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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Ab initio Folding Potentials for Proton-Nucleus Scattering based on NCSM One-Body Densities

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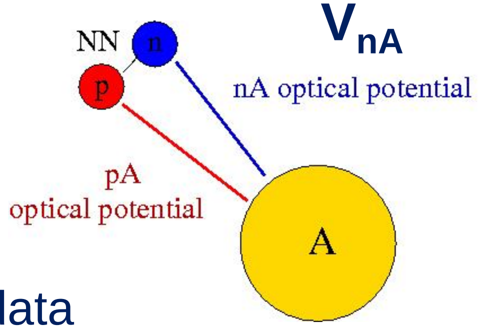
Today's challenge: Determine effective interactions V_{eff}

- **V_{eff} is effective interaction between n+A and should describe elastic scattering p+A**



- V_{nA} and V_{pA} are effective interactions
- **Mostly used: phenomenological approaches**
  - Global optical potential fits to elastic scattering data
  - Most data available for stable nuclei
  - Extrapolation to exotic nuclei questionable
- **Microscopic approaches** need to be developed or existing ones refined and adapted for exotic nuclei
  - **Microscopic approaches were developed for A being a closed shell nucleus.**

Reminder: phenomenological optical potential

e.g. Koning – Delaroche (2003)

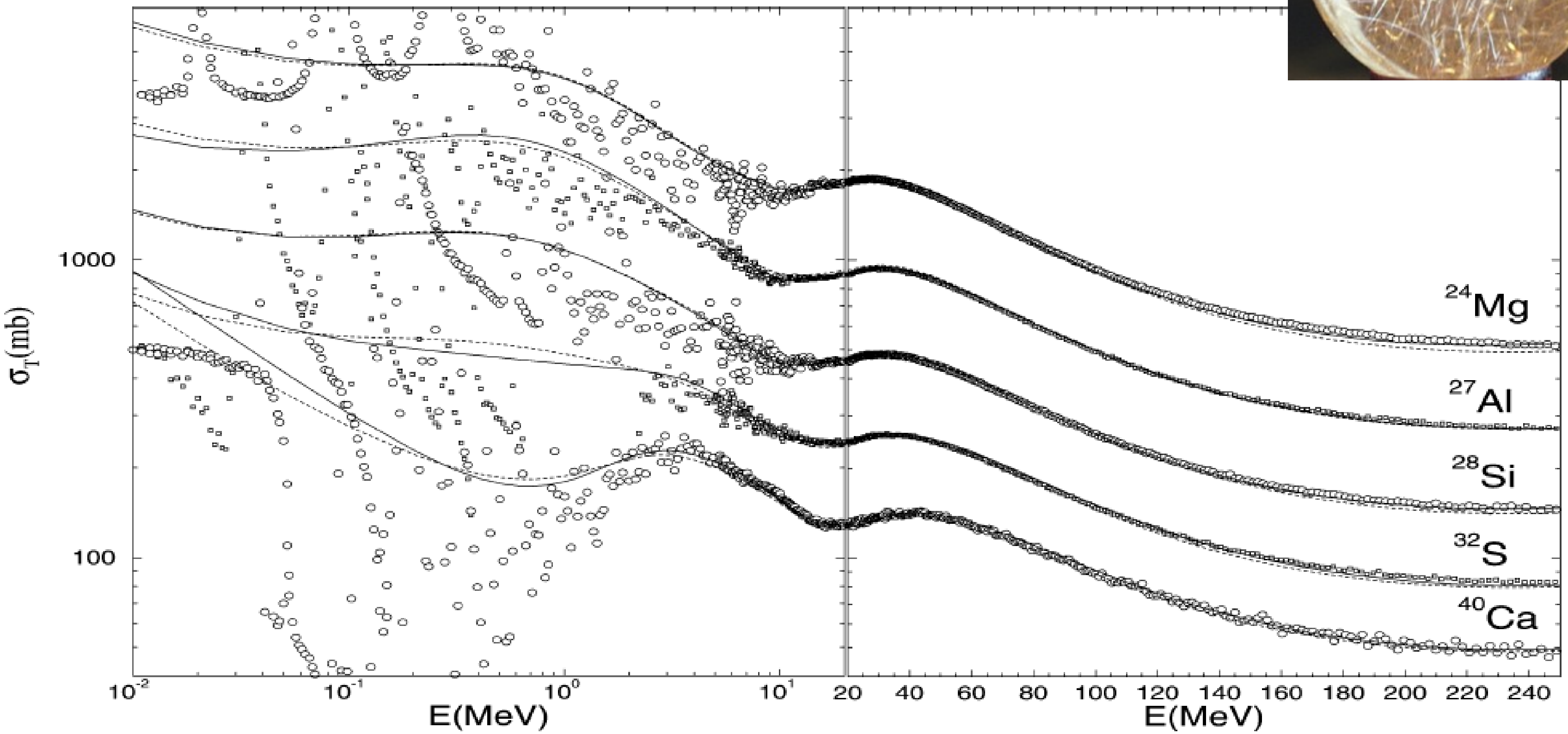


Fig. 2. Comparison of predicted neutron total cross sections and experimental data, for nuclides in the Mg–Ca mass region, for the energy range 10 keV–250 MeV.

Remark: Same importance as NN phase shift analysis

Microscopic effective Potentials

"Folding Models" for closed shell nuclei

~1990's

- **Watson Multiple Scattering**
  - Elster, Weppner, Chinn, **Thaler, Tandy**, Redish
    - Separation of p-A and n-A optical potential
    - Based on NN t-matrix as interaction input
    - Treating of interaction with (A-1)-core via mean field and as implicit three-body problem
- **Kerman-McManus-Thaler (KMT)**
  - Crespo, **Johnson**, Tostevin, Thompson
    - Based on NN t-matrix as input
    - Couple explicitly to (A-1) core
    - Introduce cluster ansatz for halo targets within coupled channels
- **G-matrix folding**
  - Arellano, Brieva, Love
    - Based on NN g-matrix
    - Improving local density approximation
  - Picked up by Amos, Karataglidis and extended to exotic nuclei

Microscopic effective Potentials

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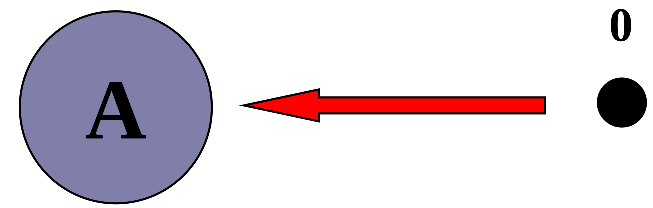
Serious obstacle in 1990's:

NO consistency between description of nuclear structure and nuclear scattering part.

Theory: `Simplest’ scattering problem:

$p \rightarrow A$



- Hamiltonian: $H = H_0 + V$
- Free Hamiltonian: $H_0 = h_0 + H_A$
- Assume: two-body interactions dominant

  (nuclear force strong and short ranged)
  - V: interactions between projectile ‘0’ and target nucleons ‘i’
  - $V = \Sigma^{A}_{i=0}\, v_{0i}$
- Transition Amplitude for scattering: $\mathbf{T = V + V\, G_0\, T}$
- **Multiple Scattering Expansion as ordering principle**

<u>Example</u>: in 3-body system Faddeev eqs “order” according to sub-systems

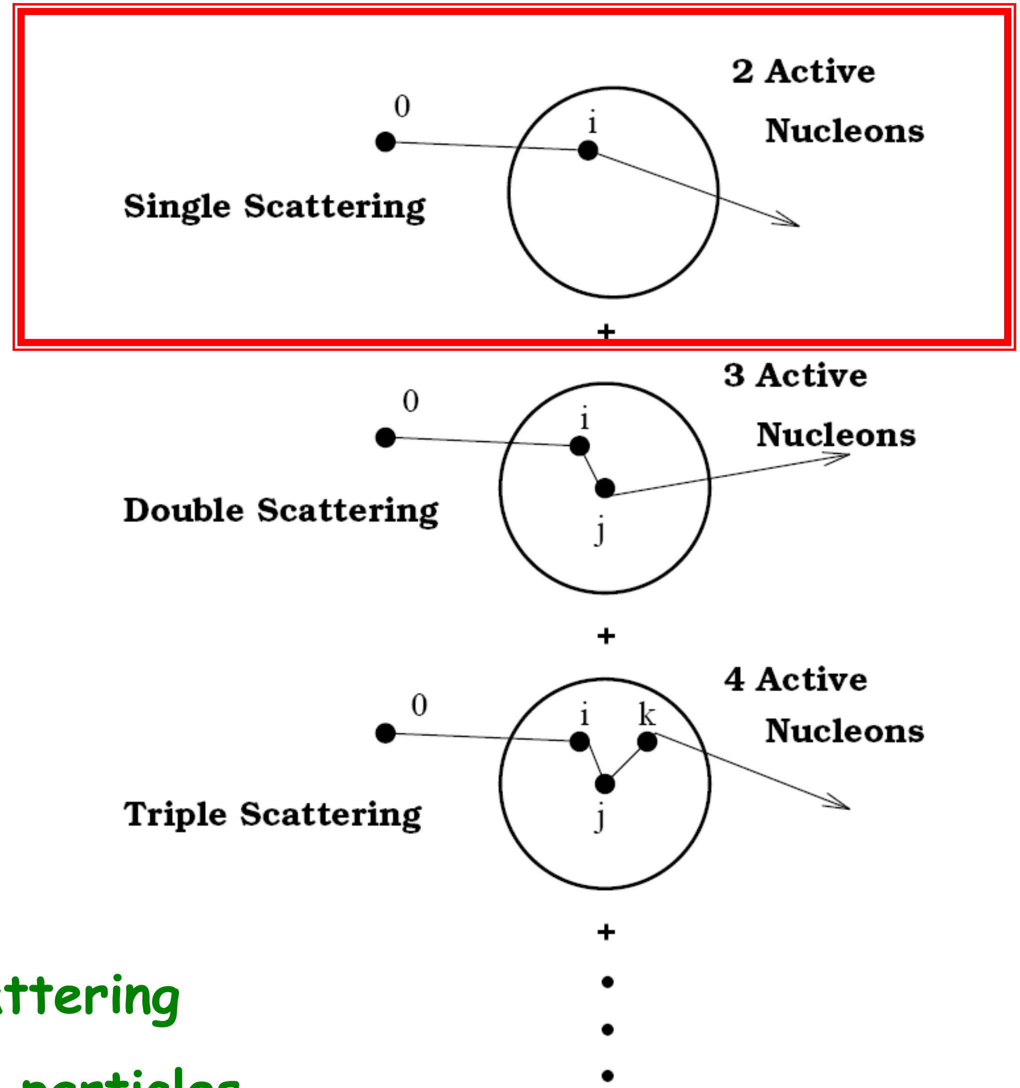
Multiple scattering approach to p+A scattering

Spectator Expansion:

Formulated by

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)



Expansion in:

- particles active in the scattering
- Antisymmetrized in active particles

Spectator Expansion in equations

$$T = \sum_{i=1}^{A} t_{0i} \quad + \quad \sum_{i<j} (t_{ij} - t_{0i} - t_{0j})$$

Scattering from pairs

$$+ \quad \sum_{i<j<k} (t_{ijk} - t_{ij} - t_{ik} - t_{jk} + t_{0i} + t_{0j} + t_{0k}) + \ldots$$

2nd order term: $t_{ij} = (v_{0i} + v_{0j}) + (v_{0i} + v_{0j}) G_0(E) t_{ij},$

Faddeev amplitudes

Single scattering term

Comment to original spectator expansion

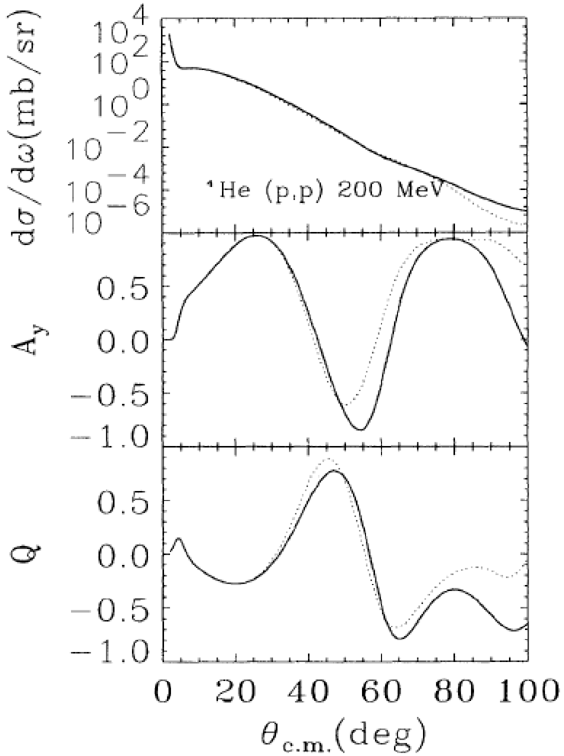
- Expands transition amplitude T   (similar to a Born expansion)
- Very useful for theoretical considerations
- Scattering with strong interactions in nonrelativistic regime:
  iterate interaction to all orders
    either in momentum space Lippmann-Schwinger eq.
    or coordinate space Schrödinger eq.

