Hot QCD: Taylor expansion and BES-II
Peter Petreczky

Goals for LQCD:
1) Provide EoS for the entire parameter space relevant for BES-II
2) Improve over existing LQCD estimates for the possible location of a critical point
3) Provide quantitative results for cumulants of charge fluctuations

Beam energy scan phase II (BES-II) at RHIC: 2019-2021
Taylor expansion of the pressure and cumulants

\[ \tilde{\chi}_{ijk}^{BQS} = \frac{\partial^{(i+j+k)}[p(\tilde{\mu}, T)/T^4]}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k} \quad \tilde{\mu} = (\mu_B, \mu_Q, \mu_S) \]

Cumulants for \( \tilde{\mu} = (\mu_B, \mu_Q, \mu_S) = 0 \) are the Taylor expansion coefficients of the logarithm of the QCD partition functions: \( \chi_{ijk}^{BQS} \equiv \tilde{\chi}_{ijk}^{BQS} |_{\tilde{\mu}=0} \)

Pressure at non-zero \( \mu_B \):

\[
\frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B(T)}{2} \left( \frac{\mu_B}{T} \right)^2 \left[ 1 + \frac{1}{12} \frac{\chi_4^B(T)}{\chi_2^B(T)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{360} \frac{\chi_6^B(T)}{\chi_2^B(T)} \left( \frac{\mu_B}{T} \right)^4 + \cdots \right]
\]

Kurtosis (\( \kappa_X \)) times variance (\( \sigma_X^2 \)):

\[
\kappa_X \sigma_X^2 = \frac{\tilde{\chi}_4^X}{\tilde{\chi}_2^X} = \frac{\chi_4^X}{\chi_2^X} + \frac{1}{2} \left( \frac{\chi_2^{BX}}{\chi_2^X} - \frac{\chi_4^X \chi_{22}^{BX}}{(\chi_2^X)^2} \right) \left( \frac{\mu_B}{T} \right)^2 + O(\mu_B^4), \quad X = Q, S
\]

for baryon number \( \chi_{24}^{BX} \rightarrow \chi_6^B, \chi_{22}^{BX} \rightarrow \chi_4^B \)

Chiral susceptibility :

\[
\frac{\chi_m}{T^2} = \frac{\partial \langle \bar{\psi} \psi \rangle / T^3}{\partial m_q / T} = \sum_{n=0}^{\infty} \frac{c_n^\chi(T)}{n!} \left( \frac{\mu_B}{T} \right)^n
\]
Progress in 2017

Calculations of 4th order cumulants on the $N_{\tau} = 12$ lattices

Physics goals:

(i) improve the calculation of the QCD equation of state at non-zero values of the conserved charge chemical potentials;

(ii) determine the crossover line of the QCD transition in the temperature and baryon chemical potential plane;

(iii) improve the characterization of the strongly interacting medium, in particular its strangeness content, in the vicinity of the transition temperature.
Equation of state at $4^{th}$ order:

Bazavov et al, PRD95 (2017) 054504

$\Rightarrow$ more reliable continuum extrapolations
Chiral crossover temperature

\[ T_{pc}(\mu_B) = T_{pc}(0) + \kappa_2^\chi \left( \frac{\mu_B}{T} \right)^2 + \mathcal{O} \left( \left( \frac{\mu_B}{T} \right)^4 \right) \]

\( \kappa_2^\chi \) is determined by \( c_2^\chi \) and its \( T \)-derivatives \( \Rightarrow \kappa_2^\chi = 0.0123(30) \)

consistent with imaginary \( \mu_B \) result: \( \kappa = 0.013(2)(1) \)  

Bonati et al, PRD 90 (2014) 114025

The chiral transition line is consistent with the freezeout line
Progress in 2017 (cont’d)

