

Configuration Interaction Studies of Nuclear Clustering

Alexander Volya

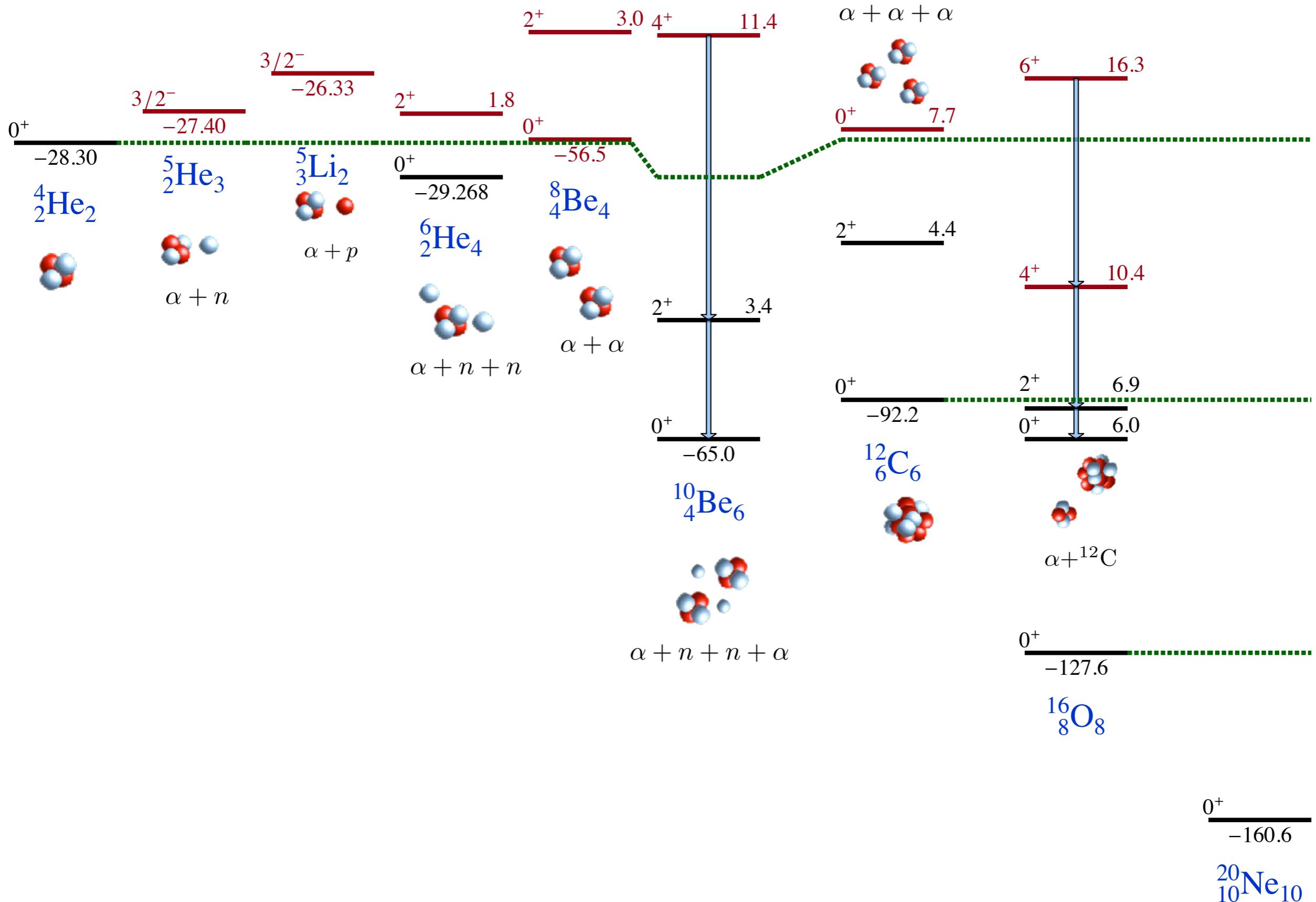
Florida State University

DOE support: DE-SC0009883

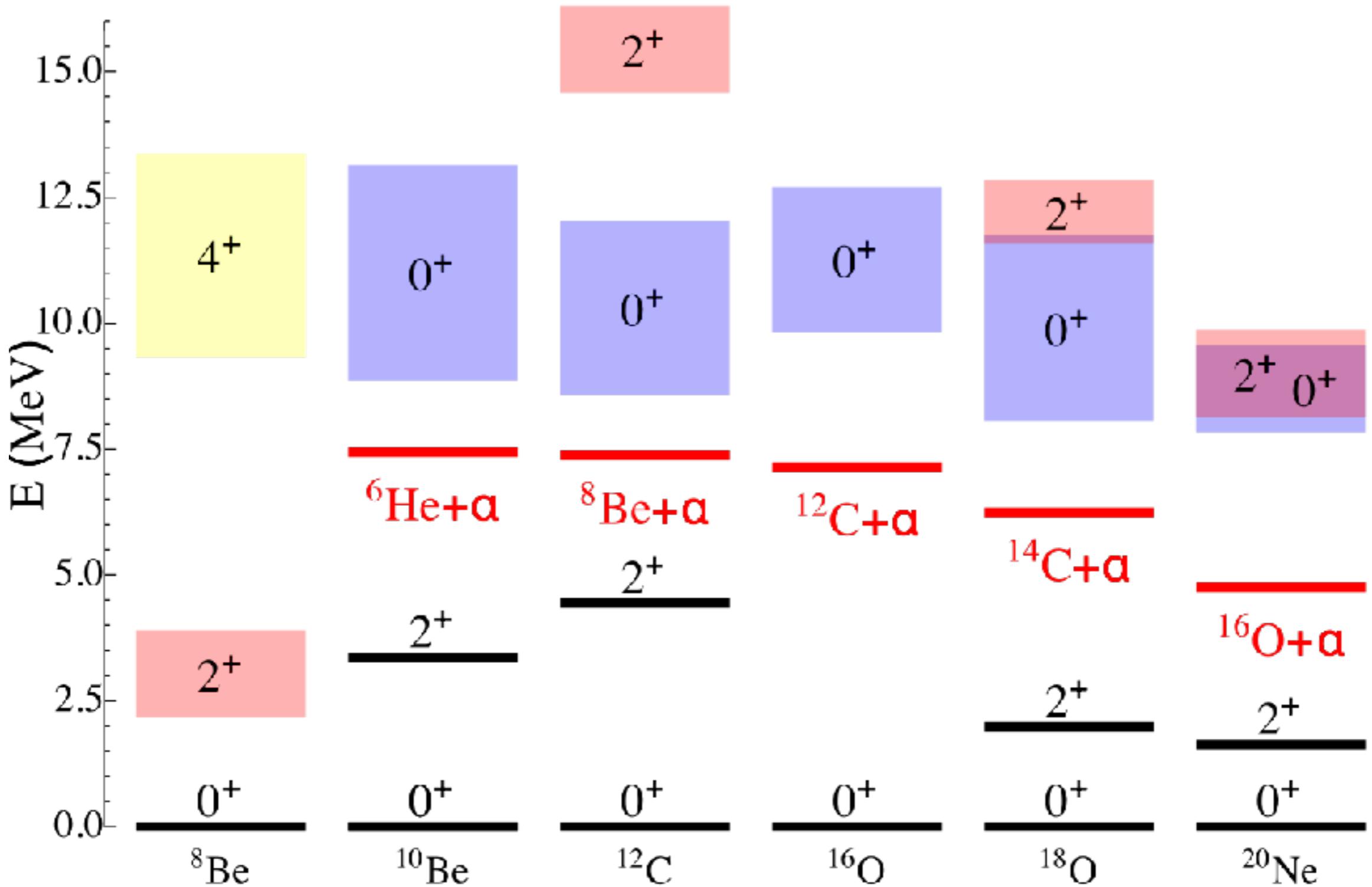
FRIB, MI

Clustering in light nuclei

Connecting bound state calculations with scattering and reactions



Clustering and continuum



Configuration Interaction

State, equivalent to operator (polymorphism)

$$|\Psi\rangle \equiv \hat{\Psi}^\dagger |0\rangle = \sum_{\{1,2,3,\dots A\}} \langle 1, 2 \dots A | \Psi \rangle \hat{a}_1^\dagger \hat{a}_2^\dagger \dots \hat{a}_A^\dagger |0\rangle$$

$$|\Psi_\alpha\rangle = \Psi_\alpha^\dagger |0\rangle = \sum_{\{m\}} X_m^\alpha a_{m_1}^\dagger a_{m_2}^\dagger a_{m_3}^\dagger a_{m_4}^\dagger |0\rangle$$

$$|\Psi_D\rangle = \Psi_D^\dagger |0\rangle = \sum_{\{m\}} X_m^D a_{m_1}^\dagger a_{m_2}^\dagger \dots a_{m_{A_D}}^\dagger |0\rangle$$

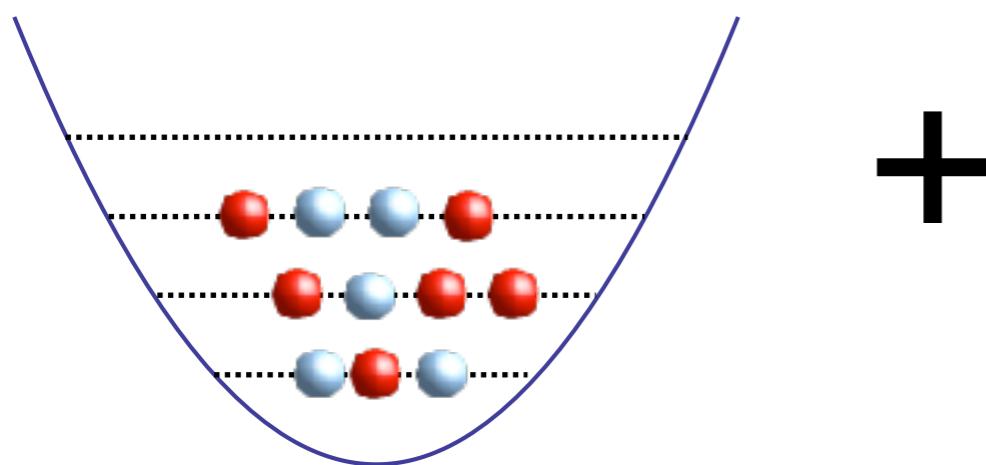
Anti-symmetrized channel
wave function components
are generated by acting
with state creation operator
and forward ordering.

