Resonances induced by Space Charge force in J-PARC MR

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Fermilab APC seminar

Space charge potential

- Space charge potential for Transverse Gaussian distribution

$$U(x,y) = \frac{\lambda_p r_p}{\beta^2\gamma^3}\int_0^\infty \frac{1-\exp\left(-\frac{x^2}{2\sigma_x^2+u}-\frac{y^2}{2\sigma_y^2+u}\right)}{\sqrt{2\sigma_x^2+u}\sqrt{2\sigma_x^2+u}}du$$

$$r_{yx} = \sigma_y^2/\sigma_x^2$$

$$\sigma_{xy}^2(s) \;=\; \beta_{xy}(s)\varepsilon_{xy} + \eta_{xy}^2(s)\delta^2$$

- Particles motion, betatron oscillation (neglect dispersion in the motion)

$$\begin{aligned} x(s',s) &= \sqrt{2J_x\beta_x(s')}\cos(\varphi_x(s',s)+\phi_x(s)) \\ y(s',s) &= \sqrt{2J_y\beta_y(s')}\cos(\varphi_y(s',s)+\phi_y(s)) \end{aligned}$$

- Space charge potential integrated along the betatron orbit

$$U(\boldsymbol{J},\boldsymbol{\phi},s) = \oint_s ds' U(x,y,s')'$$

$$\boldsymbol{J} = (J_x, J_y),\ \boldsymbol{\phi} = (\phi_x,\phi_y)$$

$$= \frac{\lambda_p r_p}{\beta^2\gamma^3}\oint_s ds' \int_0^\infty \frac{1-\exp\left(-\frac{x^2(s',s)}{2\sigma_x(s')^2+u}-\frac{y^2(s',s)}{2\sigma_y(s')^2+u}\right)}{\sqrt{2\sigma_x(s')^2+u}\sqrt{2\sigma_x(s')^2+u}}du$$

Fourier series of the space charge potential, U

- Fourier expansion of U(**J**,ϕ)

$$U(\boldsymbol{J}, \boldsymbol{\phi}) = \sum_{\boldsymbol{m}=-\infty}^{\infty} U_{\boldsymbol{m}}(\boldsymbol{J}) \exp(-i\boldsymbol{m} \cdot \boldsymbol{\phi}) \qquad \boldsymbol{m} = (m_x, m_y)$$

- Fourier coefficients are given by operating $\int \exp(-i\boldsymbol{m} \cdot \boldsymbol{\phi}) d\boldsymbol{\phi}/(2\pi)^2$

$$U_m(J_x, J_y) = \frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \int_0^\infty \frac{du}{\sqrt{2\sigma_x^2 + u}\sqrt{2\sigma_x^2 + u}}$$
$$\left[\delta_{m_x 0}\delta_{m_y 0} - \exp(w_x - w_y)(-1)^{(m_x + x_y)/2} I_{m_x/2}(w_x) I_{m_y/2}(w_y) e^{-im_x\varphi_x - im_y\varphi_y}\right]$$

$$w_x = \frac{\beta_x J_x/\sigma_x^2}{2+\eta}. \qquad w_y = \frac{\beta_y J_y/\sigma_y^2}{2+\eta/r_{yx}} \qquad r_{yx} = \sigma_y^2/\sigma_x^2 \qquad \eta = u/\sigma_x^2$$

Tune shift

- 0-th Fourier component, potential averaged by betatron phase

$$U_{00}(J_x, J_y) = \frac{\lambda_p r_p}{\beta^2\gamma^3} \oint ds \int_0^\infty \frac{d\eta}{\sqrt{2+\eta}\sqrt{2r_{yx}+\eta}} (1 - e^{-w_x - w_y} I_0(w_x) I_0(w_y))$$

- Tune shift is given by derivative of the potential for J_x、J_y.

$$\frac{\partial}{\partial J_x} = \frac{\beta_x/\sigma_x^2}{2+\eta}\frac{\partial}{\partial w_x}. \qquad \frac{\partial}{\partial J_y} = \frac{\beta_y/\sigma_x^2}{2r_{yx}+\eta}\frac{\partial}{\partial w_y}$$

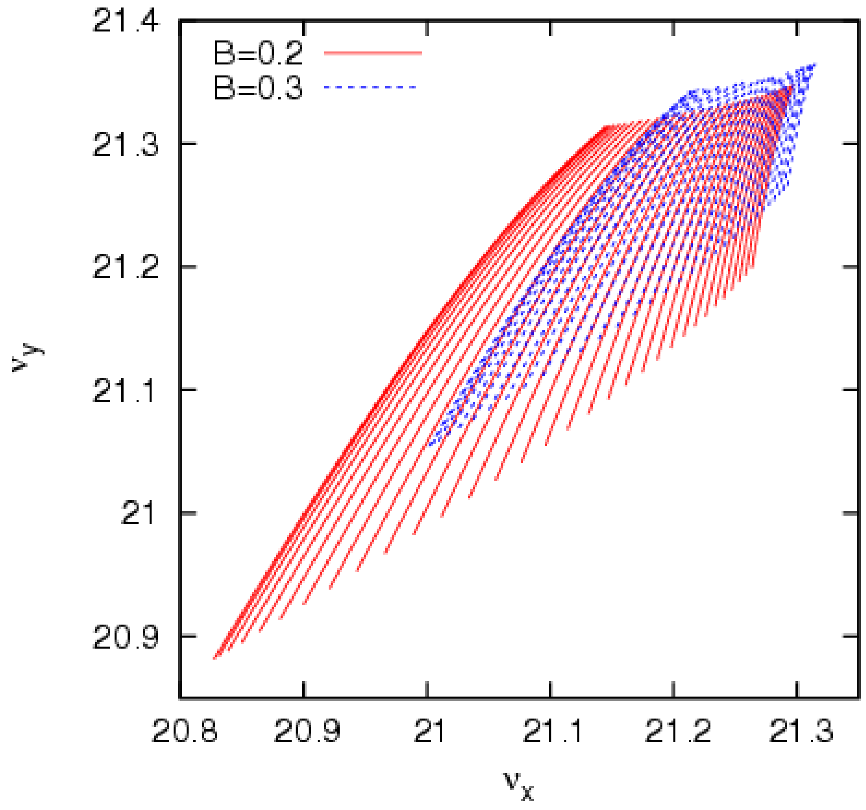
$$2\pi\Delta\nu_x = \frac{\partial U_{00}}{\partial J_x} = \frac{\lambda_p r_p}{\beta^2\gamma^3} \oint ds \frac{\beta_x}{\sigma_x^2} \int_0^\infty \frac{e^{-w_x - w_y} d\eta}{(2+\eta)^{3/2}(2r_{yx}+\eta)^{1/2}} [(I_0(w_x) - I_1(w_x)) I_0(w_y)]$$

$$2\pi\Delta\nu_y = \frac{\partial U_{00}}{\partial J_y} = \frac{\lambda_p r_p}{\beta^2\gamma^3} \oint ds \frac{\beta_x}{\sigma_x^2} \int_0^\infty \frac{e^{-w_x - w_y} d\eta}{(2+\eta)^{1/2}(2r_{yx}+\eta)^{3/2}} [I_0(w_x)(I_0(w_y) - I_1(w_y))]$$

Tune shift is function of J.

Tune shift for J-PARC MR

- Integrate space charge potential every 1m.
- $\beta_{x,y}(s)$, $\varphi_{x,y}(s)$ are given by an accelerator design code, SAD.
- Calculate tune shift integral for each J_x,J_y.



Each line is given for varying J_y with fixed J_x.

E=3.938 GeV $L = 1567.5$ m

$N_p = 3 \times 10^{13}$ N_b=8 bunches

$\varepsilon = 4 \times 10^{-6}$ m β_{xy}=12 m

Bunching factor =0.2 at injection decreases to 0.3 for a synchrotron period (ν_s=0.002)

Tune slope

- Differentiation of tune shift for J_x、J_y. Detuning for J.

$$\frac{\partial^2 U_{00}}{\partial J_x^2} = 2\pi\frac{\partial \nu_x}{\partial J_x}$$

$$= \frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds\frac{\beta_x^2}{\sigma_x^4}\int_0^\infty \frac{e^{-w_x-w_y}d\eta}{(2+\eta)^{5/2}(2r_{yx}+\eta)^{1/2}}\left[\left\{\frac{3}{2}I_0(w_x)-2I_1(w_x)+\frac{1}{2}I_2(w_x)\right\}I_0(w_y)\right]$$

$$\frac{\partial^2 U_{00}}{\partial J_x \partial J_y} = 2\pi\frac{\partial \nu_x}{\partial J_y} = 2\pi\frac{\partial \nu_y}{\partial J_x}$$

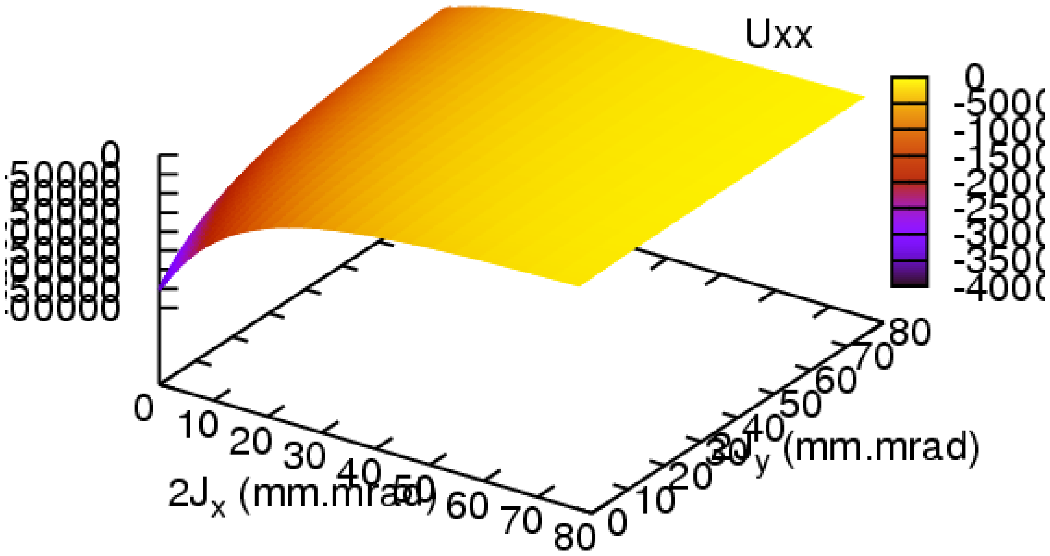
$$= \frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds\frac{\beta_x\beta_y}{\sigma_x^4}\int_0^\infty \frac{e^{-w_x-w_y}d\eta}{(2+\eta)^{3/2}(2r_{yx}+\eta)^{3/2}}[(I_0(w_x)-I_1(w_x))(I_0(w_y)-I_1(w_y))]$$

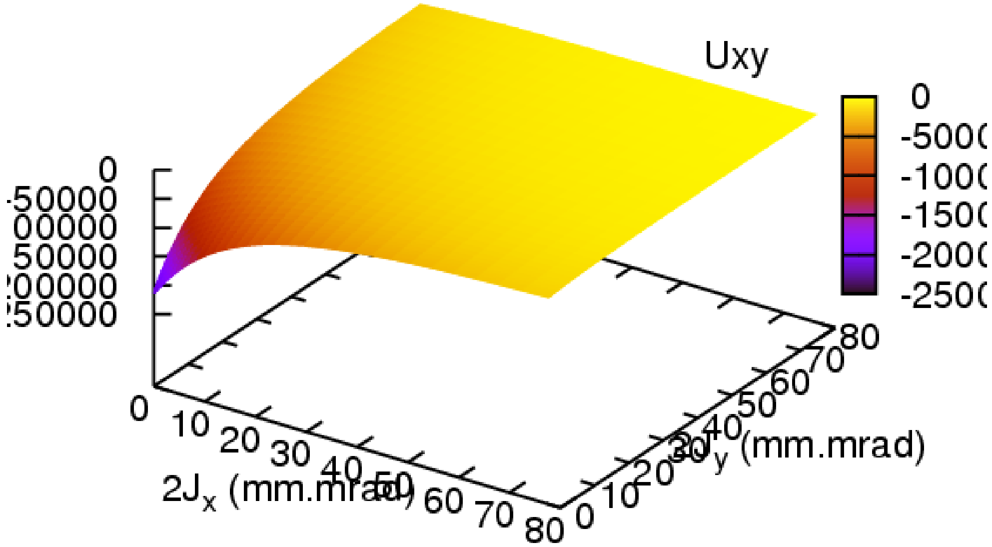
$$\frac{\partial^2 U_{00}}{\partial J_y^2} = -2\pi\frac{\partial \nu_y}{\partial J_y}$$

