RBC/UKQCD $\pi\pi$ scattering, $K \to \pi\pi$, and distillation projects

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USQCD All Hands Meeting, Fermilab

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Outline I

$\pi\pi$ scattering and $K \to \pi\pi$

Investigators: Blum (PI), Peter Boyle (Edinburgh), Norman Christ (Columbia), Daniel Hoying (UConn/BNL), Taku Izubuchi (BNL/RBRC), Luchang Jin (UConn/RBRC), Chulwoo Jung (BNL), Christopher Kelly (Columbia), Christoph Lehner (BNL), Robert Mawhinney (Columbia), Chris Sachrajda (Southampton), Amarjit Soni (BNL)

compute request: 91.2 M JPsi core-hrs on JLab or BNL KNL clusters

storage request: 200 TB disk, 200 TB tape

Motivation and background

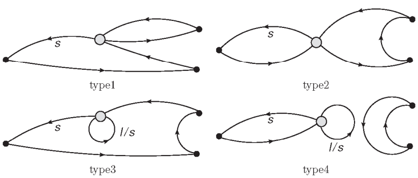
- SM extremely successful, but ...
- Direct CP violation in kaon decays offers good place to look for breakdown, *c.f.* single phase in CKM matrix must explain all violation in SM

$$
\begin{aligned}
\mathrm{Re}\,\frac{\epsilon'}{\epsilon} &= \frac{1}{6}\left(1 - \frac{\Gamma(K_S \to \pi^+\pi^-)\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^+\pi^-)\Gamma(K_S \to \pi^0\pi^0)}\right) \\
&= \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{ReA_2}} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\} \\
H_W &= \frac{G_F}{\sqrt{2}}V_{us}^* V_{ud}\sum_i \left[z_i(\mu) + \tau y_i(\mu)\right] Q_i(\mu) \\
A(K^0 \to \pi\pi)_I &= A_I e^{i\delta_I} = \langle \pi\pi | H_W | K \rangle
\end{aligned}
$$

- Experiment: $16.6(2.3) \times 10^{-4}$
- SM: $1.38(5.15)(4.59) \times 10^{-4}$ [1] (RBC/UKQCD G-parity bc project)

Methodology

Matrix elements from Euclidean correlation functions

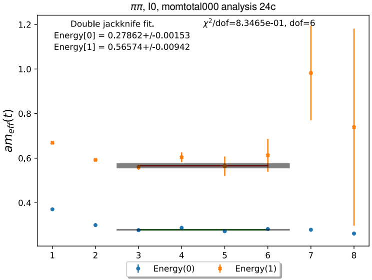


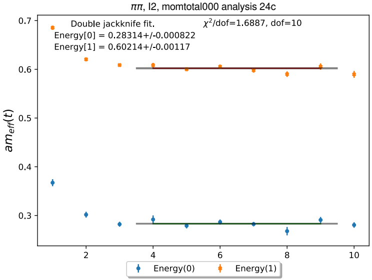
$$\langle \chi_{\pi\pi}(t) Q_i(t_{\mathrm{op}}) \chi_K^\dagger(0) \rangle = \sum_m \sum_n \langle 0|\chi_{\pi\pi}|n\rangle \langle n|Q_i|m\rangle \langle m|\chi_K^\dagger|0\rangle e^{-E_n(t-t_{\mathrm{op}})} e^{-E_m t_{\mathrm{op}}}$$

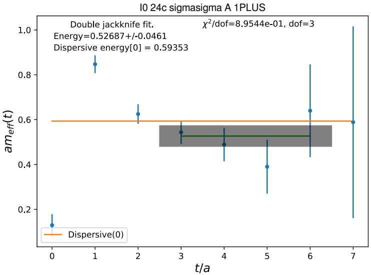
- Physical kinematics corresponds to excited state, ground state is pions at rest
- G-parity bc's (RBC/UKQCD): ground state is physical (pions at rest not allowed)
- For periodic bc's, use A2A[2]+AMA[3]+GEVP analysis to extract excited state

Preliminary results with current allocation

- 2+1 flavor, physical point, Möbius DWF, 1 GeV, 24^3 ensemble
- A2A/AMA measurements on 66 configurations, 2000 low modes, 1 hit for high







Good precision on $I = 0$ excited (physical) state, $\gtrsim 1.5$ %

Proposed calculations

2+1 flavor physical point, Möbius DWF, Iwasaki gauge action ensembles (RBC/UKQCD)

