-PDF operators $\bar{\psi}(z)\Gamma U(z,0)\psi(0)$ are **dim-3**, the lowest dimension of the quark bi-linear operators. No local operator can have even lower dimension.

X. Ji, J. Zhang, Y. Zhao, NPB924 (2017) 336, 1706.07416

- Those operators will not mix between ***different z*** and can be renormalized as

$$O_R = Z_{\bar{j}}^{-1} Z_j^{-1} e^{\delta \bar{m}|z_2 - z_1|} \overline{\psi}(z_2)\Gamma L(z_2, z_1)\psi(z_1).$$

X. Ji, J. Zhang, Y. Zhao, PRL120 (2018) 112001, 1706.08962
T. Ishikawa, Y. Ma, J. Qiu, S. Yoshida, PRD96 (2017) 094019, 1707.03107
J. Green, K. Jansen, F. Steffens, 1707.07152

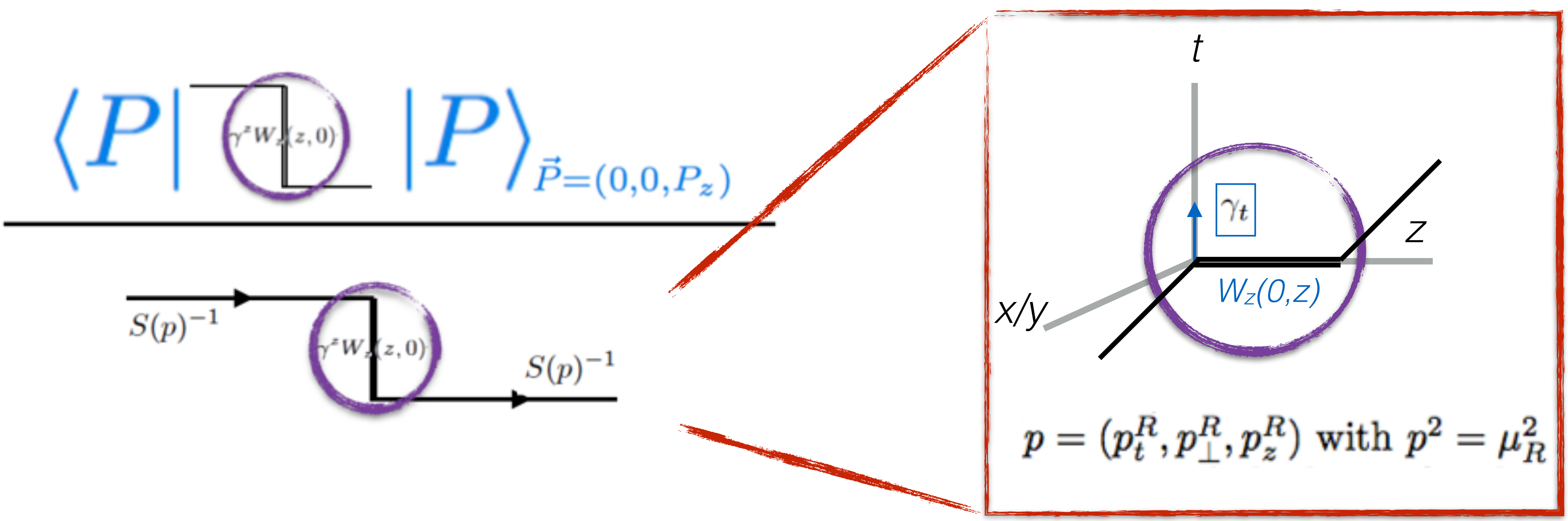
- The only concern is **whether the linear divergence can be fully removed by the non-perturbative renormalization**.

RI/MOM renormalization

The non-perturbative renormalized quasi-PDF matrix element $\tilde{h}^R$ *in the RI/MOM scheme is defined by*

$$\tilde{h}^R(z, P_z, p_z^R, \mu_R) = \tilde{Z}^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{\boxed{a\to 0}}$$

where $\tilde{h}(z, P_z, a^{-1}) = \frac{1}{2P^0}\langle P|O_{\gamma_t}(z)|P\rangle$ *is the lattice bare quasi-PDF matrix elements.*

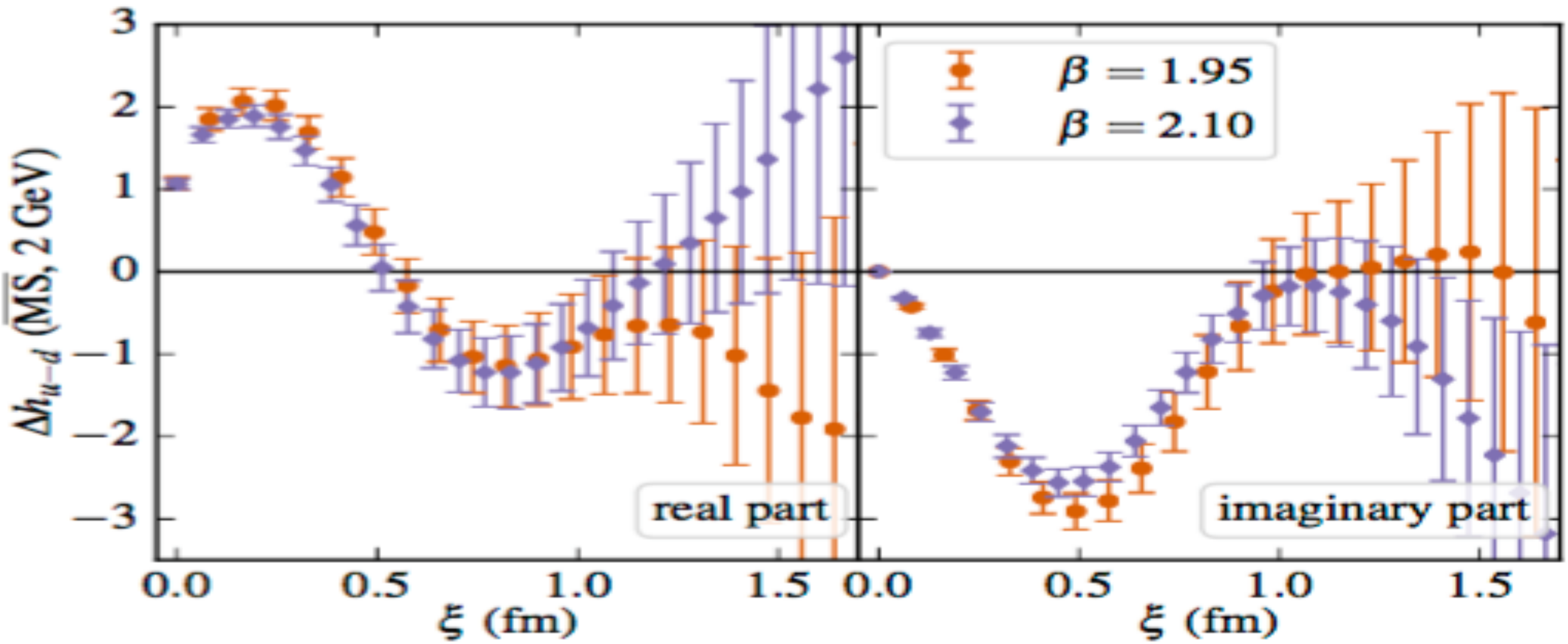


T. Ishikawa, Y. Ma, J. Qiu, S. Yoshida, PRD96 (2017) 094019, 1707.03107
LP³, PRD97 (2018) 014505, 1706.01295
LP³, 1803.04393