Instead: perform a spectator expansion in the effective interaction

Chinn, Elster, Thaler,
Phys. Rev. C47, 2242 (1993)

$$U^W = \sum_{i=1}^{A} \left[\tau_i + \tau_i G_0 Q \sum_{j \neq i} \tau_j + \cdots \right]$$

Difference visible for light nuclei



Scattering Amplitude: Lippmann-Schwinger Equation

- $T = V + V G_0 T$ (integral equation in momentum space)
- Hamiltonian: $H = H_0 + V$
- Free Hamiltonian: $H_0 = h_0 + H_A$
  - h_0: kinetic energy of projectile '0'
  - H_A: target Hamiltonian with $H_A |\Phi\rangle = E_A |\Phi\rangle$
- V: interactions between projectile '0' and target nucleons 'i' $V = \Sigma^A_{i=0} v_{0i}$ (no interactions v_{ij})
- Propagator is (A+1) body operator
  - $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

Elastic Scattering

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P = |\Phi_0\rangle\langle\Phi_0|}$
  - With **1=P+Q** and $\mathbf{[P,G_0]=0}$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$

- **Or**

  - $\mathbf{T = U + U\,G_0(E)\,P\,T}$
  - $\mathbf{U = V + V\,G_0(E)\,Q\,U}$ $\Leftarrow$ **`optical' potential**

Up to here exact

Spectator Expansion of U : $\mathbf{U^{(1)} \approx \Sigma^{A}_{i=0}\,\tau_{0i}}$ (1st order)

Chinn, Elster, Thaler, PRC 47, 2242 (1993)

$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

  ⟹ (A+1) body operator

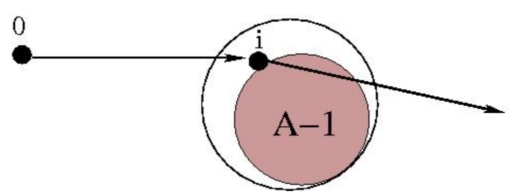
  – Standard “**impulse approximation**”:

  – Average over $H_A \Rightarrow$ constant

  $\rightarrow G_0(e)$ ==: two body operator

Going beyond impulse approximation (in 1990s):

Three-body problem with particles:



o – i – (A-1)-core

o – i : NN interaction

i – (A-1) core : e.g. mean field force **“medium modification”**

Chinn, Elster, Thaler, PRC48, 2956 (1993)
Chinn, Elster, Thaler, Weppner, PRC51 1418 (1995)

$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- Deal with **Q** **(this prevents to use NN t-matrix here)**
  - Define “two-body” operator $\mathbf{t_{0i}^{free}}$ by
  - $\mathbf{t_{0i}^{free} = v_{0i} + v_{0i}\, G_0(e)\, t_{0i}^{free}}$
  - and relate via integral equation to τ_{oi}
  - $\boldsymbol{\tau_{oi} = t_{0i}^{free} - t_{0i}^{free}\, G_0(e)\, \tau_{oi}}$ **[integral equation]**
  - keeps iso-spin character of optical potential
  - $$\mathbf{U^{(1)} = \Sigma^{A}_{i=1}\, \tau_{oi} =: N\, \tau_n + Z\, \tau_p}$$

Neutron and proton contributions are cleanly separated
Important for N≠Z nuclei

$\mathbf{t_{pp} \neq t_{np}}$ **and** $\boldsymbol{\rho_p \neq \rho_n}$

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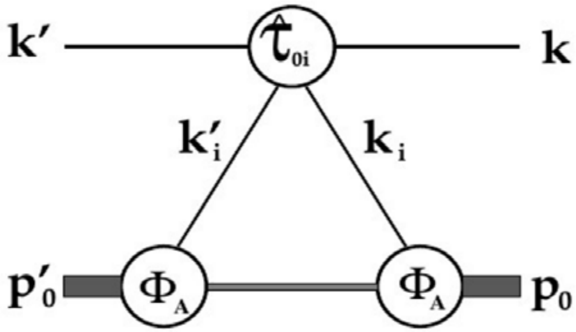
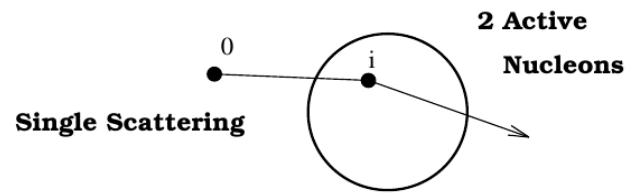
Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN interaction   *Nuclear density*

$$U_{el}(\mathbf{q},\mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\ \eta(\mathbf{q},\mathbf{K},\mathbf{P})\ \hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}), \mathcal{E}\right) \rho_i(\mathbf{P}-\frac{A-1}{2A}\mathbf{q}, \mathbf{P}+\frac{A-1}{2A}\mathbf{q})$$

$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2}$$

$$\mathbf{q} \equiv \mathbf{k}' - \mathbf{k}$$

$$\mathbf{P} \equiv \frac{\mathbf{k}_i + \mathbf{k}_i'}{2} + \frac{\mathbf{K}}{A}$$

Same NN Interaction can now be used for NN t-matrix and one-body density matrix

Elster, Cheon, Redish, Tandy, Phys. Rev. C41, 814 (1990).

Nonlocal one-body densities from the No-Core-Shell-Model (NCSM)

$$\rho_{sf}(\vec{r},\vec{r'}) = \langle \Psi' | \sum_{i=1}^{A} \delta^3(\vec{r}_i - \vec{r})\delta^3(\vec{r}'_i - \vec{r'}) | \Psi \rangle$$

$$\rho_{sf}(\vec{p},\vec{p'}) = \sum_{Kll'} (-1)^{J'-M} \begin{pmatrix} J' & K & J \\ -M & 0 & M \end{pmatrix} \mathcal{Y}^{*l'l}_{K0}(\hat{p},\hat{p}')\rho_{ll'K}(p,p')$$

$$\rho_{ll'K}(p,p') = \sum_{njn'j'} \hat{j}\hat{j}'(-1)^{\frac{l-l'}{2}}(-1)^{j+\frac{1}{2}} \begin{Bmatrix} l' & l & K \\ j & j' & \frac{1}{2} \end{Bmatrix} R_{n'l'}(p')R_{nl}(p) \left\langle A\lambda' J' \left\| (a^{\dagger}_{n'l'j'}\tilde{a}_{nlj})^{(K)} \right\| A\lambda J \right\rangle$$

Change variables to remove center-of-mass contribution:

$$\vec{q} = \vec{p}' - \vec{p} \qquad \vec{\zeta} = \frac{1}{2}(\vec{r} + \vec{r}\,') \qquad \zeta = \zeta_{rel} + \zeta_{c.m.}$$

$$\vec{\mathcal{K}} = \tfrac{1}{2}(\vec{p}' + \vec{p}) \qquad \vec{Z} = \vec{r}\,' - \vec{r}\ ,$$

$$R_{n'l'}(p')R_{nl}(p)\mathcal{Y}^{l'l}_{KM}(\hat{p},\hat{p}') = \sum_{n_q,n_{\mathcal{K}},l_q,l_{\mathcal{K}}} \langle n_{\mathcal{K}} l_{\mathcal{K}}, n_q l_q : K | n'l', nl : K \rangle_{d=1} R_{n_{\mathcal{K}} l_{\mathcal{K}}}(\mathcal{K}) R_{n_q l_q}(q) \mathcal{Y}^{l_{\mathcal{K}} l_q}_{KM}(\hat{q},\hat{\mathcal{K}})$$

Nonlocal one-body densities from NCSM

translationally invariant

$$\rho_{sf}(\vec{q},\vec{\mathcal{K}}) = \frac{1}{(2\pi)^3}\left\langle \Psi' J' M \left| \sum_{i=1}^{A} e^{-i\vec{q}\cdot(\vec{\zeta}_{rel,i}+\vec{\zeta}_{c.m.})} e^{-i\vec{\mathcal{K}}\cdot\vec{Z}_i} \right| \Psi J M \right\rangle$$

$$\rho_{sf}(\vec{q},\vec{\mathcal{K}}) = \left\langle \phi_{cm} 0s | e^{-i\vec{q}\cdot\vec{\zeta}_{c.m.}} | \phi_{cm} 0s \right\rangle \frac{1}{(2\pi)^3}\left\langle \Psi'_{ti} J' M \left| \sum_{i} e^{-i\vec{q}\cdot\vec{\zeta}_{rel,i}} e^{-i\vec{\mathcal{K}}\cdot\vec{Z}_i} \right| \Psi_{ti} J M \right\rangle$$

$$\rho_{ti}(\vec{q},\vec{\mathcal{K}}) \equiv \frac{1}{(2\pi)^3}\left\langle \Psi'_{ti} J' M \left| \sum_{i} e^{-i\vec{q}\cdot\vec{\zeta}_{rel,i}} e^{-i\vec{\mathcal{K}}\cdot\vec{Z}_i} \right| \Psi_{ti} J M \right\rangle$$

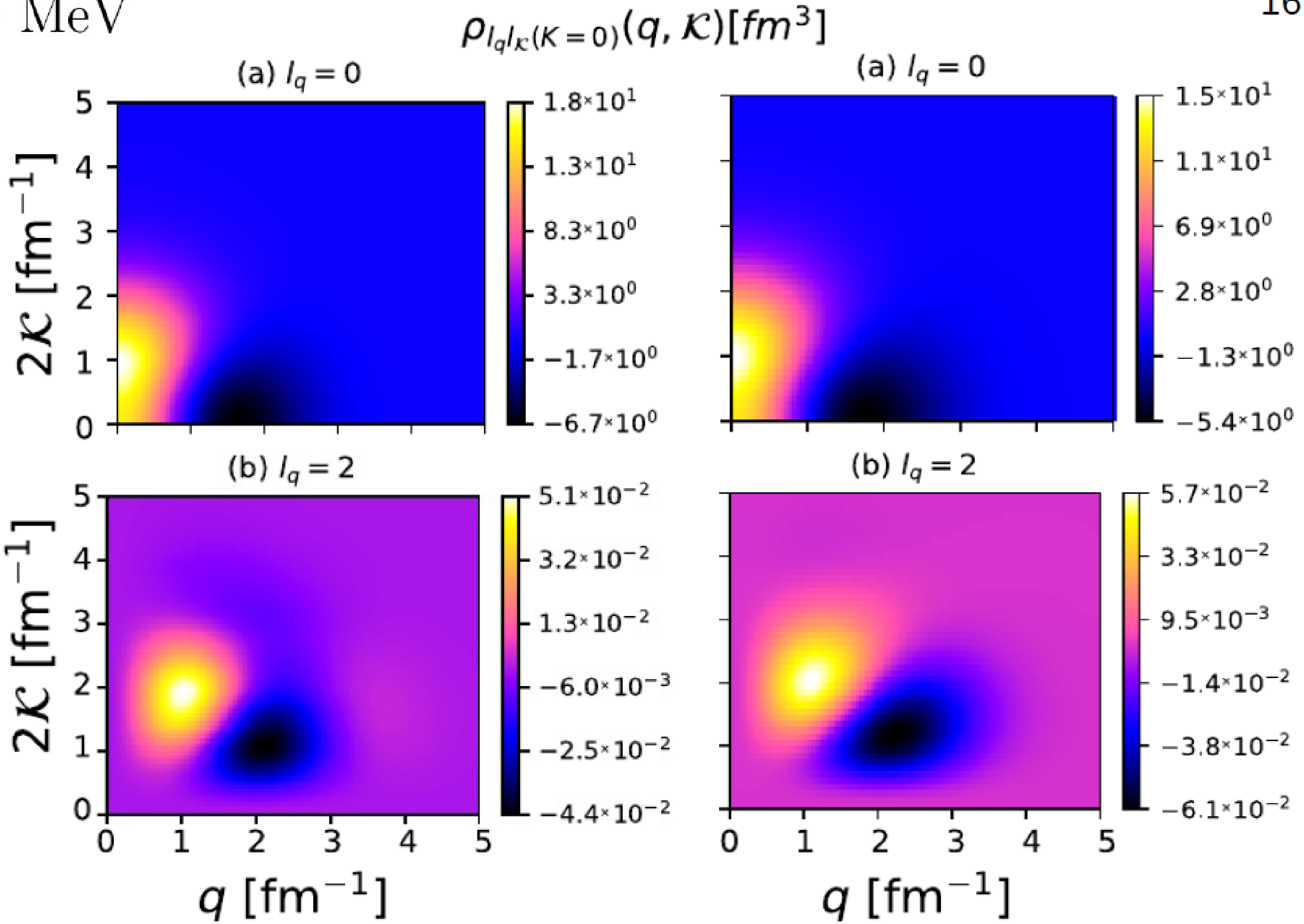
Translationally Invariant

$$\rho(\vec{q},\vec{\mathcal{K}}) = e^{\frac{1}{4A} b^2 q^2}\, \rho_{sf}(\vec{q},\vec{\mathcal{K}})$$