Table: Per-configuration cost of proposed calculations. Costs for propagators (props) are based on (z)Möbius DWF with $L_s = 12$.

type	a^{-1}	size	Cost (KNL node-hours)			configs	Total
			props	meson fields	contractions		(M core-hrs)
$K \to \pi\pi$	1	$24^3 \times 64$	72	64	739	100	16.8
$\pi\pi$, $K \to \pi\pi$	1.4	$32^3 \times 64$	171	470	202+739	100	30.4
$\pi\pi$	1	$32^3 \times 64$	114	1183	1008	100	44.0

Dominated by contractions

Outline I

QCD + QED studies using twist-averaging

Investigators: Mattia Bruno (BNL, co-PI), Xu Feng (Peking University), Taku Izubuchi (BNL/RBRC), Luchang Jin (UConn/RBRC), Christoph Lehner (BNL, PI), Aaron Meyer (BNL)

Collaborators: Tom Blum (UConn), Norman Christ (CU), Chulwoo Jung (BNL), Chris Sachrajda (Southampton), Amarjit Soni (BNL)

compute request: 59 M JPsi core-hrs on JLab or BNL KNL clusters

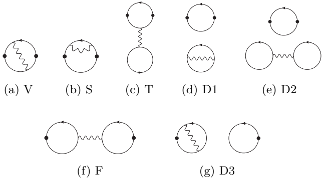
storage request: 80 TB disk

Motivation and background

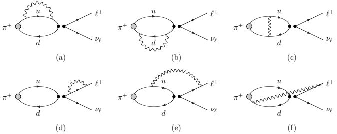
- $O(\alpha)$ isospin breaking corrections are important for many QCD observables
  - muon g-2
  - light quark masses
  - f_π
  - τ decays (dispersive treatment of muon g-2)
- 1^{st} two corrections calculated on 1.73 GeV, 48^3, physical point Möbius DWF ensemble (RBC/UKQCD)
- goal: take continuum limit

Methodology: perturbative treatment of QED @ $O(\alpha)$

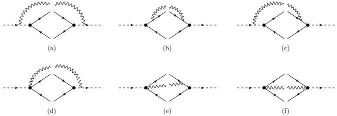
HVP



f_π



f_π EFT[5]



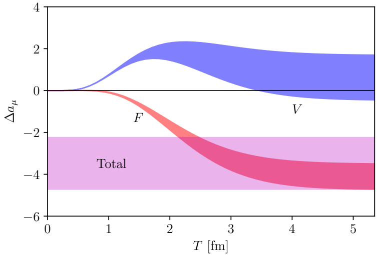
- Sample photon vertex stochastically, using importance sampling strategy

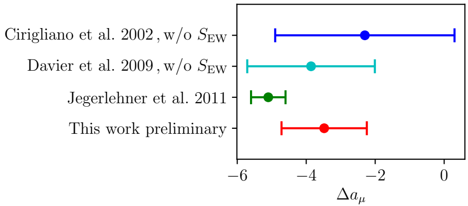
$$C_2^{ab}(z) = \langle O_a(z) O_b(0) \rangle \,,$$
$$C_3^{ab;\mu}(x,z) = \langle O_a(z) O_b(0) j_\mu(x) \rangle \,,$$
$$C_4^{ab;\mu\nu}(x,y,z) = \langle O_a(z) O_b(0) j_\mu(x) j_\nu(y) \rangle$$
$$O_a(z) = \bar{q}(z)\Gamma_a q(z) \,, \quad j_\mu(x) = \bar{q}(x)\gamma_\mu q(x) \,,$$

- Use twist averaging for photon to reduce/control FV errors [6]

Results from current allocation

- $O(\alpha)$ corrections to HVP, 1.73 GeV, physical point Möbius DWF ensemble (RBC/UKQCD) [4]
- Isospin breaking corrections in τ decays (Bruno, KEK workshop on HVP)





Proposed calculations

2+1 flavor, physical point Möbius DWF, 2.38 GeV, 64^3 ensemble (RBC/UKQCD)

12 sloppy 64^3 solves on 64 KNL nodes	600 seconds
12 exact 64^3 solves on 64 KNL nodes	2580 seconds
Number of configurations	30
Number of sloppy solves per configuration	900×12
Number of exact solves per configuration	15×12
Total computational cost on 64^3 for sloppy solves in M Jpsi-core hours	55
Total computational cost on 64^3 for exact solves in M Jpsi-core hours	4
Total request	59 M Jpsi-core hours

Table: Cost estimates for the proposed computation. We intend to use an AMA [3] setup with parameters described in this table.