Linear divergence cancellation

Example 1

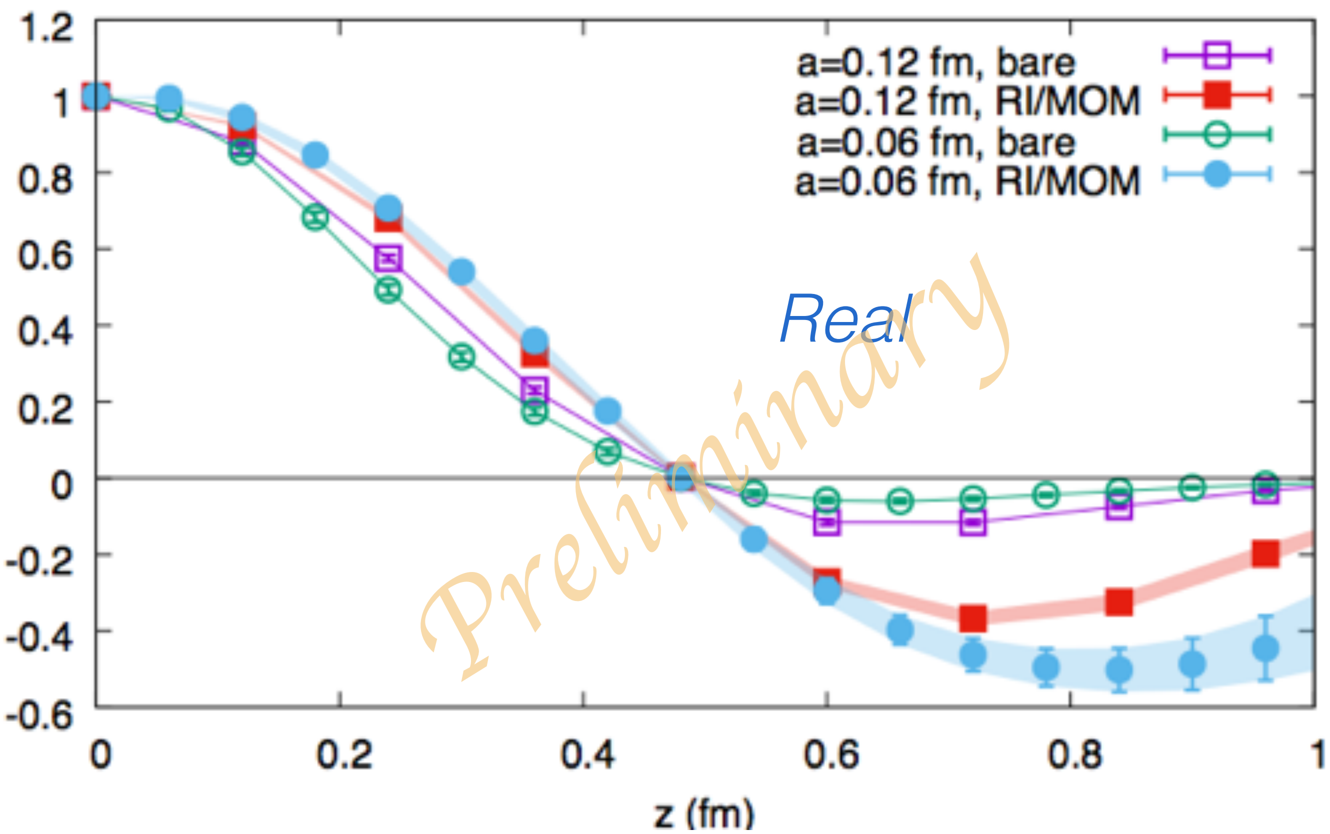
J. Green, K. Jansen, F. Steffens, 1707.07152

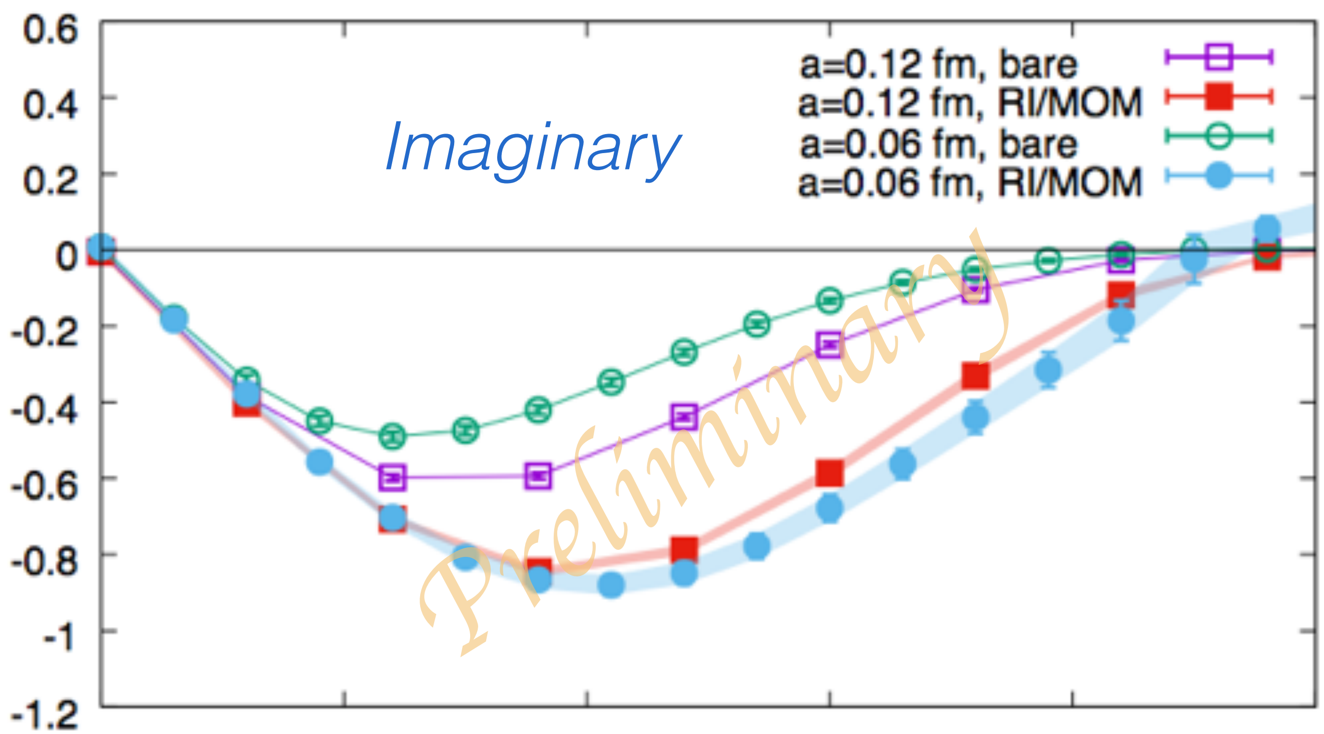


- *The quasi-PDF renormalization based the auxiliary field approach.*
- *The renormalized result at a=0.082/0.064 fm (for β=1.95 and 2.10 respectively) are consistent with each other.*