Burrows, Elster, Popa, Launey, Nogga, Maris, PRC 97, 024325 (2018)

N_{max}=8
$\hbar\omega$=20 MeV

Proton distribution
^{16}O



NNLO$_{opt}$ JISP16

Proton distribution

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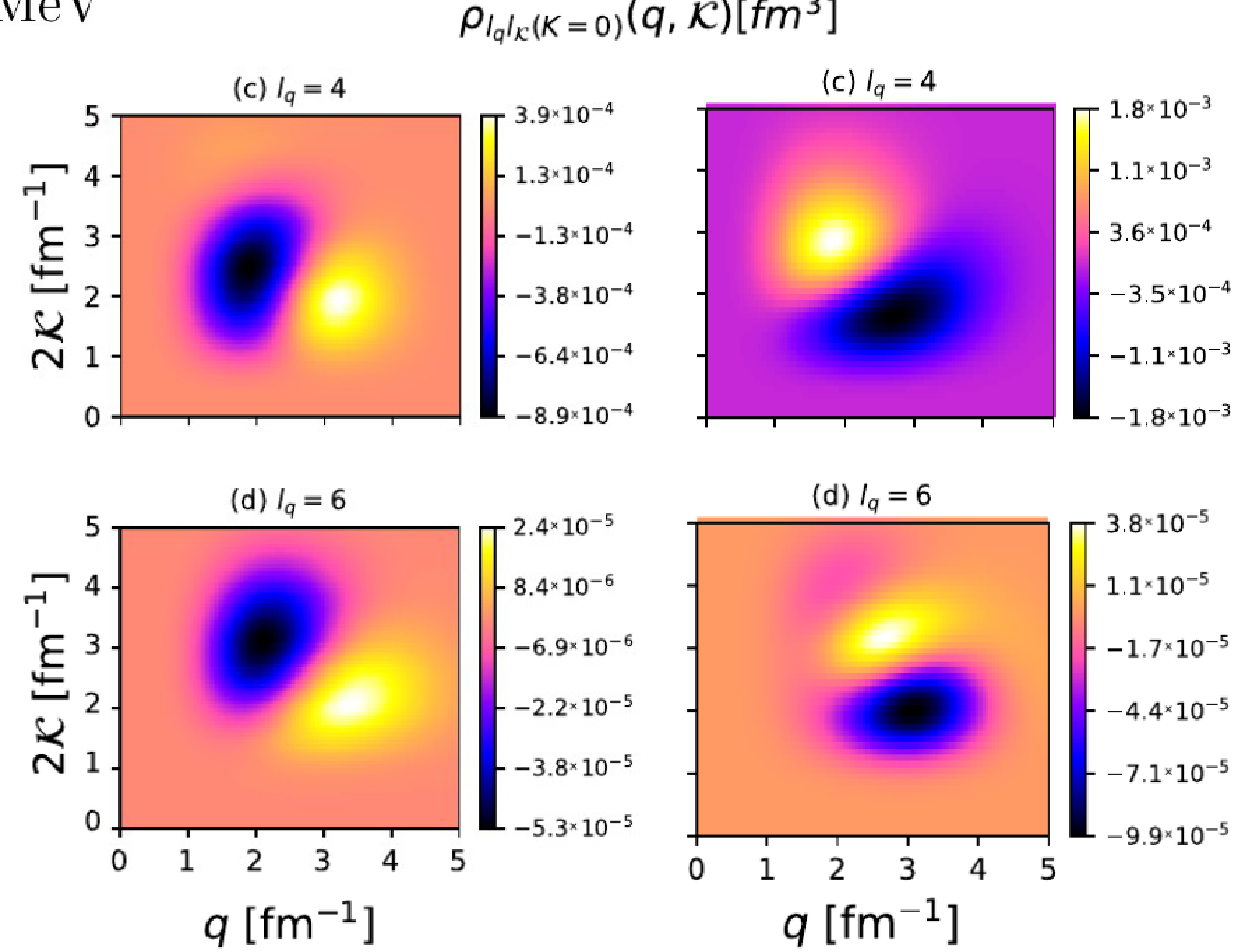

$NNLO_{opt}$ JISP16

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NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

Variables (E,k',k,φ) ⇒ (E, q, K, θ)

with $q = k' - k$

$K = ½ (k' + k)$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis

Struck nucleon "*i*" : target basis

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1}+\mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN})$$

$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]$$

Off-shell

Most general form

NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

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Struck nucleon "*i*" : target basis

Closed shell nuclei

$$
\begin{aligned}
\overline{\mathrm{M}}(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) = & \; A(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, \mathbf{1} + i \cdot C(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, (\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN} \\
& + M(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN}) \\
& + (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \\
& + (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN})
\end{aligned}
$$

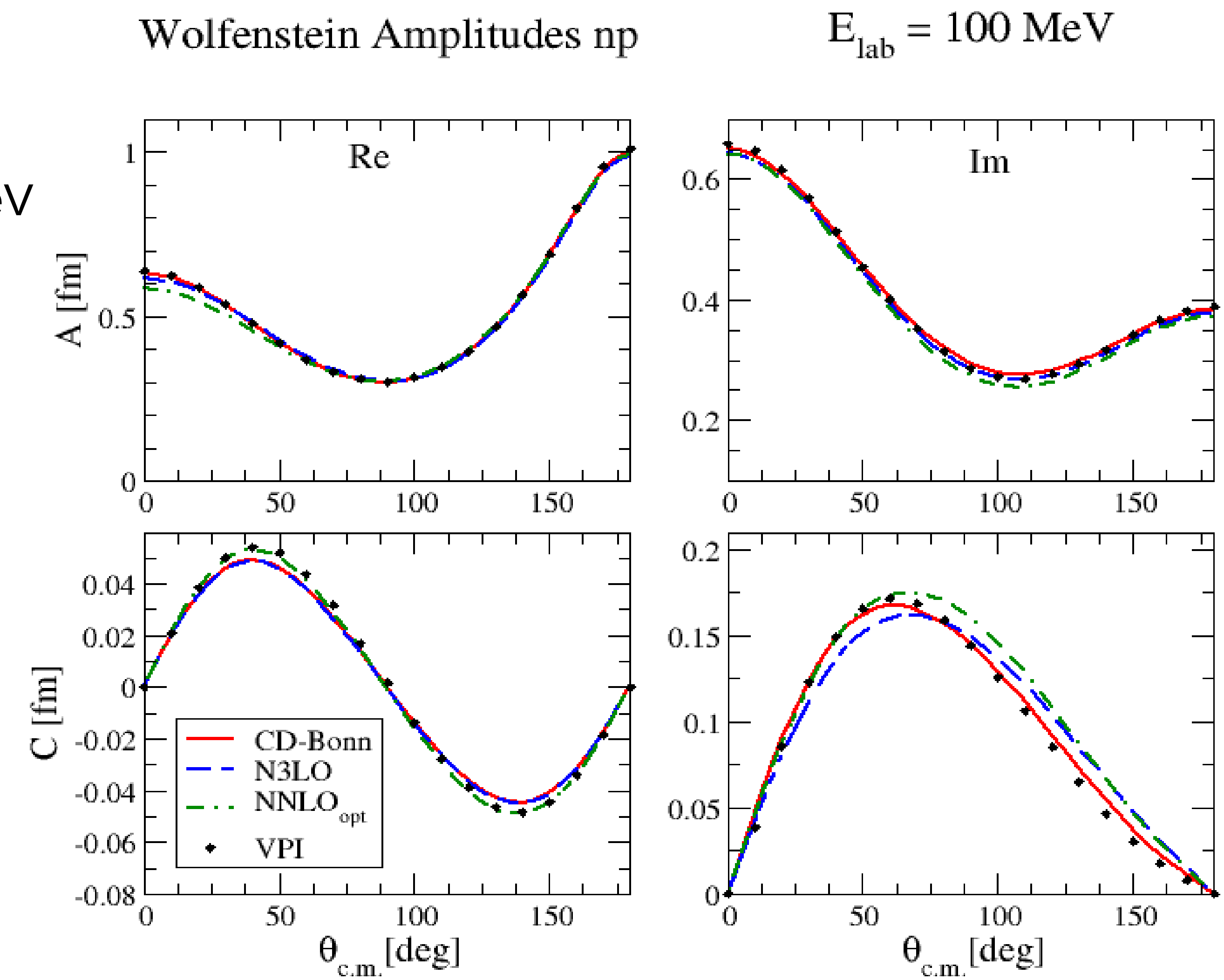
$$
+ D(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \right]
$$

Off-shell

Wolfenstein Amplitudes A and C

$NNLO_{opt}$ fitted to E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}



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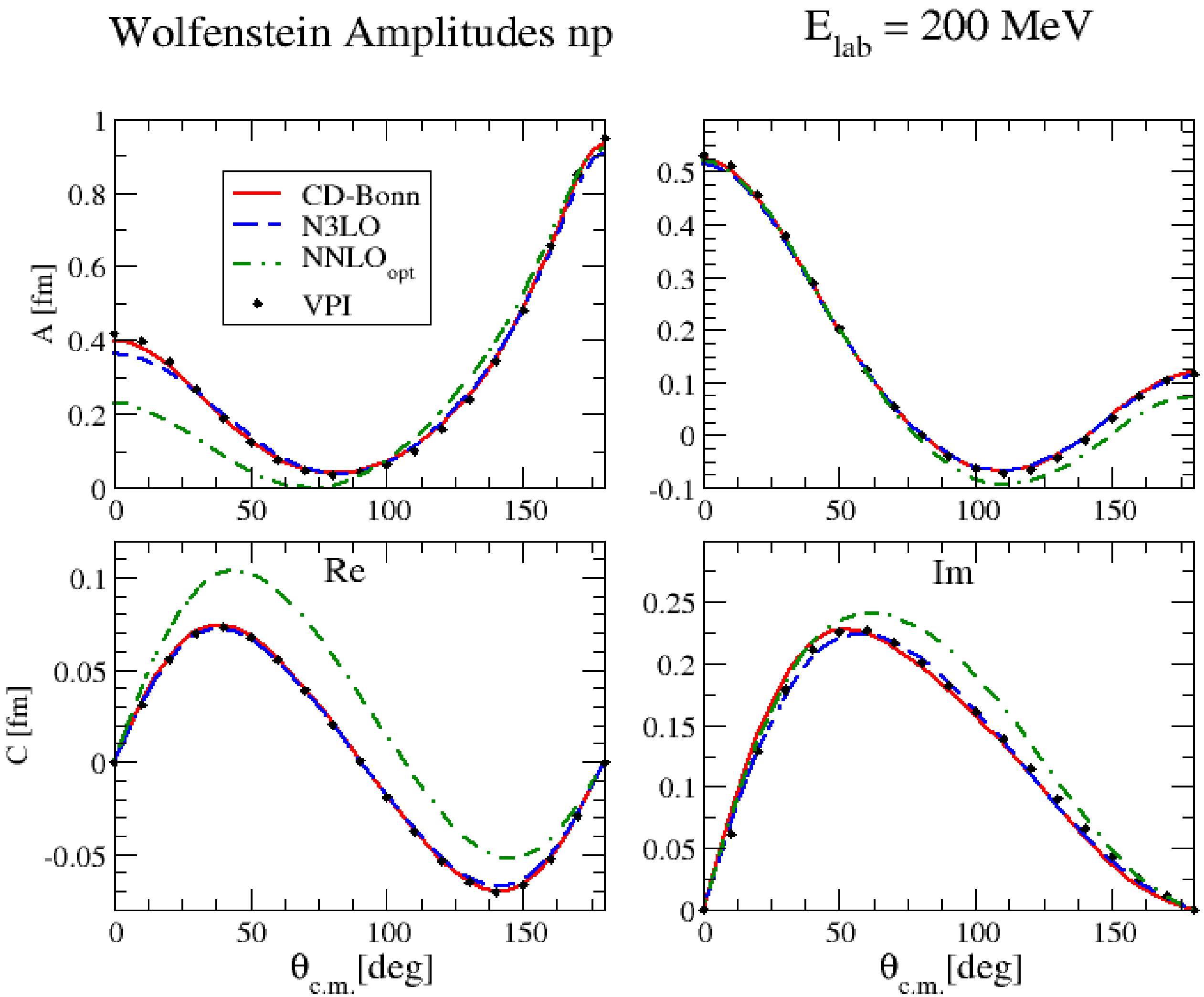
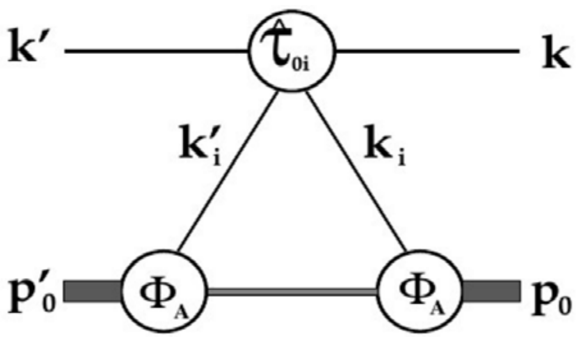
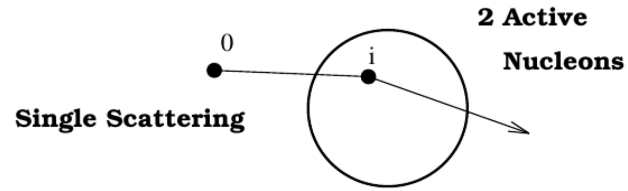


Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN interaction   *Nuclear density*

$$U_{el}(\mathbf{q},\mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\ \eta(\mathbf{q},\mathbf{K},\mathbf{P})\ \hat{\tau}_{0i}\left(\mathbf{q},\frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}),\mathcal{E}\right)\rho_i(\mathbf{P}-\frac{A-1}{2A}\mathbf{q},\mathbf{P}+\frac{A-1}{2A}\mathbf{q})$$

$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2}$$

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Elster, Cheon, Redish, Tandy, Phys. Rev. C41, 814 (1990).