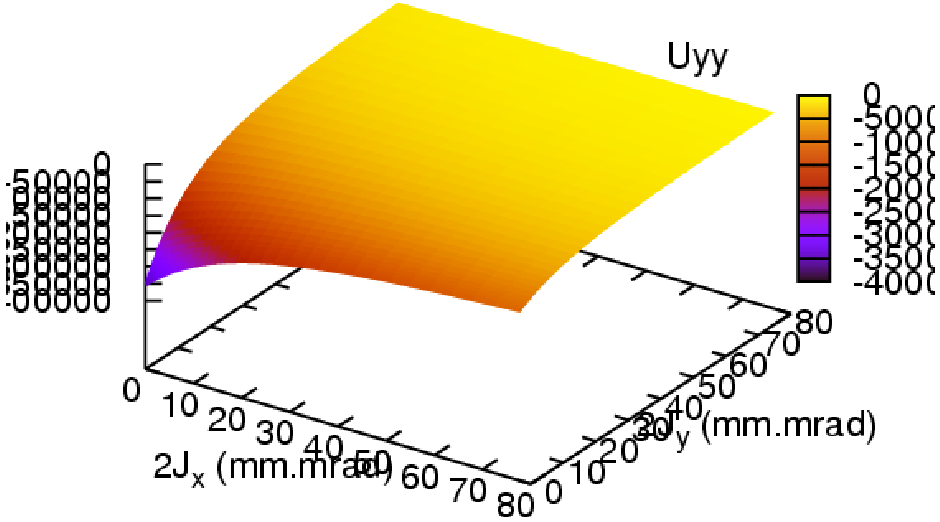
$$= \frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds\frac{\beta_y^2}{\sigma_x^4}\int_0^\infty \frac{e^{-w_x-w_y}d\eta}{(2+\eta)^{1/2}(2r_{yx}+\eta)^{5/2}}\left[I_0(w_x)\left\{\frac{3}{2}I_0(w_y)-2I_1(w_y)+\frac{1}{2}I_2(w_y)\right\}\right]$$

$$I_0(x)' = I_1(x),\ I_0(x)'' = (I_0(x)+I_2(x))/2$$

Tune slope for J-PAC MR







Resonances

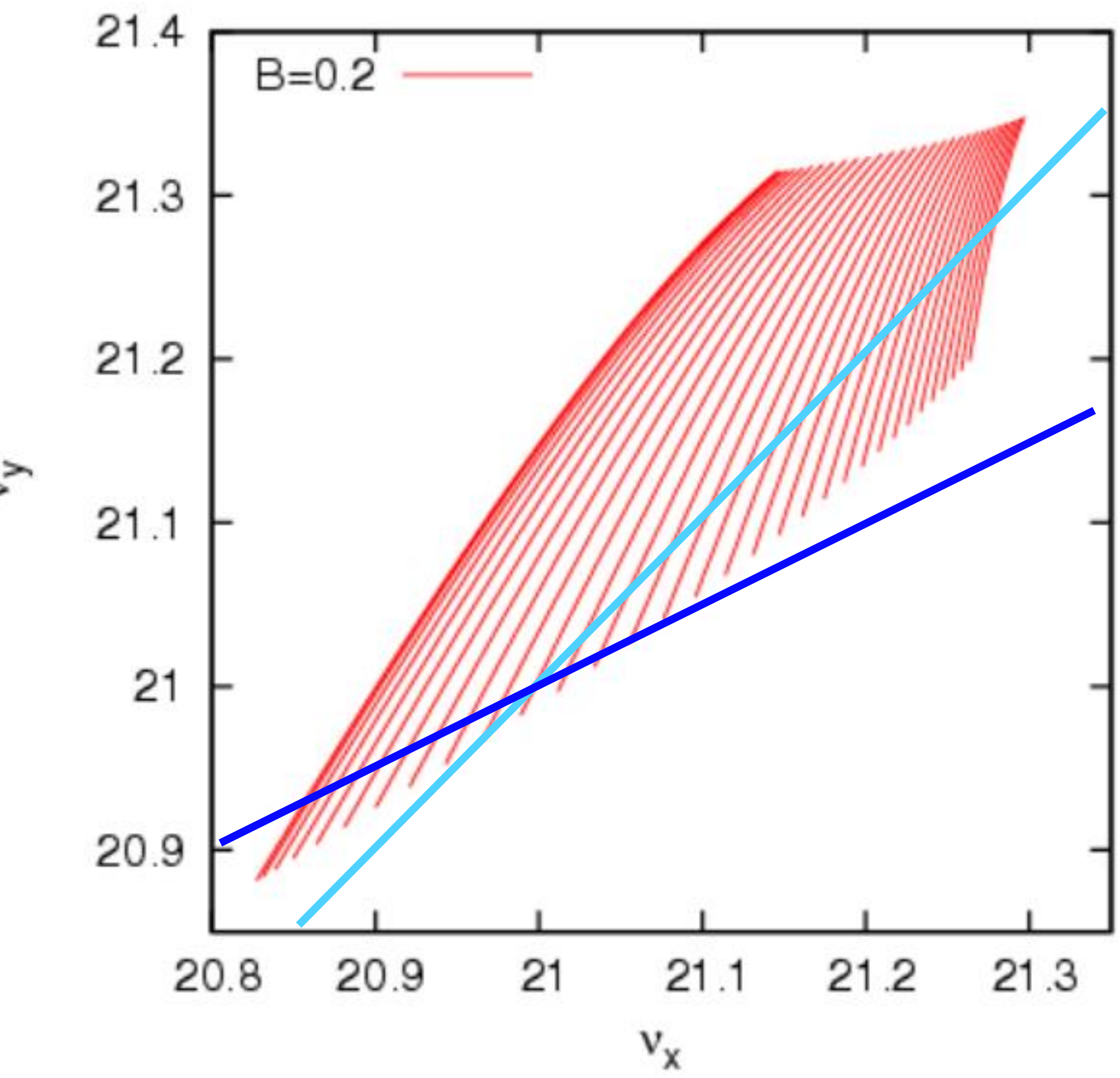
- Tune decreases due to tune shift caused by space charge force.
- The resonance condition is satisfied at certain $\mathbf{J}_R$,

$$m_x \nu_x(\boldsymbol{J}_R) + m_y \nu_y(\boldsymbol{J}_R) = n$$

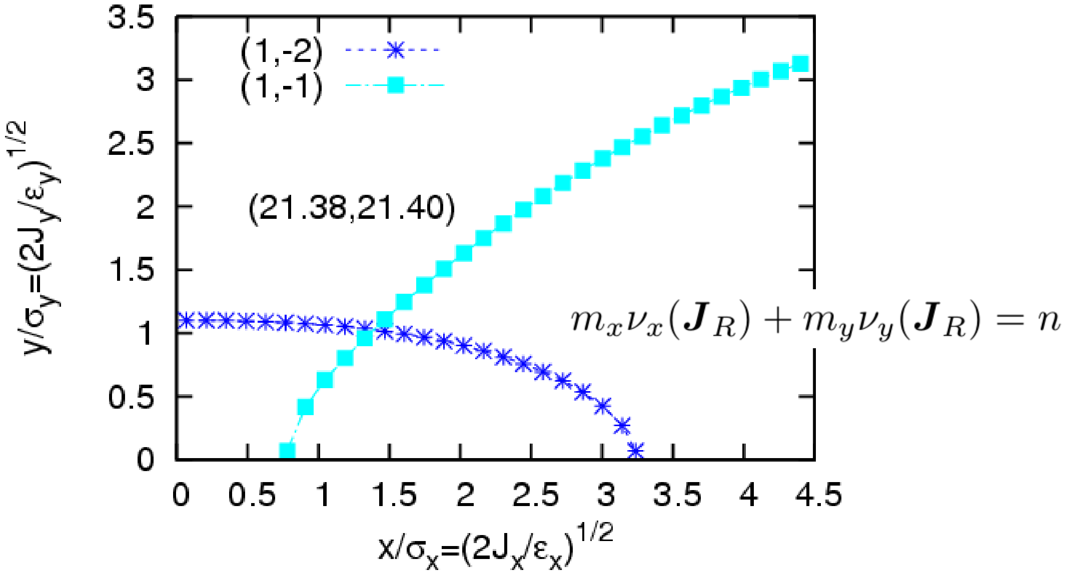
- $\mathbf{J}_R$ depicts a curve in J_x-J_y plane.
- Tune of a particle near $\mathbf{J}_R$ deviates from the above resonance condition. The particle slips with a speed linearly depend on $\mathbf{J} - \mathbf{J}_R$, but are attracted to the resonance linearly depend on the phase difference. The particle oscillates around $\mathbf{J}_R$.
- How large ($\mathbf{J} - \mathbf{J}_R$) amplitude particles are trapped in the resonance. When $\mathbf{J} - \mathbf{J}_R$ is larger than a certain number, particles motion is betatron oscillation
- Emittance growth is caused by the motion around $\mathbf{J}_R$. Synchrotron motion enhances the emittance growth due to modulational diffusion of the resonance.

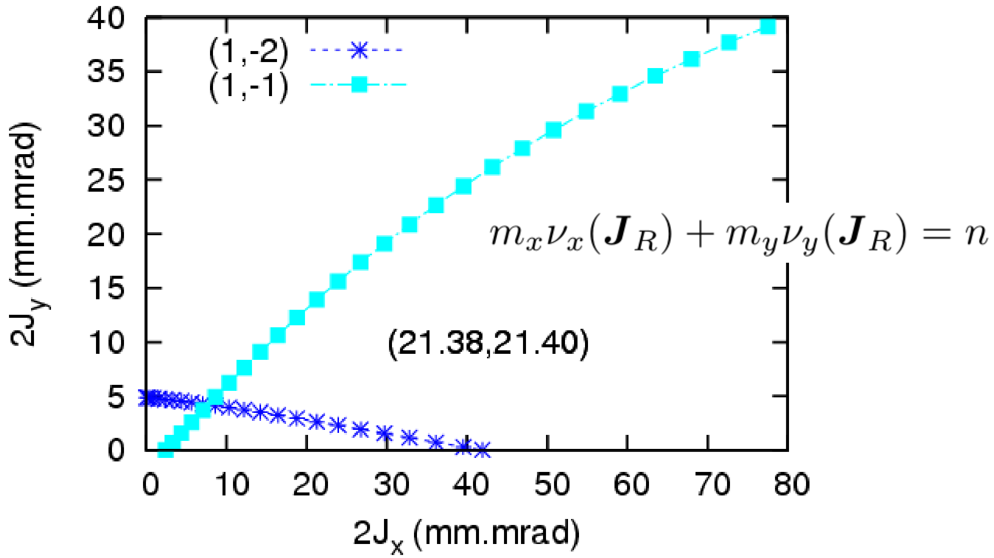
Lichtenberg, Lieberman, Regular and Chaotic Dynamics, Springer, p399.

Resonances for (21.38,21.40)



Betatron amplitudes satisfying the resonance condition





Particle motion near a resonance

- Hamiltonian

J.L. Tennyson, AIP conf. proc. 87, 345 (1982).

$$H = \mu J + U_{00}(\boldsymbol{J}) + \sum_{m_x, m_y \neq 0} U_{m_x, m_y}(\boldsymbol{J}) \exp(-i m_x \phi_x - i m_y \phi_y)$$

$$U_{00}(\boldsymbol{J}) = U_{00}(\boldsymbol{J}_R) + \left.\frac{\partial U_{00}}{\partial \boldsymbol{J}}\right|_{\boldsymbol{J}_R} (\boldsymbol{J} - \boldsymbol{J}_R) + (\boldsymbol{J} - \boldsymbol{J}_R)^t \frac{1}{2} \left.\frac{\partial^2 U_{00}}{\partial \boldsymbol{J} \partial \boldsymbol{J}}\right|_{\boldsymbol{J}_R} (\boldsymbol{J} - \boldsymbol{J}_R)$$

$$F_2(\boldsymbol{P}, \boldsymbol{\phi}) = (J_{x,R} + m_x P_1 + m_{x,2} P_2)\phi_x + (J_{y,R} + m_y P_1 + m_{y,2} P_2)\phi_y$$

$$J_x = \frac{\partial F_2}{\partial \phi_x} = J_{x,R} + m_x P_1 + m_{x,2} P_2$$

$$\psi_1 = \frac{\partial F_2}{\partial P_1} = m_x \phi_x + m_y \phi_y$$

$$J_y = \frac{\partial F_2}{\partial \phi_y} = J_{y,R} + m_y P_1 + m_{y,2} P_2$$

$$\psi_2 = \frac{\partial F_2}{\partial P_2} = m_{x,2} \phi_x + m_{y,2} \phi_y.$$

Choosing $m_{x,2} = 0, m_{y,2} = 1$,

$$P_1 = \frac{J_x - J_{x,R}}{m_x} \qquad \psi_1 = m_x \phi_x + m_y \phi_y \qquad P_2 = (J_y - J_{y,R}) - \frac{m_y}{m_x}(J_x - J_{x,R}) \qquad \psi_2 = \phi_y$$

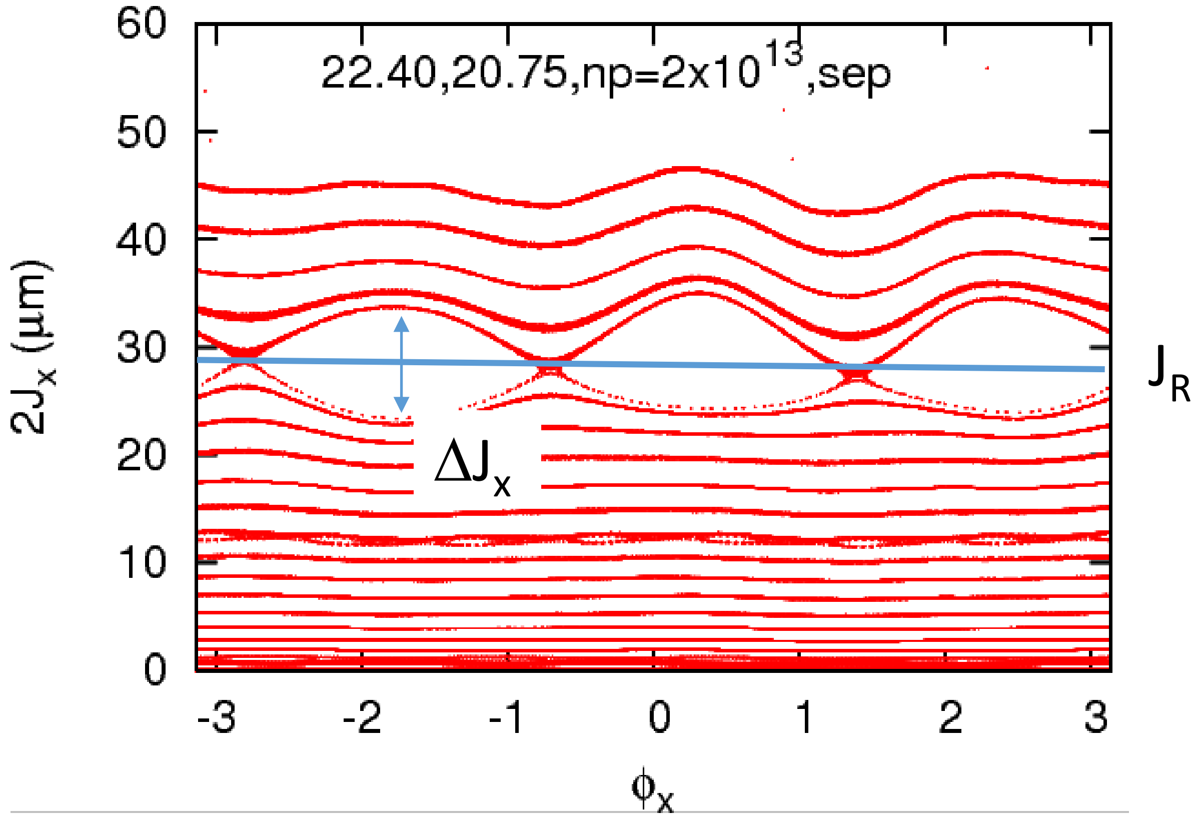
Resonance width

$$H_{00} = U_{00} = \frac{\Lambda}{2} P_1^2 + \Lambda_{12} P_1 P_2 + \frac{\Lambda_{22}}{2} P_2^2$$

$$\Lambda \equiv m_x^2 \frac{\partial^2 U_{00}}{\partial J_x^2} + 2 m_x m_y \frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{00}}{\partial J_y^2}$$

$$H = \frac{\Lambda}{2} P_1^2 + U_{\boldsymbol{m}}(\boldsymbol{J}_R) \cos \psi_1 .$$

$$\Delta P_1 = 4 \sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}} \qquad \Delta J_x = 4 m_x \sqrt{\frac{U_{\boldsymbol{m}}}{\Lambda}}$$



J-PARC MR operating points

- Bunching factor at injection 0.2 reduce to 0.3 in 1-2 synchrotron period (~500-1000 turn 2.5-5ms).