Linear divergence cancellation

Example II





- *The* ***RI/MOM renormalized*** *and normalized quasi-PDA at a=0.06/0.12 fm:*

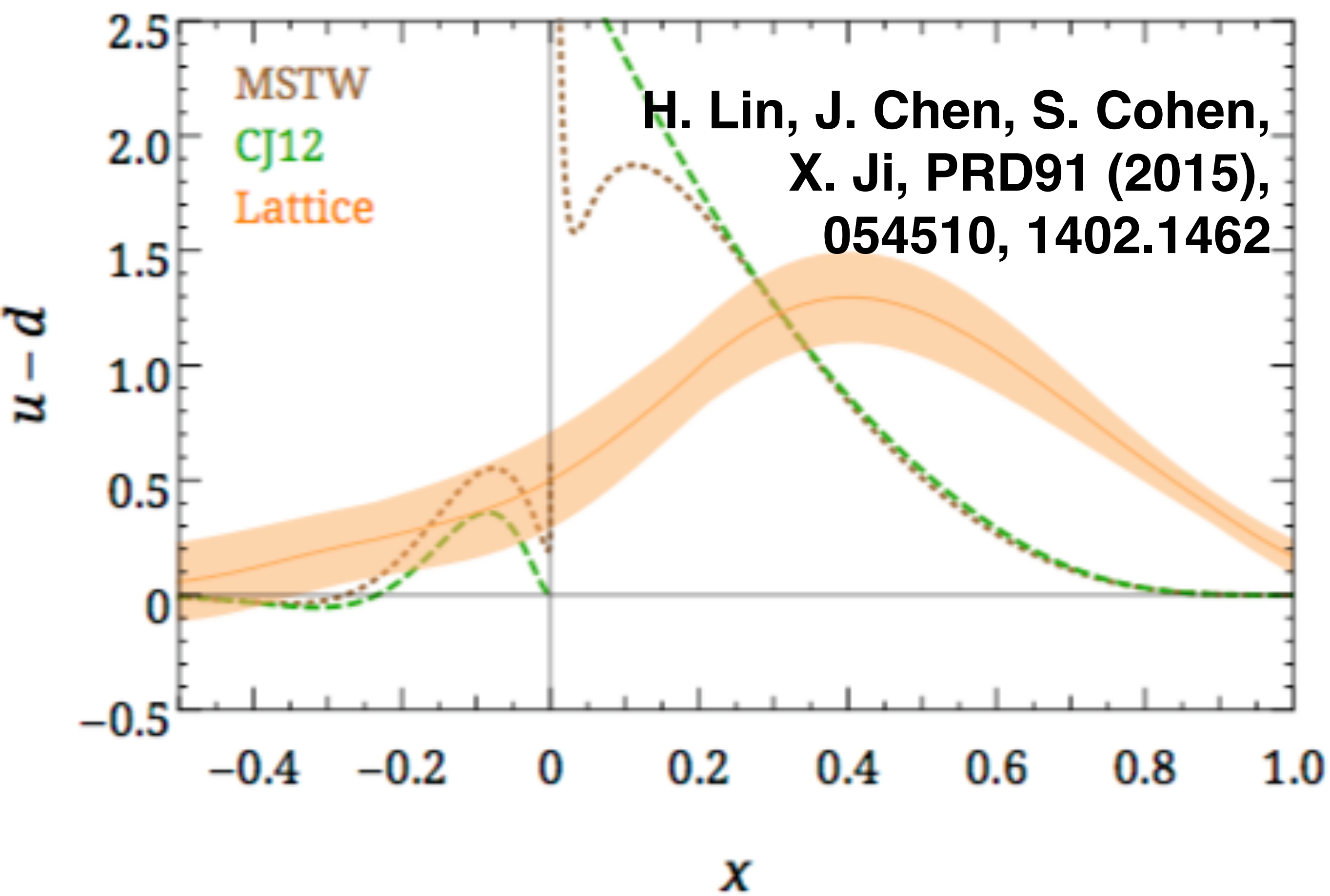
$$\langle \eta_s(P_z = 1.3\mathrm{GeV})|\bar{\psi}(z)\gamma_z\gamma_5 U_z(z,0)\psi(0)|0\rangle$$

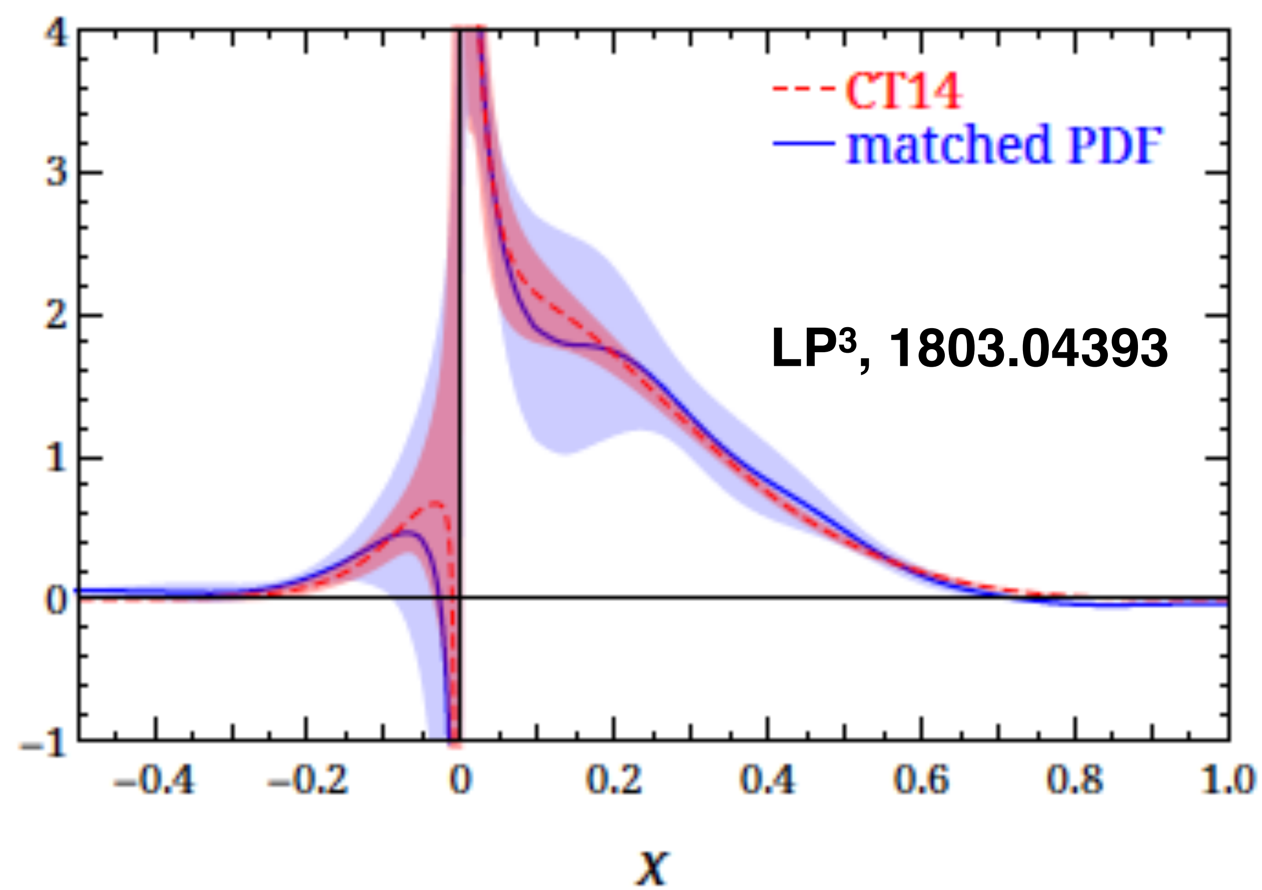
- *The renormalized results at a=0.12 fm and a=0.06 fm agree with each other well up to z~0.5 fm.*

- *The present statistics at a=0.06 fm is ~1/4 of that at a=0.12 fm. It will be improved to provide a stronger check.*

Unpolarized quasi-PDF:

from 2014 to 2018



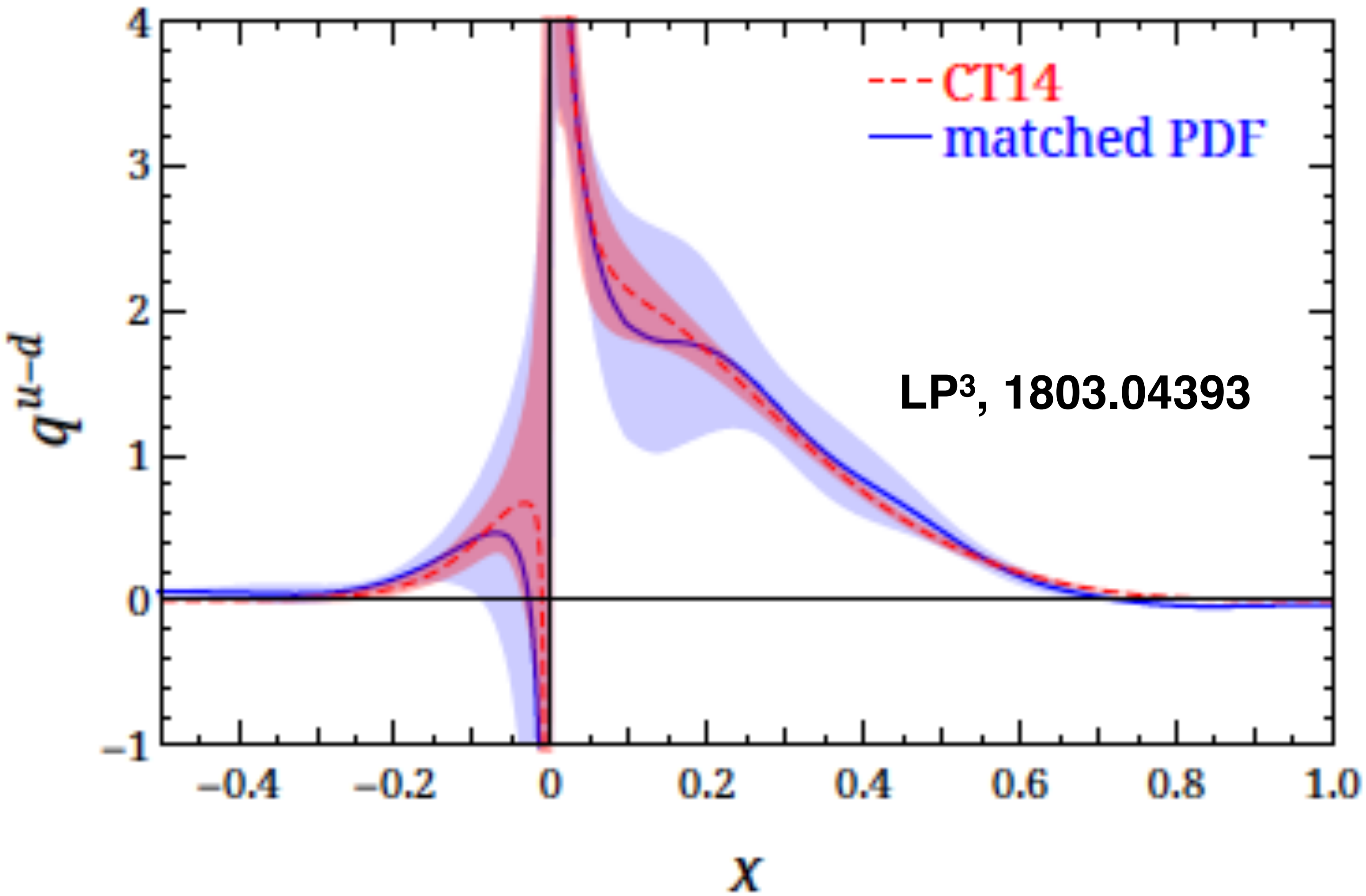


- First result at **2014**:
- P_z=1.3 GeV, m_π=310 MeV
- \+ **1-loop MS-bar matching**
- \+ **Mass correction**

- Present one at **2018**:
- P_z=3.0 GeV, m_π=130 MeV
- \+ **Modified definition with γ_t;**
- \+ **RI/MOM renormalization;**
- \+ **Complete 1-loop matching;**

Present Results

of u-d unpolarized nucleon PDF

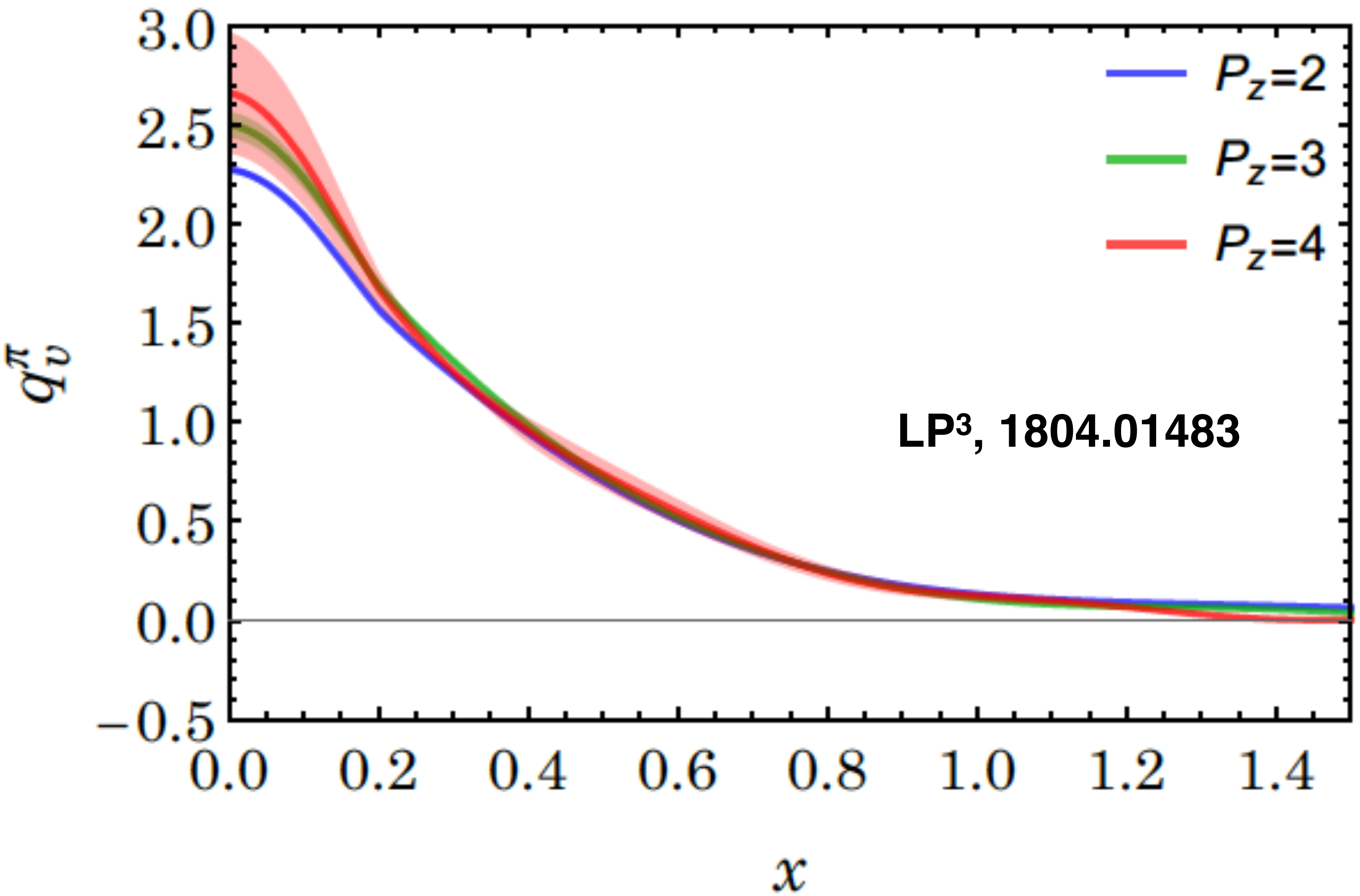


- a=0.09 fm, clover (m_{π}=130 MeV) on 2+1+1 HISQ;
- Pz=3.0 GeV, 4 $t_f \in$[0.72-1.08] fm;
- 128 measures on 309 configurations, with momentum smearing.

- Red band for the statistical uncertainties and **blue band** for the systematic uncertainties from kinds of the sources.

Present Results

of pion valence quark PDF



- a=0.12 fm, clover (m_π=310 MeV) on 2+1+1 HISQ;
- Pz=0.8-1.7 GeV, 4 t_f ∈[0.72-1.08] fm;
- 460 configurations, with momentum smearing.

- Based on the auxiliary field approach to do the renormalization;
- The RI/MOM renormalization is in progress.

Backup

The moments

of the quasi-PDF operator

- All the moment of the MS-bar renormalized quark-PDF except zero-th one.

- The first moment of the RI/MOM renormalized quasi-PDF is also finite, while the higher moments still diverge.

- **But those divergences are irrelevant to the power divergence of the lattice regularization.**

- Then the higher twist effects can be safely suppressed by large P_z.