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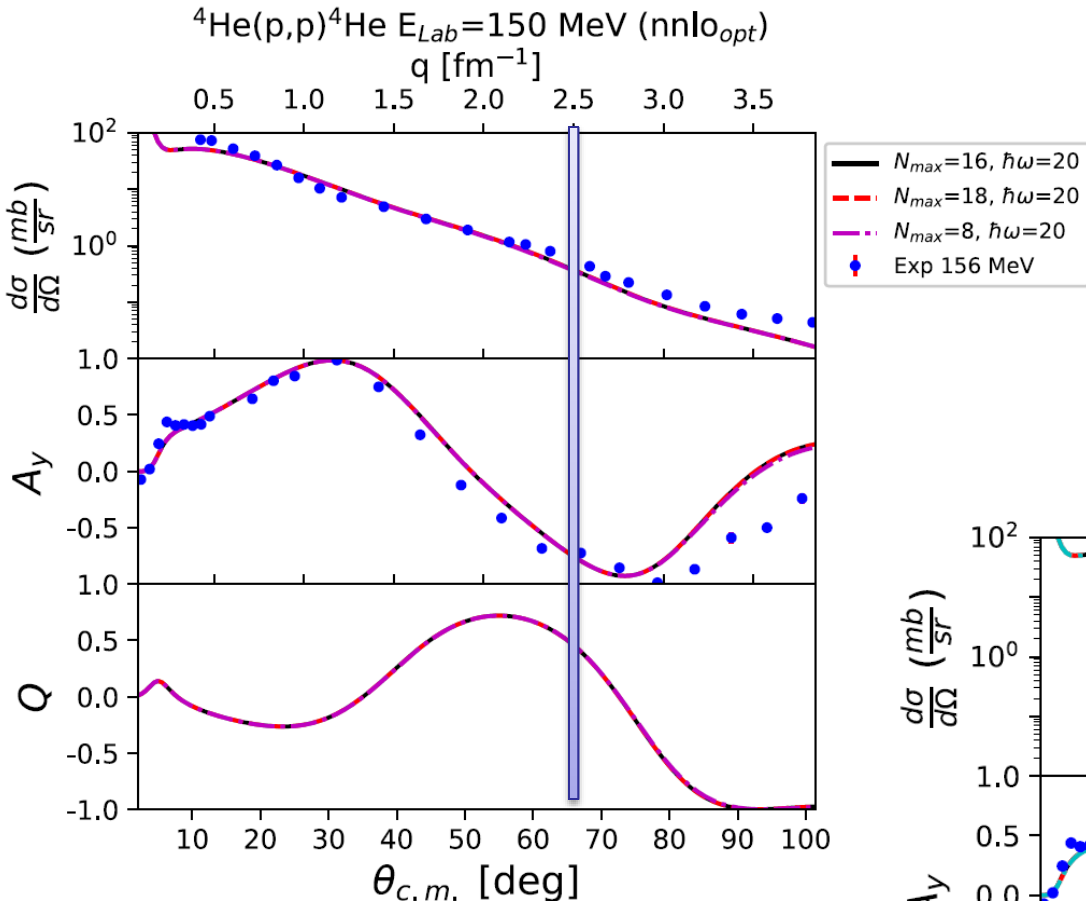
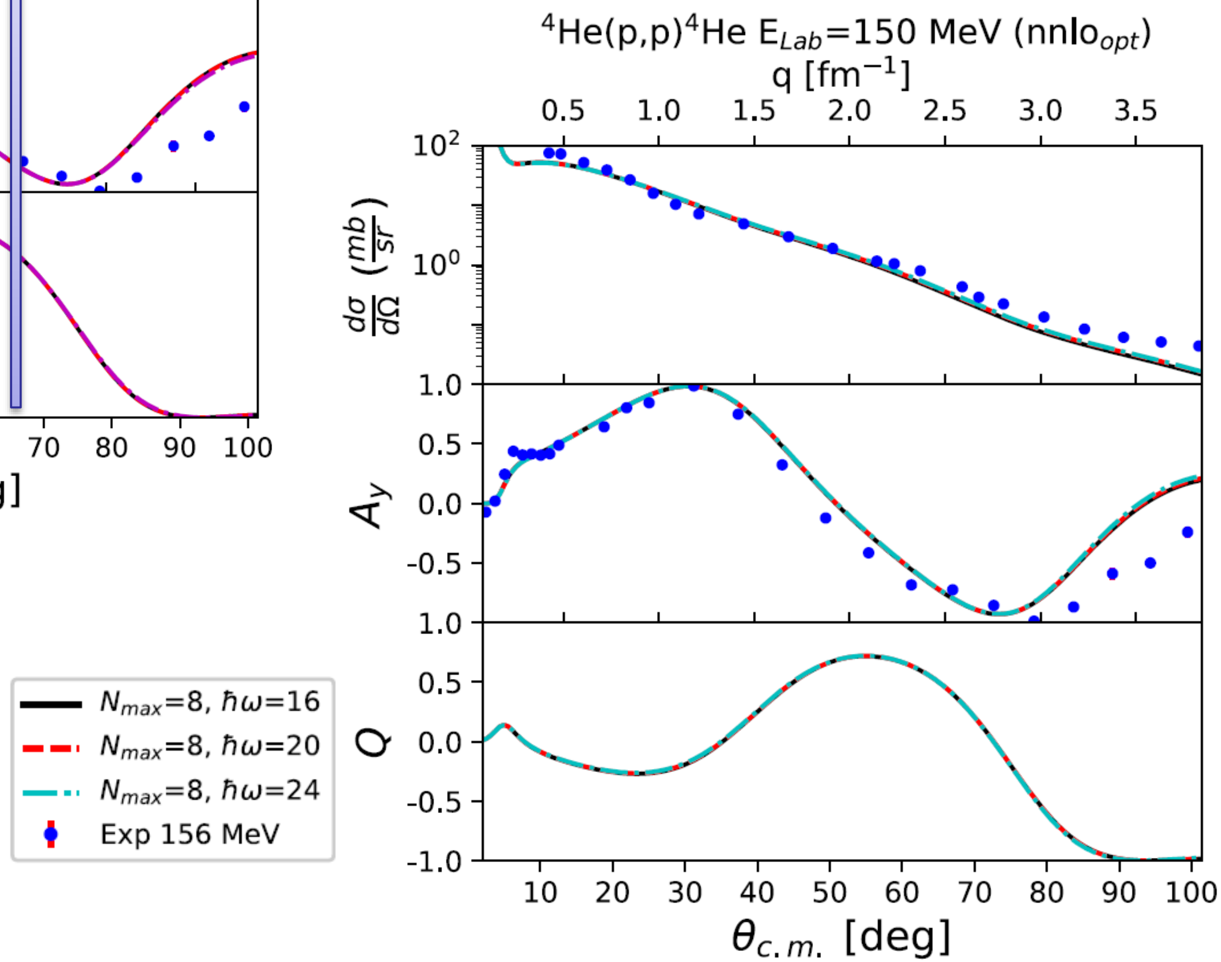


$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$



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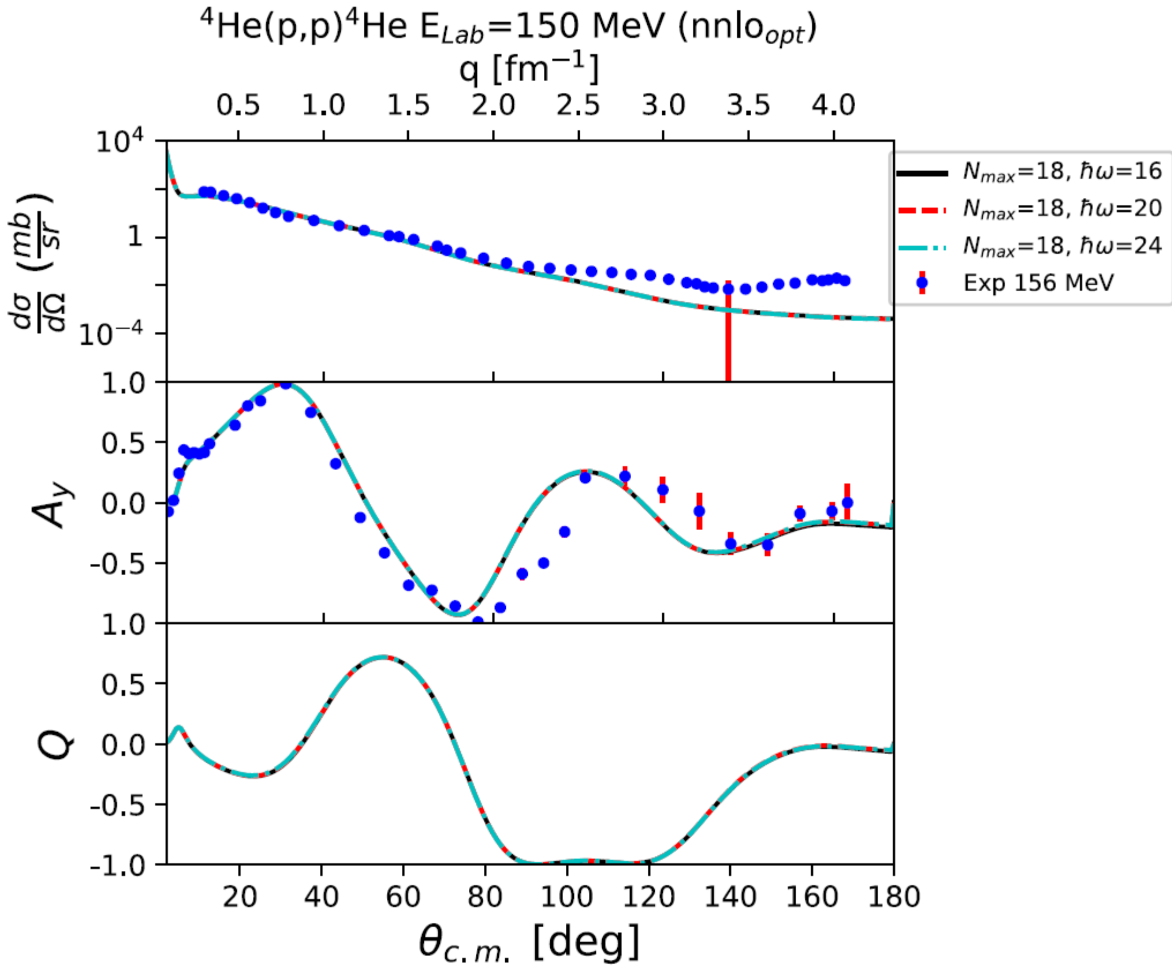

Binding Energy [MeV]	Proton RMS Radii (Point) [fm^{-1}]	N_{max}	$\hbar\omega$ [MeV]
-27.5010	1.43980	18	16
-27.5850	1.43680	18	20
-27.5940	1.43620	18	24
-24.6540	1.49210	8	16
-26.2170	1.45190	8	20
-26.9550	1.43010	8	24

