北京大学物理学院
School of Physics, Peking University

Gamow shell model with realistic nuclear forces

Furong Xu

I. Model

With-Core Gamow Shell Model (CGSM) based on realistic forces

(resonance + continuum)

II. Applications

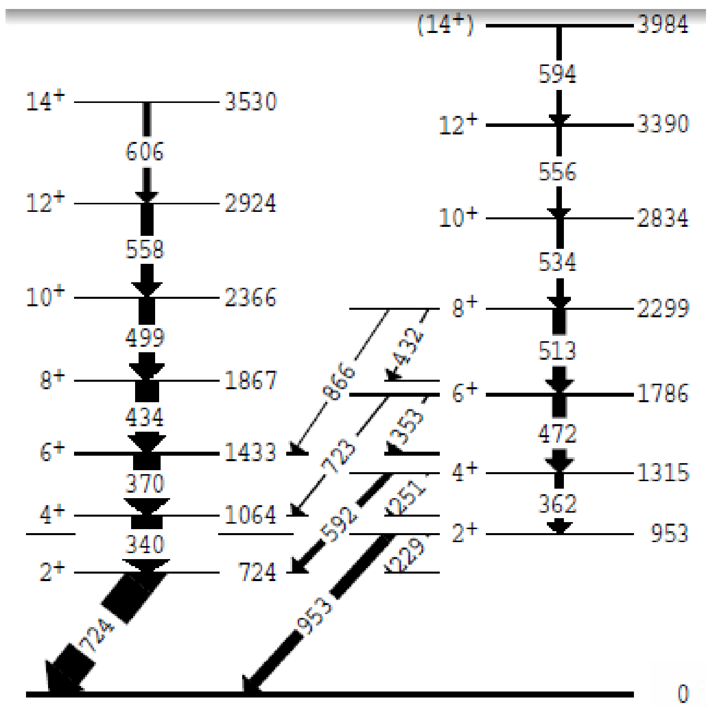
Neutron-rich oxygen isotopes

Excitation spectra

Connecting Bound States to the Continuum
Facility for Rare Isotope Beams (FRIB)
June 11-22, 2018

γ-ray spectra

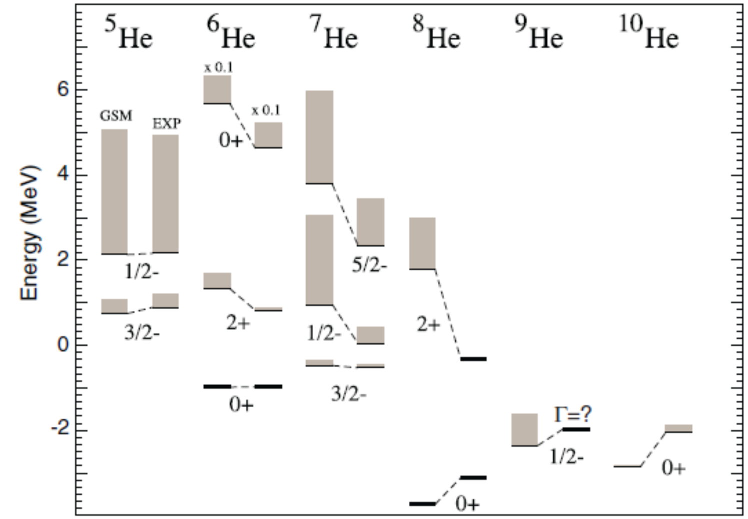
^{188}Pb: prolate and oblate bands

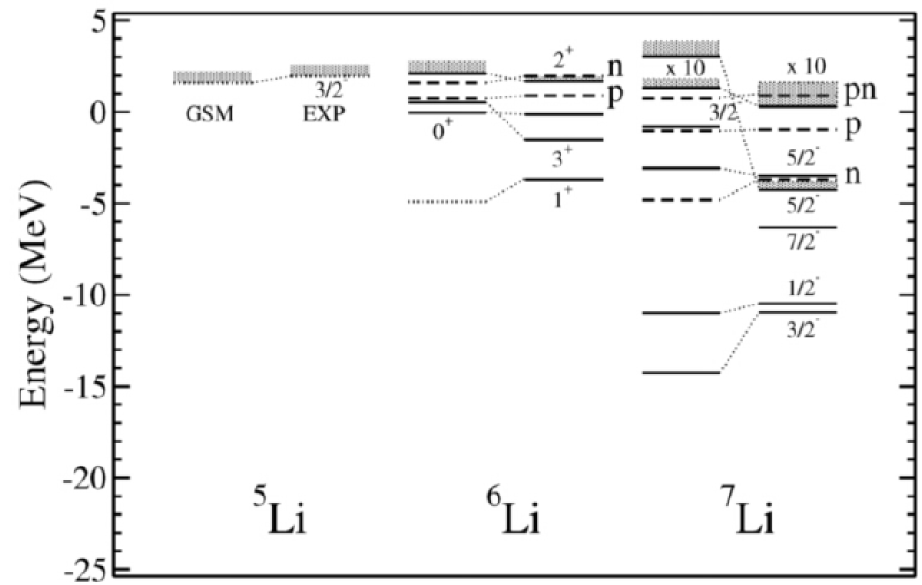


J. Pakarinen et al., PRC 72, 011304(R) (2005)

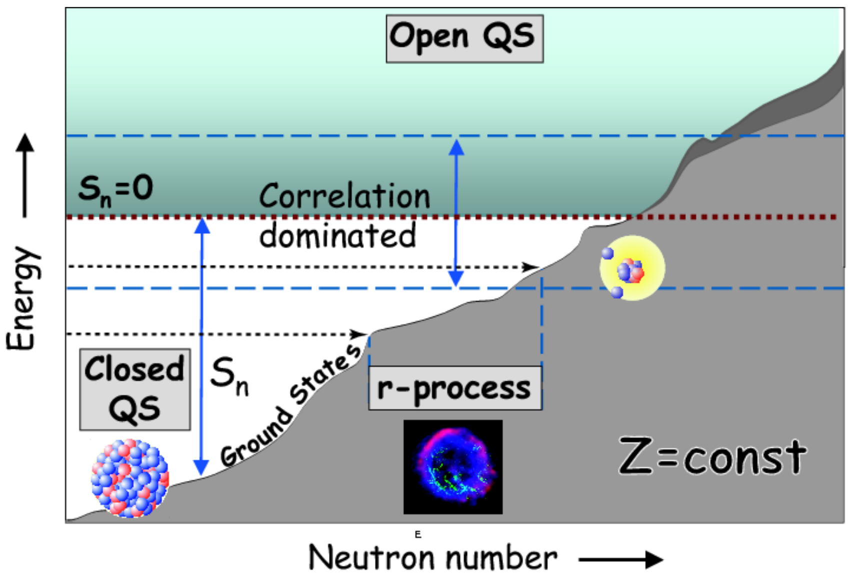
Spectra of resonance states

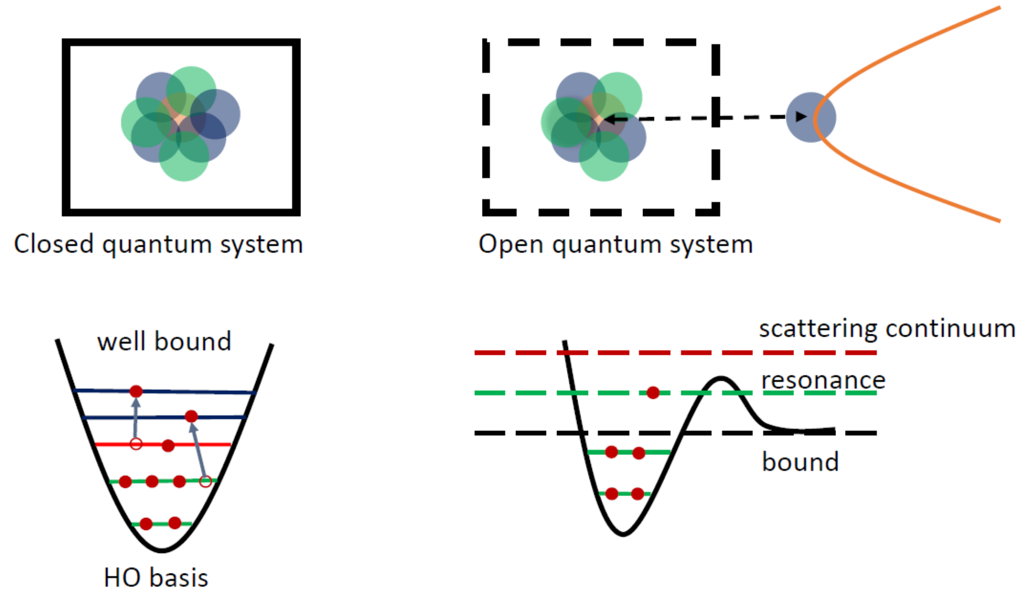
Energies and resonance widths against particle emissions