The auxiliary field approach of the renormalization

- The wilson link can be understood as a auxiliary “heavy quark” propagator:

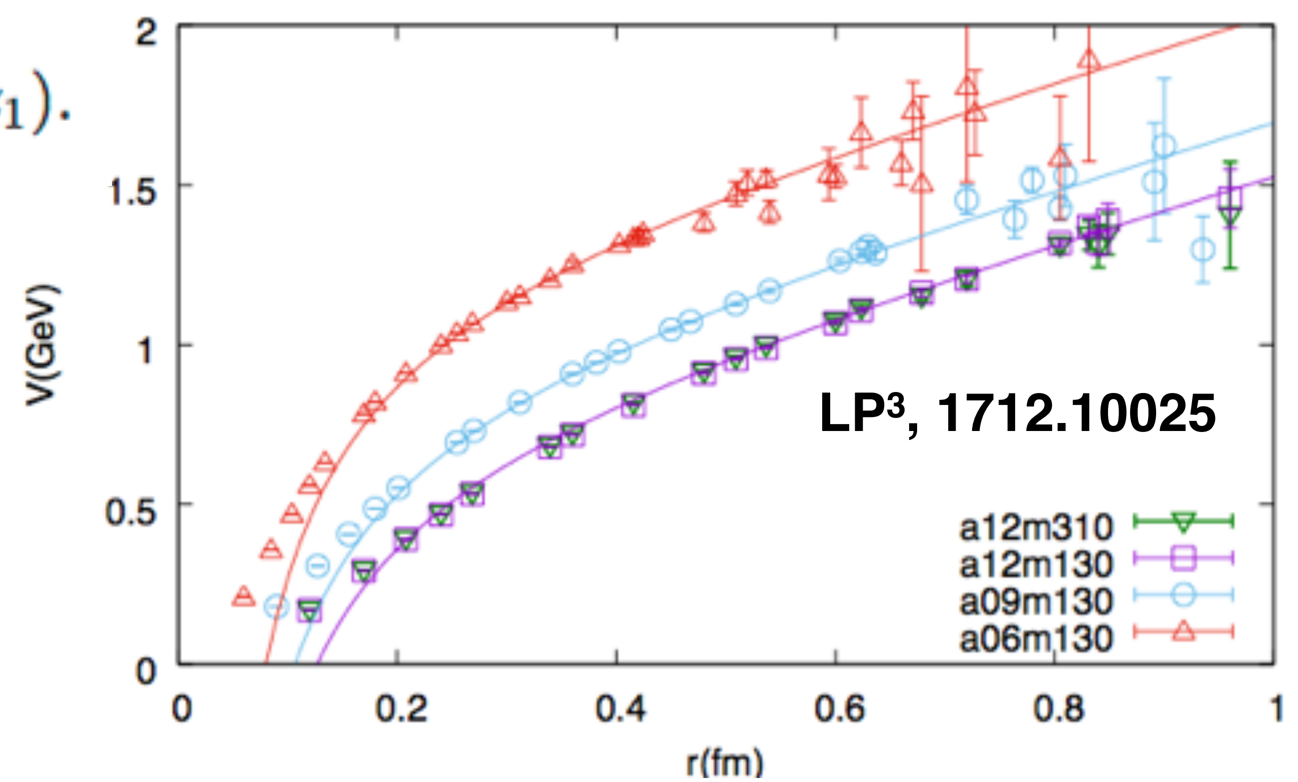
X. Ji, J. Zhang, Y. Zhao, PRL120 (2018) 112001, 1706.08962
J. Green, K. Jansen, F. Steffens, 1707.07152

$$L(x,y) = \mathcal{P}\exp\left(-ig\int_0^1 d\lambda\,\frac{dz^\mu}{d\lambda}A_\mu(z(\lambda))\right) \longrightarrow Q(x)\overline{Q}(y)$$

- Then the quasi-PDF operator become the product of the heavy-light quark bilinear operators, and then can be removed by,

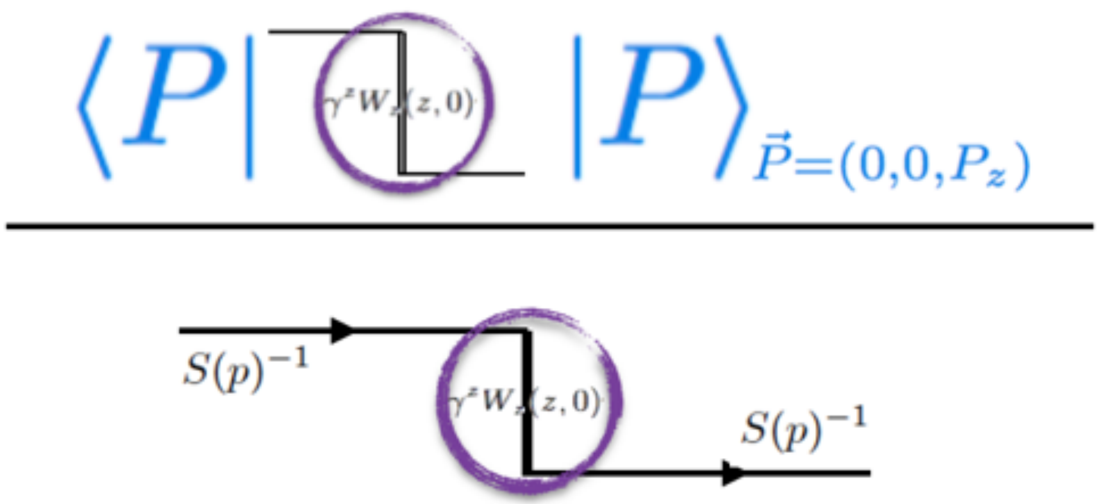
$$O_R = Z_{\bar{j}}^{-1}Z_j^{-1}e^{\delta\bar{m}|z_2-z_1|}\overline{\psi}(z_2)\Gamma L(z_2,z_1)\psi(z_1).$$

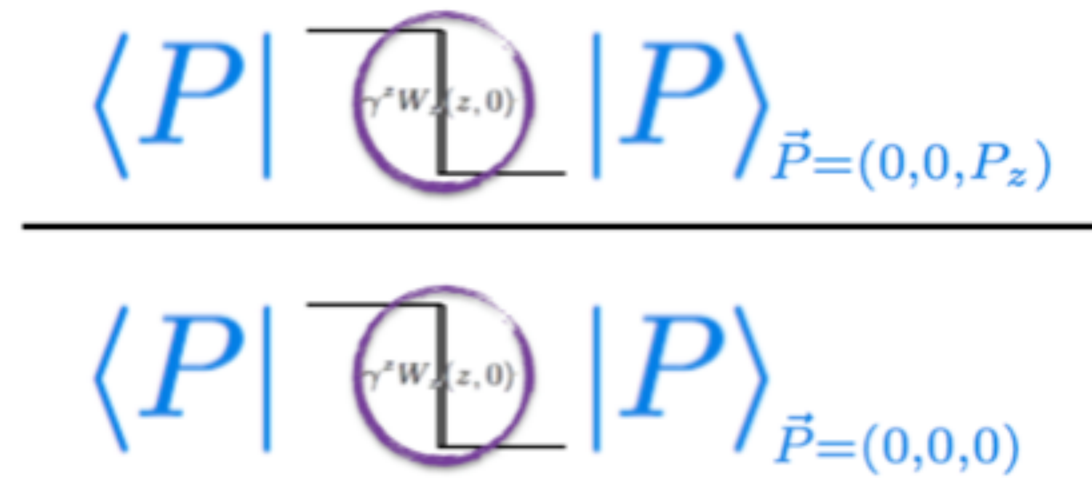
- One can determine **Z** from the normalization and **δm** from wilson loops