Tune operating point in early stage

- ν_x, ν_y=22.4,20.75

Present operating point

- 21.38, 21.40
- 21.35,21.40

Now considering

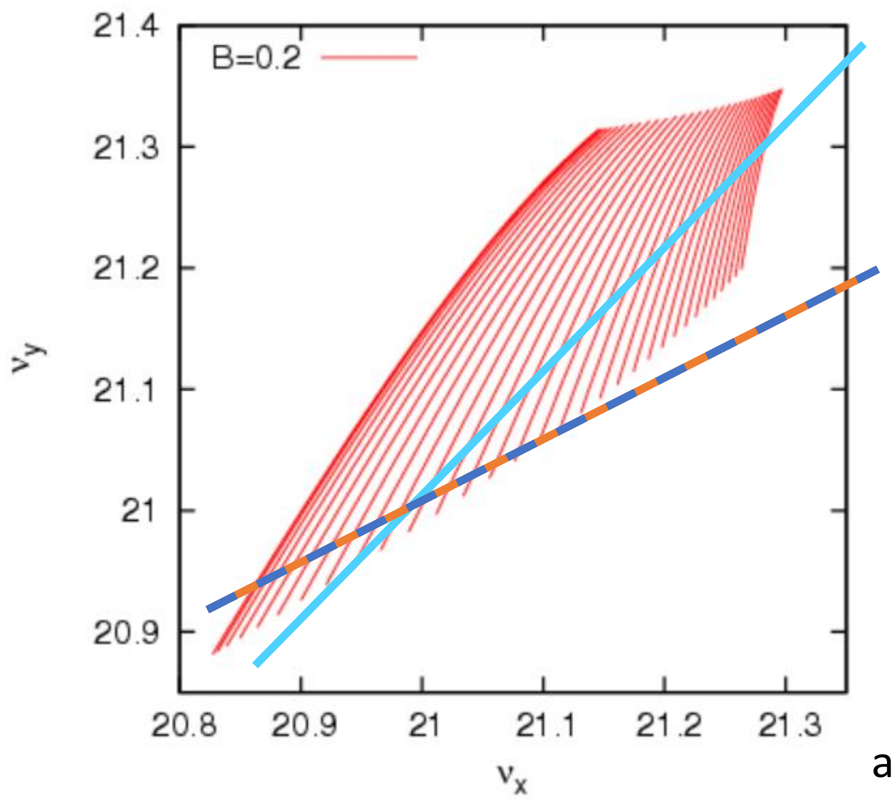
- 22.35,22.40
- 21.40,20.45

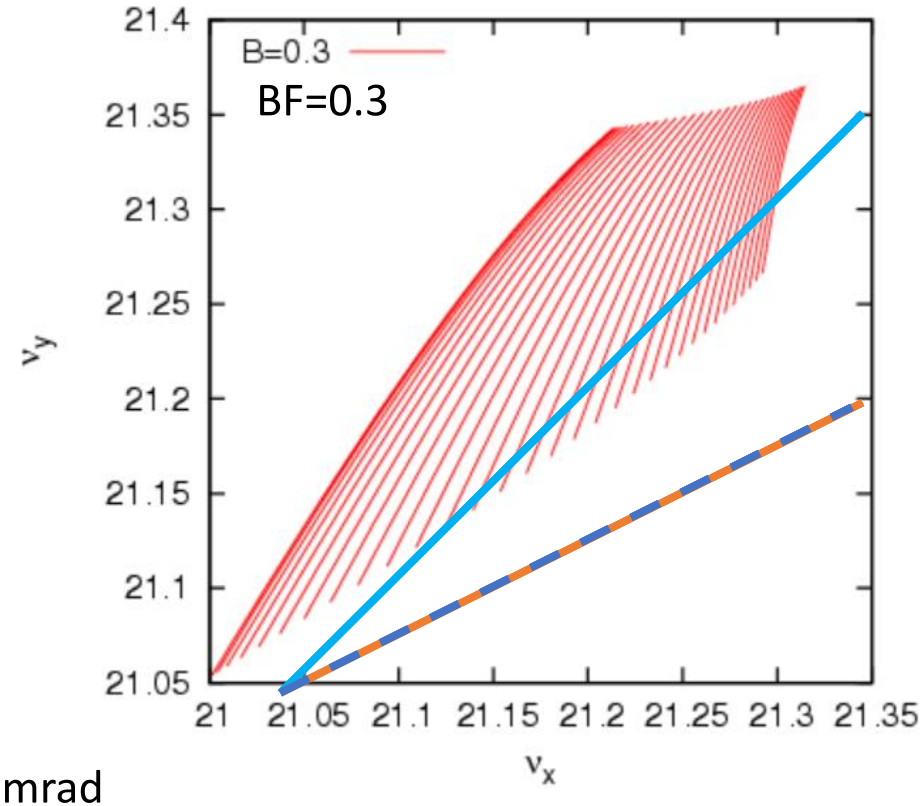
- How resonances are serious in these operating points.

How the resonances affect emittance growth and beam loss at ~(21.35,21.40)

Structure resonances

- Space charge force induces (m_x,m_y,n)=(2,-2,0) (2,-4,-42).
- Sextupole magnets induces (1,-2,-21).





aperture: 2J=60-80 mm.mrad

Space charge driven resonance

- Resonance driving term

$$U_m(J_x, J_y) = \frac{\lambda_p r_p}{\beta^2\gamma^3} \oint ds \int_0^\infty \frac{du}{\sqrt{2\sigma_x^2 + u}\sqrt{2\sigma_x^2 + u}}$$
$$\left[\delta_{m_x 0}\delta_{m_y 0} - \exp(w_x - w_y)(-1)^{(m_x + x_y)/2} I_{m_x/2}(w_x) I_{m_y/2}(w_y) e^{-im_x\varphi_x - im_y\varphi_y}\right]$$

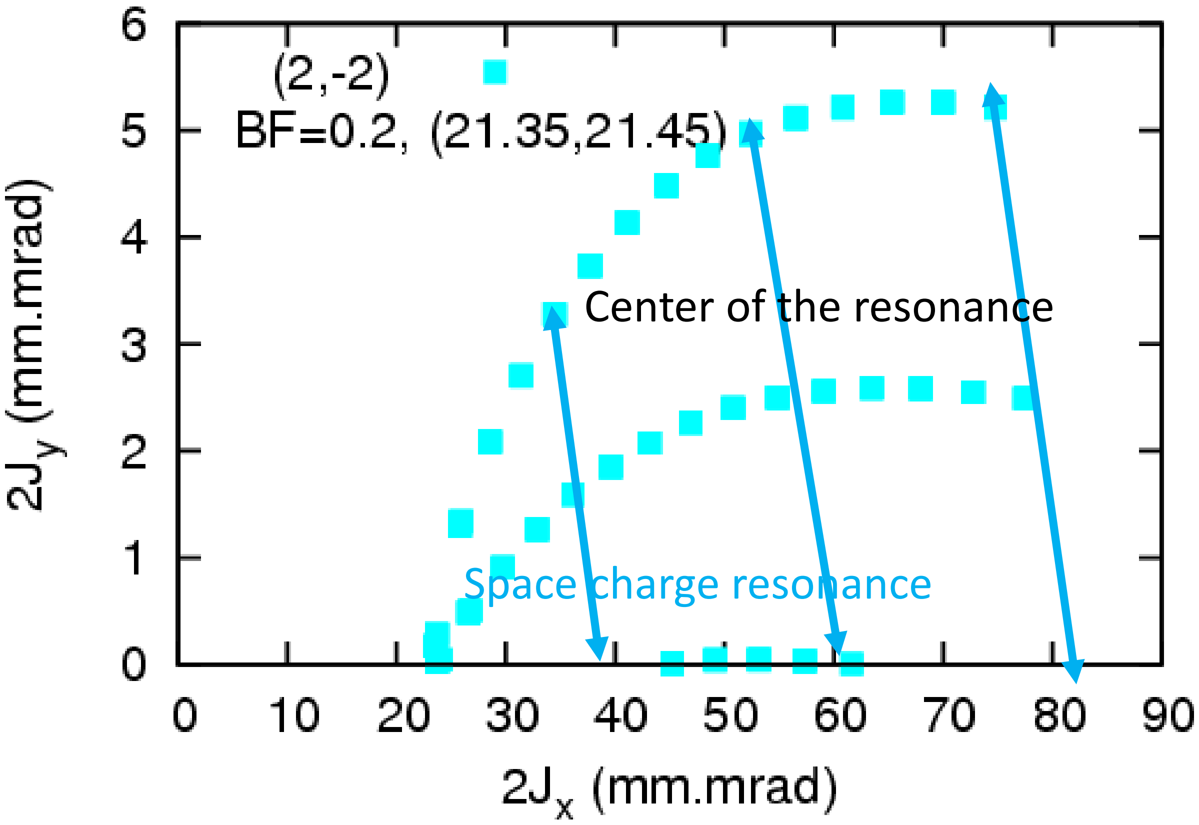
- Resonance (full) width

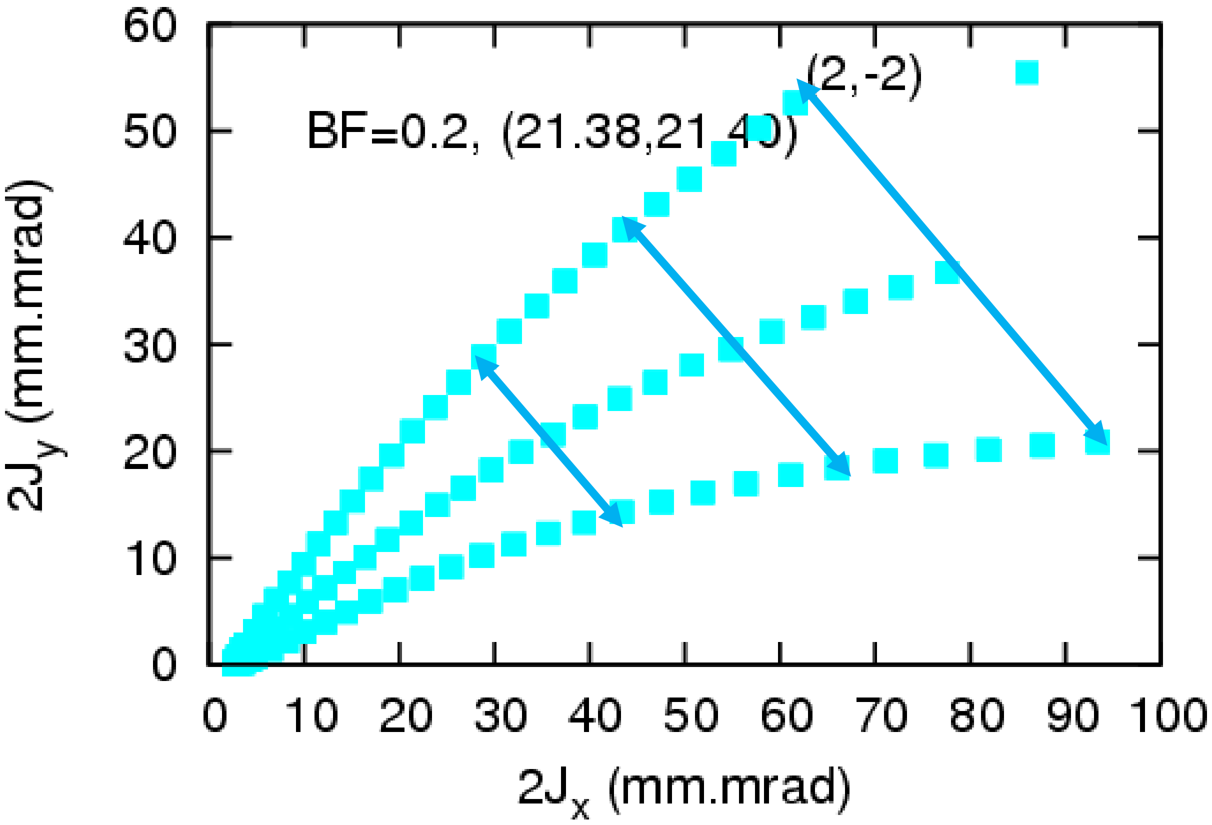
$$\Delta P_1 = 4\sqrt{\frac{\boldsymbol{Um}}{\Lambda}} \qquad \Delta J_x = 4m_x\sqrt{\frac{\boldsymbol{Um}}{\Lambda}}$$

$$\Lambda \equiv m_x^2 \frac{\partial^2 U_{00}}{\partial J_x^2} + 2m_x m_y \frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{00}}{\partial J_y^2}$$

Space charge driven Resonance (2,-2,0)