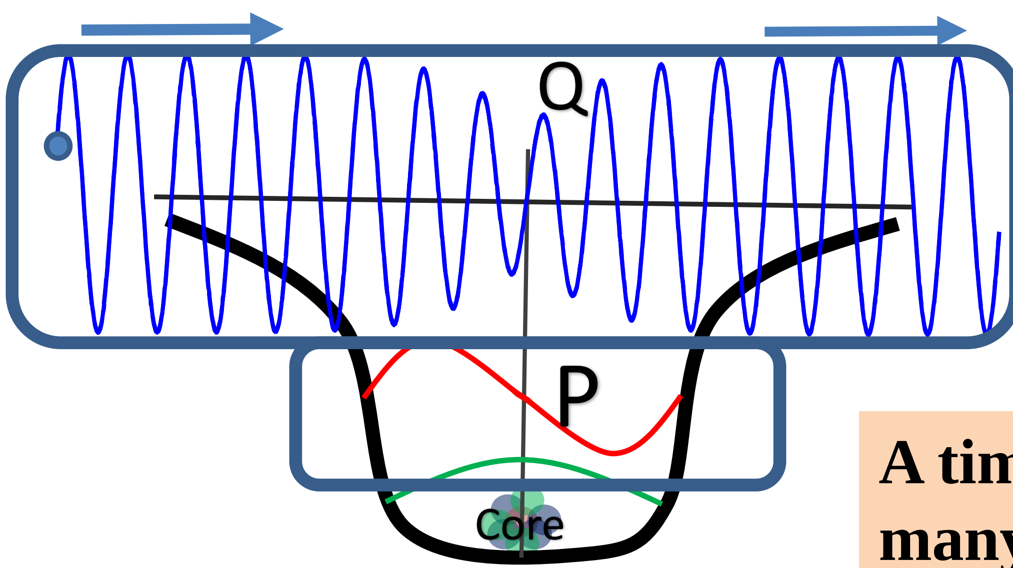




N. Michel, W. Nazarewicz, J. Okolowicz, M. Ploszjczak, Nucl. Phys. A 752, 335c (2005)







A time-dependent many-body problem

T. Berggren, Nucl. Phys. A109 (1968) 265

Single-particle basis in complex-*k* plane describe bound, resonance and scattering on equal footing.

The radial wave function $u(r)/r$

$$\frac{d^2u(k,r)}{dr^2} = \left[\frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}U(r) - k^2\right]u(k,r)$$

boundary conditions

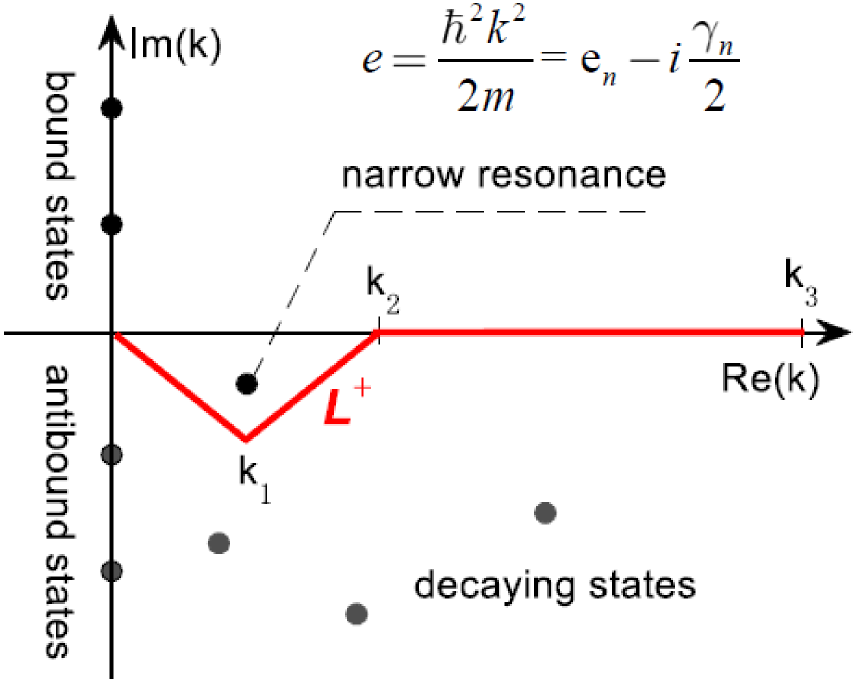
$$u(0) = 0,$$

$$u(a)O_l'(ka) - u'(a)O_l(ka) = 0$$

$$O_l(kr) \sim e^{i(kr - l\pi/2)}$$

Outgoing solution at large distance

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

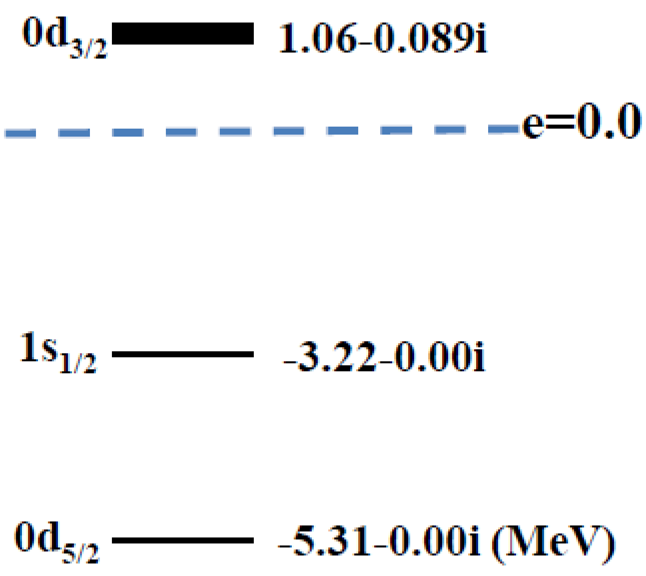


Orthogonality and Completeness

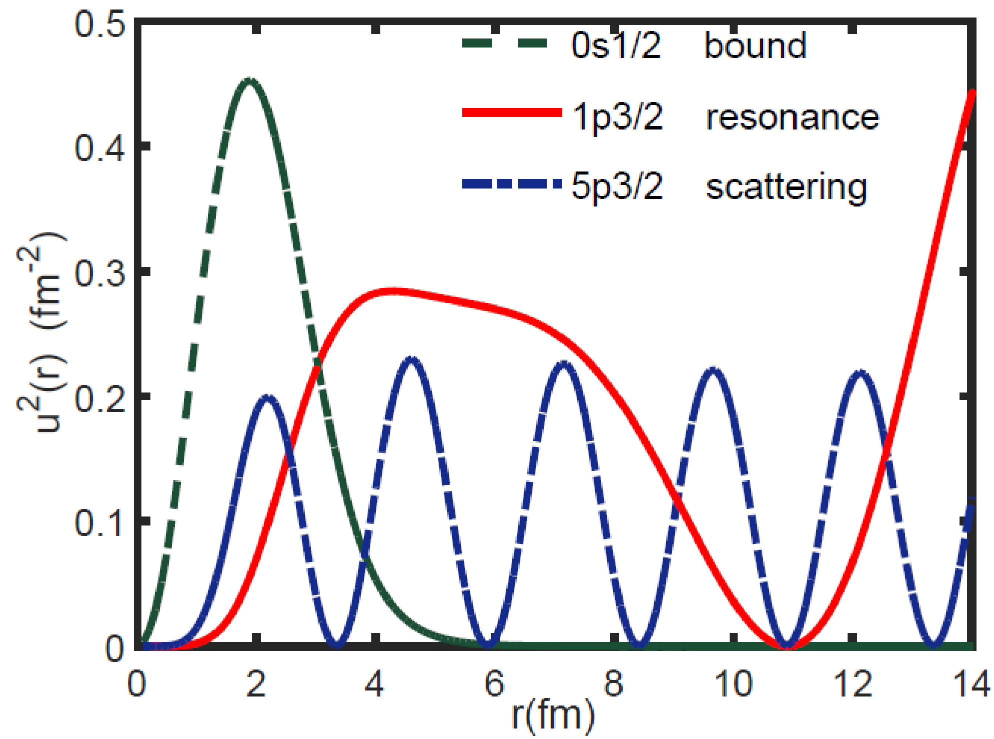
$$\delta(r - r') = \sum_n w_n(r,k_n)w_n(r',k_n) + \frac{1}{\pi}\int_{L^+} dq\, u(r,q)u(r',q)$$

Discretized

Woods-Saxon potential, CD-Bonn, ^{16}O core



WS SPE's



^{17}O

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

Details for Berggren basis, see also talks by:

Nazarewicz, Sossez, Ploszajczak, Barrett, Id Betan

R.J. Liotta *et al*., PLB 367, 1 (1996)…

used Berggren basis to describe single-particle resonance in nuclei;

later for two-particle resonance (Betan *et al*., PRL 89, 042601 (2002)

using phenomenological potential

Many-body systems

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} v_{ij}^{NN} - \frac{\boldsymbol{P}^2}{2Am}$$

$$\boldsymbol{P} = \sum_{i=1}^{A} \boldsymbol{p}_i$$

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + U + \sum_{i<j=1} \left(v_{ij}^{NN} - U - \frac{p_i^2}{2Am} - \frac{\boldsymbol{p}_i \boldsymbol{p}_j}{Am} \right)$$

$$= H_0 + V.$$

$$E = E_n - i \frac{\Gamma_n}{2}$$

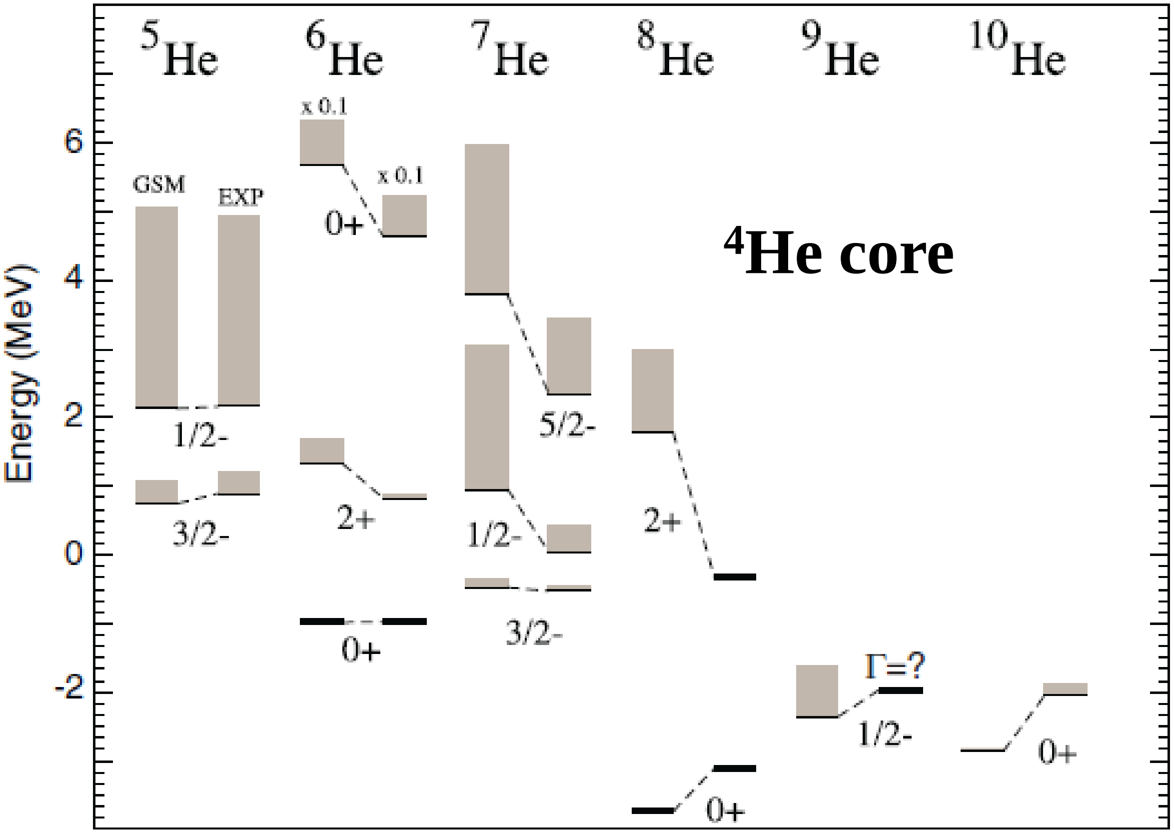
$$H_0 = \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} + U \right)$$

Michel, Nazarewicz, Ploszajczak, Rotureau *et al.*, 2003--

$$V = V_{WS} + V_{J,T}(\vec{r}_1, \vec{r}_2)$$

$$V(\mathbf{r}_i, \mathbf{r}_j) = -V_{\mathrm{SGI}}^{(J,T)} \exp\left[-\left(\frac{\mathbf{r}_i - \mathbf{r}_j}{\mu}\right)^2\right] \delta(r_i + r_j - 2R_0)$$

$V_{\mathrm{SGI}}^{(J)}$ is the strength in the JT channel

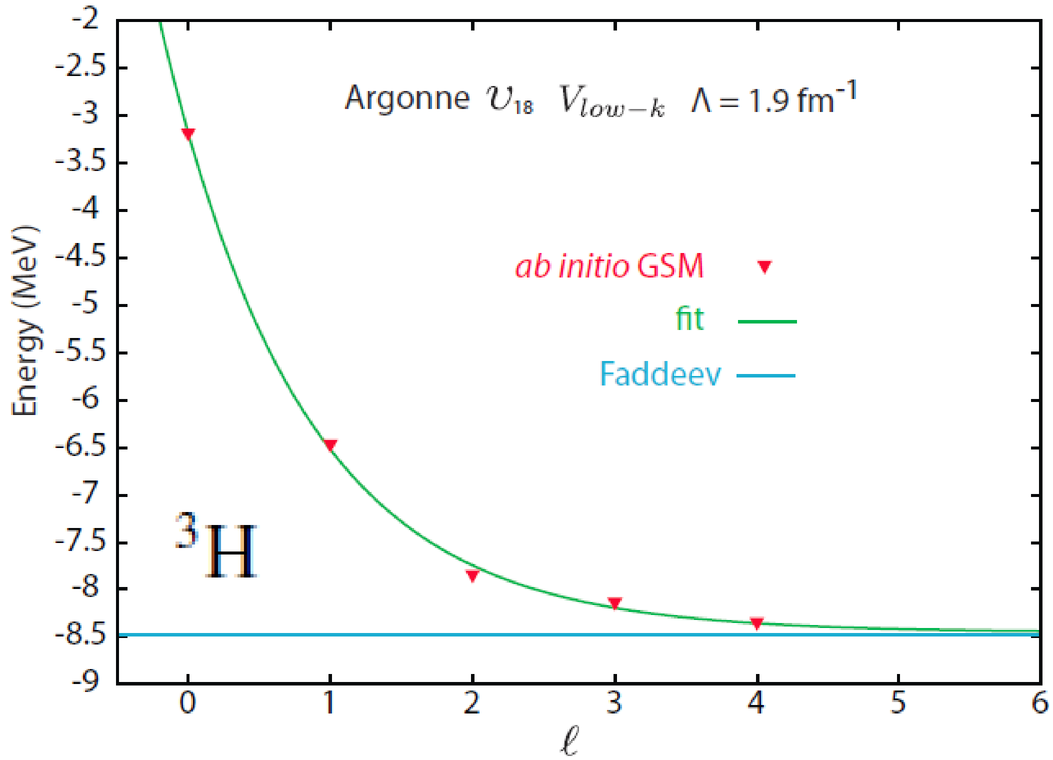


Michel, Nazarewicz, Płoszajczak, Vertse, JPG 36 (2009) 013101

- **Hagen, Hjorth-Jensen *et al.*, PRC 73, 064307 (2006): Core GSM with realistic forces, but neglecting Q-box, applied to two-particle systems (e.g., ^{18}O)**
- **Later, Tsukiyama Hjorth-Jensen, Hagen, PRC 80, 051301 (R) (2009): improving by using Q-box but no folded-diagrams.**

Papadimitriou *et al.* , Phys. Rev. C 88, 044318 (2013): realistic forces

***Ab initio* no-core Gamow shell model for light nuclei**



Gamow shell model with an inert core

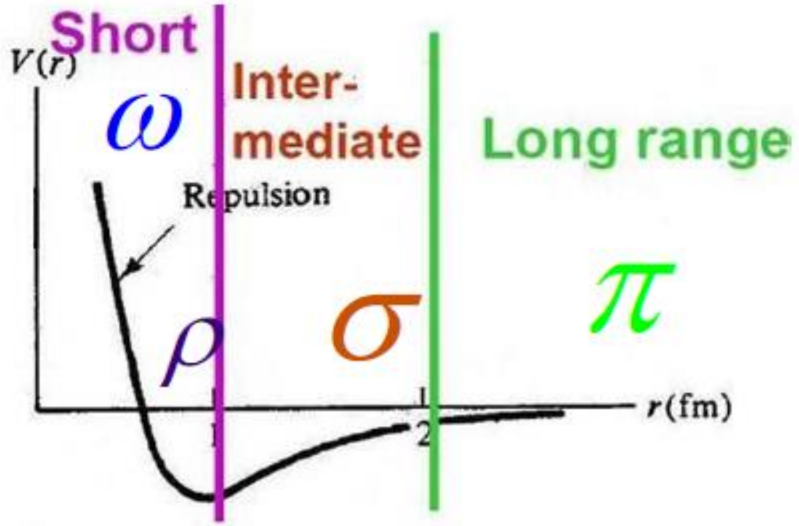
1. Start from realistic forces;

2. Take a double magic core

   Q-box + folded diagrams （MBPT）

3. Calculate resonance spectra

CD-Bonn



CGSM based on realistic nuclear forces

Realistic nuclear forces ⇨ Gamow shell model calculations

Taking a doubly closed core

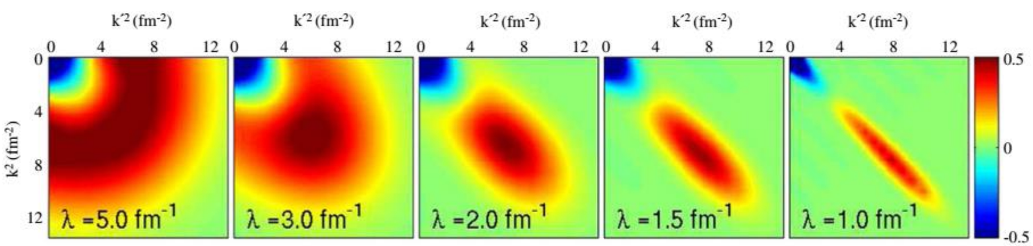
Bare forces: Strong repulsion, slow convergence