Another way

to remove the linear divergence

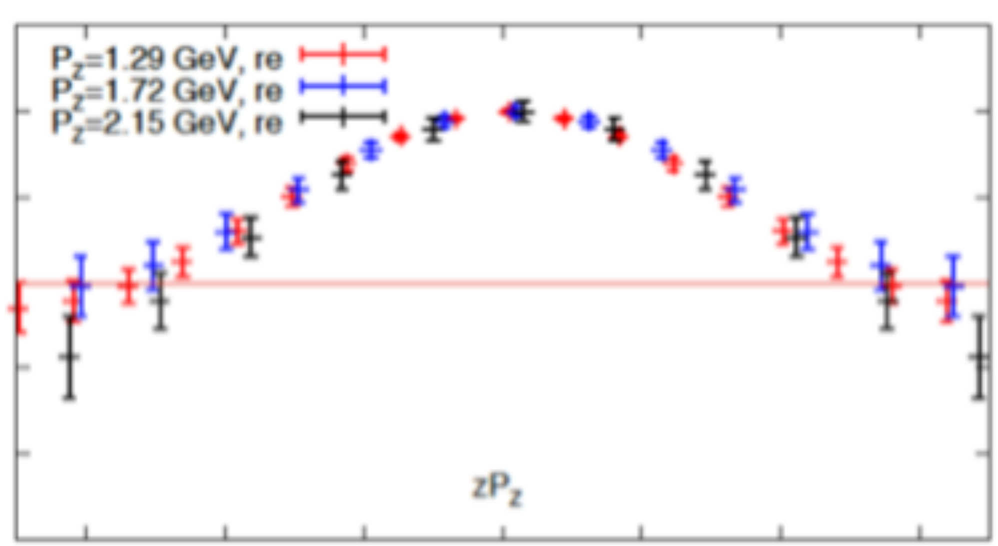




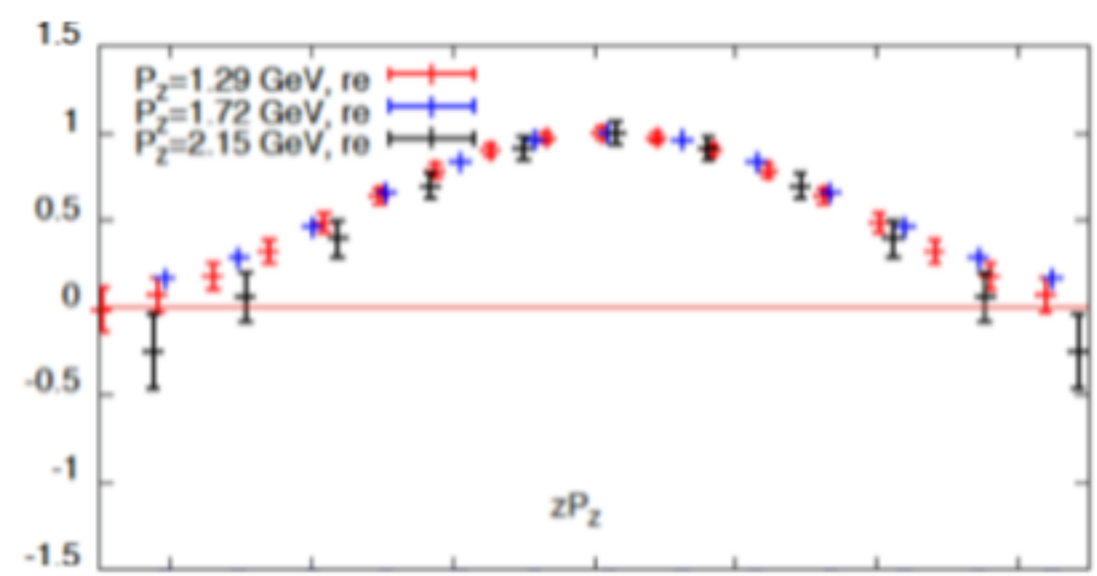
RI/MOM renormalized $\tilde{h}^R$ with $p_z^R = 0$

The results are very close to each other.