- Resonance width in amplitude space. Bunching factor 0.2

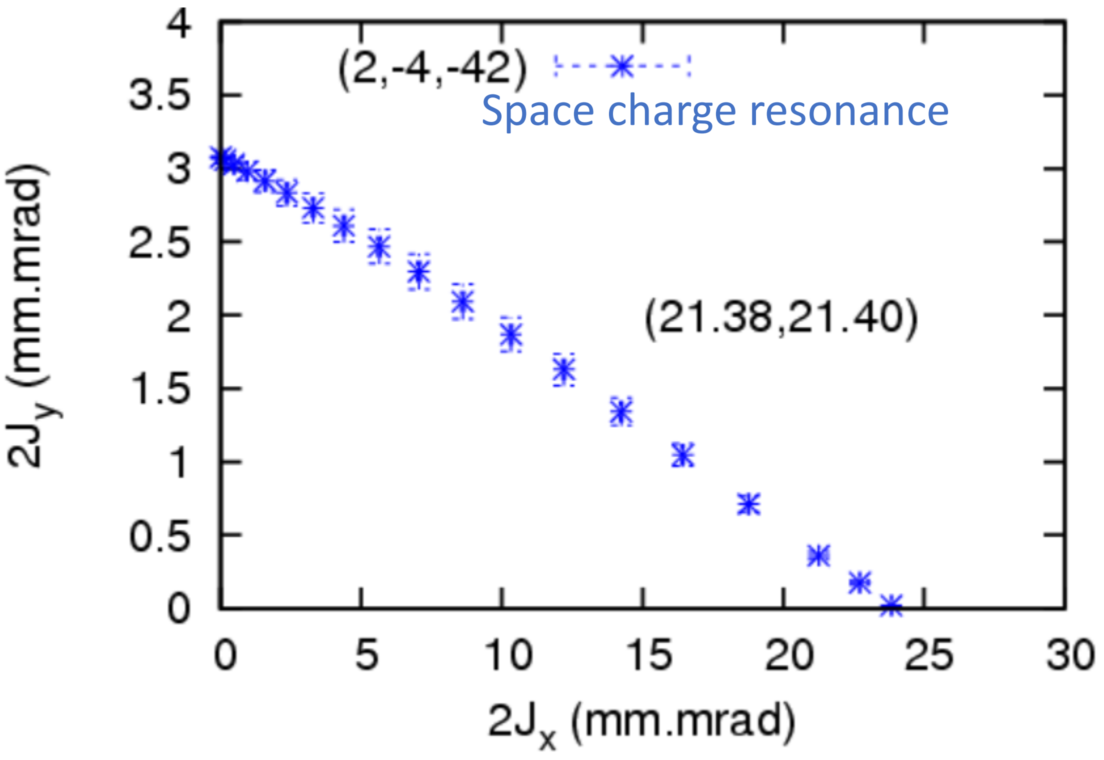


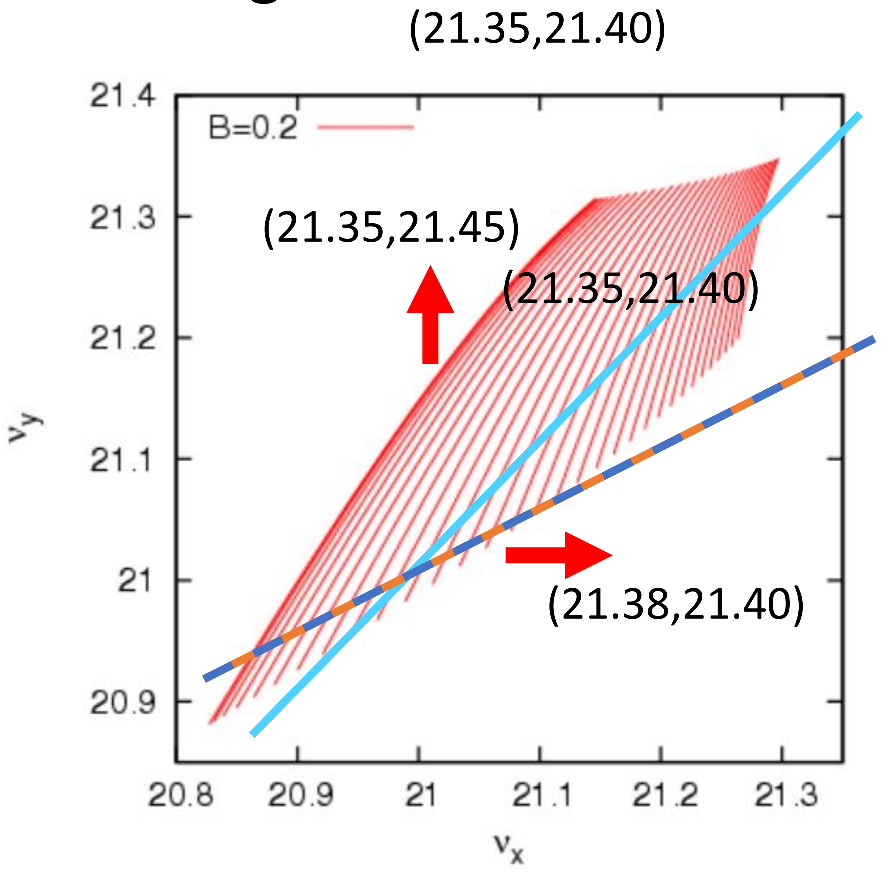


MR aperture: 2J=60-80 mm.mrad

Space charge driven Resonance (2,-4,-42)

- Resonance width in amplitude space. Bunching factor 0.2

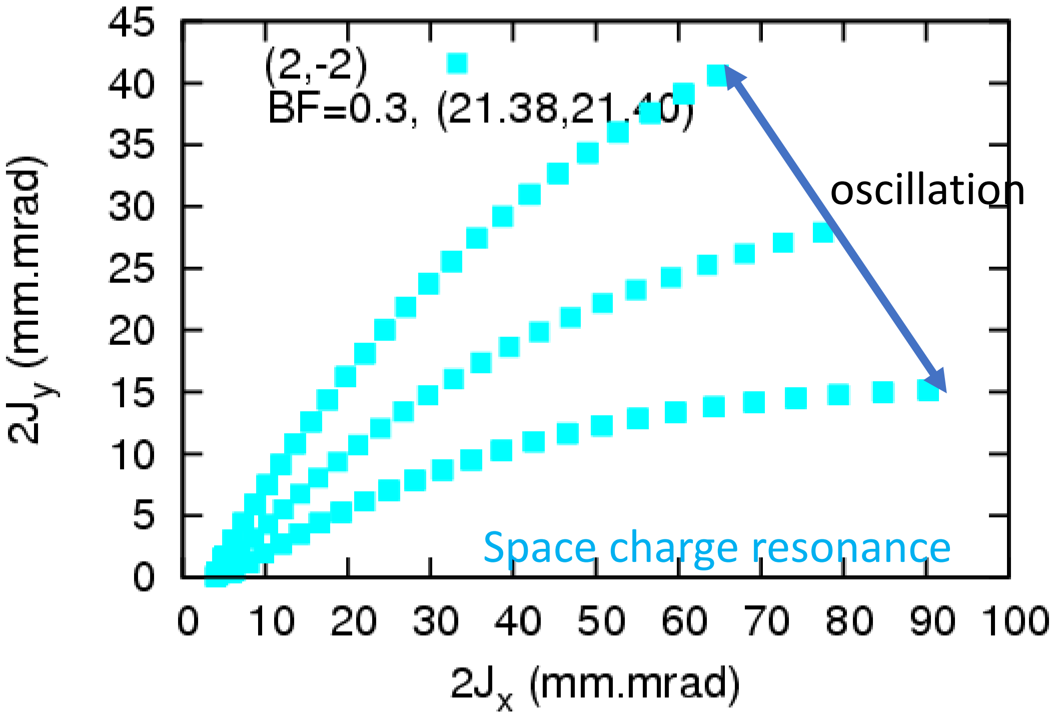


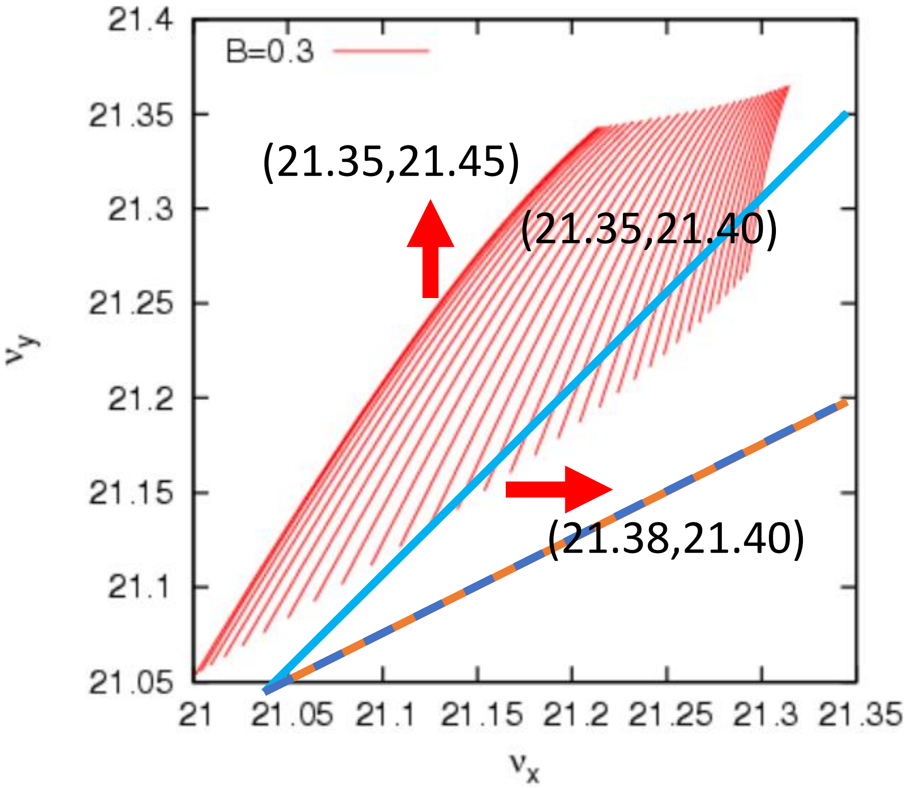


MR aperture: 2J=60-80 mm.mrad

Space charge driven Resonance (2,-2,0) for bunching factor 0.3

- Resonance width in amplitude space. Bunching factor 0.3





- Center of the (2,-2) resonance is out of tune spread area for (21.35,21.45)

MR aperture: 2J=60-80 mm.mrad

Sextupole driven resonances

$$U_s(x, y, s) = \frac{k_2(s)}{6}(x^3 - 3xy^2)$$

- Integrate over the ring. Fourier components

$$U_{3,0} = \frac{G_{3,0}}{12\sqrt{2}} J_x^{3/2} \qquad U_{1,\pm 2} = \frac{G_{1,\pm 2}}{4\sqrt{2}} J_x^{1/2} J_y$$

$$G_{3,0} = \oint_s ds' k_2 \beta_x^{3/2} e^{3i\varphi_x}$$

$$G_{1,\pm 2} = \oint_s ds' k_2 \beta_x^{1/2} \beta_y e^{i(\varphi_x \pm 2\varphi_y)}$$

Structure resonance driven by sextupole magnets (1,-2,-21)

- Super periodicity 3
- The integrals for the sextupole component should be done in each 1/3 ring.

$$G_{3,0,1/3} = \int_0^{L/3} ds' k_2 \beta_x^{3/2} e^{3i\varphi_x} \qquad G_{1,\pm 2,1/3} = \int_0^{L/3} ds' k_2 \beta_x^{1/2} \beta_y e^{i(\varphi_x \pm 2\varphi_y)}$$

- Simple summation of each 1/3 integral under the resonance condition ν_x-2ν_y=-21.

$$G_{m_x,m_y} = 3G_{m_x,m_y,1/3} \qquad G_{m_x,m_y,1/3} = G_{m_x,m_y,2/3} = G_{m_x,m_y,3/3}$$

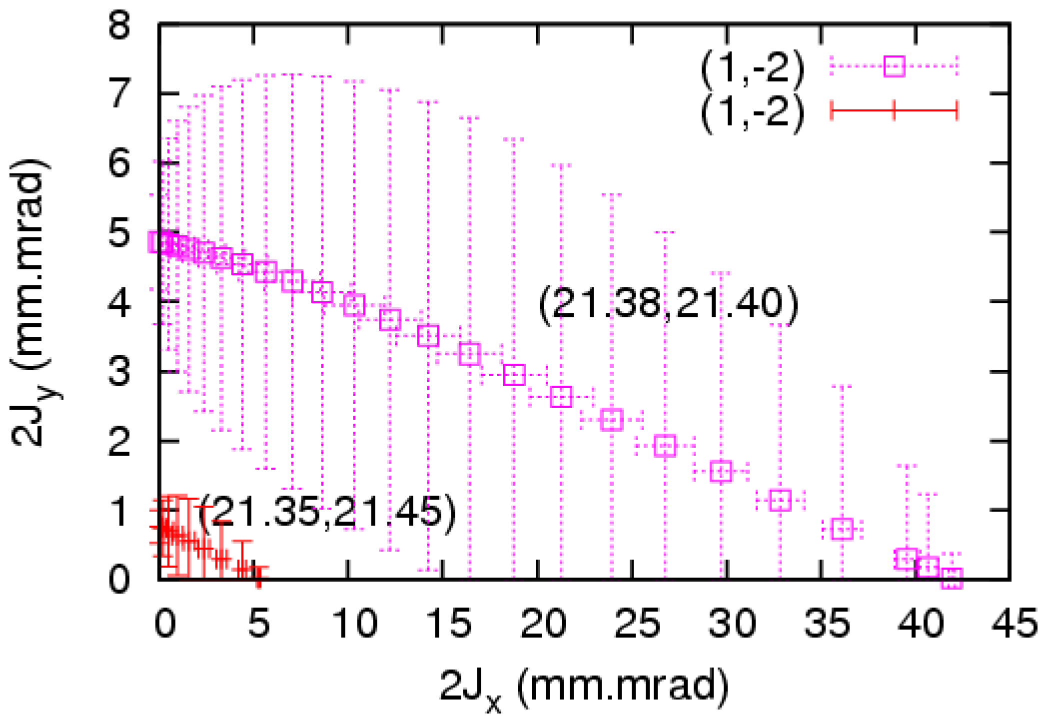
$$G_{1-2} = -39 - 8.9i$$

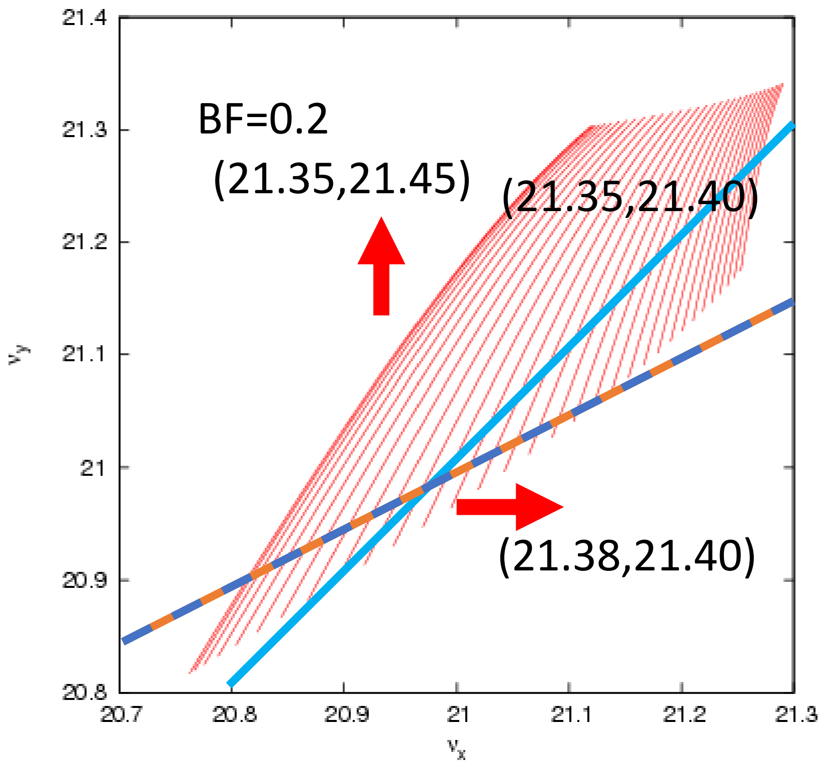
K.Ohmi et al., IPAC17

- $U_{1,\pm 2} = \frac{G_{1,\pm 2}}{4\sqrt{2}} J_x^{1/2} J_y$ $=7.1\ J_x^{1/2}J_y$