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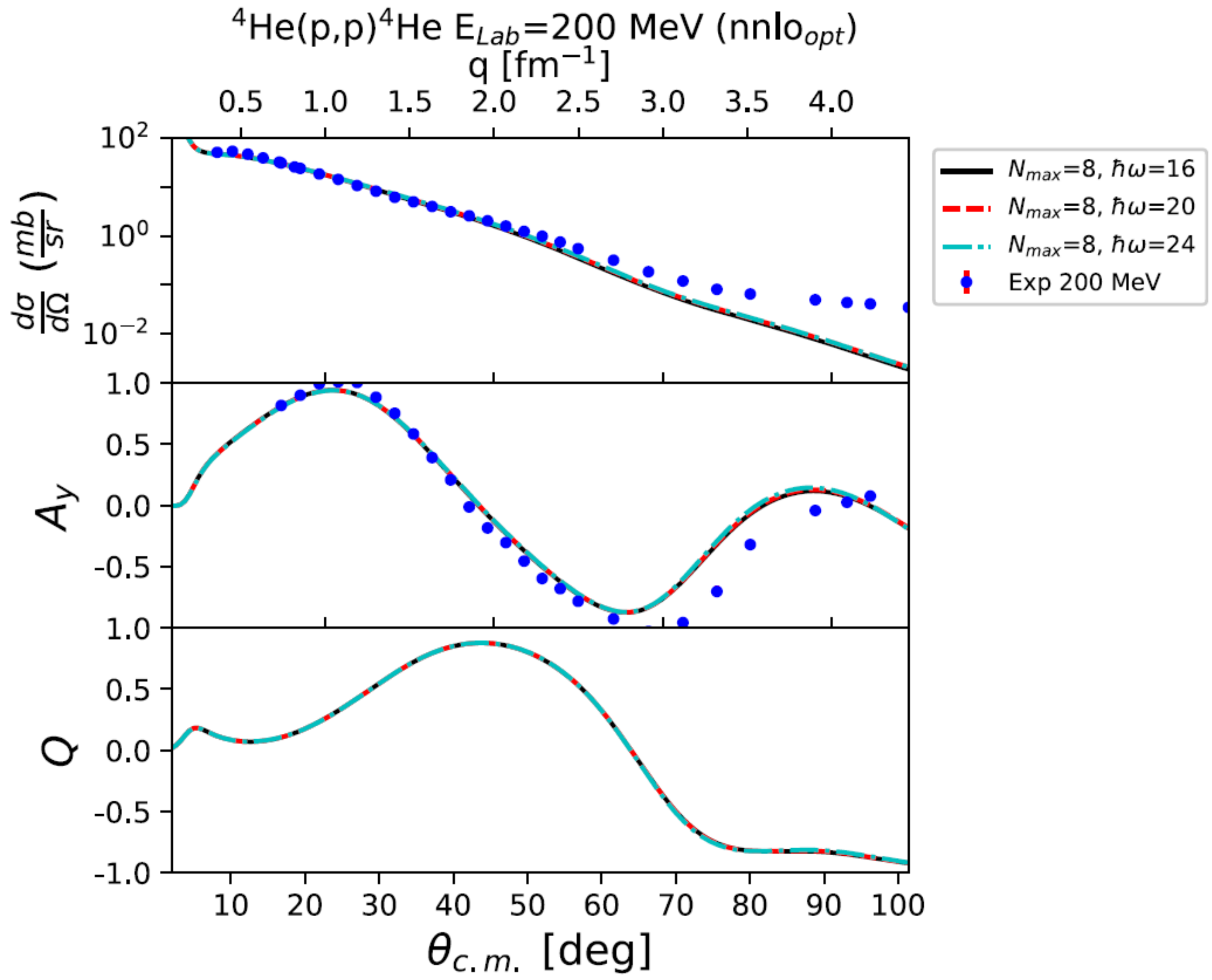
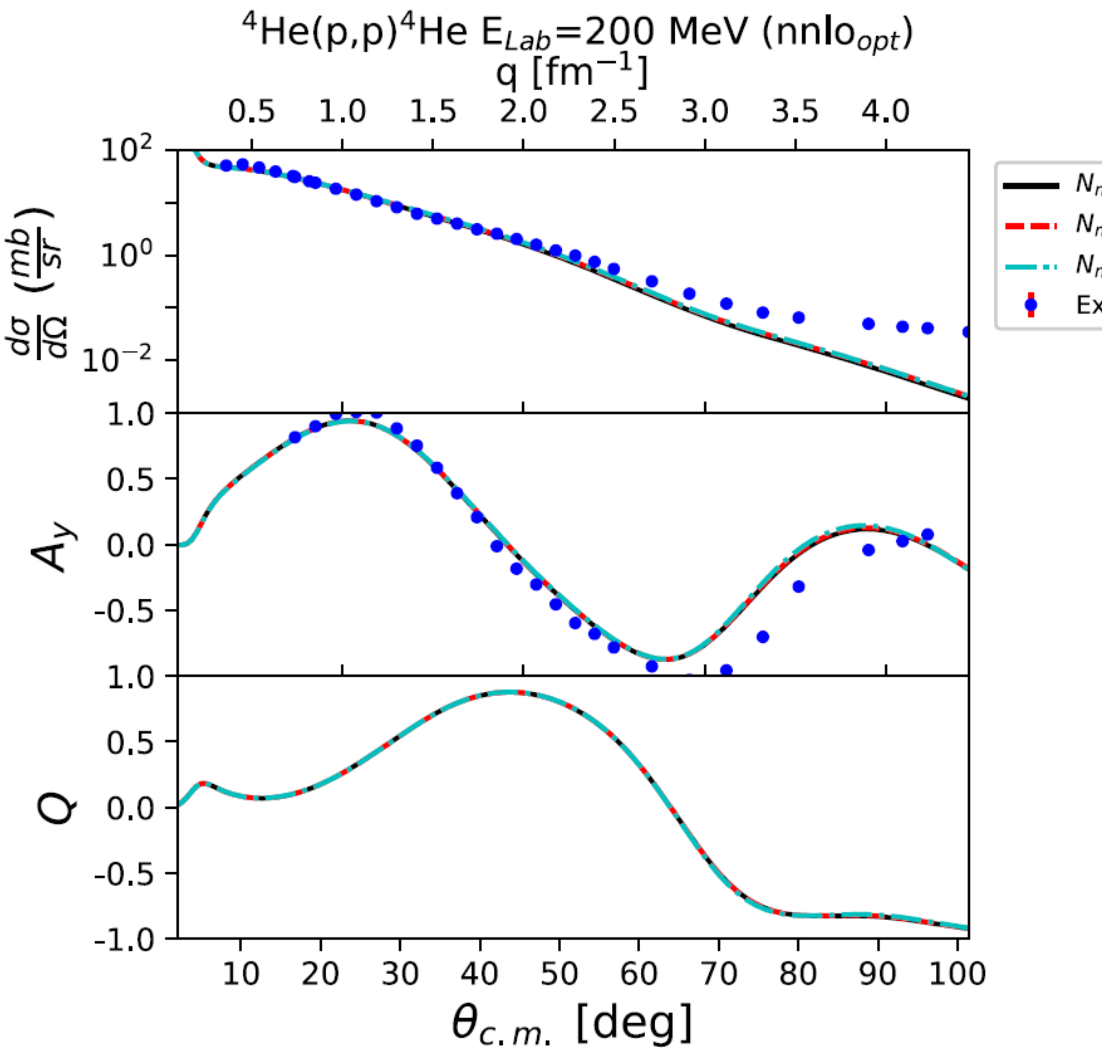
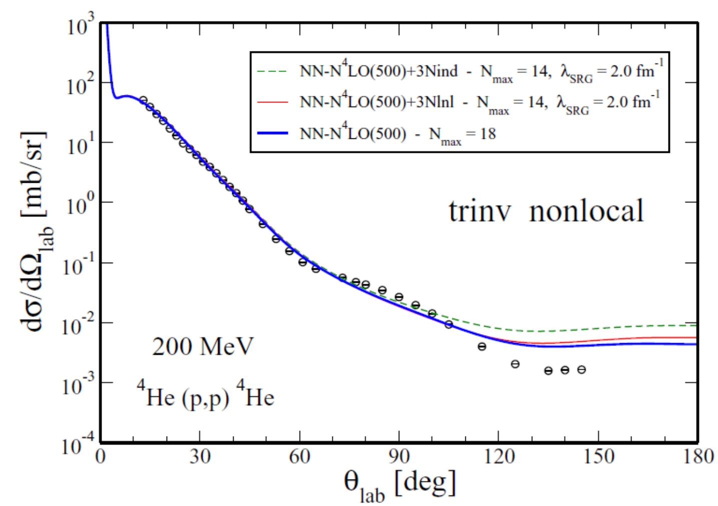
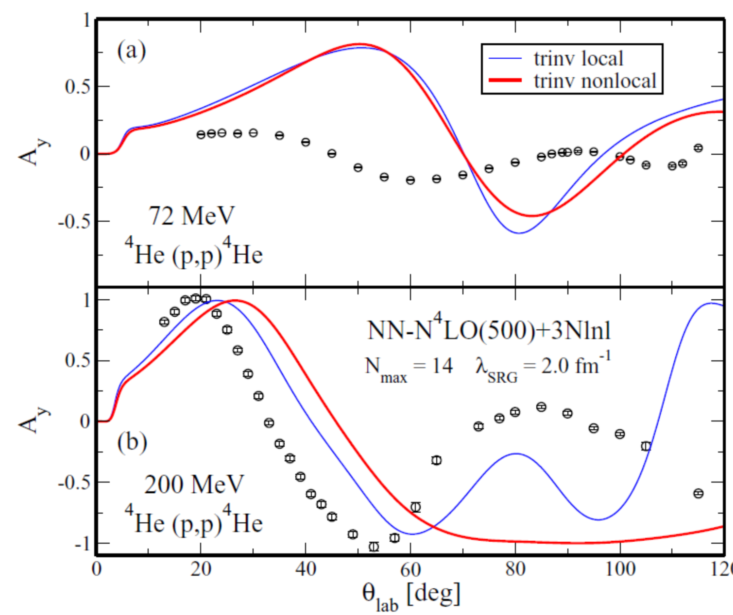


^{4}He(p,p)^{4}He E_{Lab}=200 MeV (nnlo$_{opt}$)
q [fm^{-1}]
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
10^2
10^0
10^{-2}
$\frac{d\sigma}{d\Omega}$ $(\frac{mb}{sr})$
A_y
Q
1.0 0.5 0.0 -0.5 -1.0
10 20 30 40 50 60 70 80 90 100
$\theta_{c.m.}$ [deg]
N_{max}=8, ħω=16
N_{max}=8, ħω=20
N_{max}=8, ħω=24
Exp 200 MeV

Gennari, Vorabbi, Calci, Navratil, PRC97, 034619 (2018)







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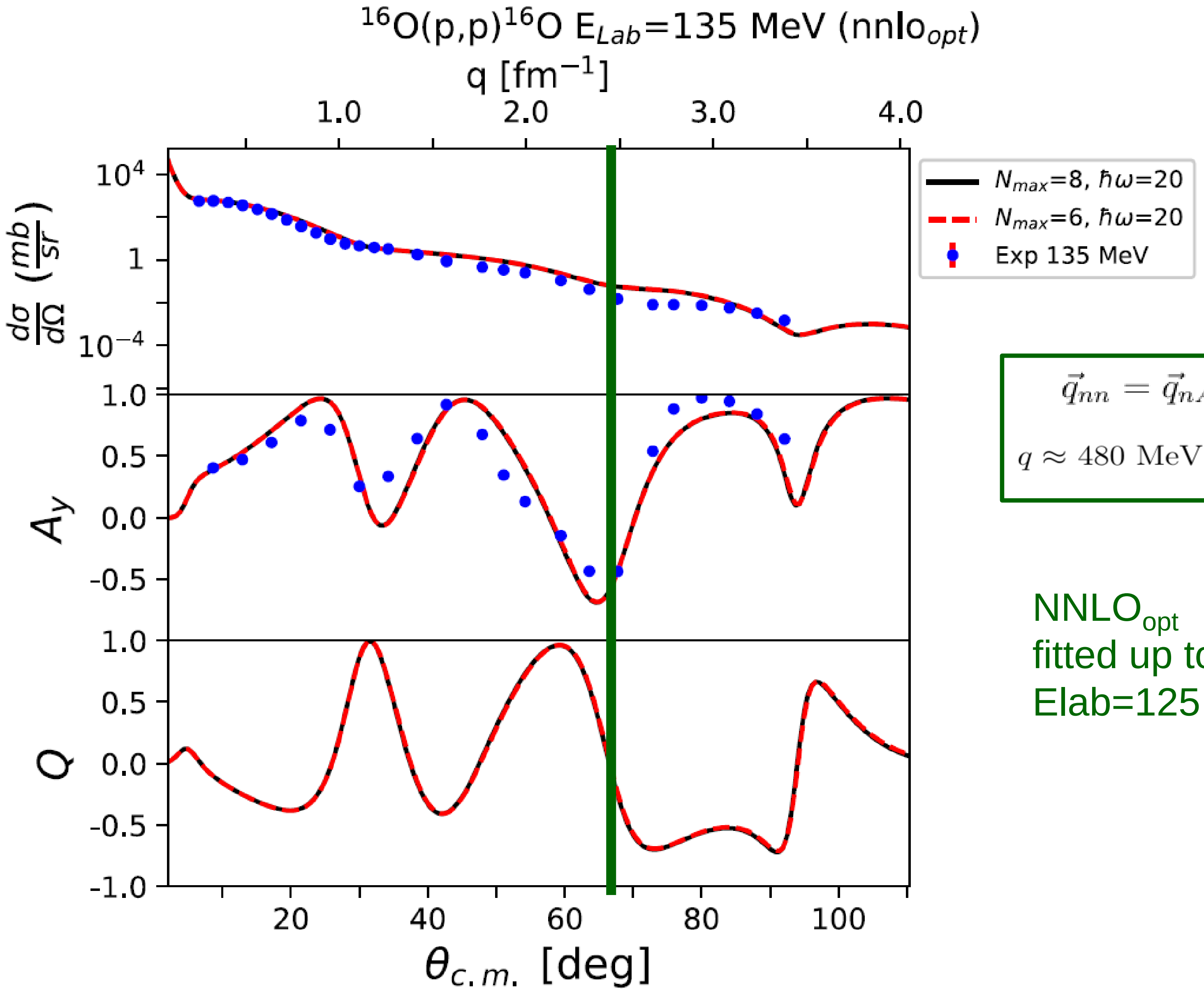