$\tilde{h}(z, P_z)/\tilde{h}(z,0)$

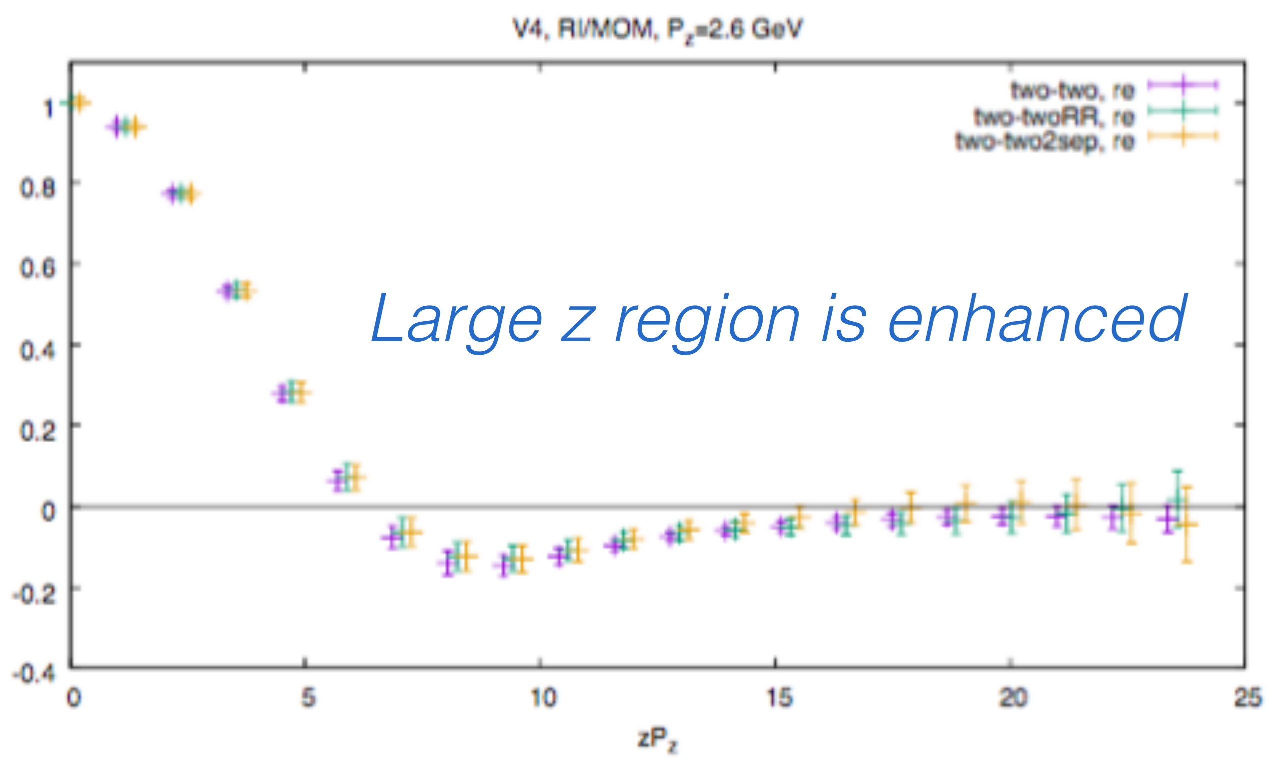


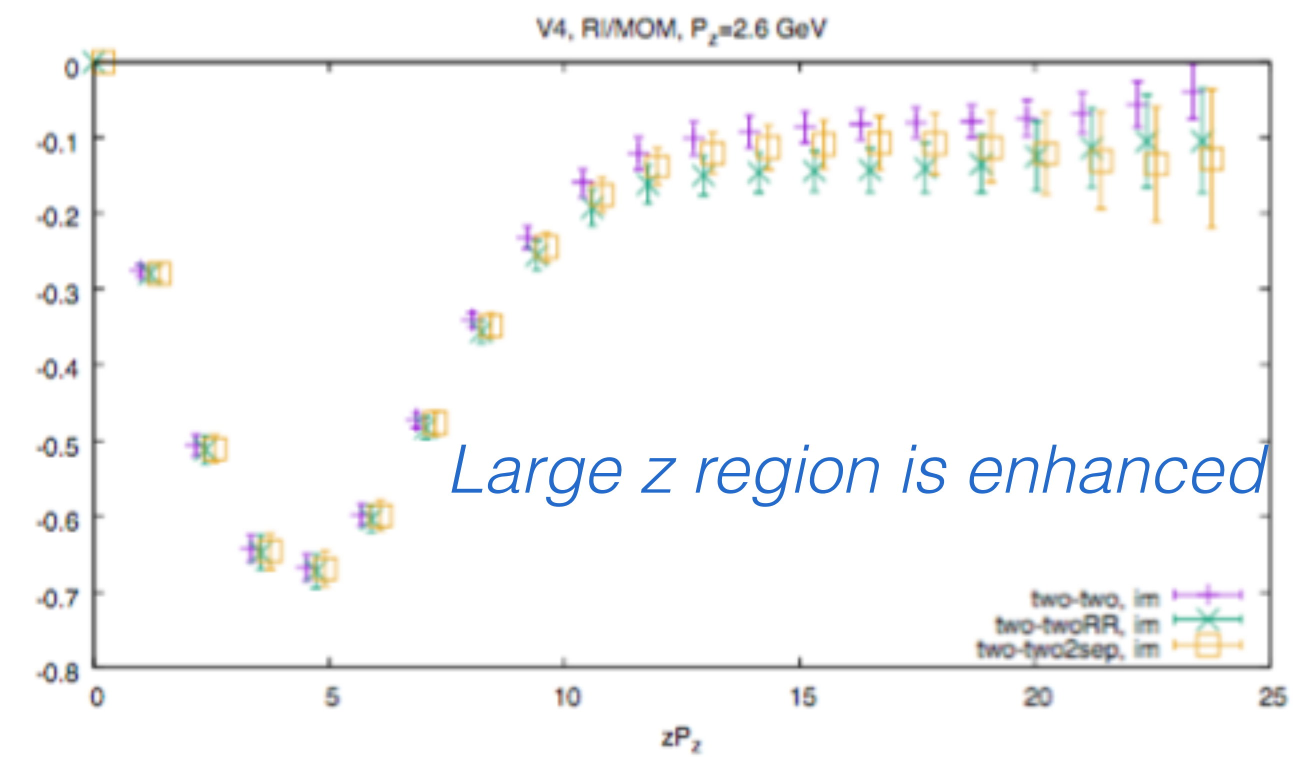
a12m310 ensemble



Lattice simulation

The excited-state contaminations





- Two-two: use all the four separations (0.72fm, 0.81fm, 0.9fm, 1.08fm), just consider the contaminations from the excited-ground states transition
- Two-twoRR: also include the contaminations from excited-excited ME.
- Two-two2sep: just use the data with the largest two separations (0.9 fm and 1.08 fm).

- **Will just use the two-twoRR results in the following discussion.**

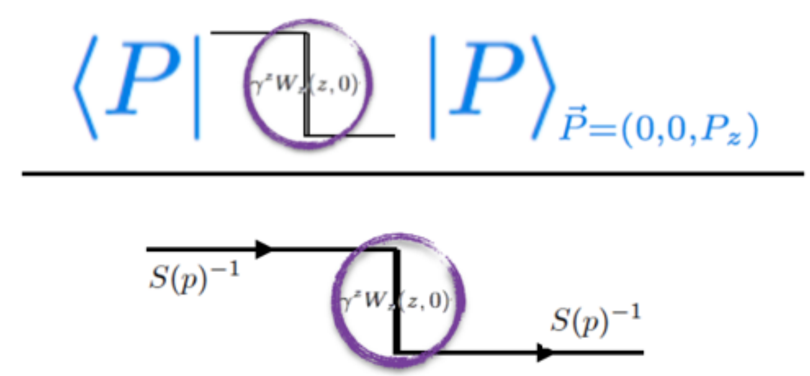
The residual

μ_R and p_z^R dependence

$$
\begin{aligned}
q(x,\mu) \; = & \int_{-\infty}^{\infty} \frac{dy}{|y|} \left\{ \delta\left(1-\frac{x}{y}\right) - \frac{\alpha_s C_F}{2\pi} C_1\left(\frac{x}{y}, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}, \frac{\mu_R^2}{p_z^{R\,2}}\right) \right\} \\
& \int_{-\infty}^{\infty} \frac{e^{iyP_z z}}{4\pi} \langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R_{p^2=\mu_R^2,\, p_z=p_z^R} \\
& + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}, \alpha_s^2\right),
\end{aligned}
$$

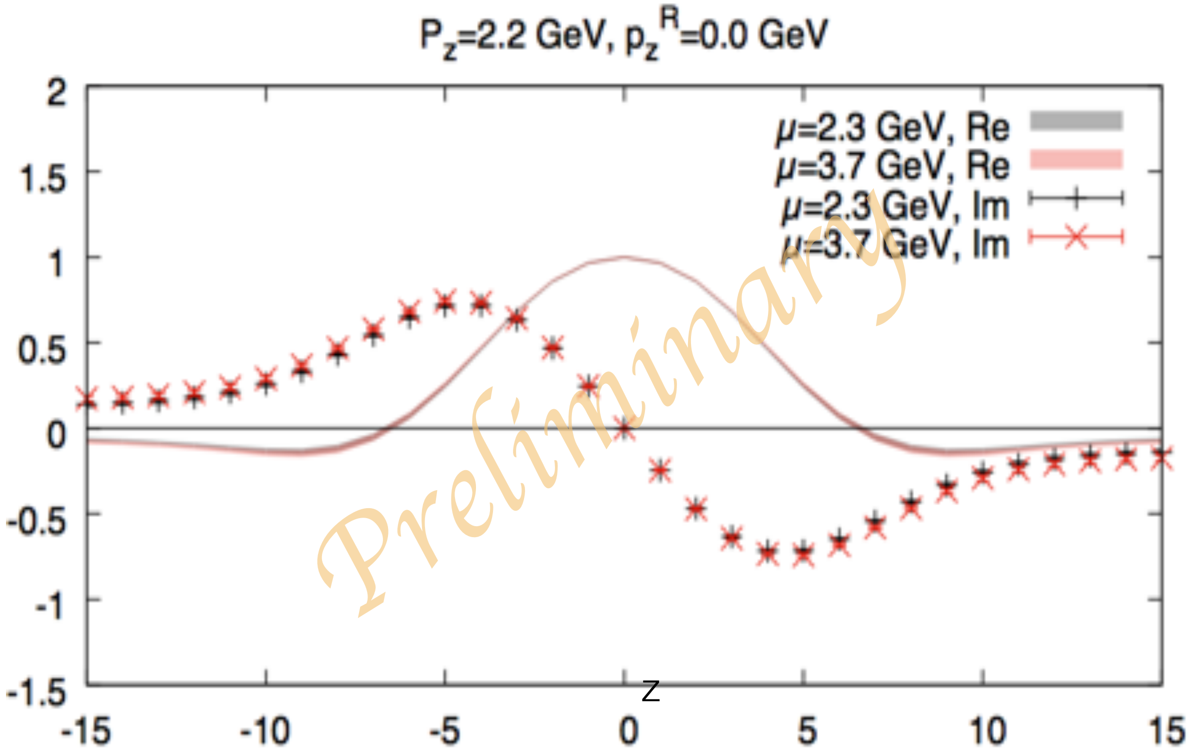
$$p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$

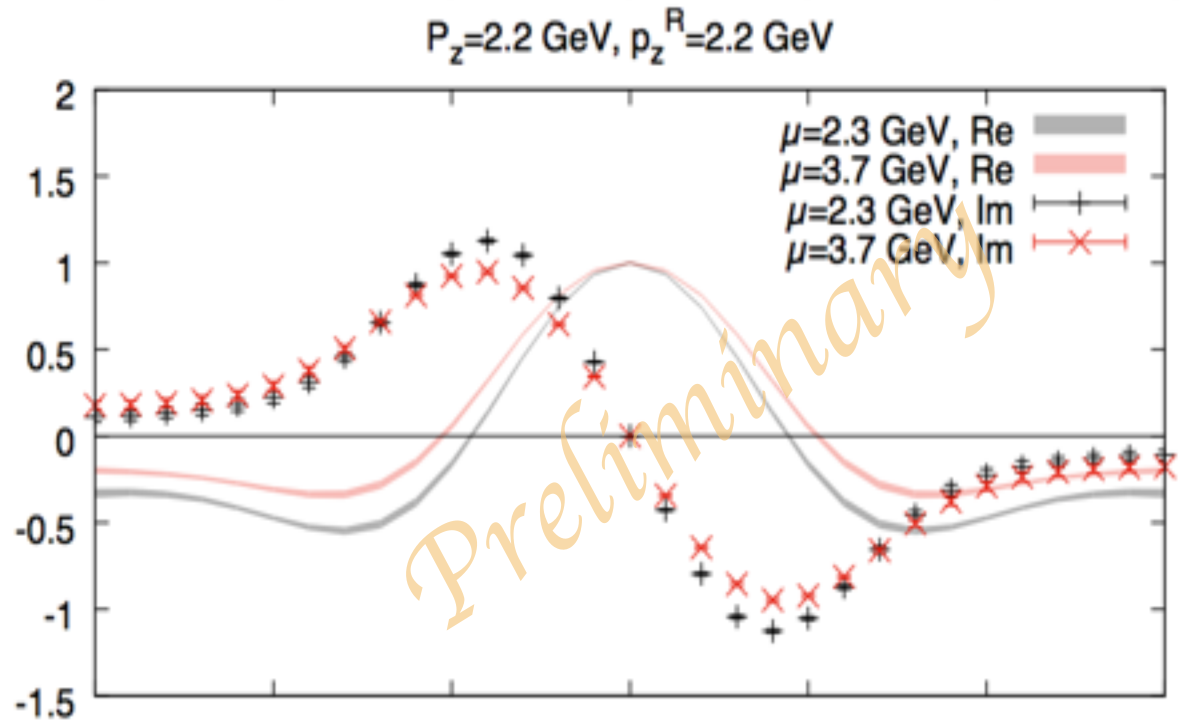
$$\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R =$$