$$|\Psi_C\rangle = \Psi_\alpha^\dagger \Psi_D^\dagger |0\rangle$$

Configuration interaction approach and clustering

Traditional shell model configuration
m-scheme

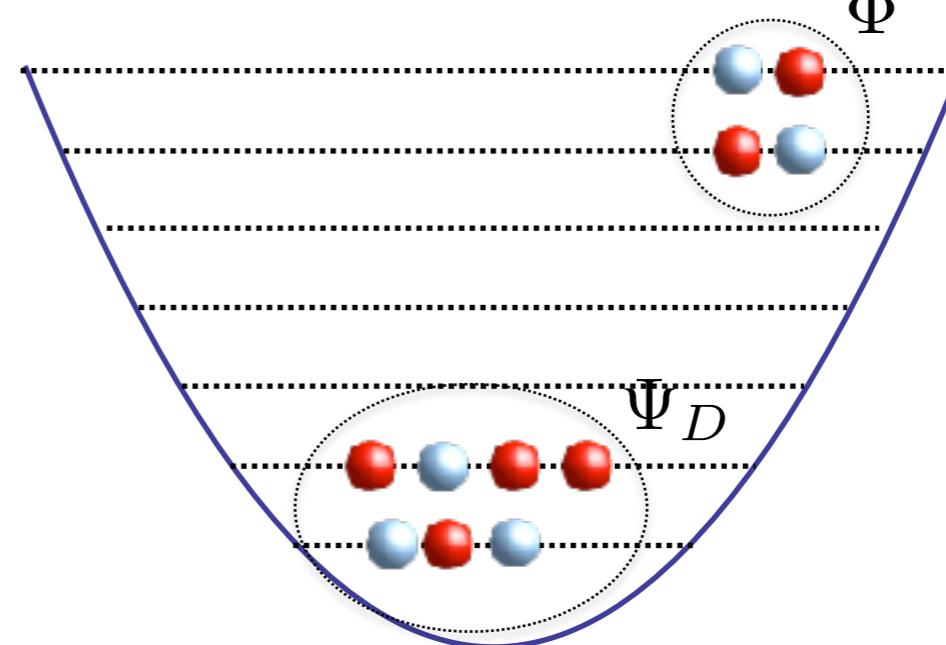
$$|\Psi\rangle = \Psi^\dagger |0\rangle \sim a_1^\dagger a_2^\dagger \dots a_A^\dagger |0\rangle$$



$$|\Psi\rangle$$

Cluster configuration
SU(3)-symmetry basis

$$|\text{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger\Psi_D^\dagger|0\rangle$$

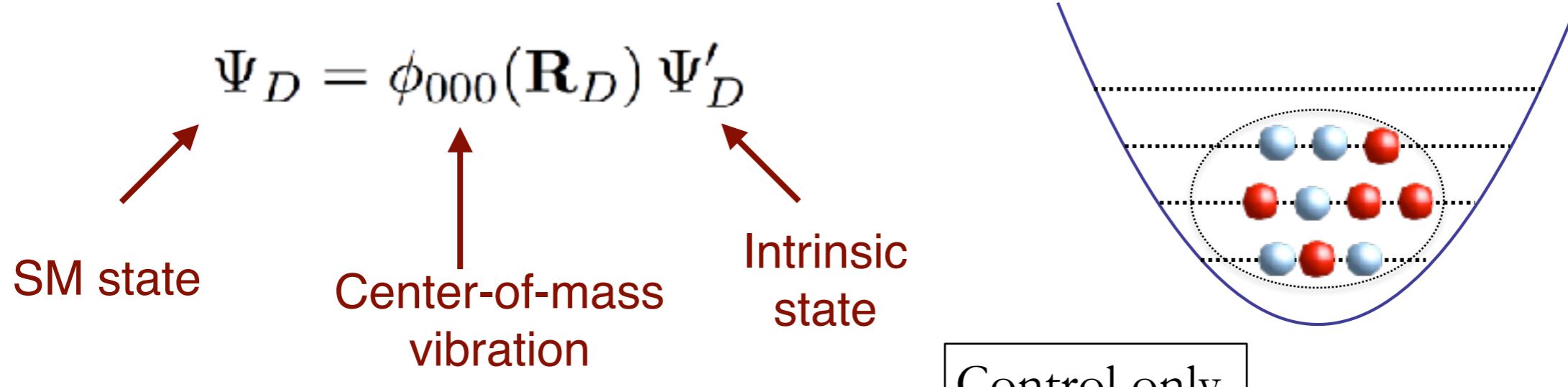


+

$$\Phi^\dagger|\Psi_D\rangle$$

Translational invariance and Center of Mass (CM)

Shell model, Glockner-Lawson procedure



Controlling CM

$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu$$

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}} (\mathcal{B}_\mu^\dagger + \mathcal{B}_\mu)$$

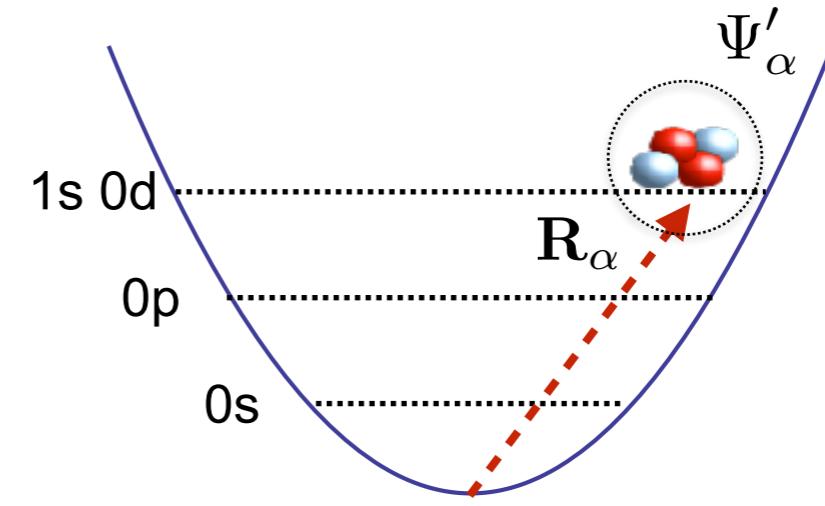
Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R}) \Psi'$$

\mathcal{B}^\dagger and \mathcal{B} CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \mathcal{B}^\dagger \cdot \mathcal{B}^\dagger \Psi_{n\ell m}$$

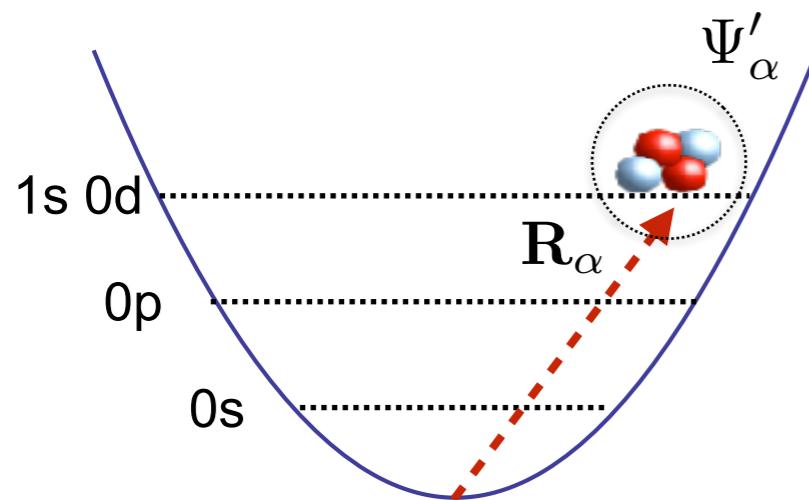
$\mathcal{B}^\dagger \times \mathcal{B}$ CM angular momentum operator



Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(p)(sd)(sdg)(pfh)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

Select configuration content of NCSM wave functions for ${}^4\text{He}$ with $\Omega = 20$ MeV boosted by 8 quanta ($L = 0$).

Approximation of $N_{\max}=0$ (s^4) Cluster coefficients for SU(3) components



Expand SU(3) 4-nucleon structure in intrinsic+ relative
all oscillator quanta of excitation are in relative motion.

$$\Psi_{n\ell m} = \sum_{\eta} X_N^{\eta} \Phi_{(N,0):\ell m}^{\eta}$$

$$X_N^{\eta} = \sqrt{\frac{1}{4^N} \frac{N!}{\prod_i (N_i!)^{\alpha_i}} \frac{4!}{\prod_i \alpha_i!}}$$

Volya and Yu. M. Tchuvil'sky, Phys. Rev. C 91, 044319 (2015).

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).

M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).