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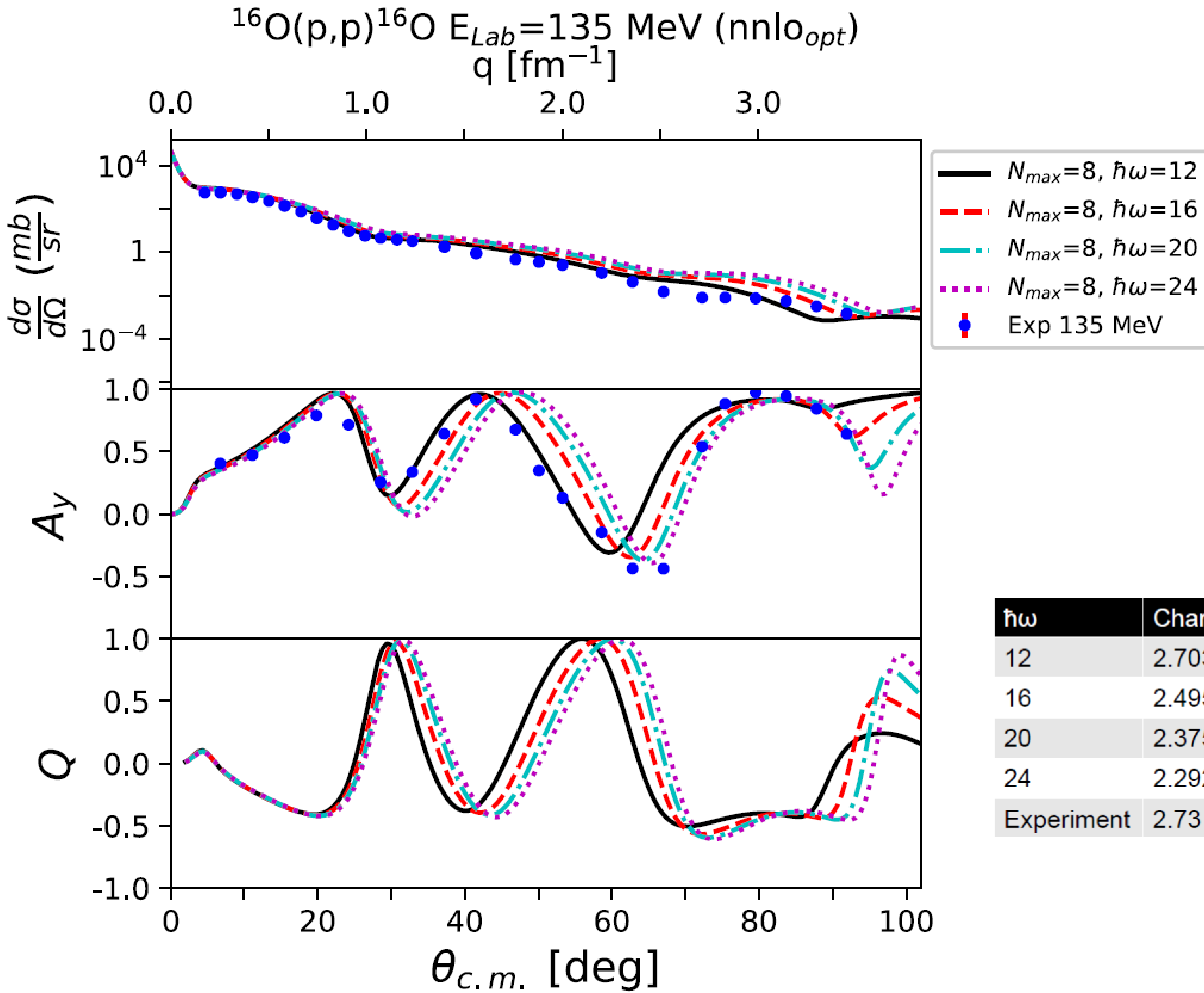
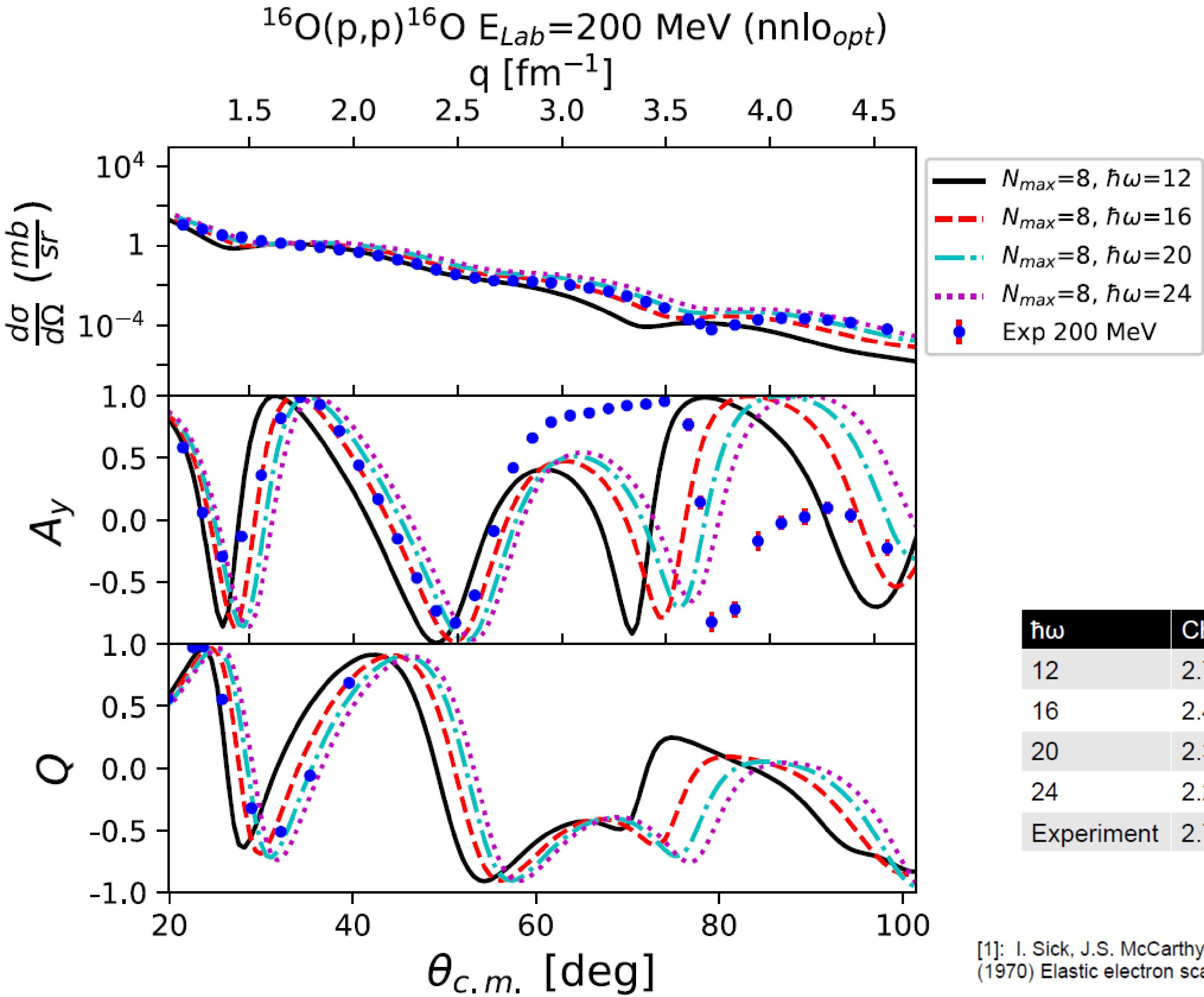
fitted up to
Elab=125 MeV



ħω	Charge Radius
12	2.70318402
16	2.49584345
20	2.37548417
24	2.29262311
Experiment	2.73 ± 0.025 fm[1]

[1]: I. Sick, J.S. McCarthy, Nucl. Phys. A150, 631–654 (1970) Elastic electron scattering from ^{12}C and ^{16}O



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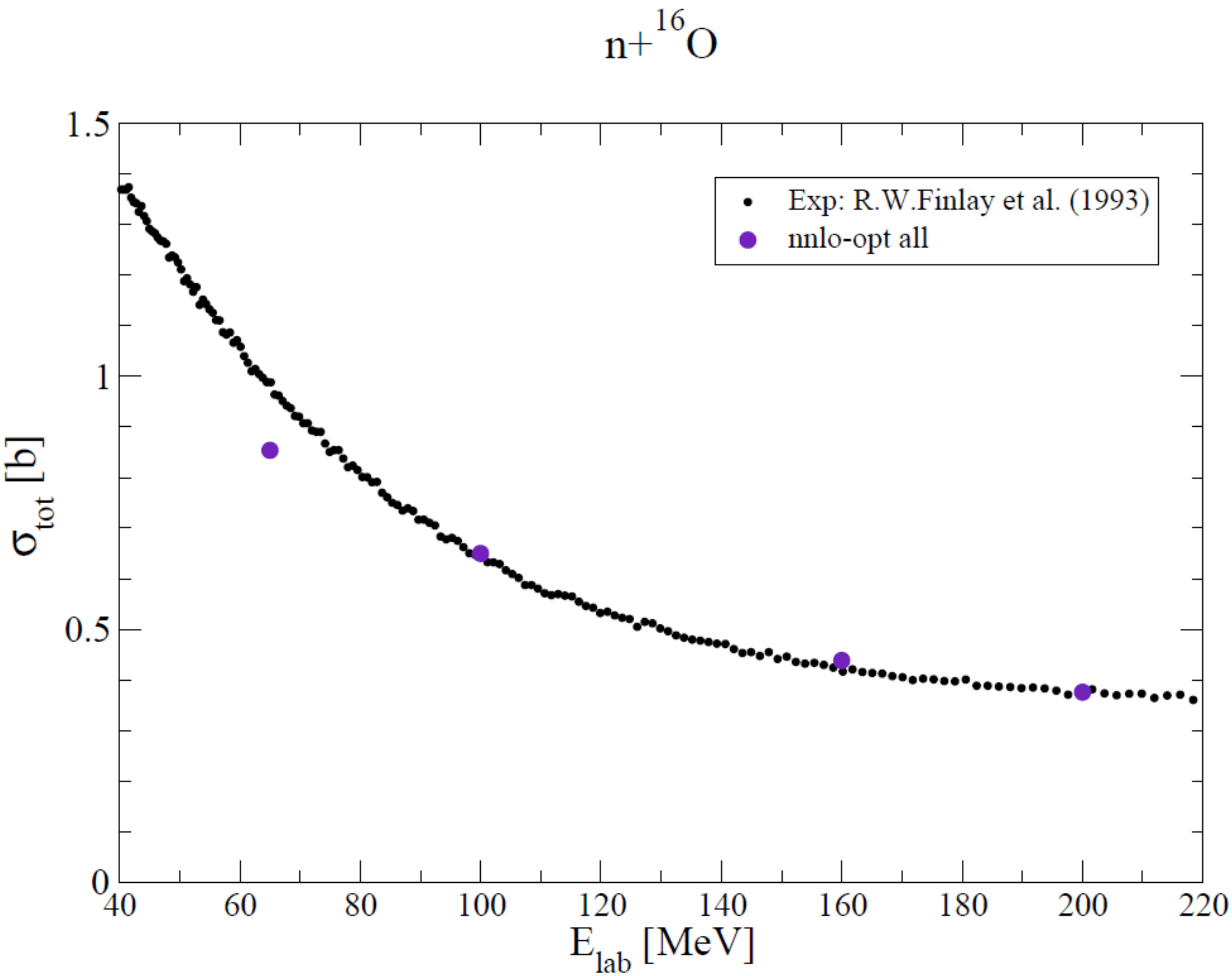
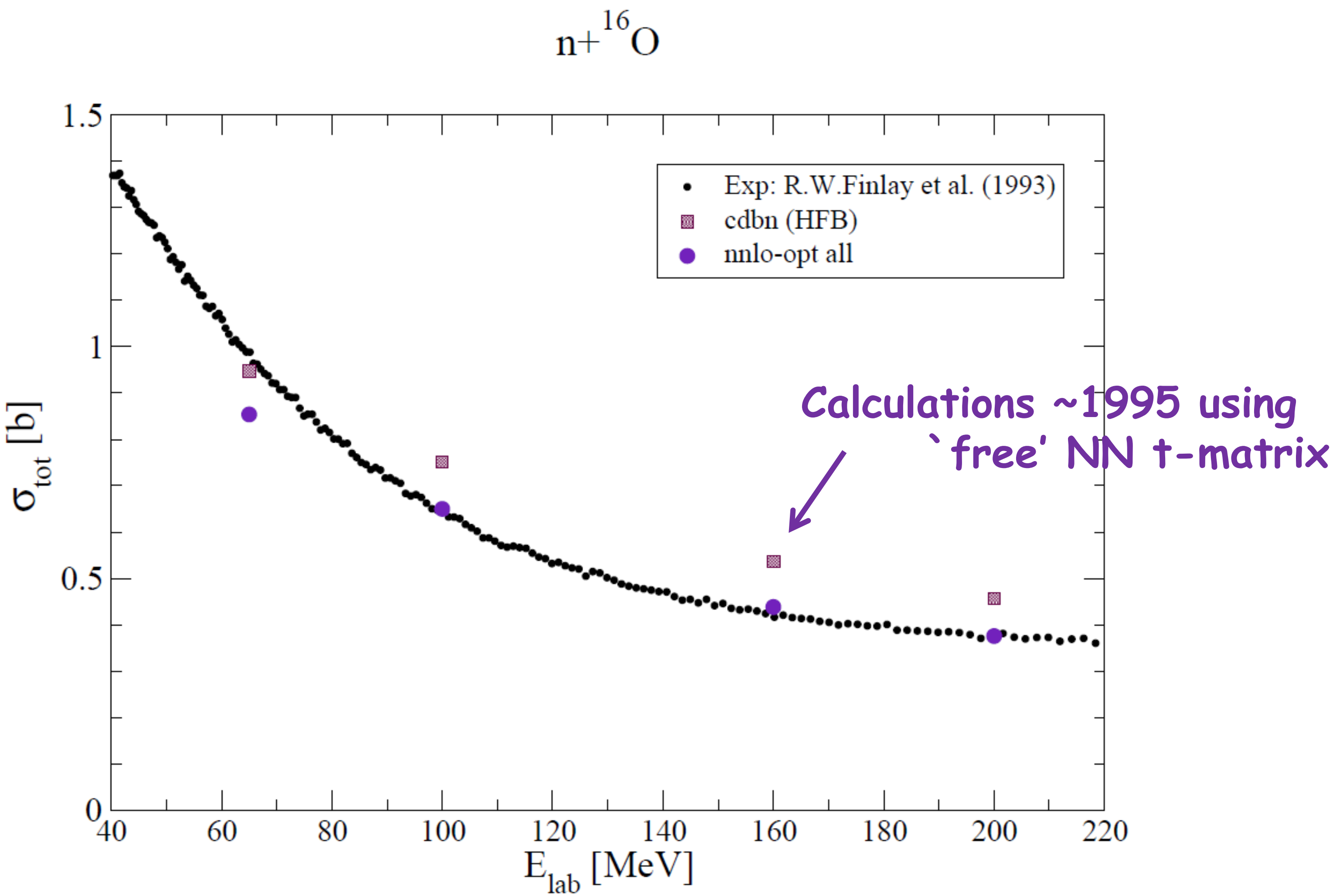
Total cross section for neutron scattering