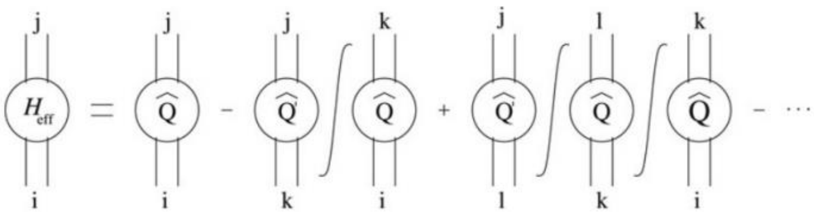
$V_{low\ k}$ or SRG

To remove hard core, but still keep good descriptions of NN scattering phase shifts

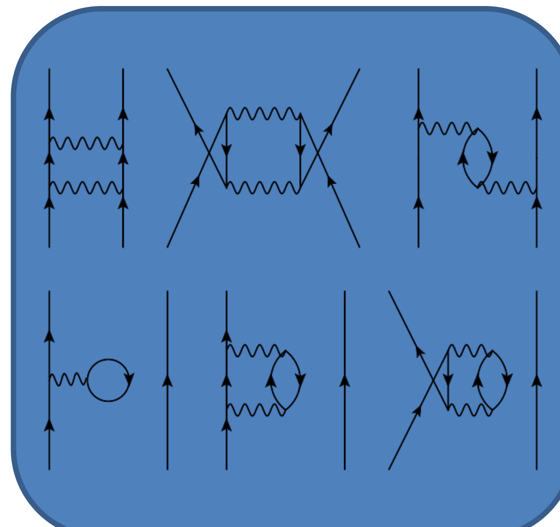
$$\langle \alpha_P | \tilde{H}_{\text{eff}} | \alpha_{P'} \rangle = \sum_{\alpha_{p''}} \sum_{\alpha_{p'''}} \sum_{kk'k'' \in \mathcal{K}} \langle \alpha_P | \tilde{k}'' \rangle \langle \tilde{k}'' | \alpha_{p''} \rangle \langle \alpha_{p''} | \tilde{k} \rangle E_k \langle \tilde{k} | \alpha_{p'''} \rangle \langle \alpha_{p'''} | \tilde{k}' \rangle \langle \tilde{k}' | \alpha_{P'} \rangle$$

$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\text{rel}}, H_\lambda], H_\lambda]$$





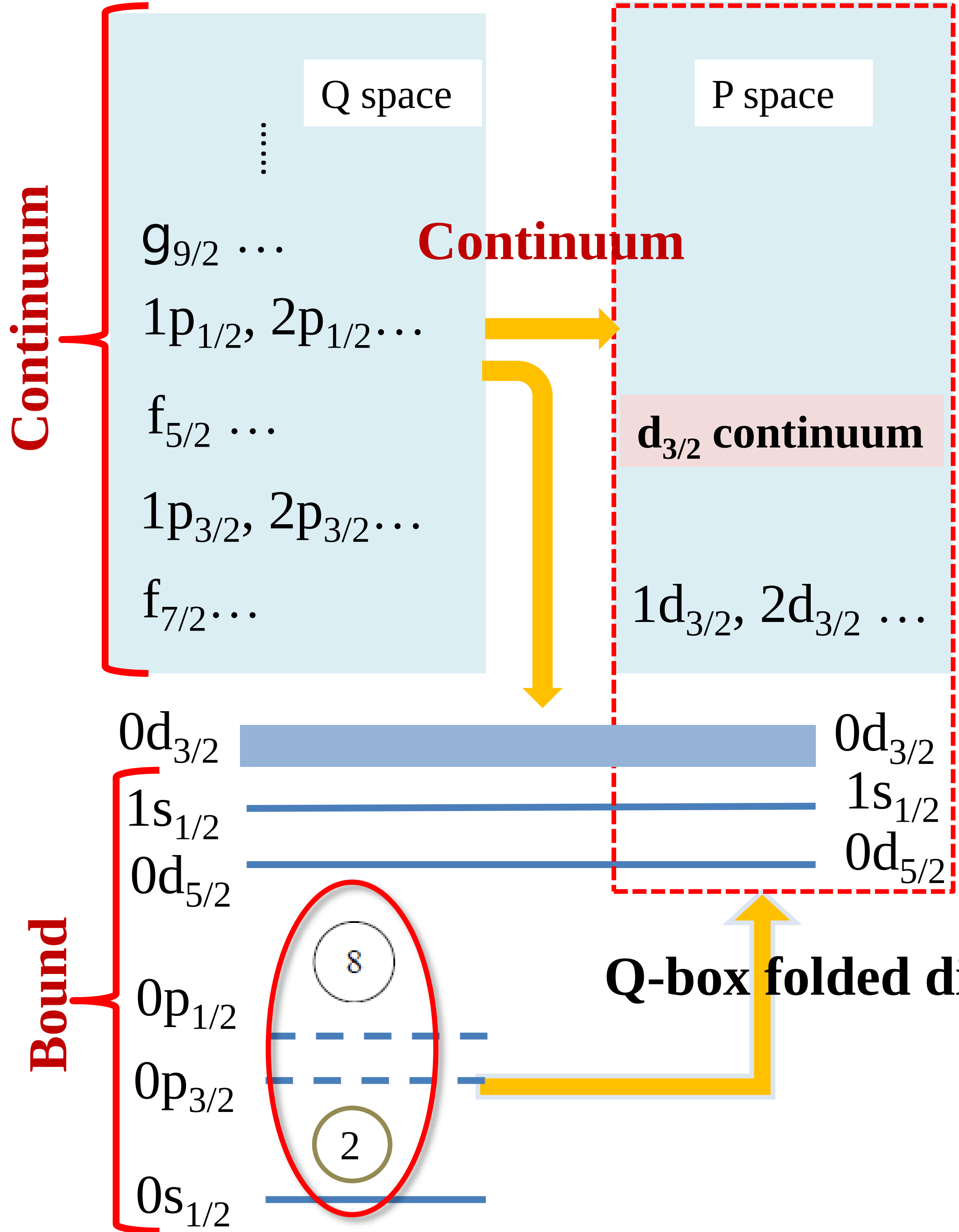
Q

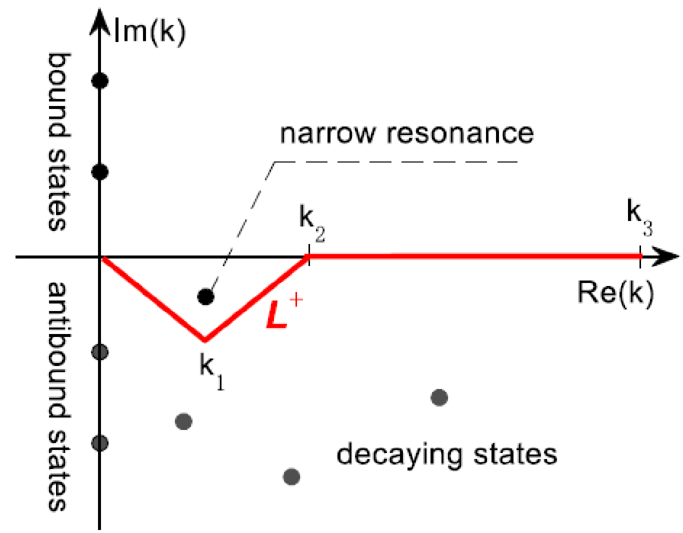


$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP,$$

Non-degenerate extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011);

Model space





For each given partial wave

Berggren Completeness Relation

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

Q-box folded diagrams in complex-k basis

$$\varepsilon_n = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

We need to establish the effective Hamiltonian in the model space P, based on realistic forces

Q-box folded diagram method in complex-energy space

1. $V_{low\text{-}k}$
2. Using Brody-Mshinsky brackets, NN interaction which is in relative and CoM coordinates is transferined into the laboratory (HO basis)

Truncated by $N_{shell} \sim 12$, an approximate completeness

$$\sum_{\alpha\leq\beta}^{\text{NShell}} |\alpha\beta\rangle\langle\alpha\beta| = \mathbf{1}$$

where $|\alpha\beta\rangle$ is the two-particle states in HO basis

In HO basis, NN matrix elements:

$$V_{osc} = \sum_{\alpha\leq\beta}\sum_{\gamma\leq\delta} |\alpha\beta\rangle\langle\alpha\beta|V_{low-k}|\gamma\delta\rangle\langle\gamma\delta|$$

In Berggren basis

$$\langle ab|V|cd\rangle \approx \sum_{\alpha\leq\beta}\sum_{\gamma\leq\delta}\langle ab|\alpha\beta\rangle\langle\alpha\beta|V_{low-k}|\gamma\delta\rangle\langle\gamma\delta|cd\rangle$$

where $|ab\rangle$ is two-particle states constructed with Berggren s.p. basis

For identical particles (pp or nn), the expansion coefficients are

$$\langle ab|\alpha\beta\rangle = \frac{\langle a|\alpha\rangle\langle b|\beta\rangle - (-1)^{J-j_\alpha-j_\beta}\langle a|\beta\rangle\langle b|\alpha\rangle}{\sqrt{(1+\delta_{ab})(1+\delta_{\alpha\beta})}}$$