O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R}) \Psi'$$

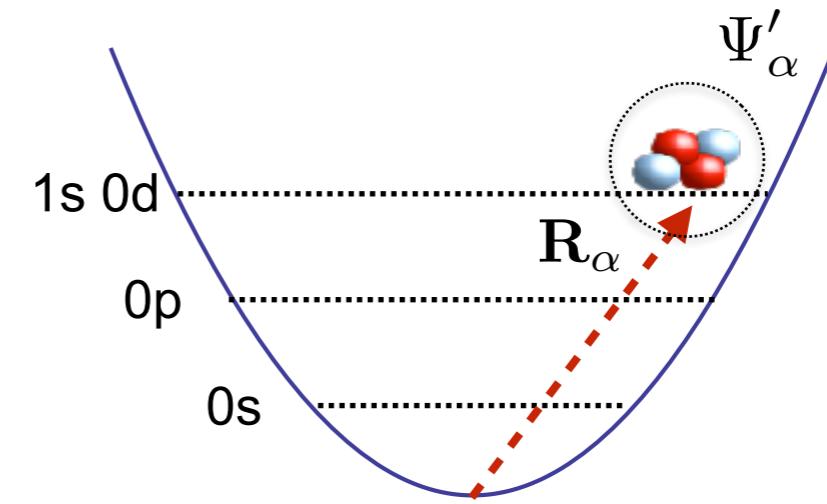
\mathcal{B}^\dagger and \mathcal{B} CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \mathcal{B}^\dagger \cdot \mathcal{B}^\dagger \Psi_{n\ell m}$$

$\mathcal{B}^\dagger \times \mathcal{B}$ CM angular momentum operator

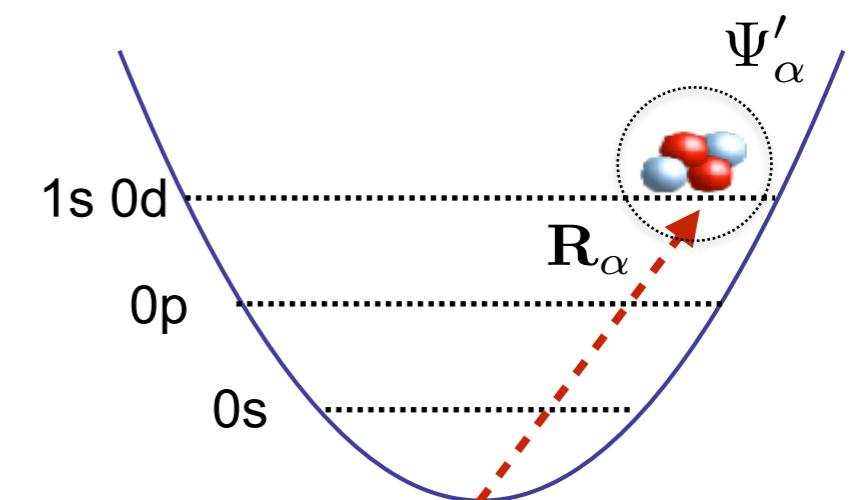
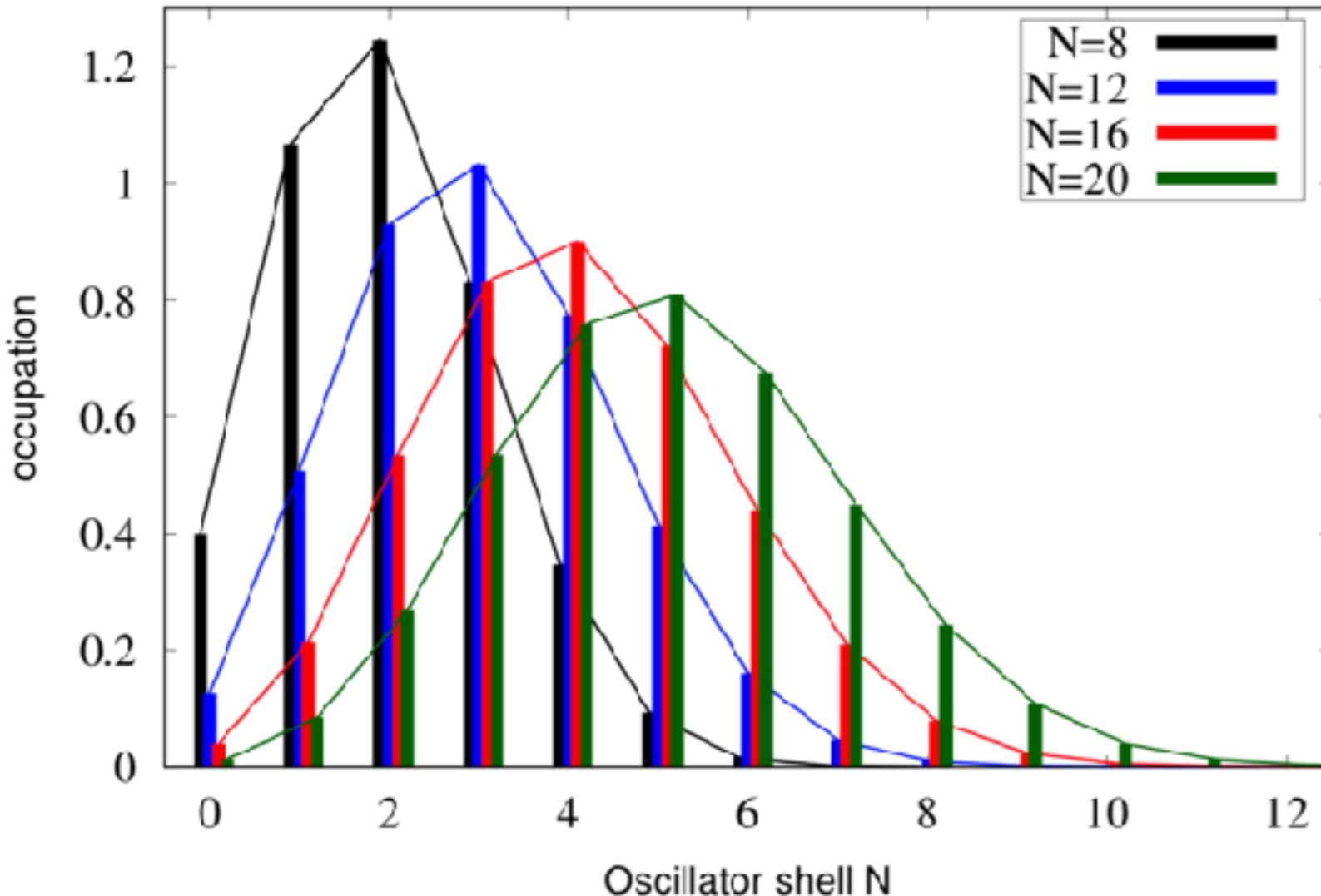
$$\Psi_\alpha = \phi_{n\ell m}(\mathbf{R}) \Psi'_\alpha = \sum_n X_{n\ell}^\eta \Phi_{(n,0):\ell m}^\eta$$

Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(p)(sd)(sdg)(pfh)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

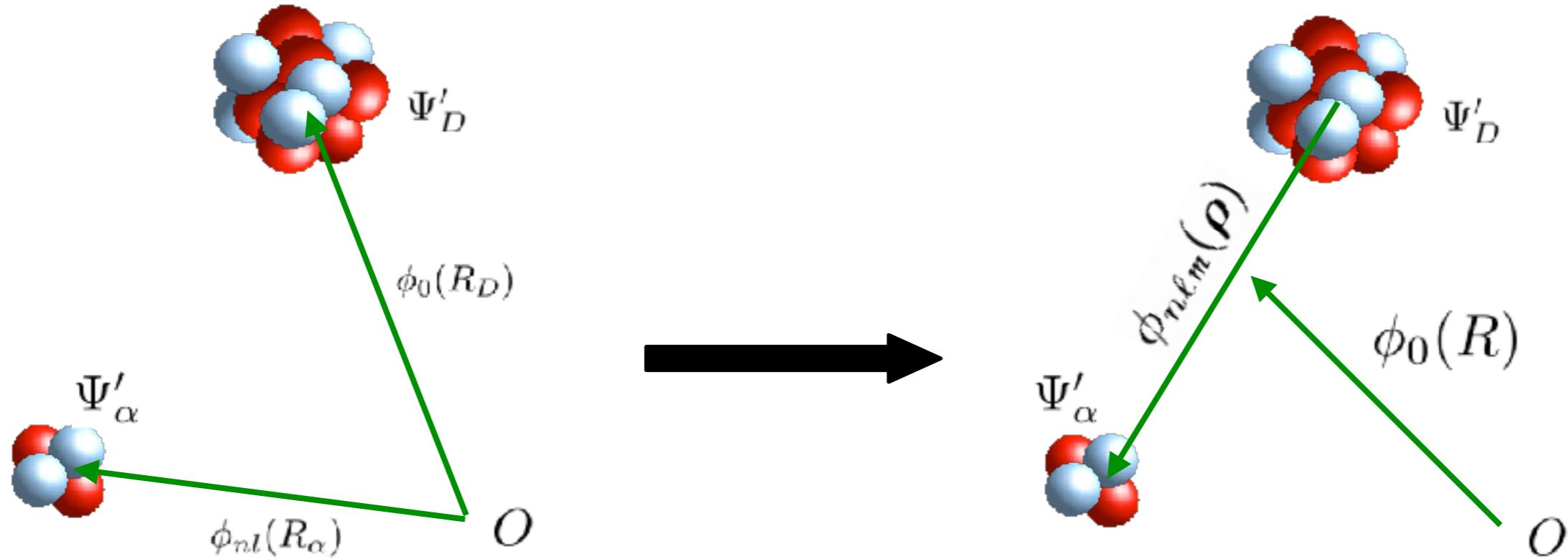


Select configuration content of NCSM wave functions for ${}^4\text{He}$ with $\Omega = 20$ MeV boosted by 8 quanta ($L = 0$).