Total cross section for neutron scattering



Chinn, Elster, Thaler, Weppner, PRC51, 1033 (1995)

Review of previous calculations

Weppner, Elster, Hüber, PRC 57, 1378 (1998)

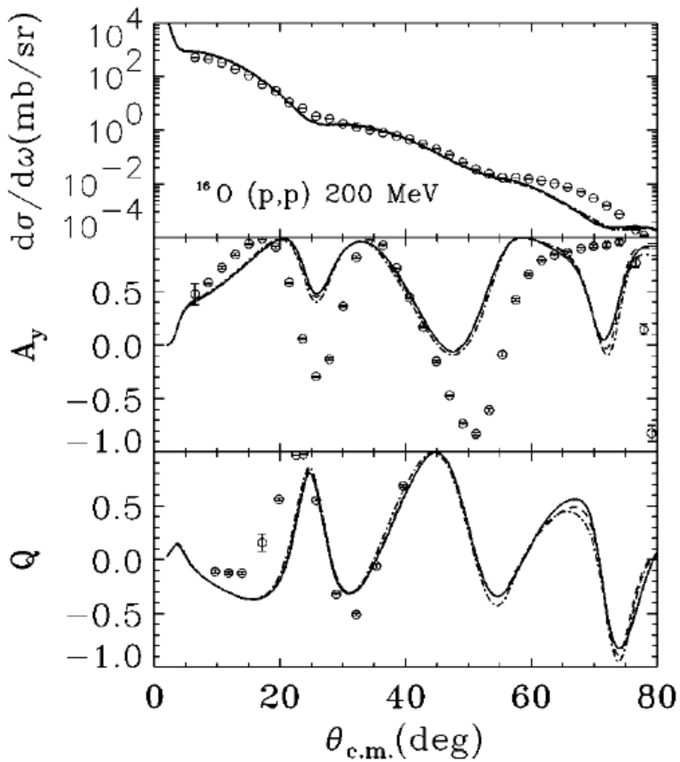


FIG. 1. The angular distribution of the differential cross section ($d\sigma/d\Omega$), analyzing power (A_y), and spin rotation function (Q) are shown for elastic proton scattering from ^{16}O at 200 MeV laboratory energy. The solid line represents the calculation performed with a first-order full-folding optical potential based on the DH density [14] and the CD-Bonn model [2]. The dashed line uses the NijmI model instead, the dash-dotted line the NijmII model [1]. The data are taken from Ref. [19].

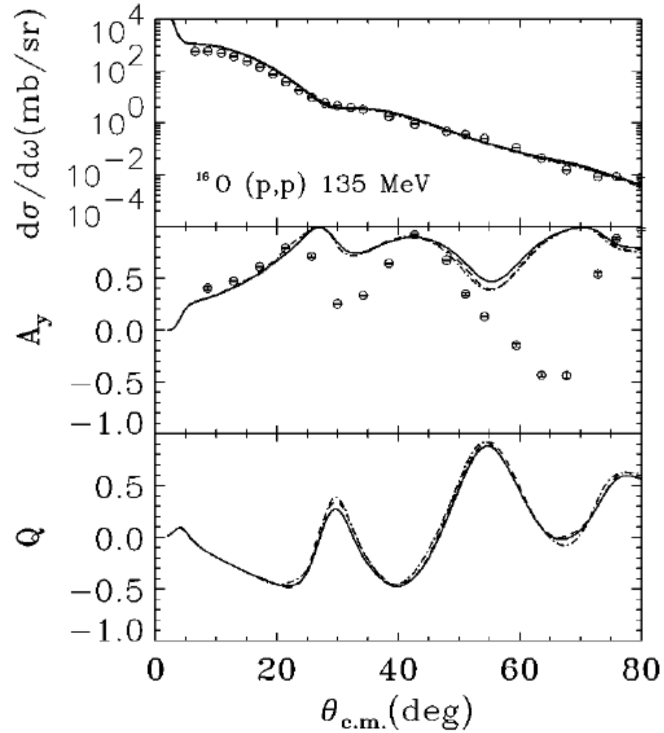


FIG. 5. Same as Fig. 1, except that the projectile energy is 135 MeV. The data are taken from Ref. [22].

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Theory in current implementation strictly valid only for closed shell nuclei

NN t-matrix in Wolfenstein representation:

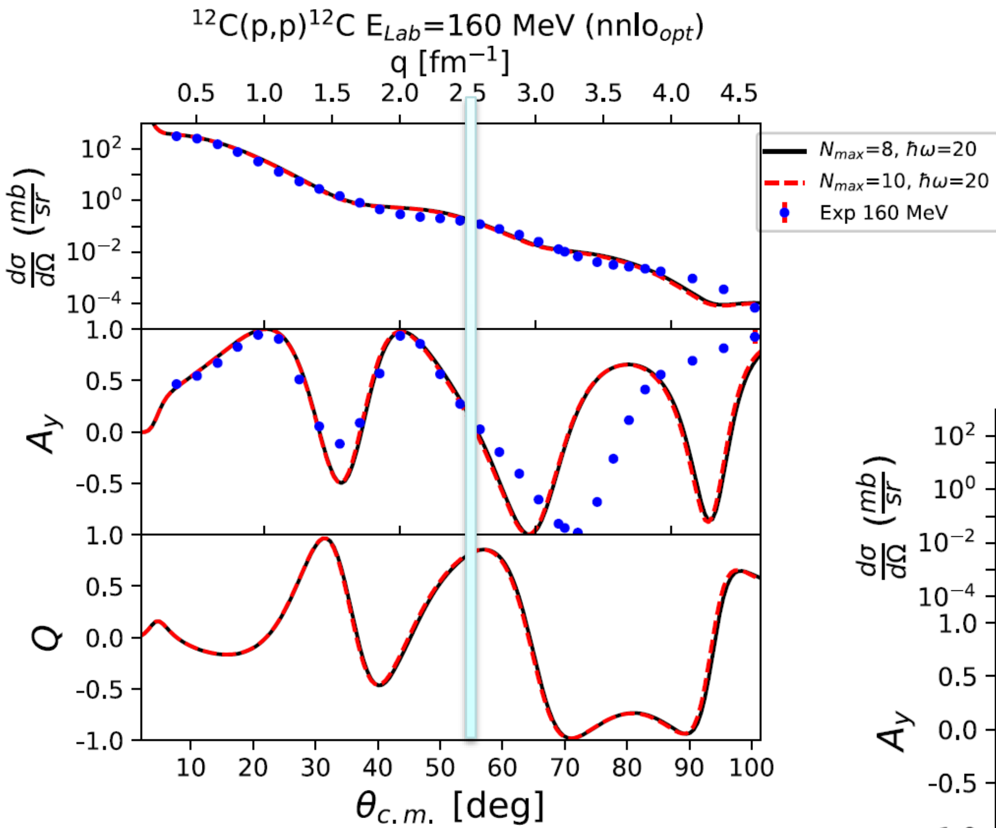
Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

Closed shell nuclei

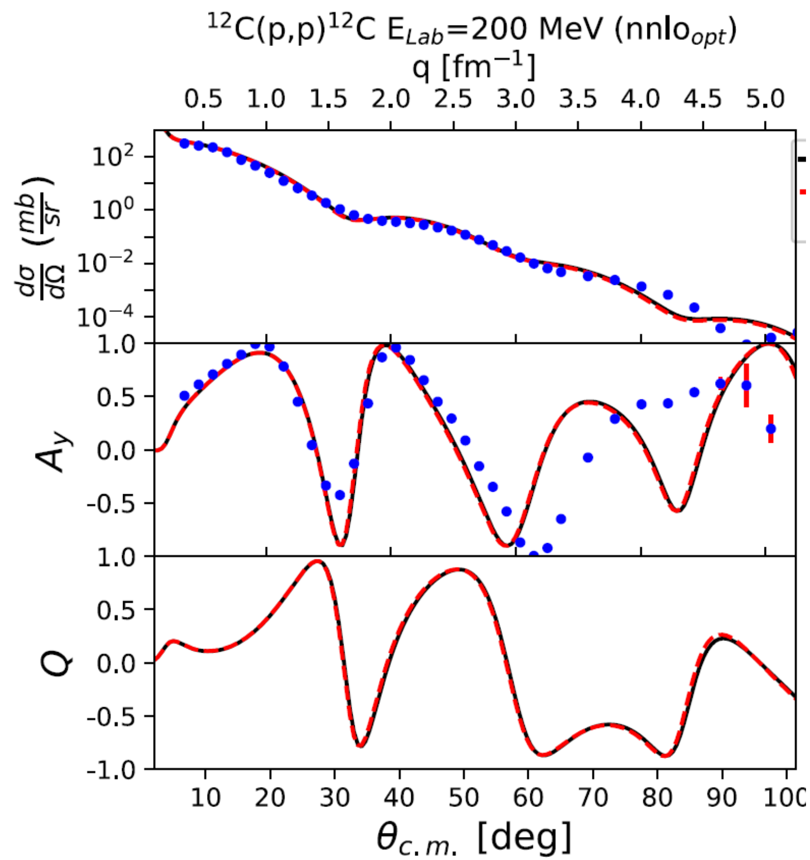
$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1} + \mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN})$$