For np: $\langle ab|\alpha\beta\rangle = \langle a|\alpha\rangle\langle b|\beta\rangle$

one-body expansion coefficients are calculated with

$$\langle a|\alpha\rangle = \int drr^2 u_a(r) R_\alpha \delta_{l_a l_\alpha}\delta_{j_a j_\alpha}\delta_{t_a t_\alpha}$$

$u_a(r)$ – Berggren basis; R_α - HO basis

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + U + \sum_{i<j=1} \left(v_{ij}^{NN} - U - \frac{p_i^2}{2Am} - \frac{\boldsymbol{p}_i \boldsymbol{p}_j}{Am} \right)$$

$$= H_0 + V.$$

$\frac{p_i^2}{2Am}$

$\frac{\boldsymbol{p}_i \boldsymbol{p}_j}{Am}$

using the exterior complex scaling technique

Q-box

$$\hat{Q}(E) = PVP + PVQ \frac{1}{E - QHQ} QVP$$

$$\widehat{Q}(E) = PVP + PV \frac{Q}{E - QH_0Q} VP + PV \frac{Q}{E - QH_0Q} VP \frac{Q}{E - QH_0Q} VP + \dots$$

2nd order perturbation 3rd order perturbation

In a degenerate s.p. space, E can be assumed approximately, $2e_i$, E is the starting energy

Q-box folded diagrams

$$V_{eff} = \widehat{Q}(\varepsilon_0) - \widehat{Q}'(\varepsilon_0) \int \widehat{Q}(\varepsilon_0) + \widehat{Q}'(\varepsilon_0) \int \widehat{Q}(\varepsilon_0) \int \widehat{Q}(\varepsilon_0) \dots.$$

$$V_{eff} = \hat{Q}(\varepsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\varepsilon_0) [V_{eff}]^k \qquad \varepsilon_0 = \varepsilon_c + \varepsilon_d$$

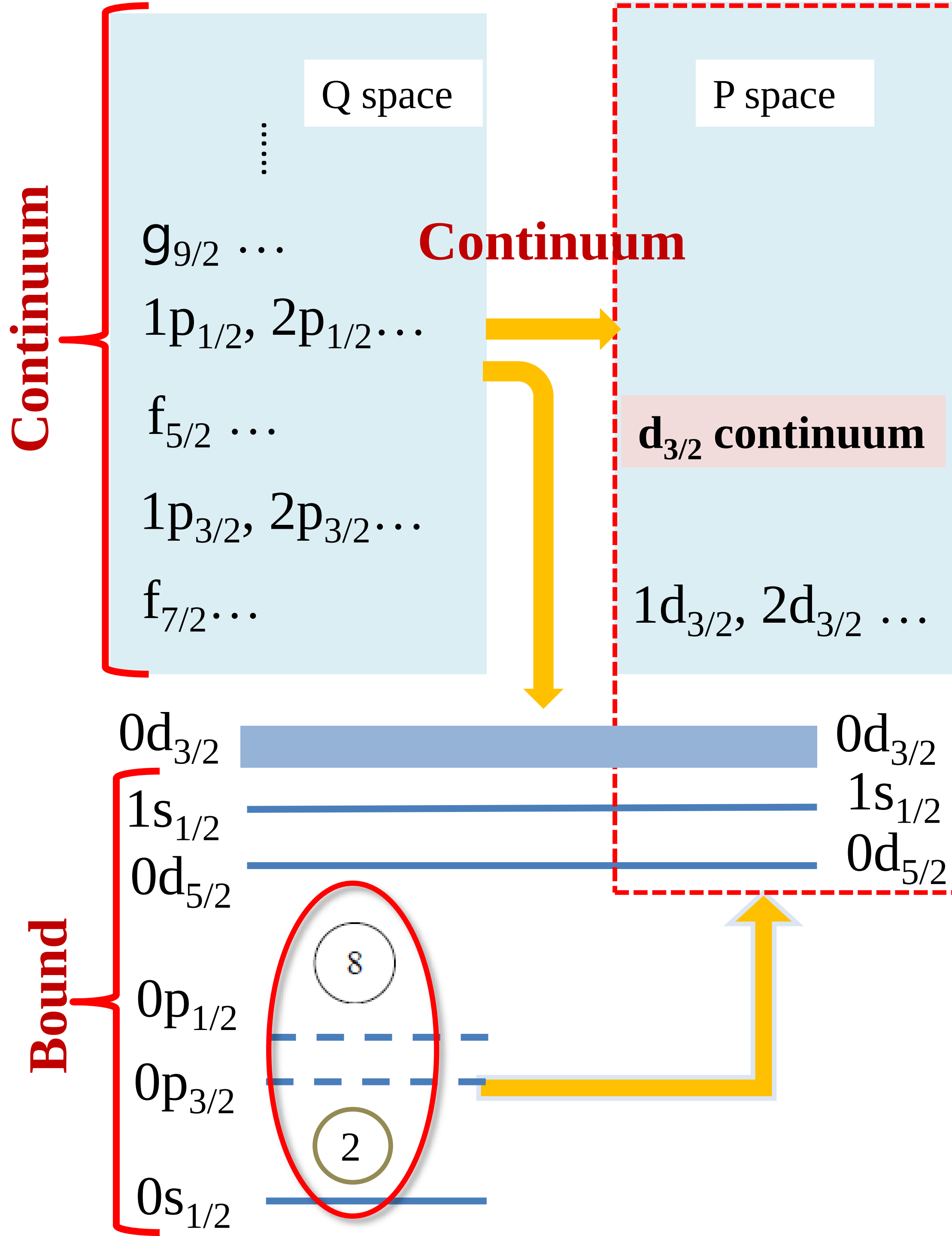
(i.e., the starting energy E)

Q-box derivatives

$$\hat{Q}_k(E) = \frac{1}{k!} \frac{d^k \hat{Q}(E)}{dE^k} = (-1)^k PVQ \frac{1}{(E - QHQ)^{k+1}} QVP$$

Kuo-Krenciglowa (KK) method

Model space



$$P + Q = 1$$

P is the model space

Q is the excluded space (including the core)

The Berggren space must be nondegenerate

Q-Box folded diagrams for nondegenerate space: Extended Kuo-Krenciglowa (EKK)

$$H_{eff} = PH_0P + PH_1Q\frac{1}{E - QHQ}QH_1P$$

$$H_{eff} - E_x = PH_0P + V_{eff} - E_x$$ (Meaning: expanded around E_x)

$$\tilde{H}_{eff} = PH_0P - E_x + \hat{Q}(E_x) + \sum_{k=1}^{\infty}\hat{Q}_k(E_x)[\tilde{H}_{eff}]^k$$

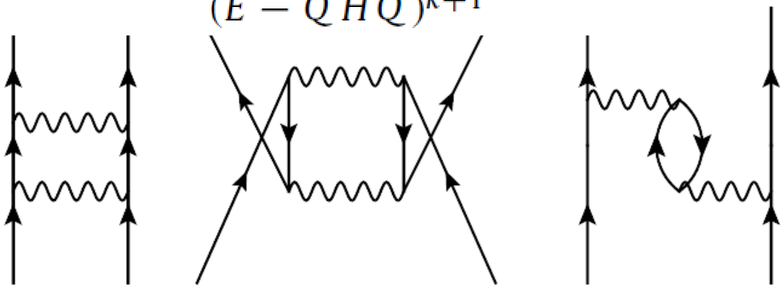
$$PH_0P - E_x = e(a) + e(b) - w$$

ω - starting energy

$$\tilde{H}_{eff}^{\{n\}} = \tilde{H}_{BH}(E) + \sum_{k=1}^{\infty}\hat{Q}_k(E)\{\tilde{H}_{eff}^{\{n-1\}}\}^k$$

$$\hat{Q}_k(E) = \frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k} = (-1)^k PVQ\frac{1}{(E - QHQ)^{k+1}}QVP$$

$\tilde{H}_{\text{eff}}^{\{n\}}$ stands for $\tilde{H}_{\text{eff}} = H_{\text{eff}} - E$ at the n-th iteration

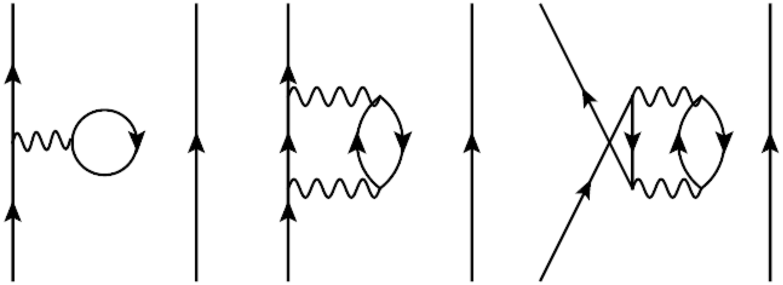


$\tilde{H}_{\text{BH}} = H_{\text{BH}}(E) - E$ is the Block–Horowitz Hamiltonian an energy E, with