CM-boosted configuration from shell model perspective



Recoil Recoupling

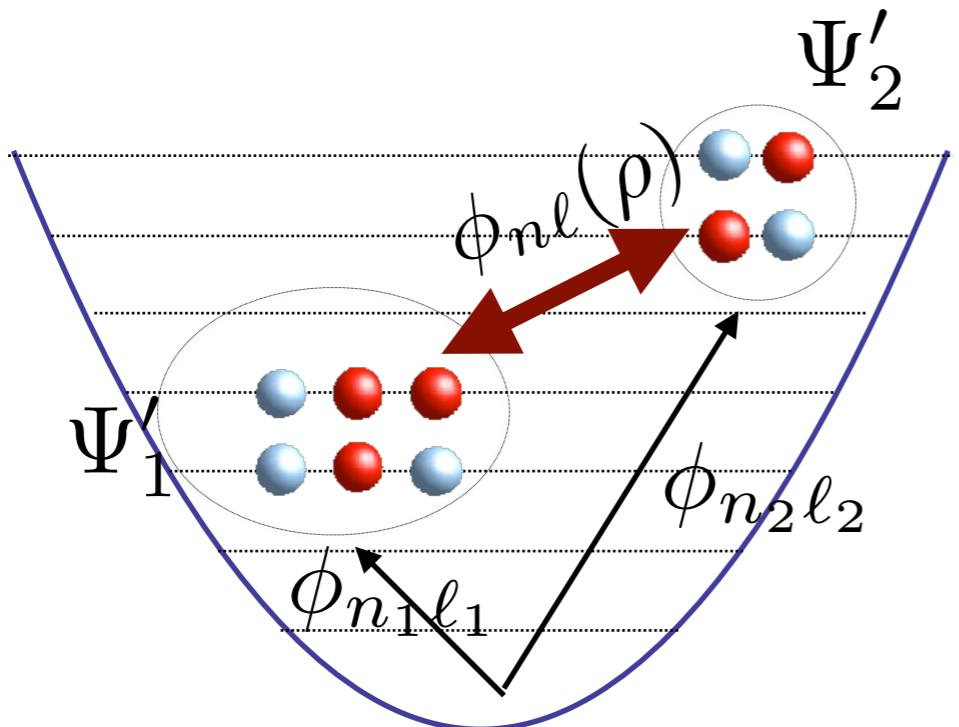


- Recoupling is done with Talmi-Moshinsky brackets

$$\Phi_{n\ell m} = \mathcal{A} \left\{ \phi_{000}(\mathbf{R}) \phi_{n\ell m}(\boldsymbol{\rho}) \Psi'^{(1)} \Psi'^{(2)} \right\}$$

Center-of-mass recoil correction

Channel of relative motion



$$\Psi = \phi_{000}(\mathbf{R}) \Psi'$$

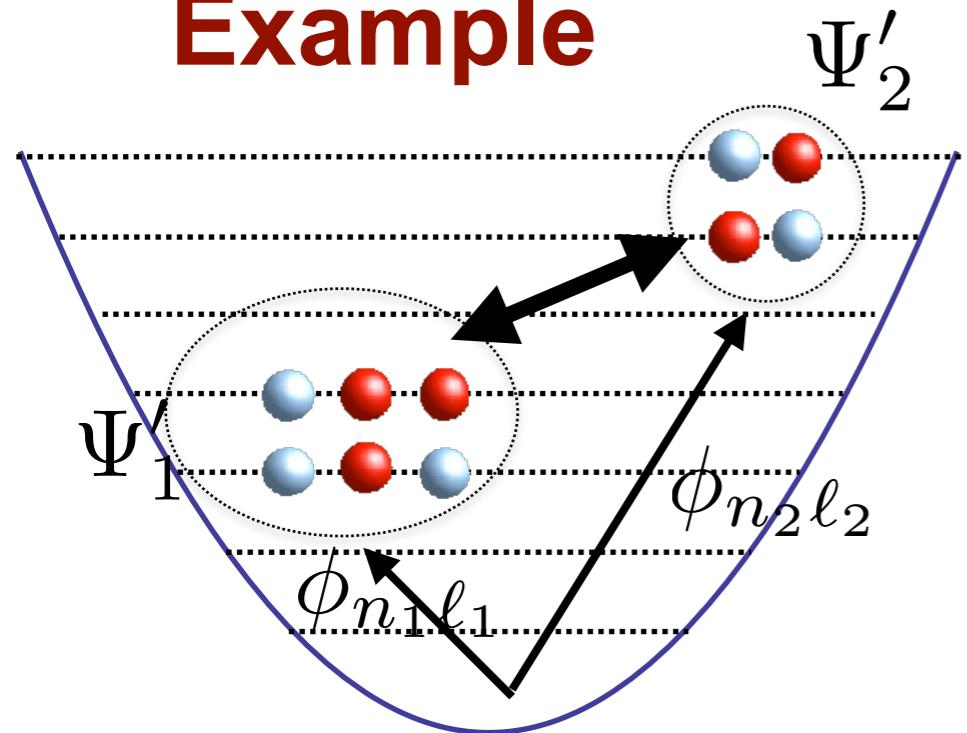
Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R}) \Psi'$$

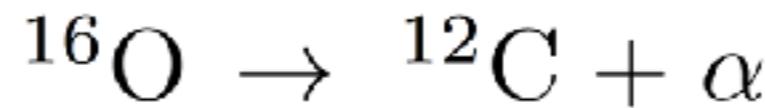
CM-Recouple

$$\Phi_{n\ell}^\dagger = \sum_{\substack{n_1\ell_1 \\ n_2\ell_2}} \mathcal{M}_{n_1\ell_1 n_2\ell_2}^{n\ell 00;\ell} \left[\Psi_{n_1\ell_1 m_1}^\dagger \times \Psi_{n_2\ell_2 m_2}^\dagger \right]_\ell$$

Example



Channel of relative motion



$$\langle \Psi^{(16\text{O})} | \Phi_{20} \rangle = \sum_{\substack{n_1 \ell_1 \\ n_2 \ell_2}} \mathcal{M}_{n_1 \ell_1 n_2 \ell_2}^{2000;0} \langle \Psi^{(16\text{O})} | \Psi_{n_1 \ell_1 m_1}^\dagger \Psi_{n_2 \ell_2 m_2}^\dagger | 0 \rangle$$

n_1	ℓ_1	n_2	ℓ_2	$\mathcal{M}_{n_1 \ell_1 n_2 \ell_2}$	$(0p_{3/2})^8$	SU(3)
0	0	2	0	$\frac{9}{16} \approx 0.563$	0.0761	$\sqrt{3/32}$
0	1	1	1	$-\frac{3\sqrt{3}}{8} \approx -0.650$	-0.0878	$-1/\sqrt{8}$
0	2	0	2	$\sqrt{\frac{3}{32}} \approx 0.306$	0.0414	$1/\sqrt{32}$
1	0	1	0	$\sqrt{\frac{15}{128}} \approx 0.342$	0.0463	$\sqrt{5}/12$
1	1	0	1	$-\frac{\sqrt{3}}{8} \approx -0.217$	-0.0293	$-1/\sqrt{72}$
2	0	0	0	$\frac{1}{16} \approx 0.063$	0.0085	$1/\sqrt{864}$

Exact SF

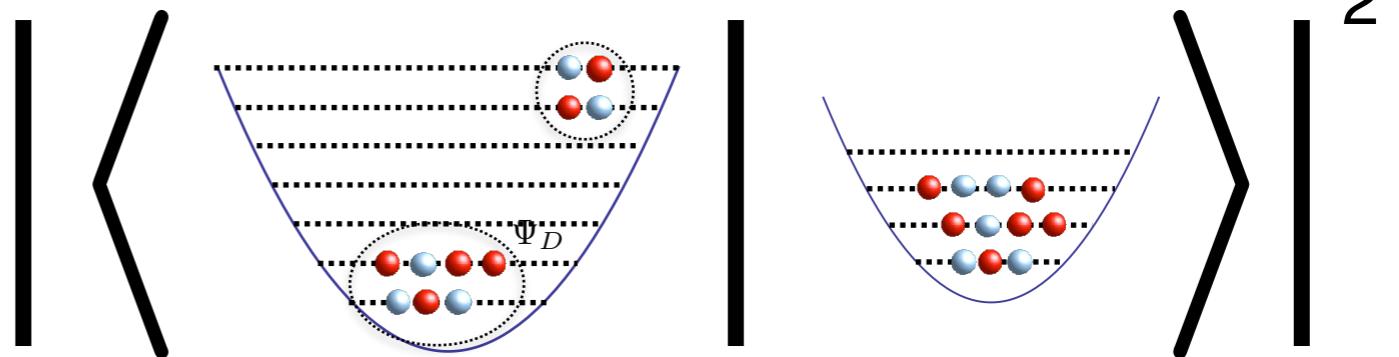
0.0183

8/27=0.296

Cluster Spectroscopic Characteristics

Traditional spectroscopic factor

$$S_{n,\ell} \equiv \left| \langle \Psi^{(A)} | \Phi_{n\ell} \rangle \right|^2 =$$



$$\Phi_{n\ell}^\dagger = \sum_{\substack{n_1 \ell_1 \\ n_2 \ell_2}} \mathcal{M}_{n_1 \ell_1 n_2 \ell_2}^{n\ell 00; \ell} \left[\Psi_{n_1 \ell_1 m_1}^\dagger \times \Psi_{n_2 \ell_2 m_2}^\dagger \right]_\ell$$

$$\mathcal{M}_{n_1 \ell_1 n_2 \ell_2}^{n\ell 00; \ell} \langle \Psi^{(A)} | \Phi_{n\ell} \rangle = \langle \Psi^{(A)} | \left[\Psi_{n_1 \ell_1 m_1}^\dagger \times \Psi_{n_2 \ell_2 m_2}^\dagger \right]_\ell \rangle$$

$$\langle \Psi^{(A)} | \Phi_{n\ell} \rangle = R \langle \Psi^{(A)} | \left[\Psi^\dagger \times \Psi_{n\ell m}^\dagger \right]_\ell | 0 \rangle$$