Structure resonance driven by sextupole

- Resonance width in amplitude space

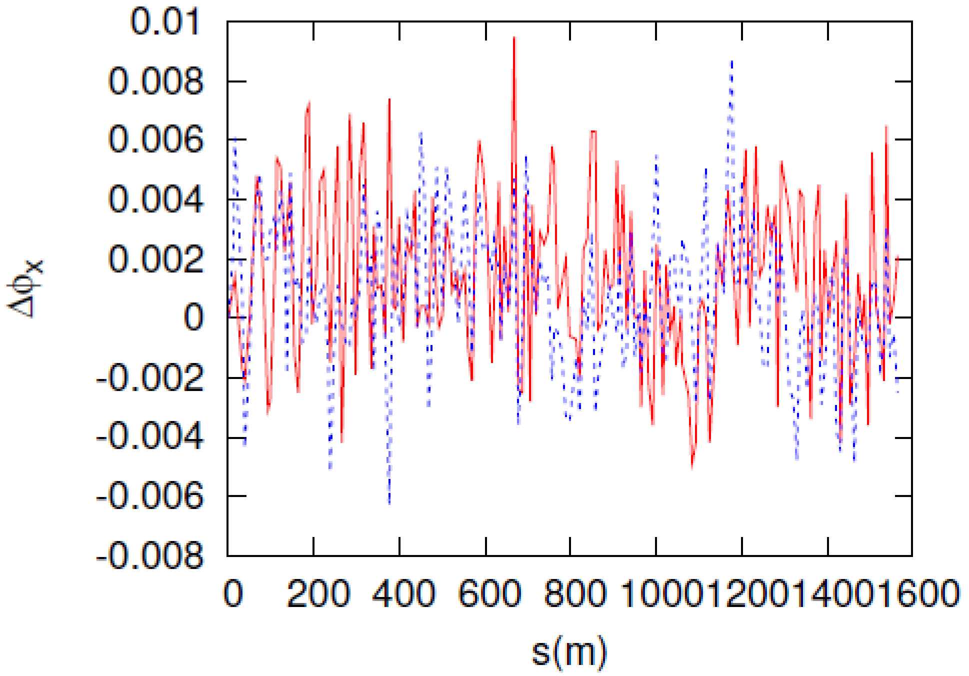


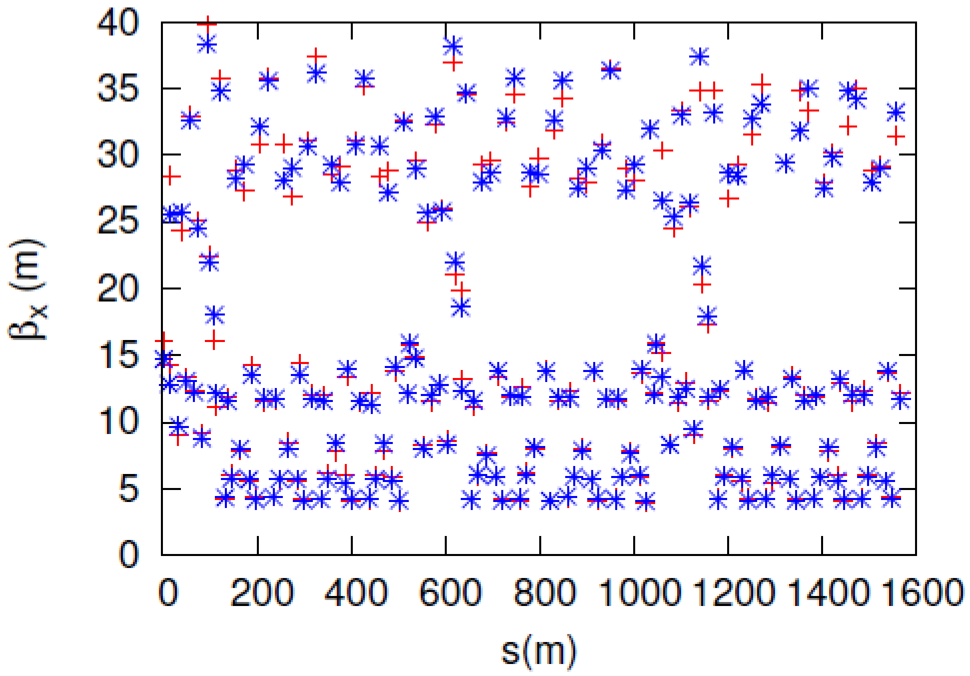


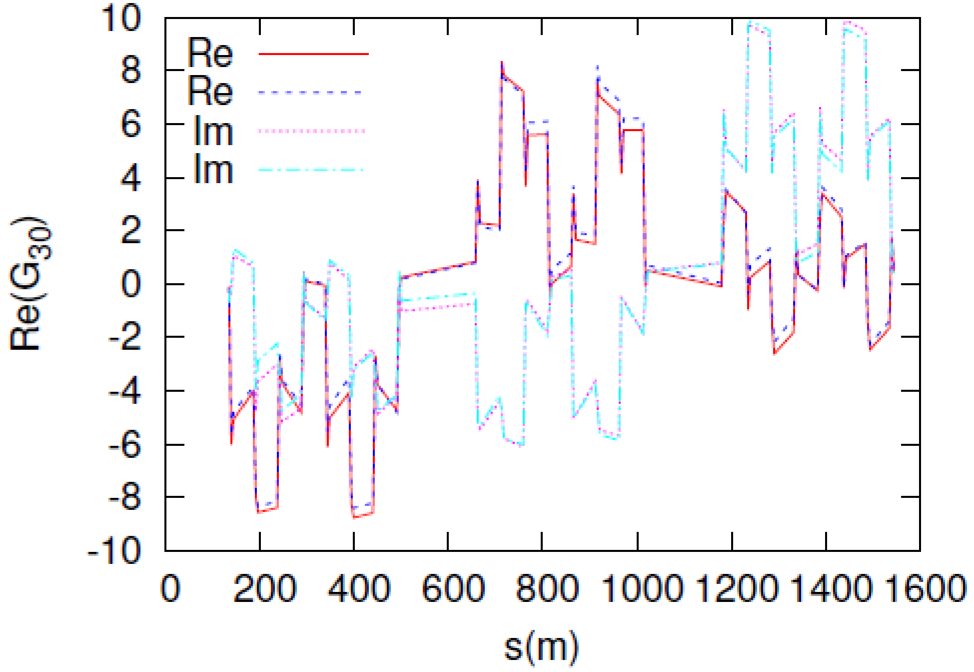
sextupole resonance

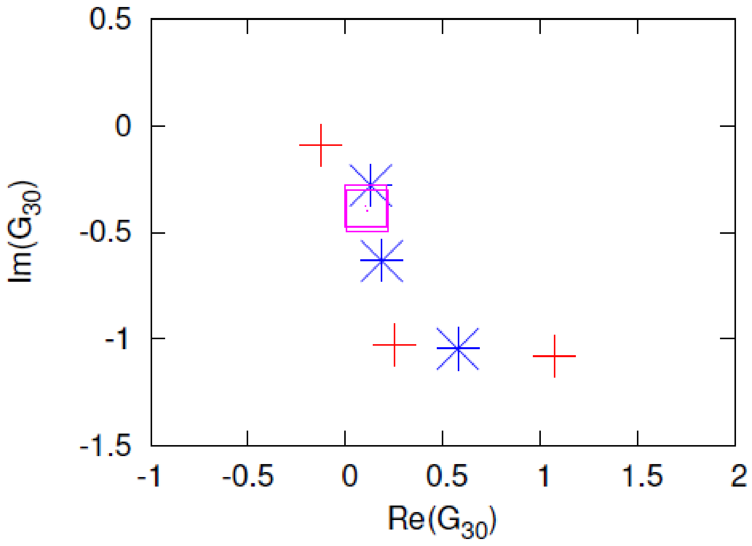
Non-structured resonance

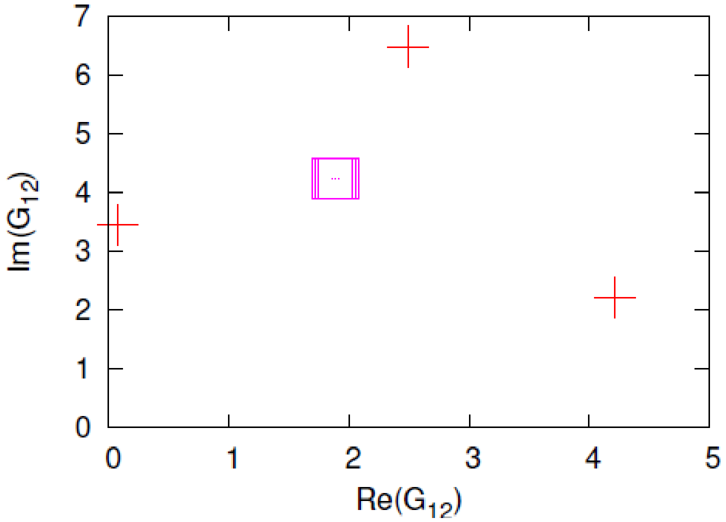
- Symmetry breaking induced by lattice errors excites non-structures resonances
- Asymmetry in beta function/phase measurements.











- G_{30} and G_{12} for each 1/3 using measured β and phase are plotted red and blue.
- G's are the same without errors (magenta points)

Resonance width for non-structured resonances induced by sextupoles with lattice errors

$$G_{30} = G_{30,1/3} + e^{i2\pi/3} G_{30,2/3} + e^{i4\pi/3} G_{30,3/3}$$
$$G_{1+2} = G_{1+2,1/3} + e^{i2\pi/3} G_{1+2,2/3} + e^{i4\pi/3} G_{1+2,3/3}$$

K.Ohmi et al., IPAC17

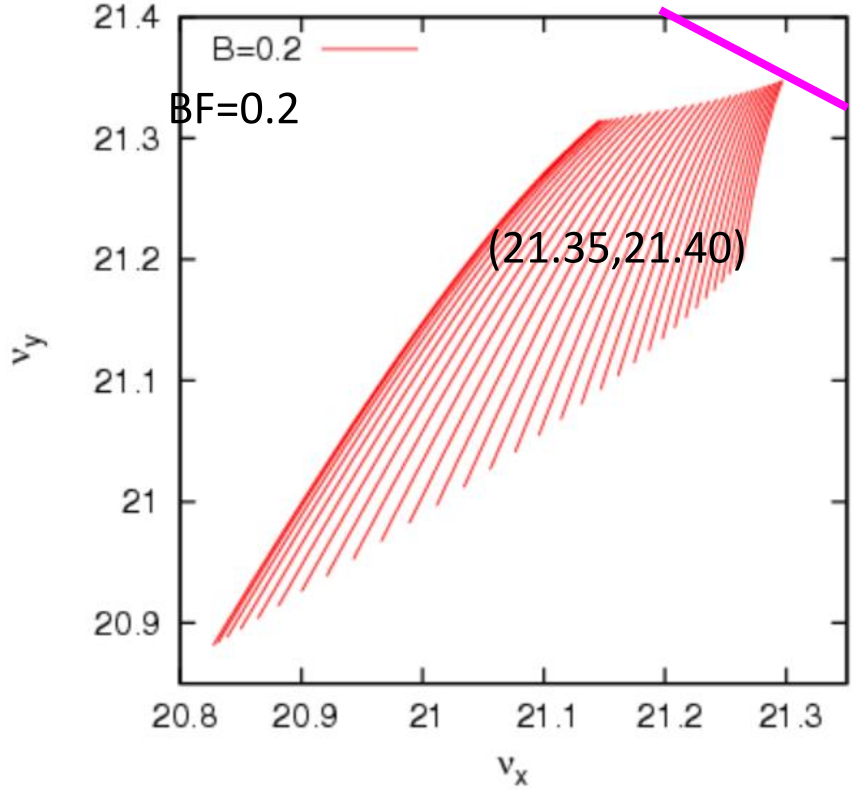
$$G_{30} = 0.63{+}1.63i \quad G_{1+2} = -7.0{-}3.9i$$

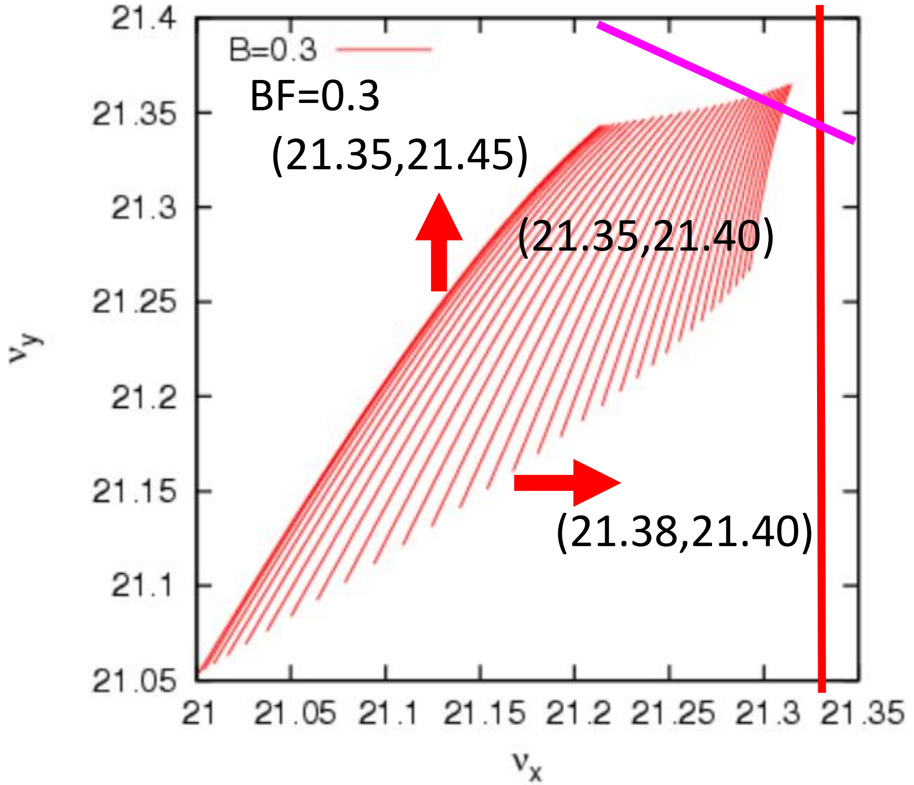
- $U_{3,0} = \frac{G_{3,0}}{12\sqrt{2}} J_x^{3/2}$ $=0.10\ J_x^{3/2}$
- $U_{1,\pm 2} = \frac{G_{1,\pm 2}}{4\sqrt{2}} J_x^{1/2} J_y$$=1.4\ J_x^{1/2} J_y$