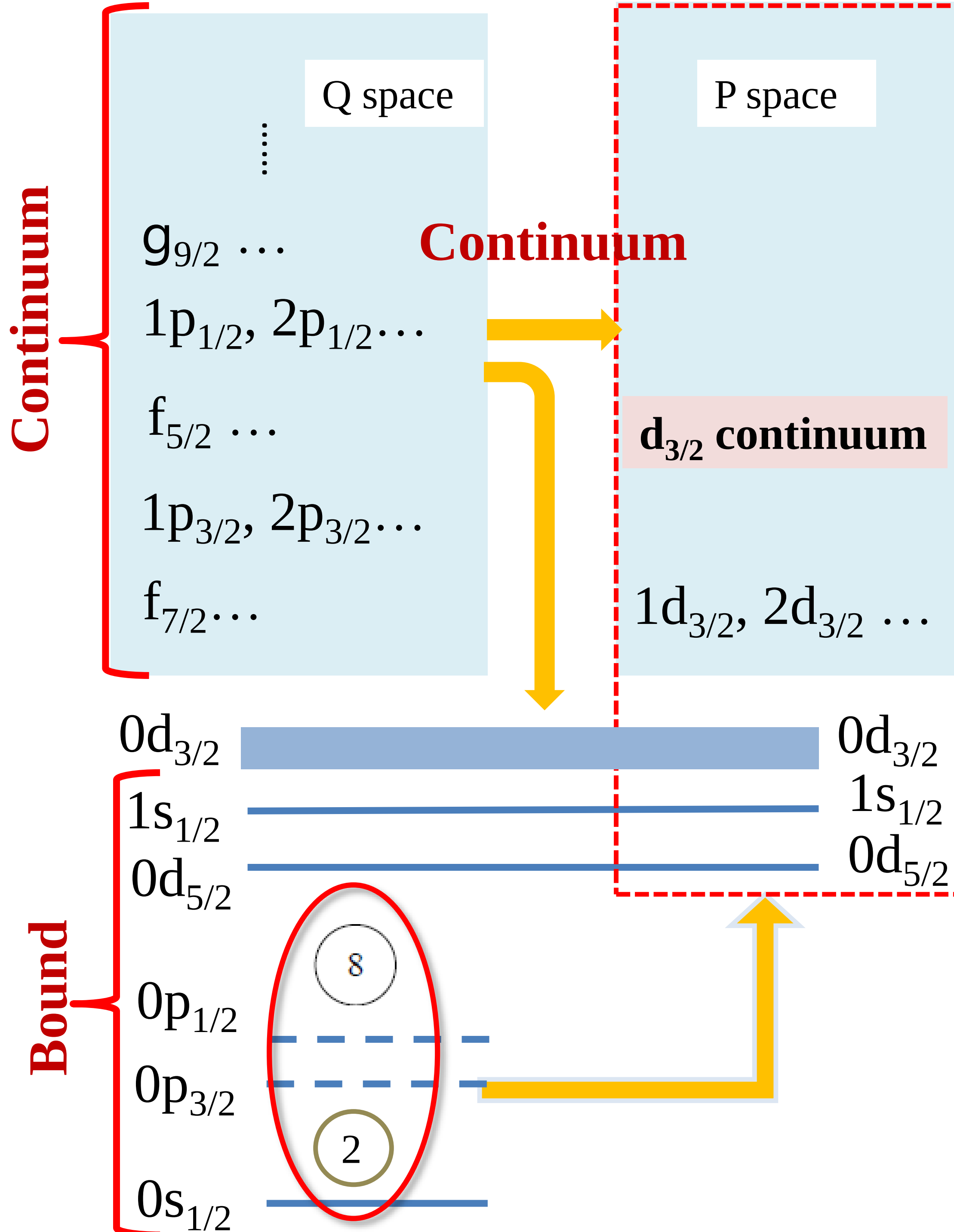
$$H_{BH} = PHP + PHQ\frac{1}{E - QHQ}QHP$$

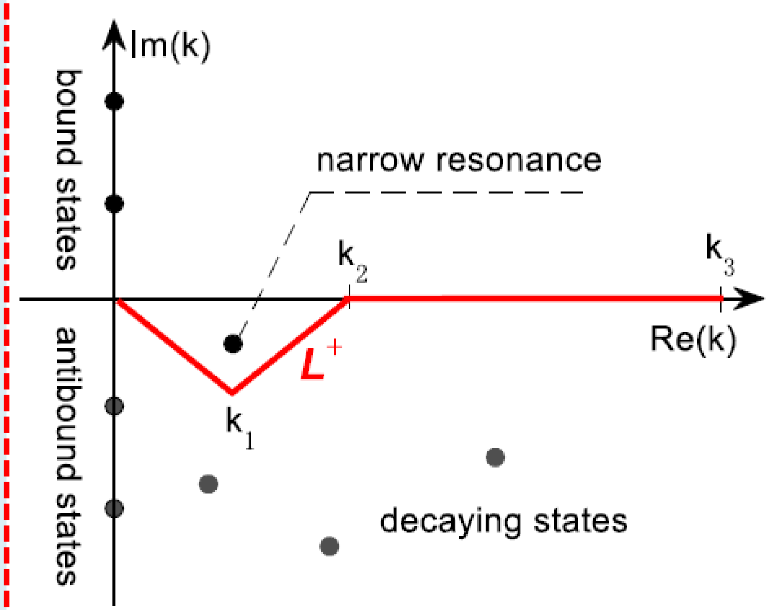
The effective Hamiltonian is obtained by $H_{\text{eff}} = \tilde{H}_{\text{eff}} + E$

effective interaction



May see talk by Morten

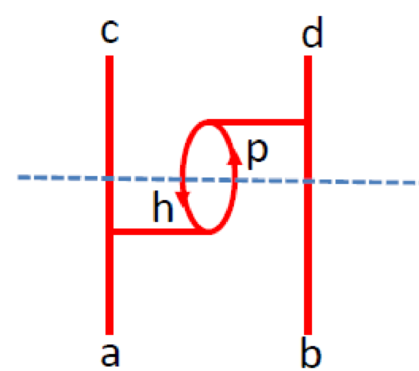




$$H_{eff} = PH_0P + PH_1Q\frac{1}{E - QHQ}QH_1P$$

$$\frac{1}{E-[(e_c+e_b)+(e_p-e_h)]}$$

$$= \frac{1}{[E-(e_c+e_b)]+(e_h-e_p)}$$



CoM correction

$$H = \sum_{i=1}^{A} \left(\frac{p_i^2}{2m} + U \right) + \sum_{i<j} \left(v_{ij} - U - \frac{p_i^2}{2Am} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{Am} \right)$$

Lawson method is no longer valid

Wave functions?

In cluster orbital coordinates (COSM): R, r_i

Y. Suzuki, K. Ikeda, RC 38, 410 (1988).

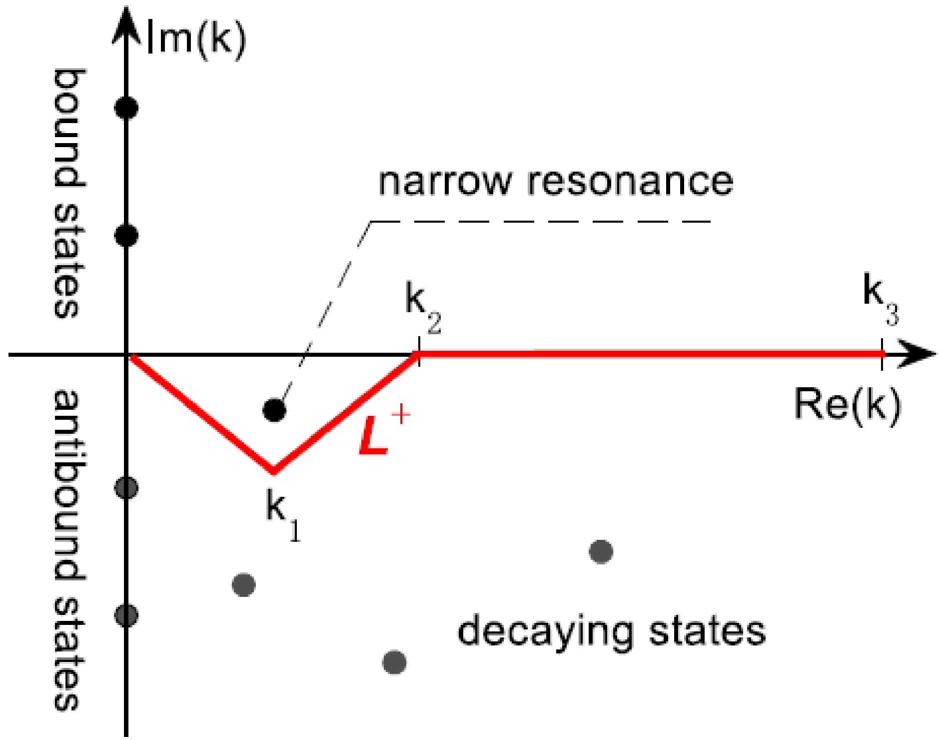
But with realistic forces:

$$\langle ab|V|cd\rangle \approx \sum_{\alpha\leq\beta} \sum_{\gamma\leq\delta} \langle ab|\alpha\beta\rangle \langle \alpha\beta|V_{low-k}|\gamma\delta\rangle \langle\gamma\delta|cd\rangle$$

$$\langle ab|\alpha\beta\rangle = \langle a|\alpha\rangle \langle b|\beta\rangle \qquad \langle a|\alpha\rangle = \int drr^2 u_a(r) R_\alpha \delta_{l_a l_\alpha} \delta_{j_a j_\alpha} \delta_{t_a t_\alpha}$$

In our CGSM calculations, for low-lying states we assume small CoM effects due to wave functions expressed in the laboratory coordinates.