Recoil Factor

$$R = \left(\mathcal{M}_{00n\ell}^{n\ell 00; \ell} \right)^{-1} = (-1)^{N_{\text{rel}}} \left(\frac{A}{A_1} \right)^{N_{\text{rel}}/2}$$

$$S_\ell = \sum_n S_{n,\ell}$$

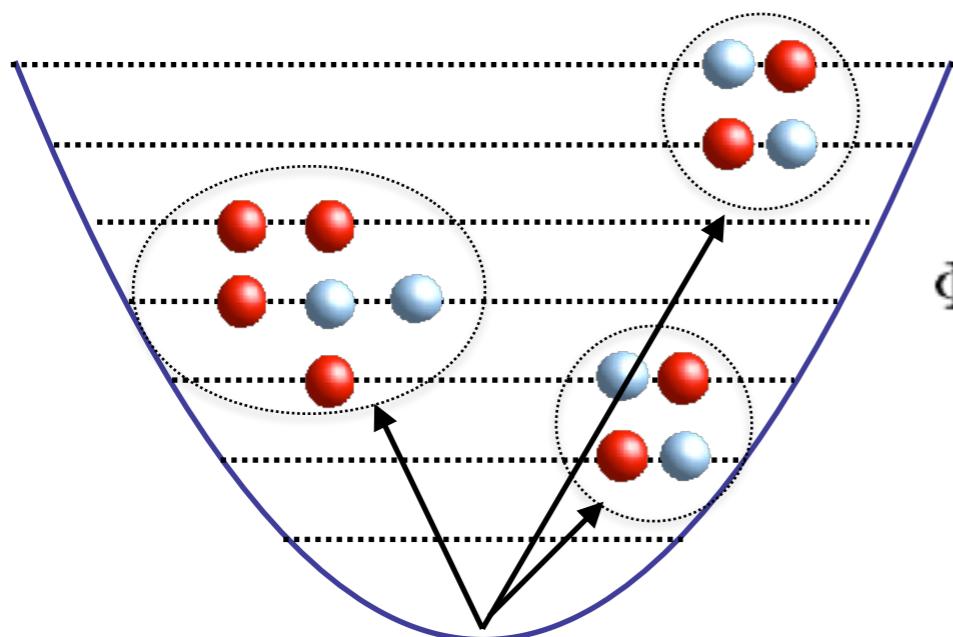
²

Channels, spectroscopic factors examples

parent	channel	N_c	$ \langle \Psi \Phi_{nl} \rangle $	$\langle \Phi_{nl} \Phi_{nl} \rangle$
$^{16}\text{O}[0]$	$^{12}\text{C}[(0, 4)] + \alpha[0]$	4	$\sqrt{8/27}$	$8/27$
$^{16}\text{O}[0]$	$^{12}\text{C}[\text{p}_{3/2}^8] + \alpha[0]$	4	0.135	0.018
$^{16}\text{O}[0]$	$^{12}\text{C}[\text{p}_{3/2}^8] + \alpha[4]$	4	0.130	0.017
$^8\text{Be}[(4, 0)]$	$\alpha[0] + \alpha[0]$	4	$\sqrt{3/2}$	$3/2$
$^8\text{Be}[0]$	$\alpha[0] + \alpha[0]$	4	1.160	$3/2$
$^8\text{Be}[4]$	$\alpha[0] + \alpha[0]$	4	0.984	$3/2$
$^8\text{Be}[4]$	$\alpha[0] + \alpha[0]$	6	0.644	$15/8$
$^8\text{Be}[4]$	$\alpha[2] + \alpha[2]$	4	0.981	1.492
$^{12}\text{C}[\text{p}_{3/2}^8]$	$\alpha[0] + \alpha[0] + \alpha[0]$	8	$1/4$	$81/80$
$^{16}\text{O}[0]$	$(\alpha[0])^4$	12	$\sqrt{3/10}$	$3/10$

$|=0$ spectroscopic amplitudes of base

Multi-cluster states



$$\Phi_{n\ell}^\dagger = \sum_{\substack{n_1\ell_1 \\ n_2\ell_2 \\ n_3\ell_3}} \mathcal{M}_{n_1\ell_1 n_2\ell_2 n_3\ell_3}^{n\ell} \Psi_{n_1\ell_1 m_1}^\dagger \Psi_{n_2\ell_2 m_2}^\dagger \Psi_{n_3\ell_3 m_3}^\dagger$$

n Internal channel basis label

l Asymptotic channel quantum numbers

Normalized Spectroscopic Factors

$$\mathcal{N}_{nn'}^{(\ell)} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle \quad \text{Norm kernel}$$

$$|\Psi_\nu^{(\ell)}\rangle = \sum_n \left(\frac{1}{\sqrt{\mathcal{N}^{(\ell)}}} \right)_{\nu n} |\Phi_{n\ell}\rangle \quad \text{Orthonormalized basis}$$

$$S_\ell^{(\text{new})} \equiv \sum_\nu \left| \langle \Psi^{(A)} | \Psi_\nu^{(\ell)} \rangle \right|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

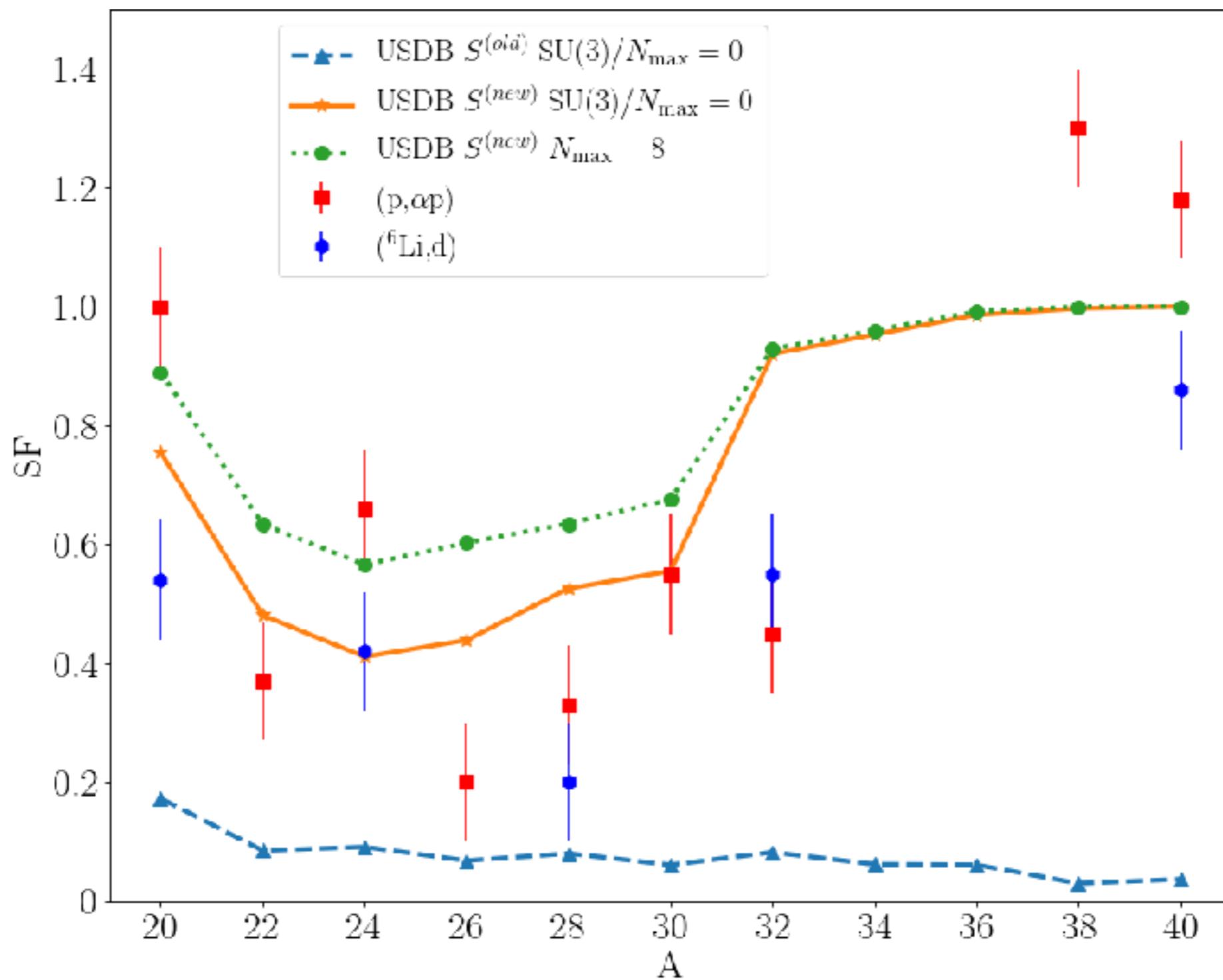
R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)