Outline I

Exclusive Study of $(g-2)_\mu$ HVP and Nucleon Form Factors with Distillation

Investigators: A. S. Meyer (PI), M. Bruno, T. Izubuchi, Y. C. Jang, C. Jung, and C. Lehner

compute request: 46.7 M JPsi core-hrs on JLab or BNL KNL clusters

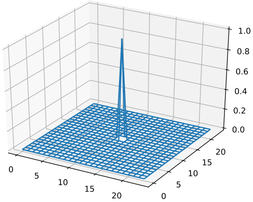
storage request: 50 TB disk

Motivation and background

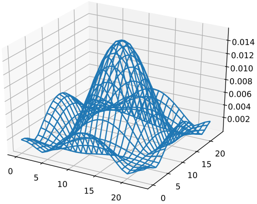
- muon g-2 experiment E989 at Fermilab
  - Error on HVP contribution to $g-2$ desired at sub-percent level
  - Long distance part of correlation function is noisy, dominates error
  - use exclusive $\pi\pi$ channel(s) to improve "bounding method" [4], significantly reduce statistical error
- ν oscillation experiments NOνA, DUNE, and HyperK
  - precision measurements of mass-squared splittings, mixing angles, CP-violating angle in the lepton sector
  - need accurate/precise nucleon axial-vector form factor calculations

Distillation Method (JLab/Trinity [7])

Eigenvectors of 3D laplacian act as a projection that smears quark fields in space



$\Longrightarrow$



9 evecs (57 equiv), $\sum_i p_i^2 \leq 2$

Eigenvectors used as sources, contracted at sink to create "perambulators"

$$M^{ji}_{t,\beta\alpha} = \sum_{xy} \sum_{ab} \langle j^{b}_{t;y} | \left(D^{ba}_{yx,\beta\alpha}\right)^{-1} | i^{a}_{0;x} \rangle$$

Meson correlation functions constructed from tracing over perambulators

$$C(t) = \mathrm{tr}[\Gamma M(t,t')\Gamma' M(t',t'')\Gamma'' M(t'',t''')\ldots]$$

Generalized EigenValue Problem

Vector current operator:

- Local $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x)$

Two 2π operators with different momenta

$$\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z-y)\psi(y)\right|^2 :$$

- $\mathcal{O}_1 : \frac{L}{2\pi}\vec{p}_\pi = (1,0,0)$
- $\mathcal{O}_2 : \frac{L}{2\pi}\vec{p}_\pi = (1,1,0)$

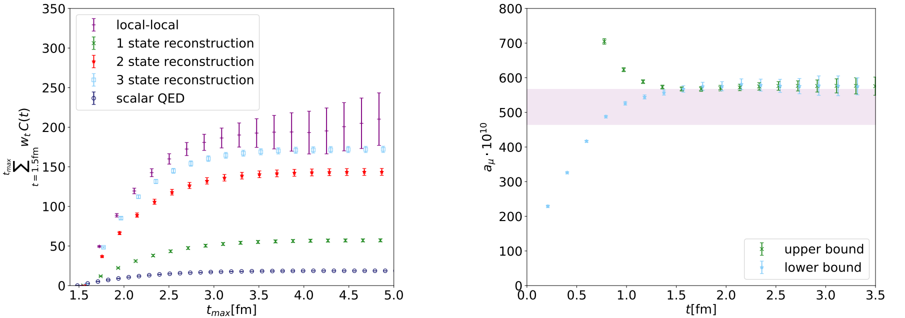
Correlators arranged in a 3×3 symmetric matrix:

	$\mathcal{O}_0$	$\mathcal{O}_1$	$\mathcal{O}_2$
$\mathcal{O}_0$	$C^{(2)}_\rho$	$C^{(3)}_{\rho\to\pi\pi}$	$C^{(3)}_{\rho\to\pi\pi}$
$\mathcal{O}_1$		$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$
$\mathcal{O}_2$			$C^{(4)}_{\pi\pi\to\pi\pi}$

Analyze with Generalized EigenValue Problem (GEVP) method:

$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n\delta t}$$

Results - HVP Bounding Method



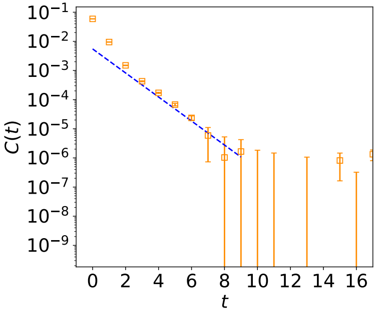
$a^{-1} = 1.015\ \mathrm{GeV}$ $24^3 \times 64$ physical mass ensemble

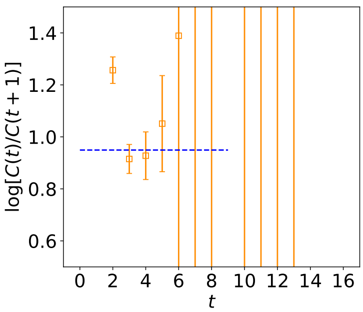
Precise reconstruction of long-distance contribution to HVP down to $1.5\ \mathrm{fm}$

No bounding method (purple band): $a_\mu^{HVP} = 516(51)$
Start bounding method at $t = 1.6\ \mathrm{fm}$, 1 state reconstruction: $a_\mu^{HVP} = 570.2(8.3)$

Factor > 5 improvement in statistical precision

Results - Nucleon Two-Point





Can compute nucleon form factors $\implies$

- g_A, $F_A(Q^2)$
- $F_V(Q^2)$
- $F_{N\to\Delta}(Q^2)$

Useful for neutrino physics:

Axial form factor a dominant source of systematic uncertainty in ν oscillation experiments

Proposed calculations

2+1 flavor, physical point, Möbius DWF, 1.73 GeV, 48^3 ensemble (RBC/UKQCD)

Table: Compute costs

Configurations	15
Eigenvectors	60
Timeslices(Sloppy)	96
Timeslices(Exact)	16
Sloppy Solves [x1000]	172.8
Exact Solves [x1000]	43.2
Time/sloppy solve [Jpsi corehr]	53.7
Time/exact solve [Jpsi corehr]	488.0
Total Time [M Jpsi corehr]	46.7

Outline I

Precise scale setting for $(g-2)_\mu$

Investigators: Mattia Bruno(PI), Taku Izubuchi, Christoph Lehner, Aaron Meyer

Collaborators: Thomas Blum, Norman Christ, Luchang Jin, Chulwoo Jung, Chris Kelly, Amarjit Soni

compute request: 47 M JPsi core-hrs on JLab or BNL KNL clusters

Motivation and background

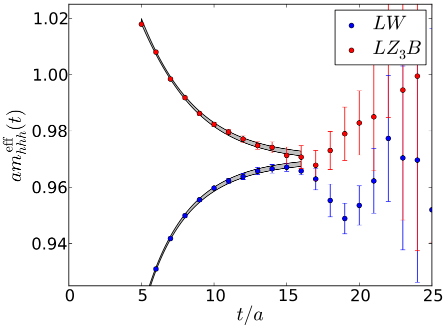
- Per-mille determination of lattice spacing needed for muon g-2 calculations
- use distillation+AMA+GEVP to
- Ω^- mass sets scale
  - ideal, isospin breaking (QED, non-degenerate quark masses) small [4]
- Demonstrate method on 1.73 GeV ensemble, then on to 2.38 GeV, take continuum limit

Methodology

Distillation with 60 modes of 3D Laplacian
$\rightarrow$ full volume average $\oplus$ optimize smearing function

AMA (2000 low-modes) sloppy inversions on 96 time slices; exact on 16
$\rightarrow$ Master-Field error analysis, other physics goals

Large basis of operators to control excited states (e.g. GEVP)
$\rightarrow$ different spin matrices and non-zero angular momentum



(RBC/UKQCD [8])

Proposed calculations

2+1 flavor, physical point, Möbius DWF, 1.73 GeV, 48^3 ensemble (RBC/UKQCD)

Table: Compute costs

single sloppy inversion on 32 KNL nodes	32 secs
single exact inversion on 32 KNL nodes	286 secs
sloppy time slices	96
exact time slices	16
cost for a single distillation mode	272 KNL node-hours
distillation eigenvectors	60
cost per configuration	3.1 M JPsi core-hrs
number of configurations	15
Total computational request	47 M Jpsi core-hrs

Outline I

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