Convergence against discretization number N_L

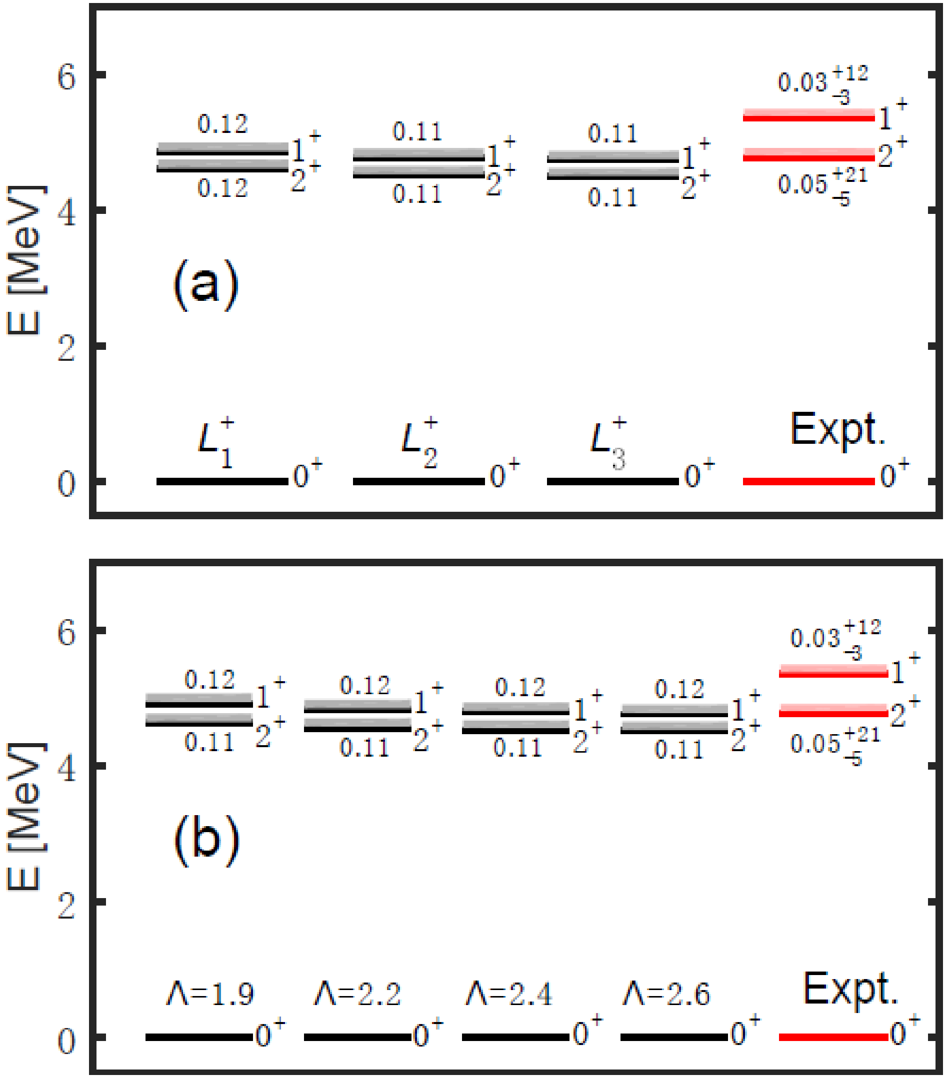


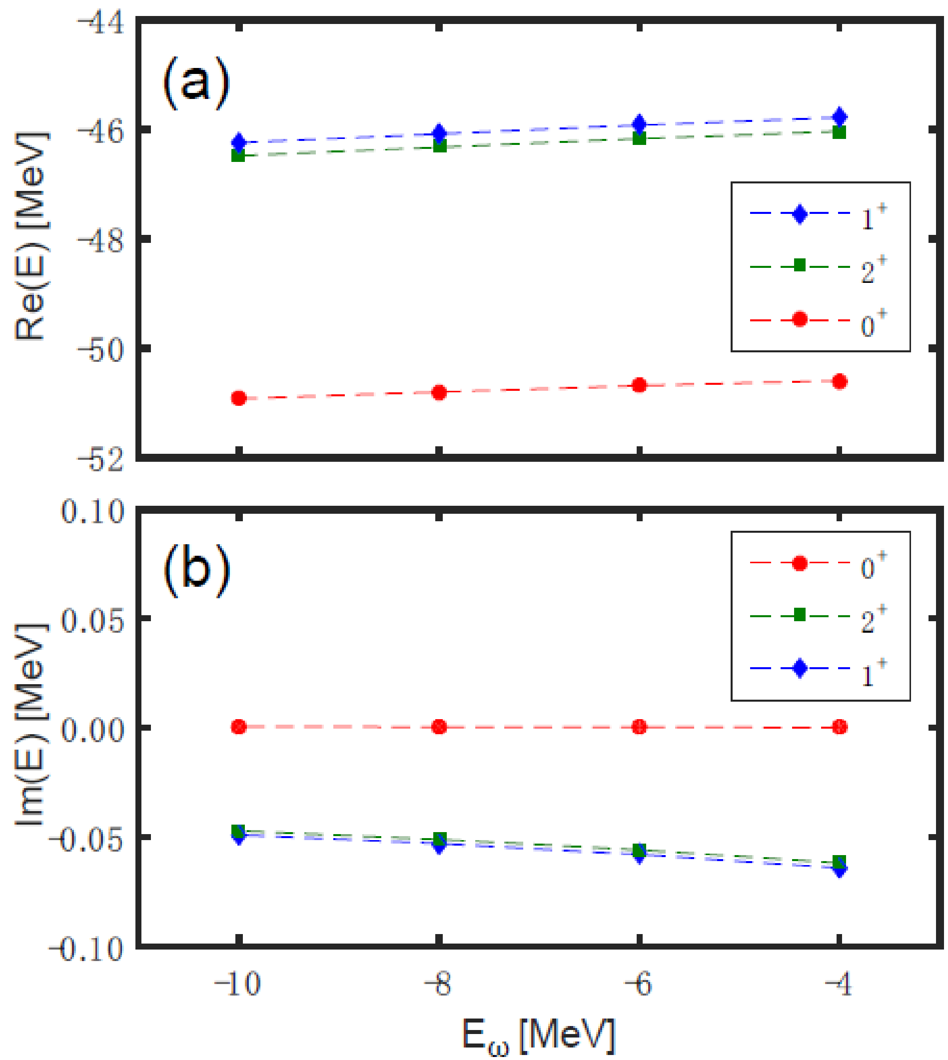
$\tilde{E}_n = E_n - i\Gamma/2$ $\quad$ ^{24}O $\quad$ $\Lambda = 2.6\ \mathrm{fm}^{-1}$

N_L	0^+	2^+	1^+
16	$-50.642 + 0.013i$	$-46.172 - 0.004i$	$-45.922 - 0.009i$
18	$-50.716 + 0.002i$	$-46.262 - 0.046i$	$-46.017 - 0.049i$
20	$-50.711 - 0.001i$	$-46.219 - 0.054i$	$-45.976 - 0.056i$
22	$-50.712 + 0.000i$	$-46.218 - 0.053i$	$-45.974 - 0.056i$

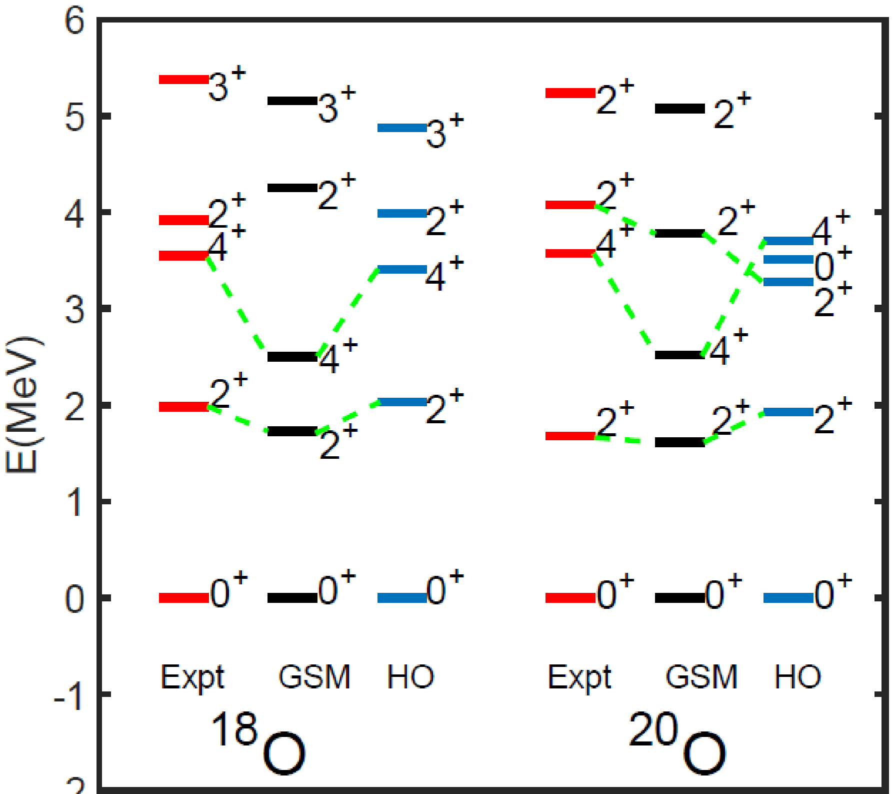
Convergences of spectroscopic calculations