How the resonances affect emittance growth and beam loss at ~(21.35,21.40)

Non-Structure resonances

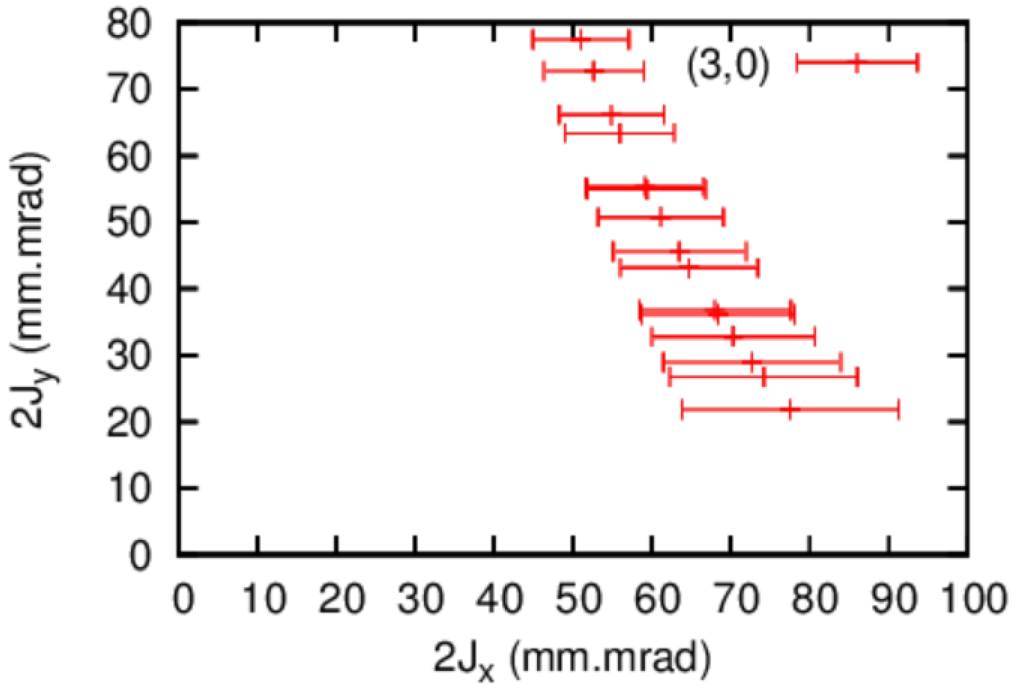
- Sextupole magnets under optics error induces (3,0,64).
- Sextupole magnets under optics error induces (1,2,64).

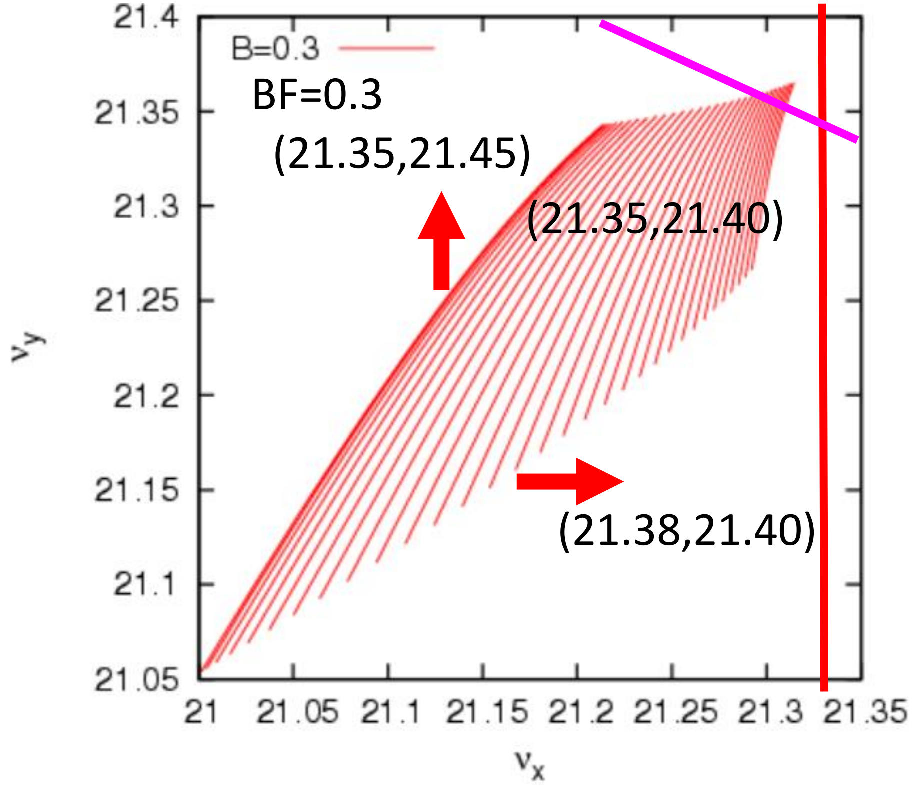




Nonstructure (3,0,64) resonance

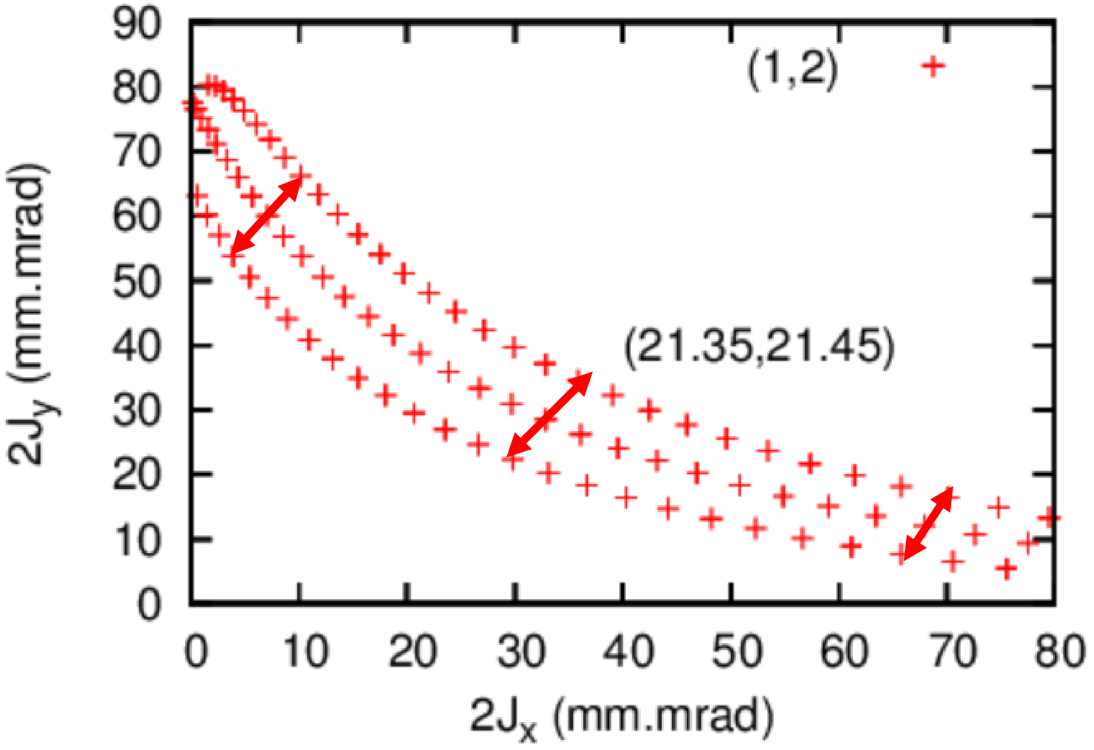
- Resonance width in amplitude space. BF=0.3, (21.38,21.40)

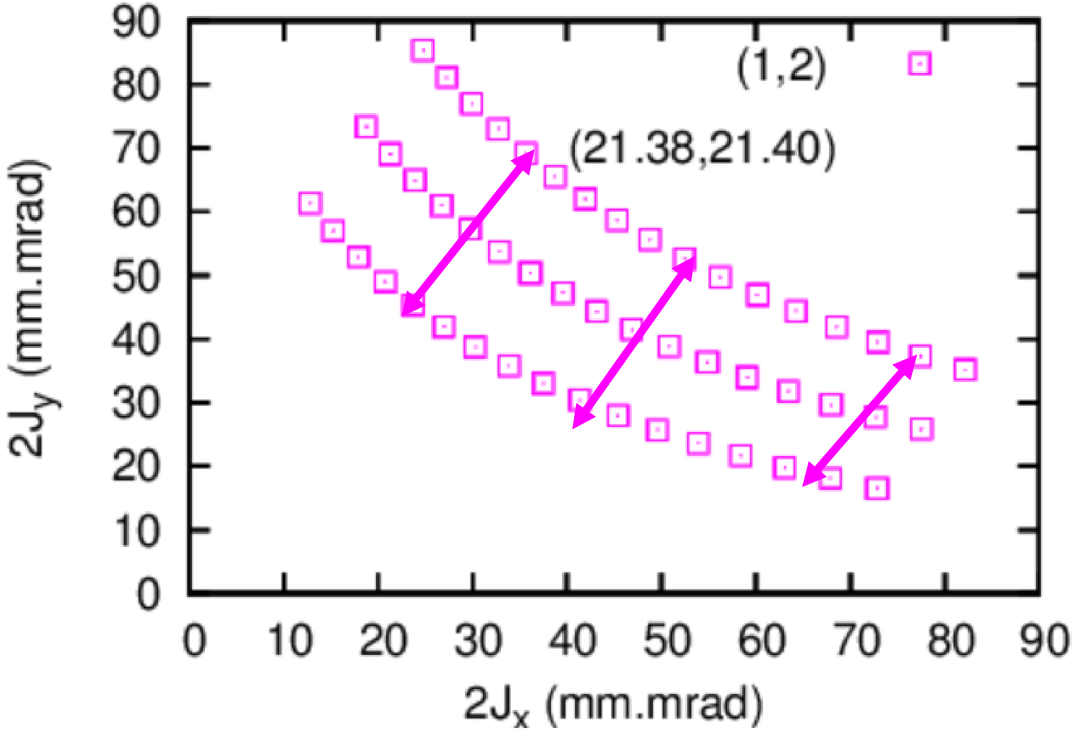




Nonstructure (1,2,64) resonance

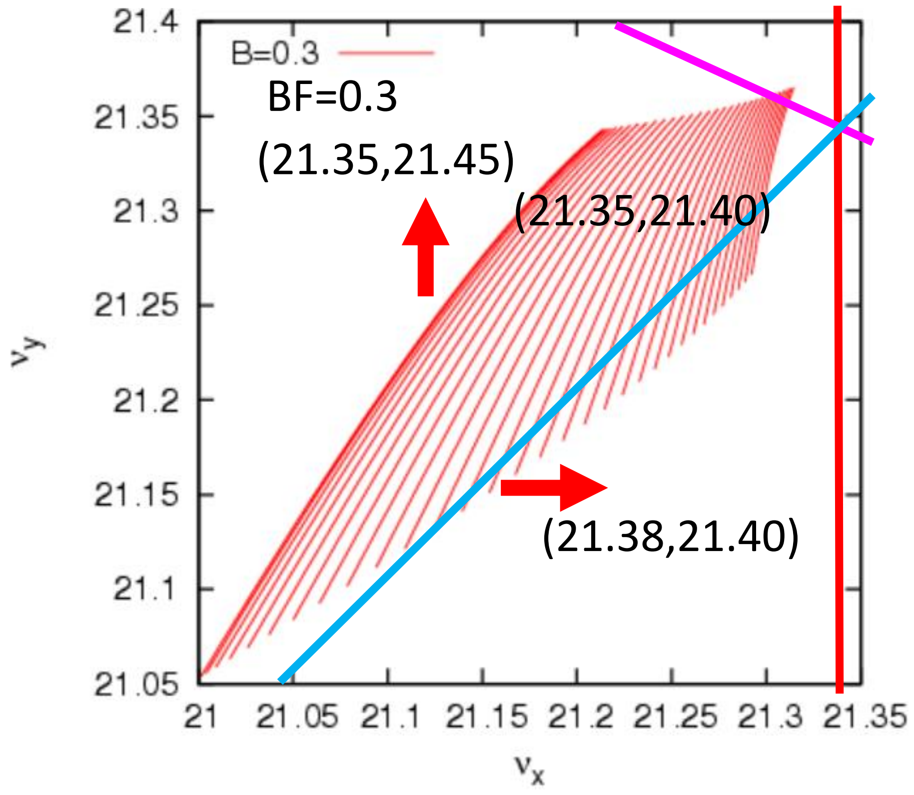
- Resonance width in amplitude space. BF=0.3

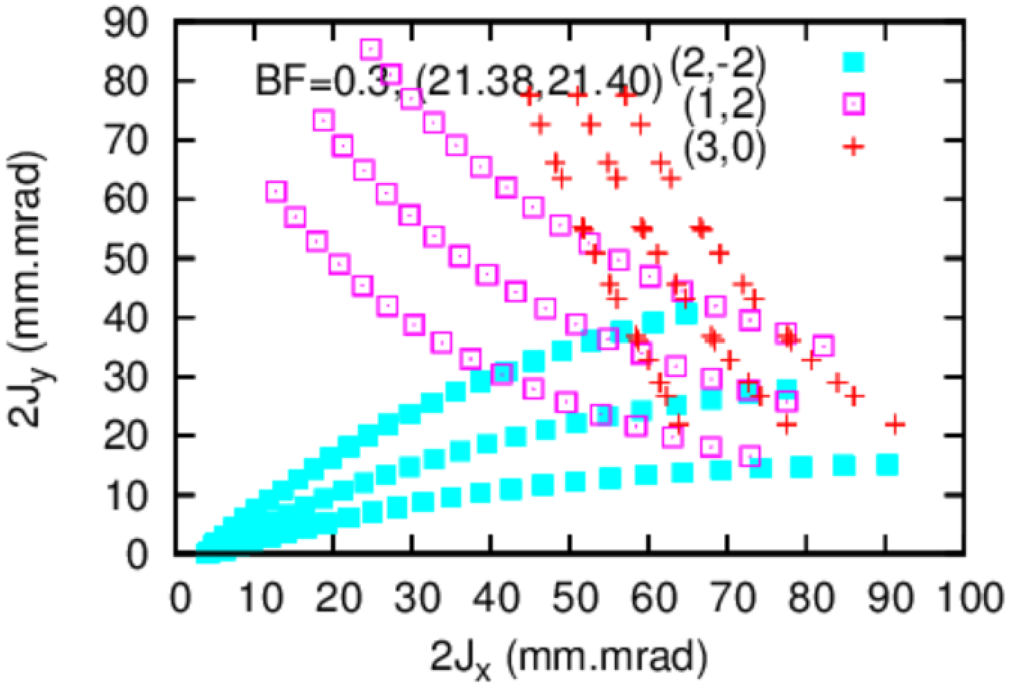




Resonance overlap (large amplitude)

- Overlap occurs at BF=0.3 and (ν_x, ν_y)=(21.38,21.40).