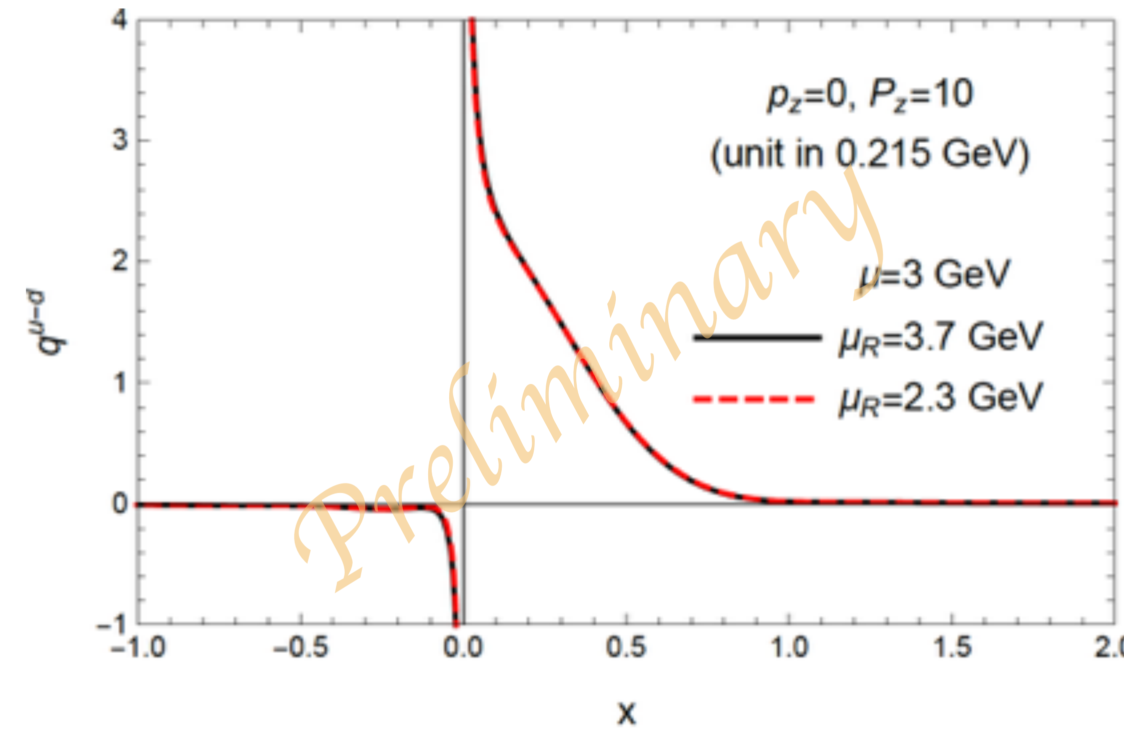


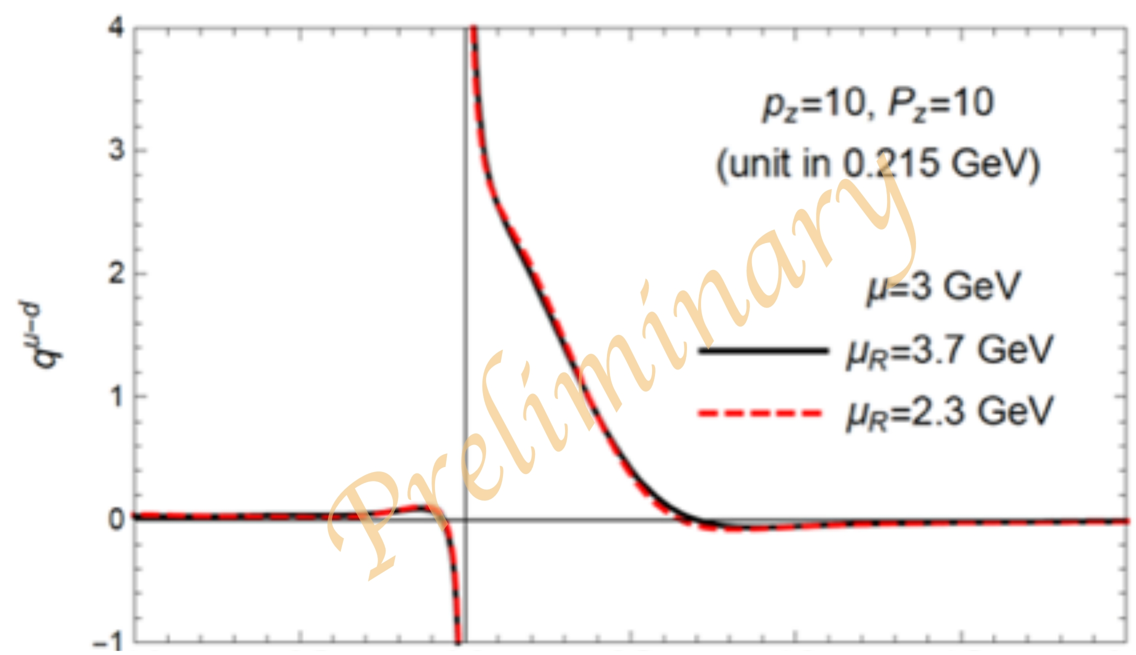
- ***The μ_R and p_z^R dependence should be cancelled with the matching in the continuum;***
- *But 1-loop matching may not be good enough to reach the goal.*

Before/after matching









- ***The μ_R dependence are cancelled after the matching;***
- *The residual p_z^R dependence will be considered as the systematic uncertainties.*

Questions from SPC

for the proposal

"Three-Dimensional and Flavor Structure of the Nucleon"

Q: b) In the description of the proposed calculation of GPDs, you discuss studying both GPDs and their quasi counterparts. We were confused by this statement; does this mean you are proposing to calculate the moments of GPDs in the "traditional" manner in addition to the x-dependent GPDs? More generally, please describe the relationship between the moment calculations and the direct calculation of GPDs, and how you would combine the two calculations.

A: For the next 5 years, our focus will be in focusing on the large-x distribution and make comparison with the upcoming experimental data. Our study on GPD can make immediately impact on the large-x region. For example, the 12-GeV upgrade at JLab will allow access to larger x region than the previous facilities, and LQCD on GPD will be valuable theoretical prediction.

Questions from SPC

for the proposal

"Three-Dimensional and Flavor Structure of the Nucleon"

Q: c) You note that the "derivative method" in ref. [21] should allow you to reach smaller values of Bjorken x. The small-x behavior is governed by Regge behavior. Do you expect your calculations to be sensitive to that?

A: Without the "derivative method", one will get the parton distribution distorted; i.e. the x-dependent shapes is dominated by the Fourier Transformation truncation errors.

There is a strong sensitivity on the smallest |x| region that one can recover and it's a function of P_z. To reach even smaller-x (without worrying about these truncation artifacts), we will still have to go to smaller lattice spacing and pushing for larger boosted momentum for lattice calculation.

Questions from SPC

for the proposal

"Three-Dimensional and Flavor Structure of the Nucleon"

Q: g) What is the long term plan? Will you need to take the continuum limit? What kind of precision, and over what region of Bjorken x, is needed to be useful to the experimental program? How long might it take to achieve that?

A: The long term plan is to take the continuum limit on isovector PDF, GPD, and the flavor-dependent distribution. If by then, there is a well-defined TMD functions (other than the transversity), we will be exploring these possibilities too.

Even with ONLY the isovector PDF calculation, if LQCD can provide 15% accuracy (with total errors, etc), it will make dramatic improvement in the least known anti-u or anti-d quark distribution by at least 20% at large x (beyond the reach of any planned experiments).