S. G. Kadomenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).

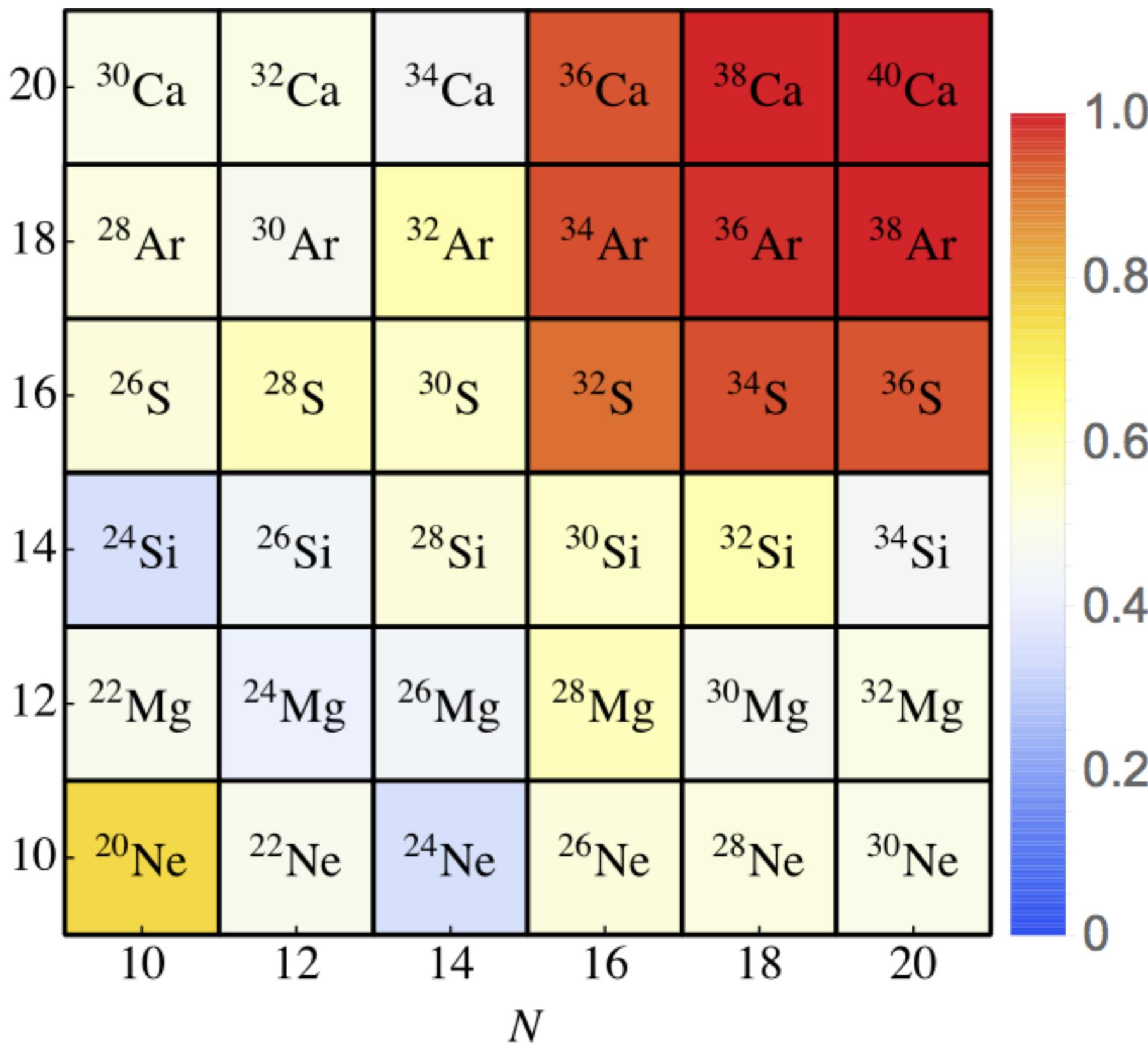
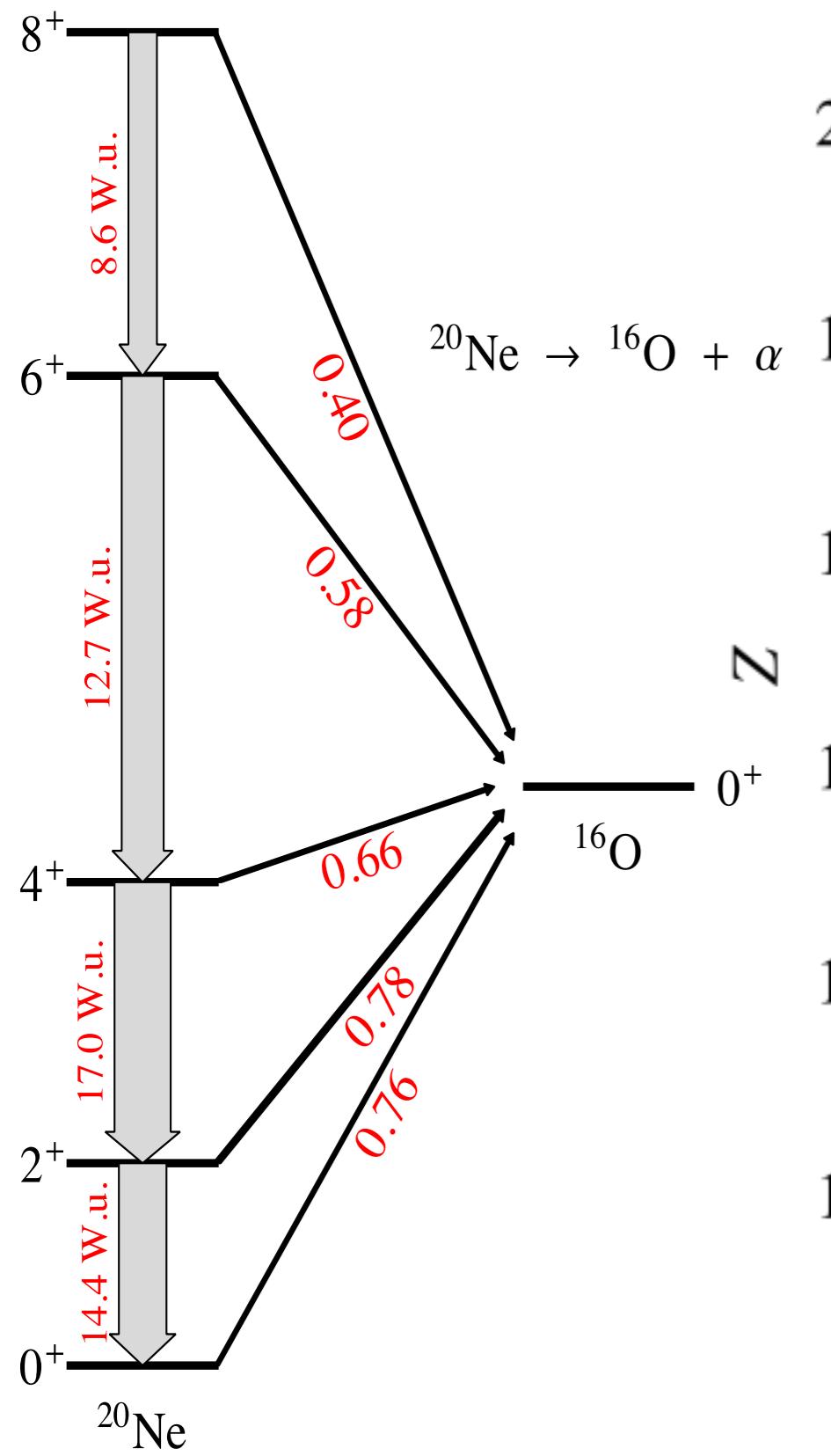
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.

T. Fliessbach and H. J. Mang, Nucl. Phys. A 263, 75–85 (1976).

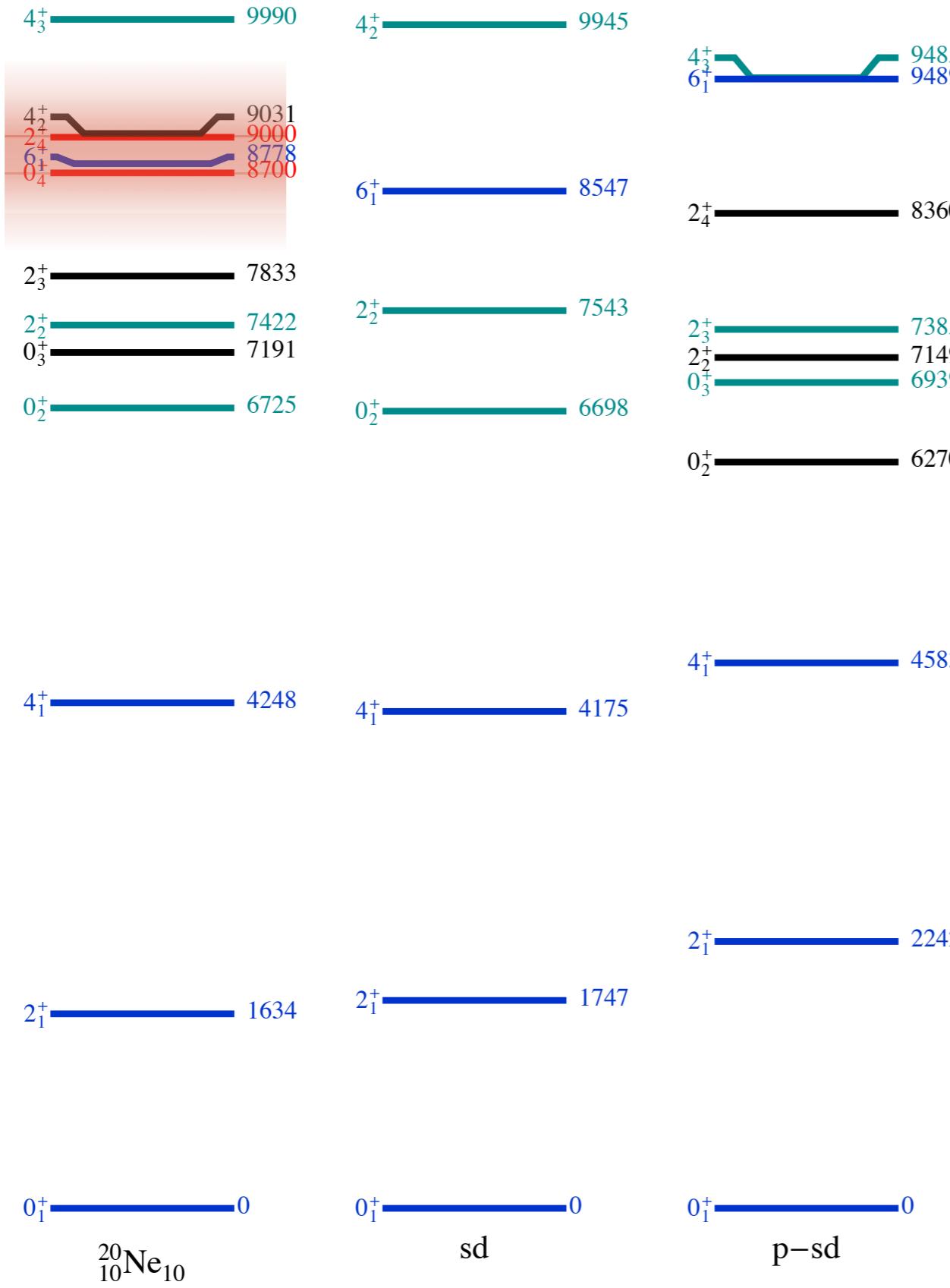
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286