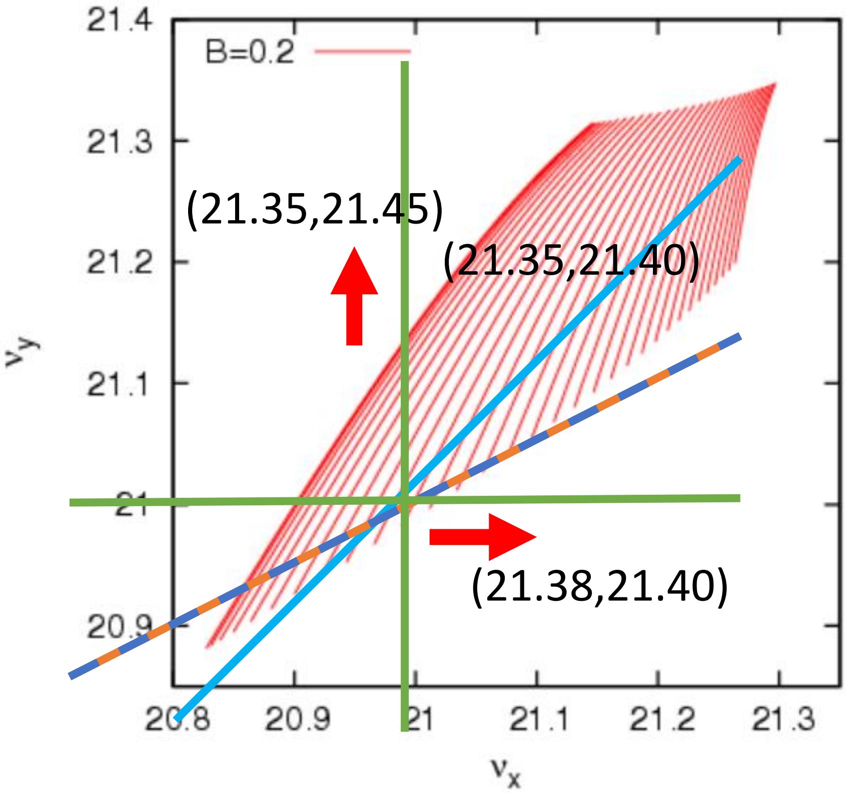


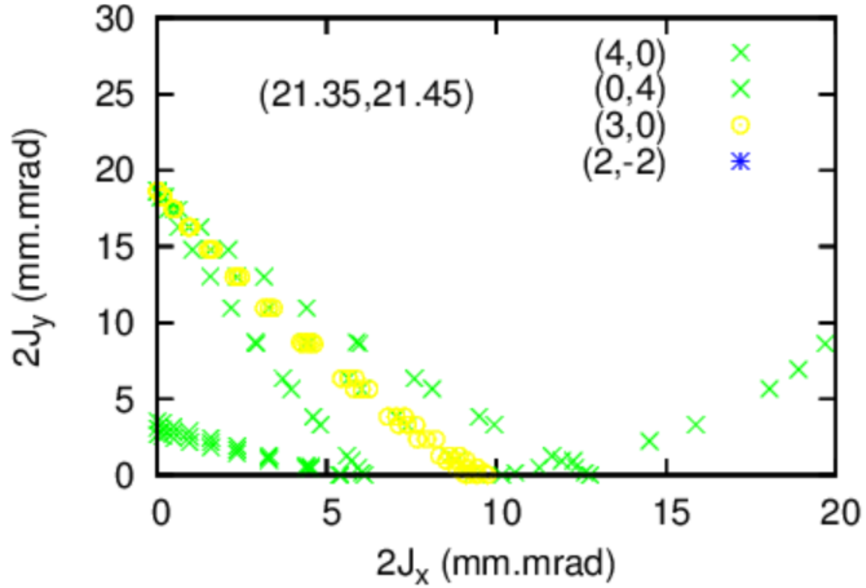


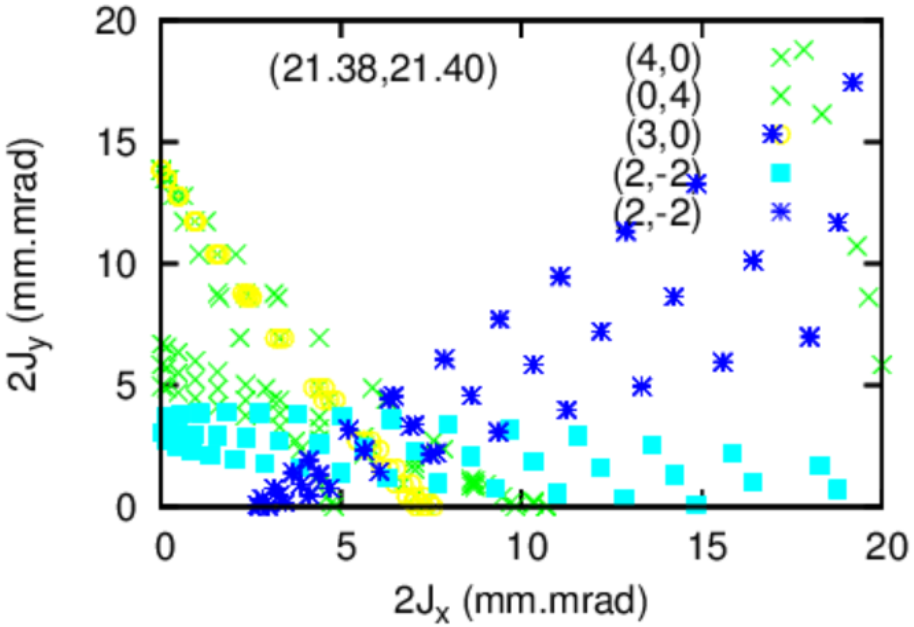
- Only (1,2) resonance is seen at (21.35,21.45).

Resonance overlap (small amplitude)

- BF=0.2







Synchrotron motion

- $\sigma_z >> \sigma_{xy}$, longitudinal Gaussian distribution

Space charge potential

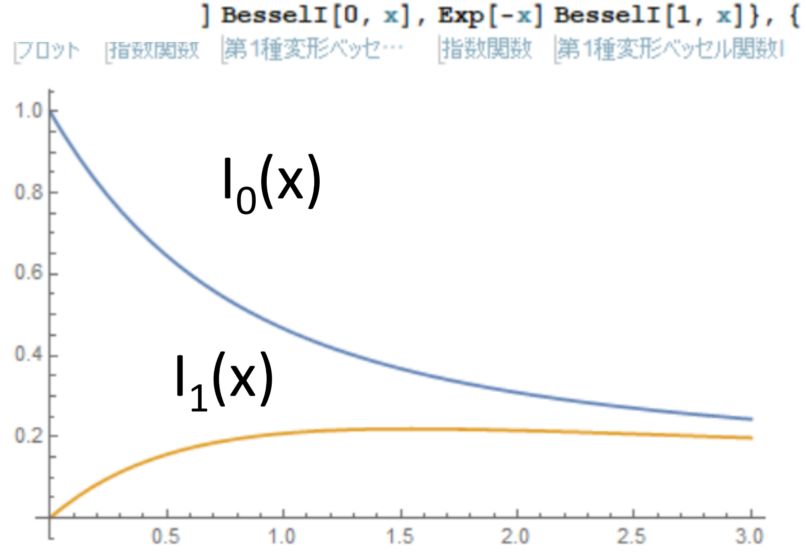
$$U^{(3)}(x,y,z) = \lambda_p(z) U(x,y) \qquad \lambda_p(z) = \frac{N_p}{\sqrt{2\pi}\sigma_z} e^{-z^2/2\sigma_z^2}$$

$$U^{(3)}(\boldsymbol{J},\boldsymbol{\phi}) = \lambda_p(J_z,\phi_z) U(J_x,\phi_x,J_y,\phi_y) \qquad \boldsymbol{J} = (J_x,J_y,J_z)$$

$$U^{(3)}(\boldsymbol{J},\boldsymbol{\phi}) = \sum_{\boldsymbol{m}} U^{(3)}_{\boldsymbol{m}}(\boldsymbol{J}) \exp(-i\boldsymbol{m}\cdot\boldsymbol{\phi}) \qquad \boldsymbol{m} = (m_x,m_y,m_z)$$

$$U^{(3)}_{\boldsymbol{m}}(\boldsymbol{J}) = e^{-J_z/2\varepsilon_z}(-1)^{m_z/2} I_{m_z/2}\left(\frac{J_z}{2\varepsilon_z}\right) U_{m_x,m_y}(J_x,J_y)$$

- U_0,U_m for m_z=2 are a factor smaller.
- For small J_z, U_0, tune shift does not change, but Um is small.
- $J_z \sim \varepsilon_z$, U_0,U_m is smaller e^{-1}.



Tune shift and resonance condition

- Tune shift depends on the longitudinal amplitude, Jz.

$$2\pi\Delta\nu^{(3)}_{x(y)}(\boldsymbol{J}) = e^{-J_z/2\varepsilon_z} I_0\left(\frac{J_z}{2\varepsilon_z}\right) 2\pi\Delta\nu_{x(y)}(J_x, J_y)$$

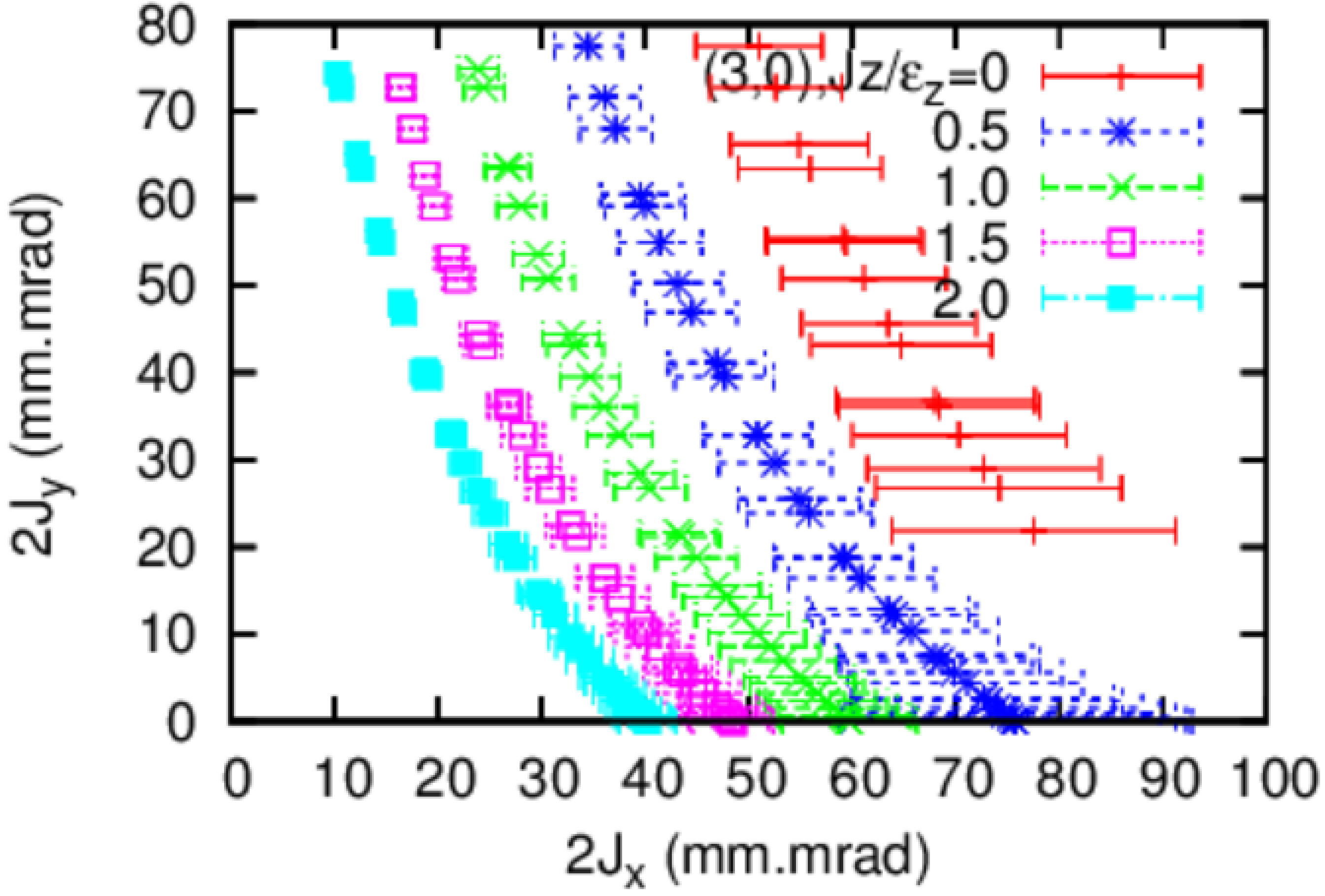
- Resonance condition. Synchrotron motion/tune is independent of the transverse motion/amplitude, approximately.

$$m_x\nu_x(\boldsymbol{J}_R) + m_y\nu_y(\boldsymbol{J}_R) + m_z\nu_z = n$$

- Resonance width of the synchrotron side-band.
- For magnets, U_m with m_z=0 is significant.