$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]$$

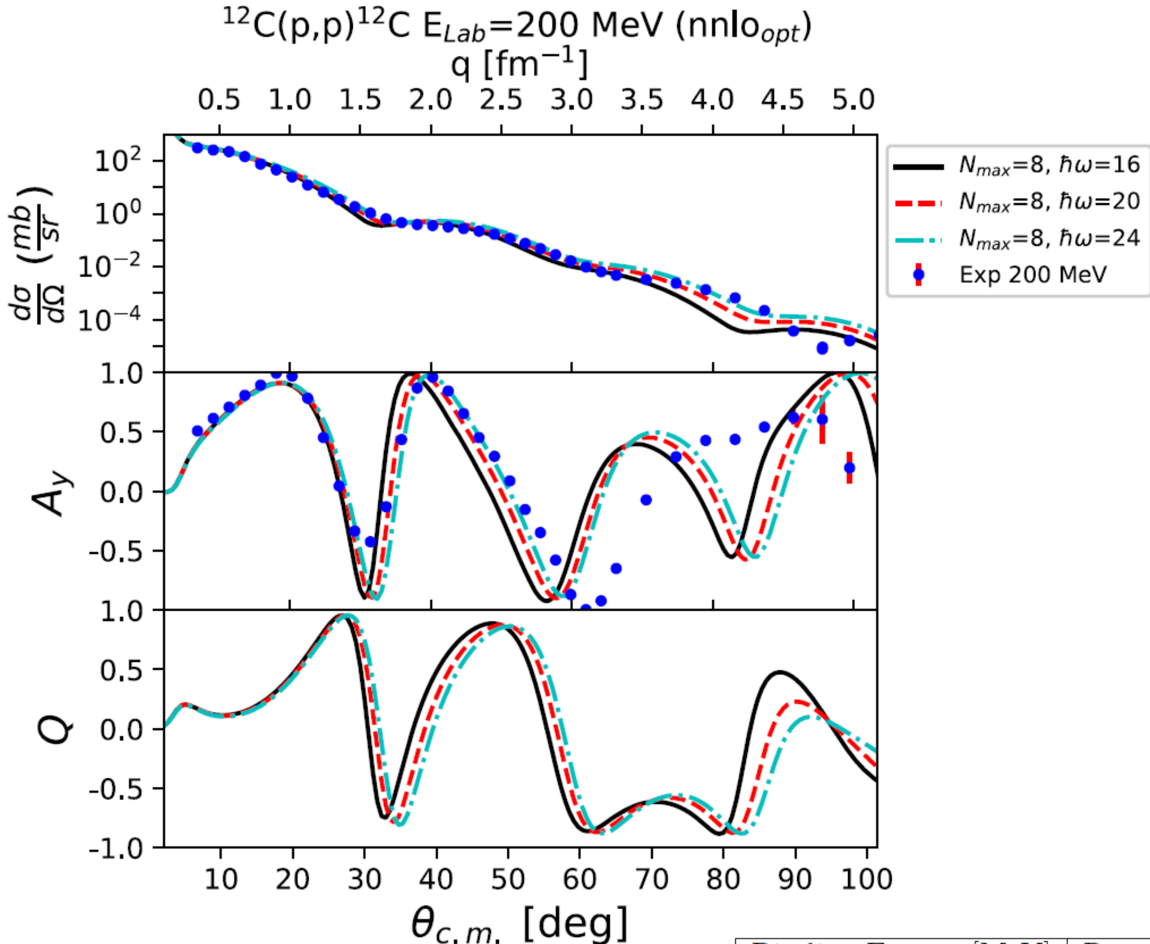
Off-shell



12C(p,p)12C E_Lab=160 MeV (nnlo_opt)
q [fm^-1]
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5
dσ/dΩ (mb/sr)
10^2 10^0 10^-2 10^-4
A_y
Q
1.0 0.5 0.0 -0.5 -1.0
10 20 30 40 50 60 70 80 90 100
θ_c.m. [deg]
N_max=8, ħω=20
N_max=10, ħω=20
Exp 160 MeV

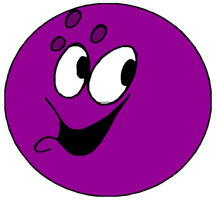


12C(p,p)12C E_Lab=200 MeV (nnlo_opt)
q [fm^-1]
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
dσ/dΩ (mb/sr)
10^2 10^0 10^-2 10^-4
A_y
Q
1.0 0.5 0.0 -0.5 -1.0
10 20 30 40 50 60 70 80 90 100
θ_c.m. [deg]



Binding Energy [MeV]	Proton RMS Radii (Point) [fm^{-1}]	N_{max}	$\hbar\omega$ [MeV]
-68.7400	2.30490	8	16
-74.9570	2.30400	10	16
-74.2340	2.18040	8	20
-79.5700	2.20220	10	20
-75.2760	2.09480	8	24
-80.2870	2.13450	10	24

p+A and n+A effective interactions (optical potentials) for closed shell nuclei



- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes
- **First consistent calculation of first order term in spectator expansion in impulse approximation carried out (Same NN interaction for structure and NN t-matrix)**

- Promising calculations for ^{4}He and ^{16}O for proton (neutron) scattering between 100 and 200 MeV proton energy
- Reasonable results for ^{12}C
- **Further studies needed what determines quality of pA observables**
- **Role of SRG evolved interactions in this approach.**
- Investigate energy dependence in NN t-matrix

$$\mathcal{E}=E_{NA}-\frac{[(A-1)/A\mathbf{K}+\mathbf{P}]^2}{4m_N}.$$

Full folding integral has same character as integral in Faddeev kernel

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Extension:
p+A and n+A effective interactions (optical potentials) for open shell nuclei

NSCM OBDM (current):

$$\left\langle n'(l's')j' \left|\left| \frac{\delta(r_i - r_s)}{r_s^2} \frac{\delta(r_i' - r_s')}{r_s'^2} \mathcal{Y}_K^{\mu\mu'}(\hat{r}_i, \hat{r}_i') \right|\right| n(ls)j \right\rangle =$$

$$\hat{K}\hat{j}\hat{j}' \begin{Bmatrix} l' & l & K \\ s' & s & 0 \\ j' & j & K \end{Bmatrix} \left\langle n'l' \left|\left| \frac{\delta(r_i - r_s)}{r_s^2} \frac{\delta(r_i' - r_s')}{r_s'^2} \mathcal{Y}_K^{\mu\mu'}(\hat{r}_i, \hat{r}_i') \right|\right| nl \right\rangle \langle s' || \hat{1} || s \rangle$$

Scalar in spin space
$\rightarrow$ closed shell nuclei

Needed: vector in spin space $\left\langle S\, m_s \,|\, \tau^{(i)}_{k_s q_s}(S) \,|\, S'\, m_s' \right\rangle$

$$\tau^{(i)}_{k_s, q_s}\left(S = \tfrac{1}{2}\right) : \quad \tau^{(i)}_{10} = 2s_z$$

$$\tau^{(i)}_{1,\pm 1} = \mp \frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

Formulation proposed in Orazbayev, Elster, Weppner, PRC 88, 034610 (2013)

Could not be implemented due to microscopic (NCSM) density not being available at the time. Now we can be serious about it.

Theory in current implementation strictly valid only for closed shell nuclei

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

Closed shell nuclei

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i\cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)}\otimes\mathbf{1} + \mathbf{1}\otimes\boldsymbol{\sigma}^{(i)})\cdot\hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{n}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN})$$

$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathrm{K}}_{NN})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathrm{q}})\right]$$

Off-shell

p+A and n+A effective interactions (optical potentials)

- **In the multiple scattering approach not even the first order term is fully explored: all work concentrates on closed-shell nuclei**

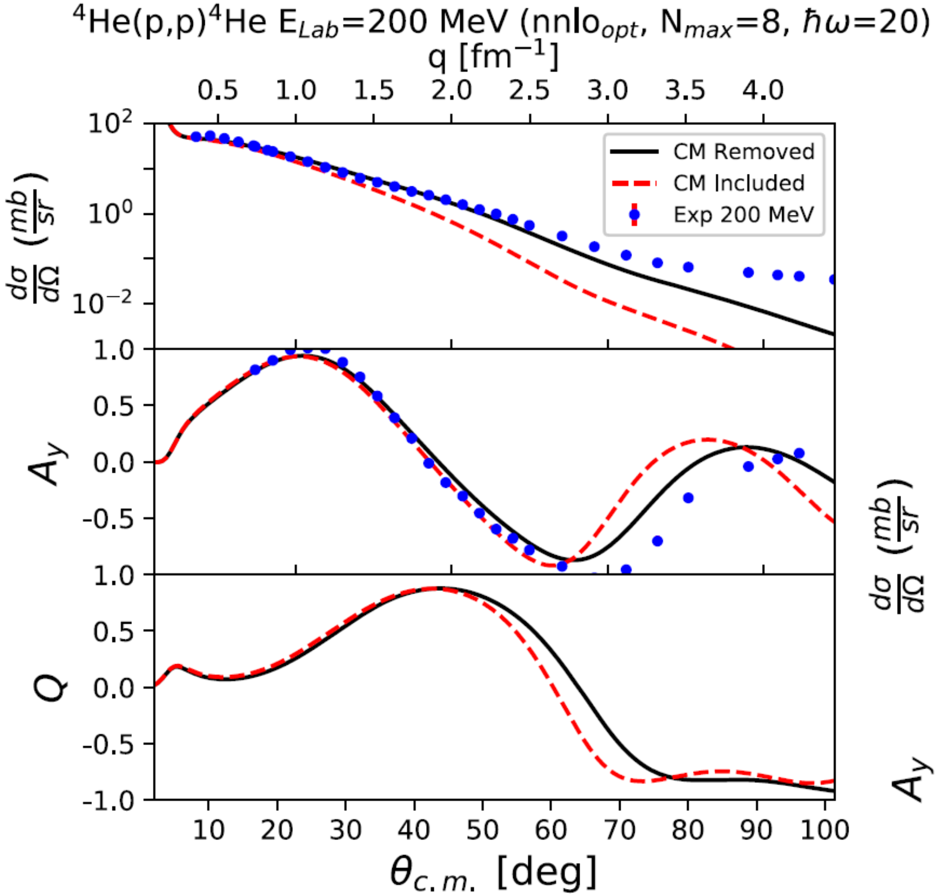
- **Today one can start to explore importance of open-shells in light nuclei: full complexity of the NN interactions enters**

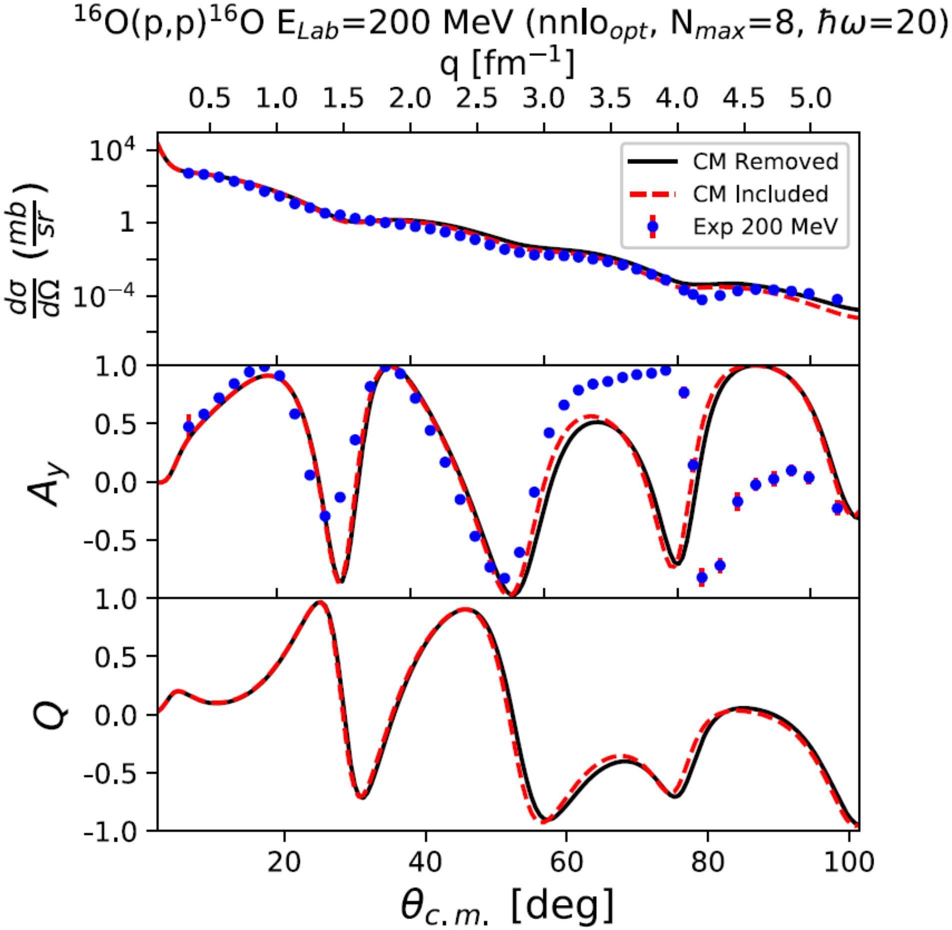
- **Longer term future (regime ~ 50 MeV lab kinetic energy):**

  Re-thinking of propagator modification in first order term

  Exploring of 2nd order term: three-body calculation with two-body density matrix (Faddeev type calculations)

Influence of CM on scattering observables





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