- [1] T.A. Carey, P.G. Roos, N.S. Chant, A. Nadsen, H.L. Chen, Phys. Rev. C 23, 576(R) (1981)
[2] N. Anantaraman et al. Phys. Rev. Lett. 35, 1131 (1975)

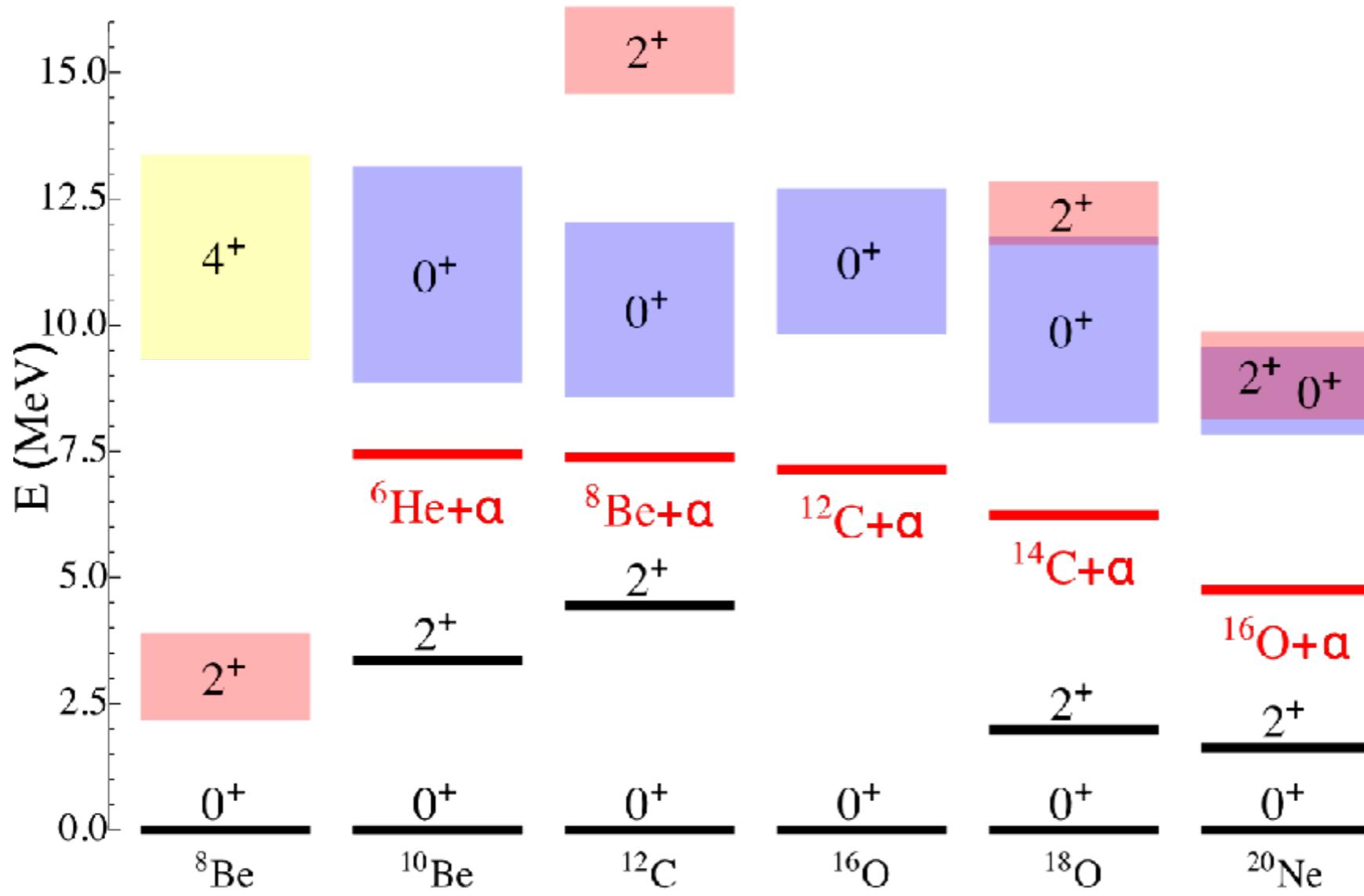


Clustering and superradiance



J	E MeV	Γ width	SF ex	SF th.
0^+	0	0		0.73
2^+	1.63	0		0.67
4^+	4.25	0		0.62
0^+	6.73	19	0.47	0.46
0^+	7.19	3.4	0.02	0.10
2^+	7.42	15	0.19	0.12
2^+	7.83	2	0.01	0.09
0^+	8.7	800	0.3	
6^+	8.78	0.11	0.5	0.51
2^+	9.00	800	0.86	

Searching for clustering states



Resonating group method

$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$

$$\sum_n \mathcal{H}_{nn'}^{(\ell)} \chi_{n'} = E \sum_n \mathcal{N}_{nn'}^{(\ell)} \chi_{n'}$$

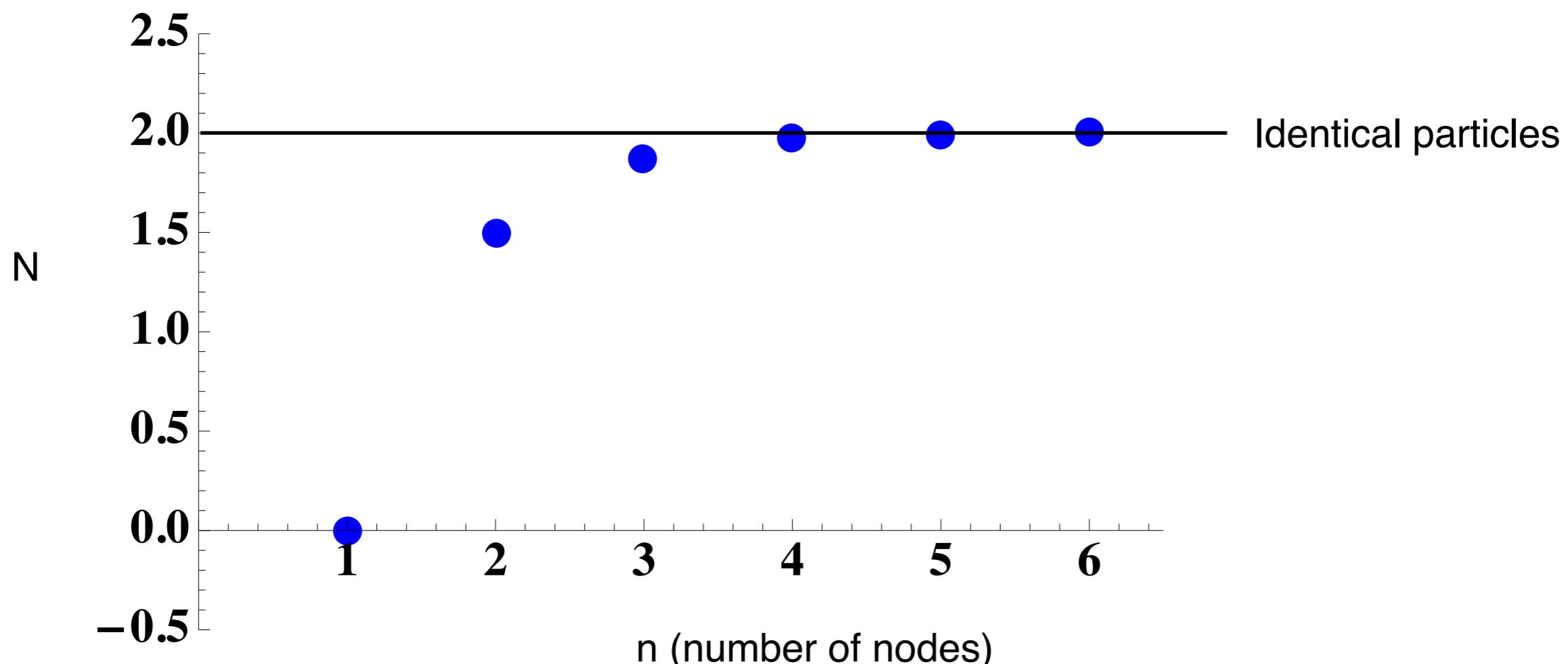
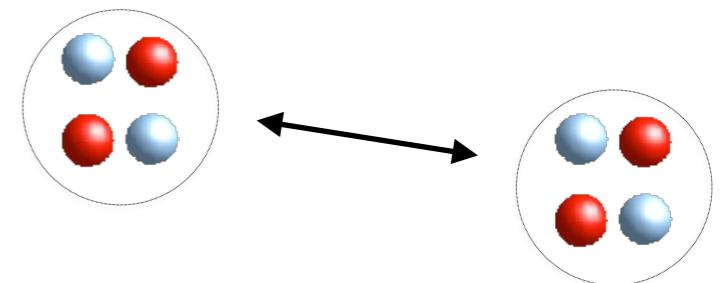
$$\mathcal{H}_{nn'}^{(\ell)}=\langle\Phi_{n\ell}|H|\Phi_{n'\ell}\rangle\qquad\qquad\mathcal{N}_{nn'}^{(\ell)}=\langle\Phi_{n\ell}|\Phi_{n'\ell}\rangle$$