Baryon number and strangeness cumulants

Evidence for missing strange baryons

Deviations of the skewness and kurtosis ratios of net proton-number fluctuations from unity in observed experiment is consistent with the $\mu_B$-dependence $S_B \sigma_B = \chi_3^B(T, \mu_B)/\chi_2^B(T, \mu_B)$ and $\kappa_B \sigma_B^2 = \chi_4^B(T, \mu_B)/\chi_2^B(T, \mu_B)$, calculated in an $O(\mu_B^2)$ Taylor expansion.
Type A proposal: Non-Gaussian cumulants of conserved charges fluctuations
F. Karsch (PI), H.-T. Ding, Swagato Mukherjee, P. Petreczky, C. Schmidt, S. Sharma
and P. Steinbrecher

Computational resources: 1.95M GPU-h and 56.4M KNL core-h = 169.2 M Jpsi core h

Storage: 100 TB disk and 90 TB tape

Readiness: ready to run!
1/3 of gauge configurations
is available, 2/3 will be generated
in the next 6 month;
the code works well and has
been benchmarked also on Skylake:
(2xSKL 8168) ≈ KNL
in terms of core h
Goal for 2018/2019

Controlled continuum extrapolations for $\mu_B$ dependence of kurtosis of electric charge and strangeness up to $\mathcal{O}(\mu_B^4) \Rightarrow$ need reliable results for the 6th order +100K configurations (200K in total); the long term goal is 400K configurations

$$T = 151, 157, 162 \text{ MeV}$$

$$r_{42}^{X,2} = \frac{1}{2} \left( \frac{\chi_{24}^{BX}}{\chi_2^X} - \frac{\chi_4^X \chi_{22}^{BX}}{(\chi_2^X)^2} \right) X = Q, S$$

large deviations from HRG
Response to SPC questions

a) Since you are asking for 90 Tbyte of tape storage, please specify how much of this is new for the analysis in this project, and how much is existing tape storage? This will help us understand what new tape resources are needed.

As specified in the proposal the 90 TByte of tape storage needed for this project summarizes the new, additional tape storage we need on top of what we already have on tape in previous projects. Our current tape storage amounts to about 460 Tbyte.

b) Please specify your quarterly run plan for your proposed project and a readiness statement. Are there any components that are needed for running this project that are still under development?

We have 100K data sets which we can start analyzing already now. We thus can start running immediately in the new funding period.

We can distribute the analysis uniformly over the next funding period, i.e. each quarter of the funding period we could use 1/4 of the allocated resources.

However, we are interested in running as much as possible in the early part of the funding period. If resources are not under too large demand we thus can imagine to use during each of the first three quarters of the allocation period about 1/3 of the allocated resources.
c) We assume that the gauge generations already exist; please confirm.

As mentioned in the proposal we will generate the gauge configurations using other NON-USQCD resources. We plan to analyze in total 300K configurations. At present 100K configurations have already been generated and can be analyzed immediately. Non-USQCD resources (PRACE) have been approved in the meantime for members of our collaboration so that the generation of additional 200K configurations will proceed during the next 6 months. This insures that the run plan outlined in (b) can be realized.

d) We are a little uncertain as to the relative performance of the Skylake and KNL nodes. You note in the text in page 12 that the performance per core is identical on the KNL and Skylake nodes since it is dominated by the CG inverter. The JPsi-equivalent core hour? equivalent of the the KNL and Skylake nodes in the USQCD call for proposals are the same, but you are arguing that the per core performance is the same for your code, even though the KNL has many more but slower cores? Please qualify.

In our tests, the Skylake node had two sockets (same as at BNL). Both of them were equipped with SKL 8168 which has 26 cores, i.e. in total a node has 52 cores. The performance of this node (2x SKL 8168) is shown in Figure 7 of our proposal. For instance, for 25 right hand sides it can be seen from the figure that this Skylake node leads to a sustained performance of 640 GFlop/s. The KNL node gives 850 GFlop/s. The KNL node has 64 cores while the two socket Skylake node has 52 cores. The ratio of nodes thus is KNL/Skylake = 64/52 = 1.23. The ratio of performance on these nodes is KNL/Skylake = = 850/640=1.33

This is the basis for our statement that the per core performance of our code is (almost) identical.