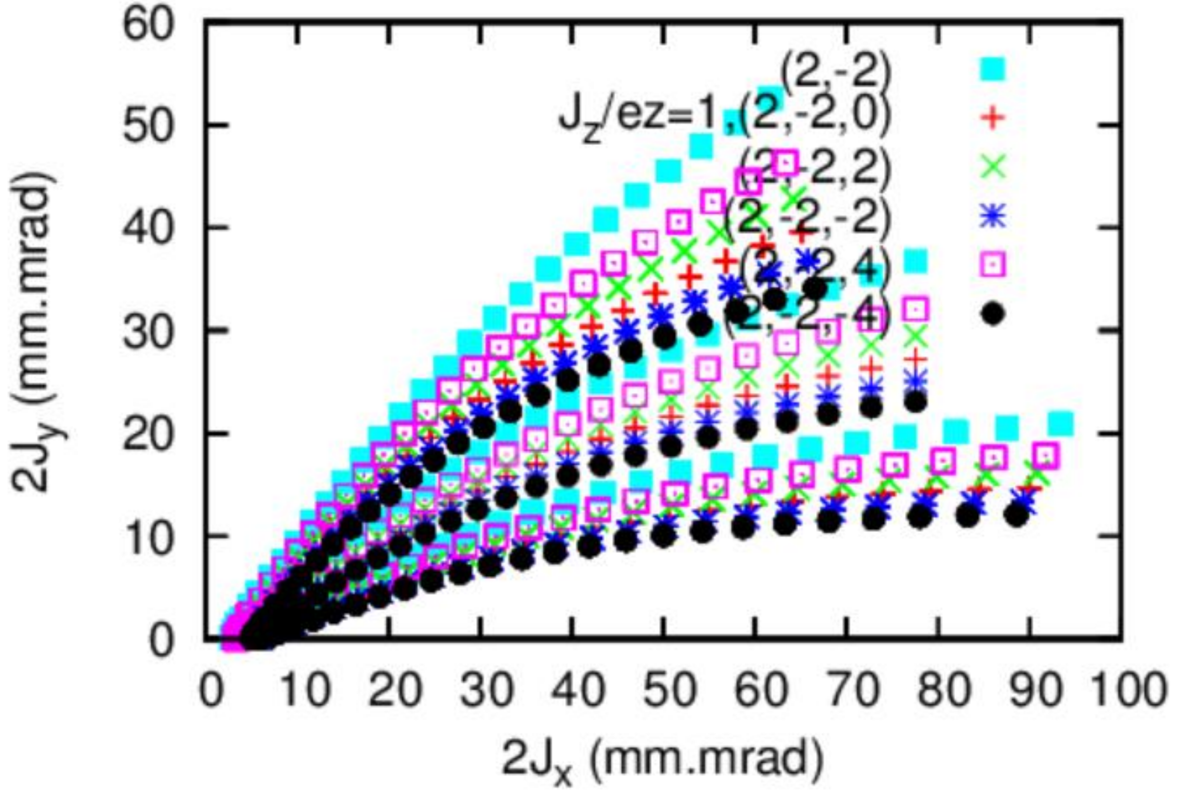
Resonance and its width for Jz

- Non-structure resonance (3,0) induced by sextupole+optics error.



Overlap of synchrotron sidebands

- BF=0.2, resonance=(2,-2,0), tune=(21.38,21.40)



- Above (1,1) line, diffusion to lower Jy with keeping Jx+Jy=const is seen.

Synchrotron motion as an external modulation

- $\sigma_z >> \sigma_{xy}$, longitudinal Gaussian distribution

Space charge potential

$$U^{(3)}(x,y,z) = \lambda_p(z) U(x,y) \qquad \lambda_p(z) = \frac{N_p}{\sqrt{2\pi}\sigma_z} e^{-z^2/2\sigma_z^2}$$

$$U^{(3)}(\boldsymbol{J},\boldsymbol{\phi},z) = \lambda_p(z) U(\boldsymbol{J},\boldsymbol{\phi}) \qquad \boldsymbol{J} = (J_x, J_y) \text{ and } \boldsymbol{\phi} = (\phi_x, \phi_y)$$

$$U^{(3)}(\boldsymbol{J},\boldsymbol{\phi},z) = \lambda_p(z) \sum_{\boldsymbol{m}} U_{\boldsymbol{m}}(\boldsymbol{J}) \exp(-i\boldsymbol{m}\cdot\boldsymbol{\phi}) \qquad \boldsymbol{m} = (m_x, m_y)$$

- Potential for magnets is independent of z.

$$H = \boldsymbol{\mu}\cdot\boldsymbol{J} + \bar{U}_0 + \delta U_0(\boldsymbol{J},t)$$

$$\bar{U}_0 = \bar{\lambda}(J_z) U_0 = e^{-J_z/2\varepsilon_z} U_0(\boldsymbol{J})$$

$$\delta U_0(\boldsymbol{J},t) = e^{-J_z/2\varepsilon_z} \left[\sum_{k=-\infty}^{\infty} I_k \left(\frac{J_z}{2\varepsilon_z}\right) e^{-2ik\phi(t)} - 1 \right] U_0(\boldsymbol{J})$$

- Resonance condition for particle with Jz, $\boldsymbol{m}\cdot(\boldsymbol{\mu} + e^{-J_z/2\varepsilon_z}\Delta\boldsymbol{\mu}(\boldsymbol{J}_R)) = n$.

Map modulated by external synchrotron oscillation

- t: turn

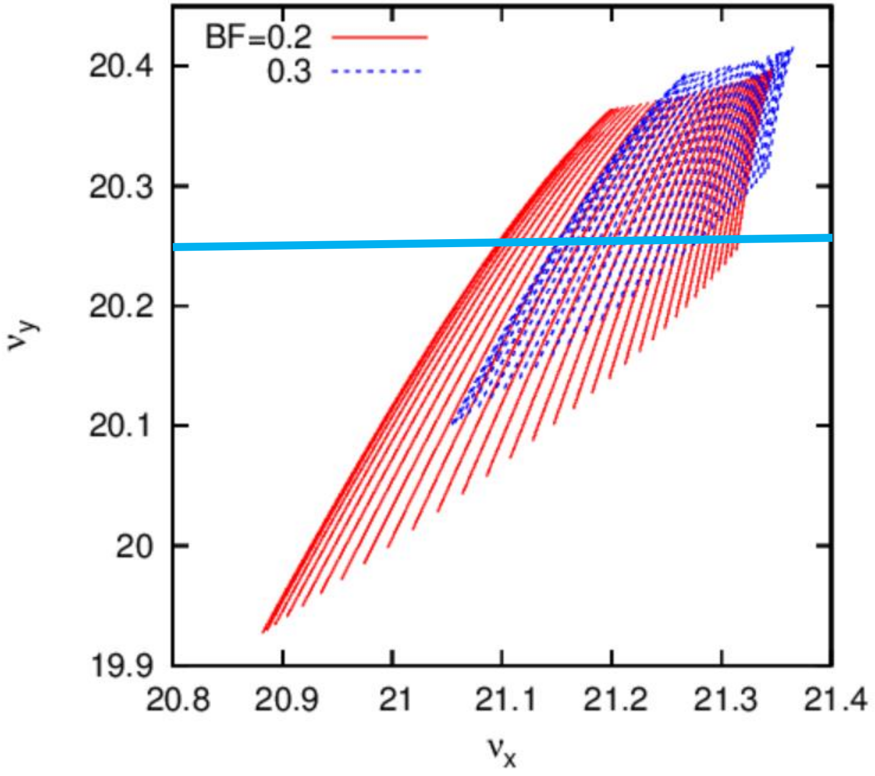
$$\begin{aligned} P_{1,t+1} &= P_{1,t} + U\boldsymbol{m}\sin\psi_{1,t} \\ \psi_{1,t+1} &= \psi_{1,t} + \Lambda P_{1,t+1} + f(\mu_s t) \end{aligned}$$

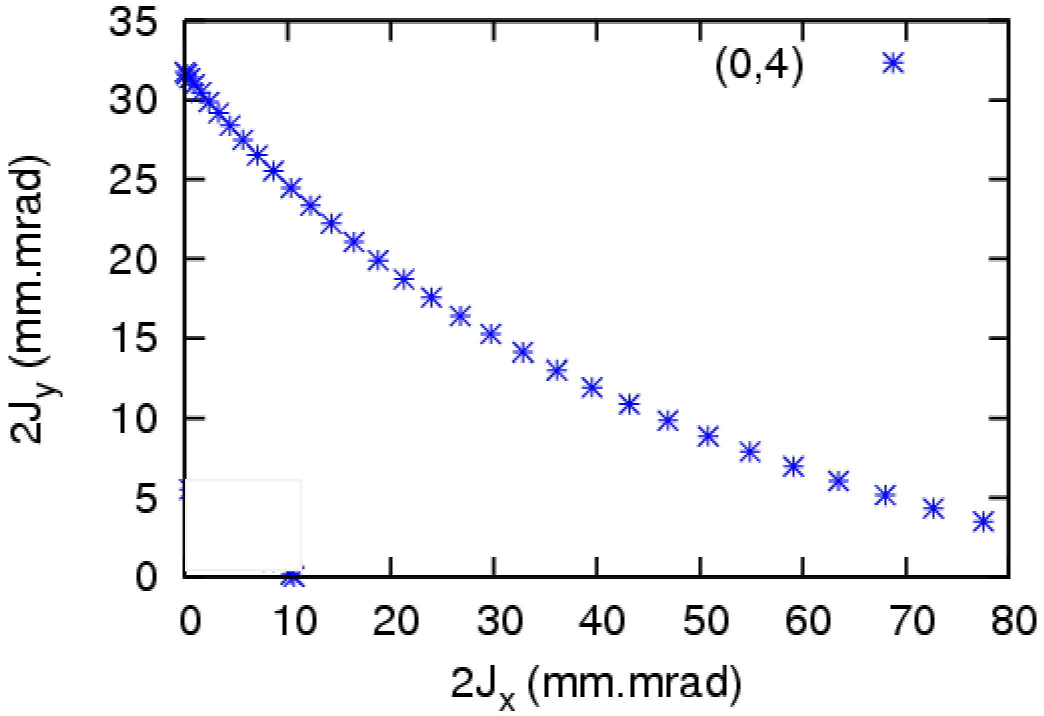
$$f(\mu_s t) = \boldsymbol{m}\cdot\delta\boldsymbol{\mu}(\boldsymbol{J}_R) \times e^{-J_z/2\varepsilon_z}\left[\sum_{k=-\infty}^{\infty} I_k\left(\frac{J_z}{2\varepsilon_z}\right)e^{-2ik\mu_s t} - 1\right]$$

- This is typical map for separatrix crossing (Lichtenberg, Lieberman, p. 365)
- Resonances related to magnets should show modulational diffusion.

Resonances for (21.40,20.45)

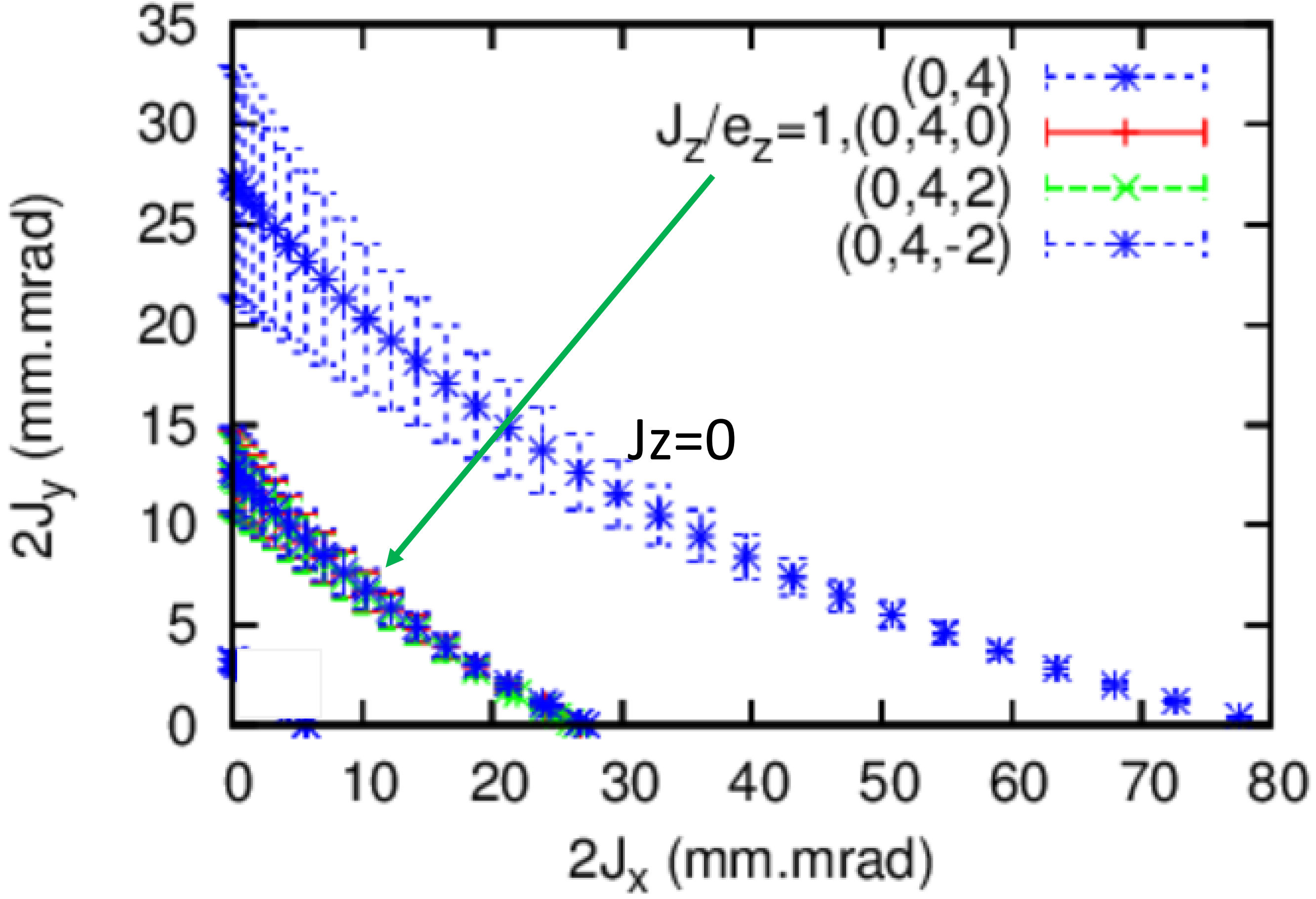
- Structure resonance driven by space charge at ν_y=20.25, (0,4,81)





MR aperture: 2J=60-80 mm.mrad

Resonance width, synchrotron sideband for space charge driven (0,4,81)



- Synchrotron sidebands overlap.

MR aperture: 2J=60-80 mm.mrad

Summary

- Resonances induced by space charge force have been studied for J-PARC MR operated at tune around (21.35,21.4), where super-periodicity 3.
- Structure Resonances are driven by space charge force and magnet nonlinearity, ν_x-ν_y=0, ν_x-2ν_y=-21.
- Non-structure resonances are driven by optics error at nonlinear magnets, 3ν_x=63, ν_x-2ν_y=-21.
- Resonance width for the structure and non-structure resonances is evaluated.
- (21.35,21.45) is less resonances compare with (21.38,21.40).
- Resonance overlap of synchrotron sidebands (under-studying).