Resonating group method ${}^8\text{Be}$

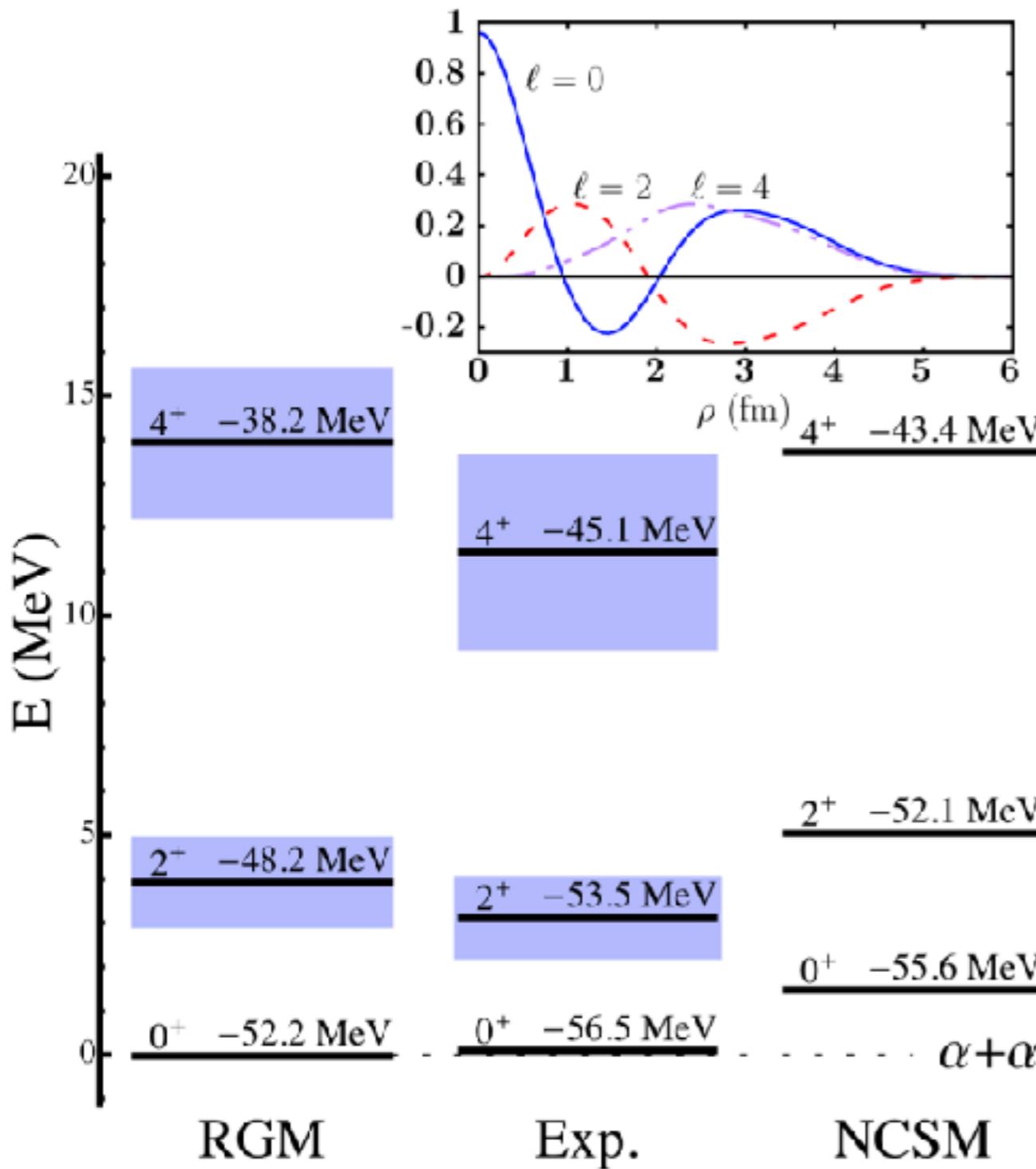
$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$

$$\sum_n \mathcal{H}_{nn'}^{(\ell)} \chi_{n'} = E \sum_n \mathcal{N}_{nn'}^{(\ell)} \chi_{n'}$$

$$\mathcal{H}_{nn'}^{(\ell)} = \langle \Phi_{n\ell} | H | \Phi_{n'\ell} \rangle \quad \mathcal{N}_{nn'}^{(\ell)} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$



Resonating group method ${}^8\text{Be}$ results

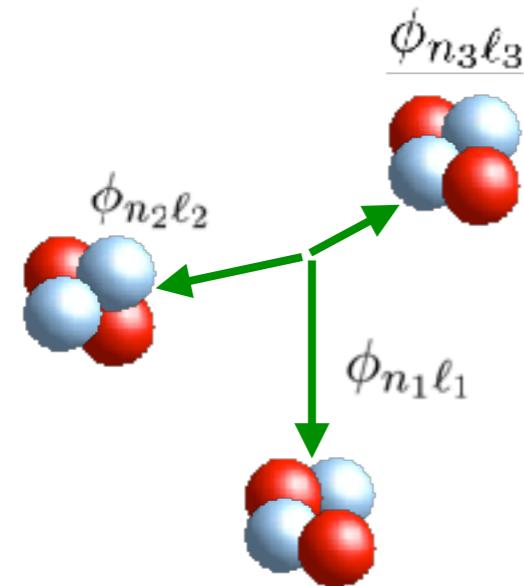


$$\hbar\Omega = 25 \text{ MeV}$$

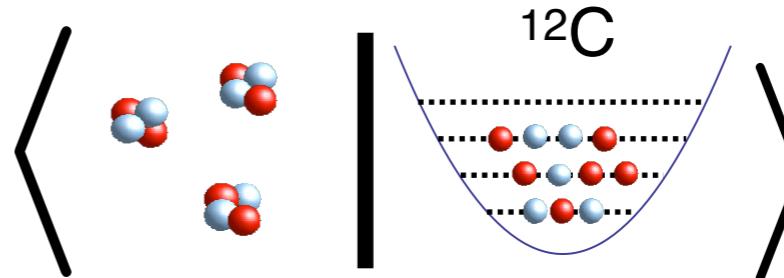
		Theory	Exp.
I=0	ev	8.7	5.6
I=2	MeV	1.3	1.5
I=4	MeV	2.1	3.5

$$\Gamma = 2P_L(\rho_c)|g(\rho_c)|^2$$

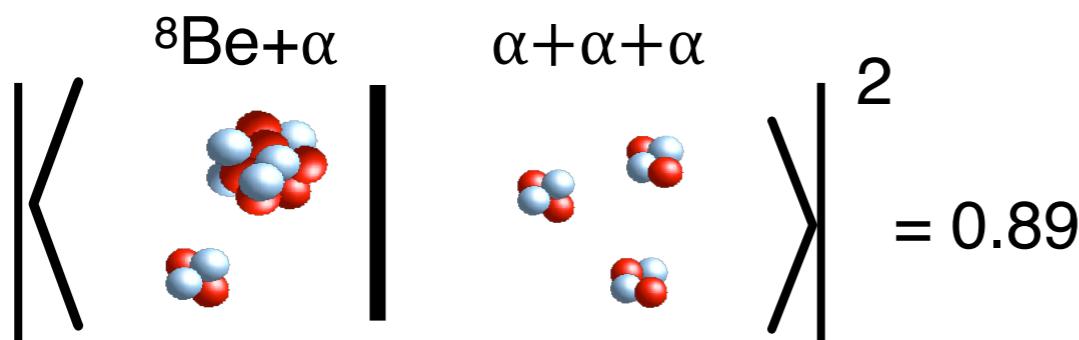
Ttriple-alpha RGM



$N_{\max(\text{rel})}=12$



parent	channel	overlap
$^{12}\text{C}[4](0_1^+)$	$\alpha[0] + \alpha[0] + \alpha[0]$	0.841
$^{12}\text{C}[4](0_2^+)$	$\alpha[0] + \alpha[0] + \alpha[0]$	0.229



Acknowledgements:

K. Kravvaris.

Yu. Tchuvil'sky, T Dytrych, A. Shirokov, J. Vary, G. V. Rogachev, V. Z. Goldberg.

Funding: U.S. DOE contract DE-SC0009883.

Publications:

K Kravvaris and A. Volya, Phys.Rev.Lett, 119(6), 062501 (2017); Journal of Phys 863, 012016 (2017)

K Kravvaris Doctoral dissertation, Florida State University (2018)

D. K. Nauruzbayev, V. Z. Goldberg, A. K. Nurmukhanbetova, M. S. Golovkov, A. Volya, G. V. Rogachev, and R. E. Tribble, Phys. Rev. C **96**, 014322 (2017)

A. Volya and Y. M. Tchuvil'sky, Phys.Rev.C 91, 044319 (2015); J. Phys. Conf. Ser. 569, 012054 (2014); (World Scientific, 2014), p. 215.

M. L. Avila, G. V. Rogachev, V. Z. Goldberg, E. D. Johnson, K. W. Kemper, Y. M. Tchuvil'sky, and A. Volya, Phys. Rev. C 90, 024327 (2014).

A. M. Long, T. Adachi, M. Beard, G. P. A. Berg, Z. Buthelezi, J. Carter, M. Couder, R. J. deBoer, R. W. Fearick, S. V. Förtsch, J. Görres, J. P. Mira, S. H. T. Murray, R. Neveling, P. Papka, F. D. Smit, E. Sideras-Haddad, J. A. Swartz, R. Talwar, I. T. Usman, M. Wiescher, J. J. Van Zyl, and A. Volya
Phys. Rev. C 95, 055803

Resources: <https://www.volya.net/> (see research, clustering)