^{24}O

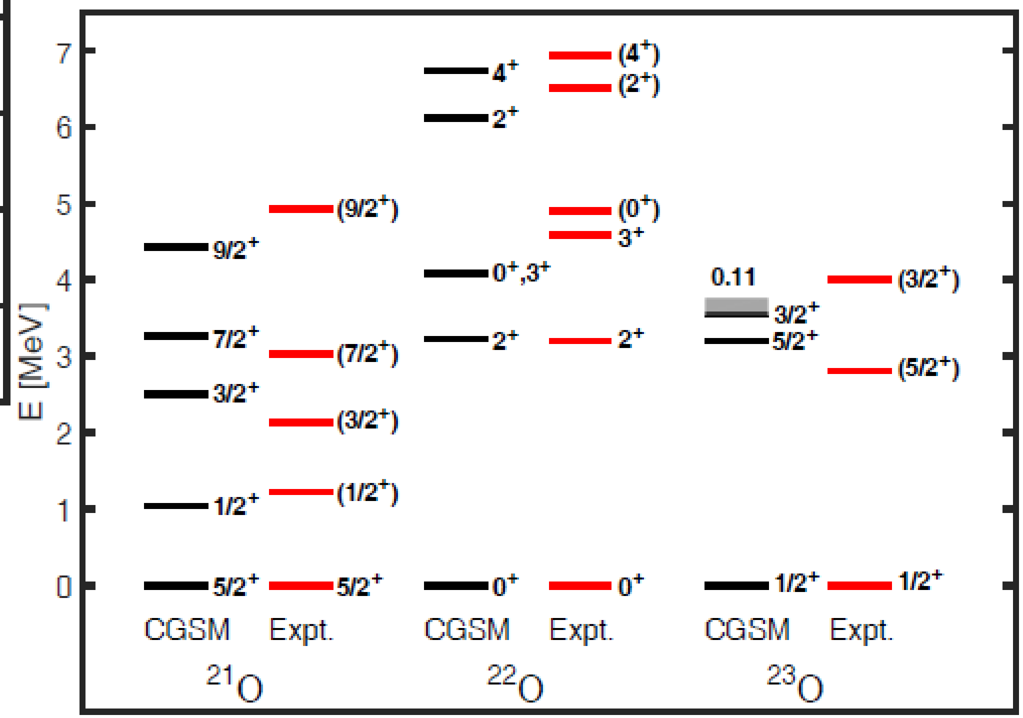


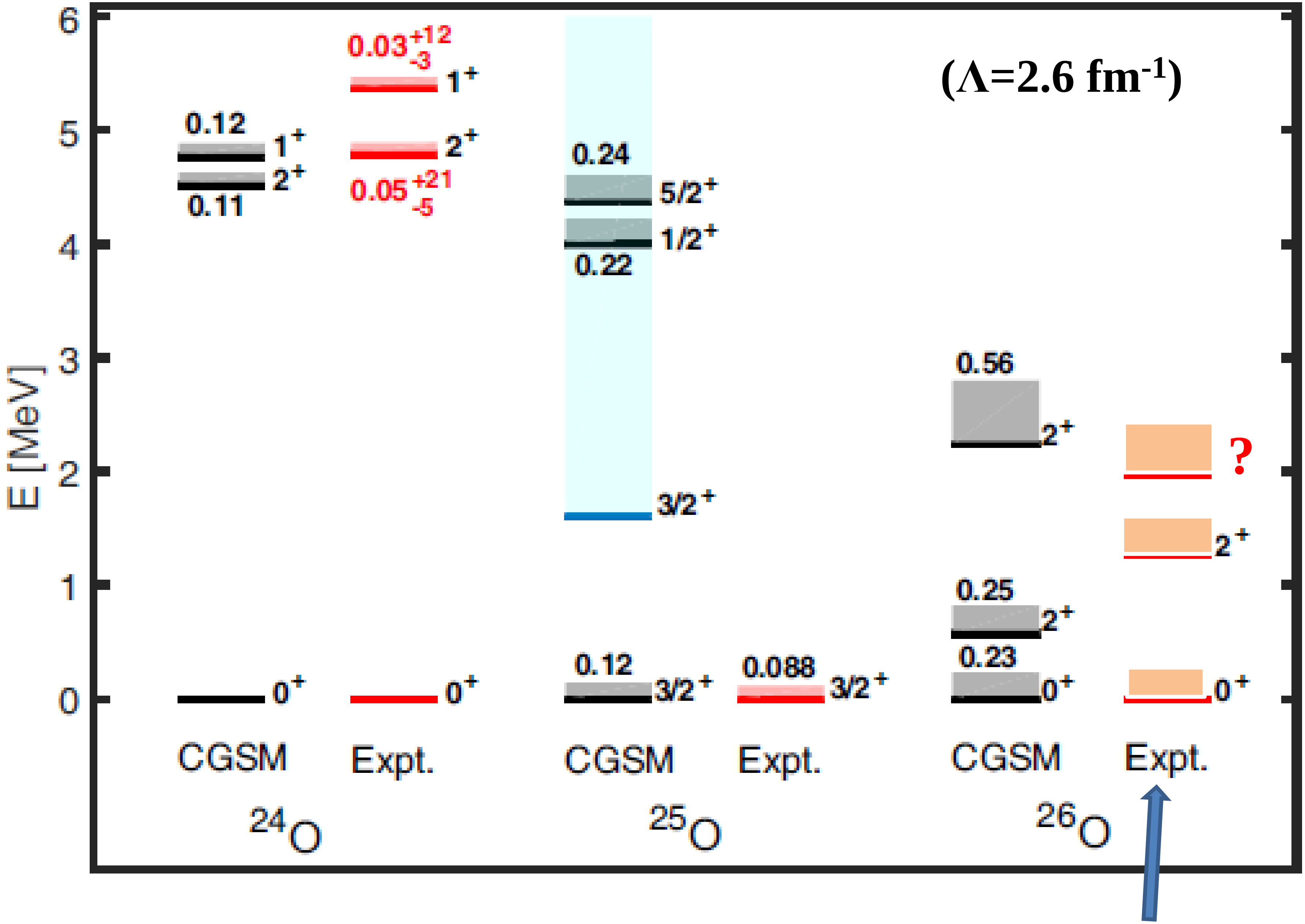


CD-Bonn CGSM, compared with conventional H.O. SM



Hard cutoff Λ=2.6 fm^{-1} to reduce 3NFs





Y. Kondo *et al.*, PRL 116, 102503 (2016)

Binding energies, one-neutron separation energies

$$\tilde{e}_n = e_n - i\gamma_n/2$$

$0d_{3/2}$ **1.06-0.089i** | **0.94-0.048i**

e=0.0

$1s_{1/2}$ **-3.22-0.00i** | **-3.27-0.00i**

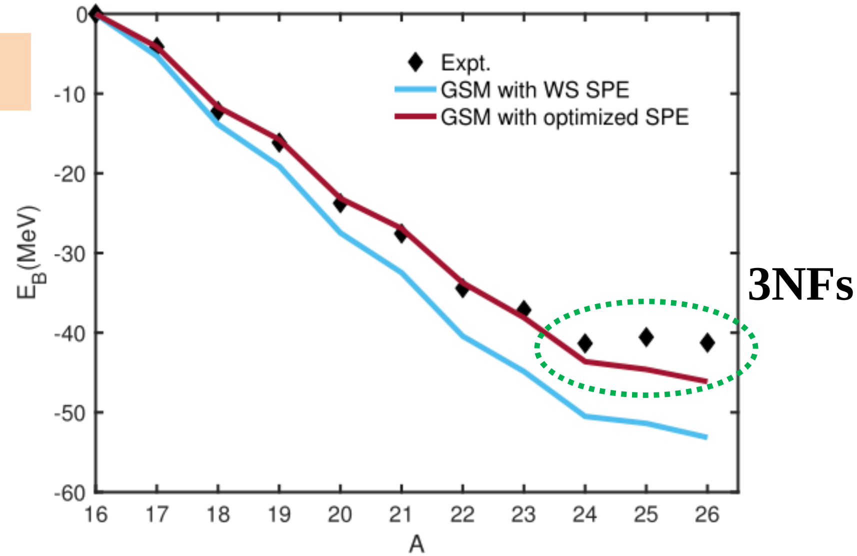
-4.14-0.00i

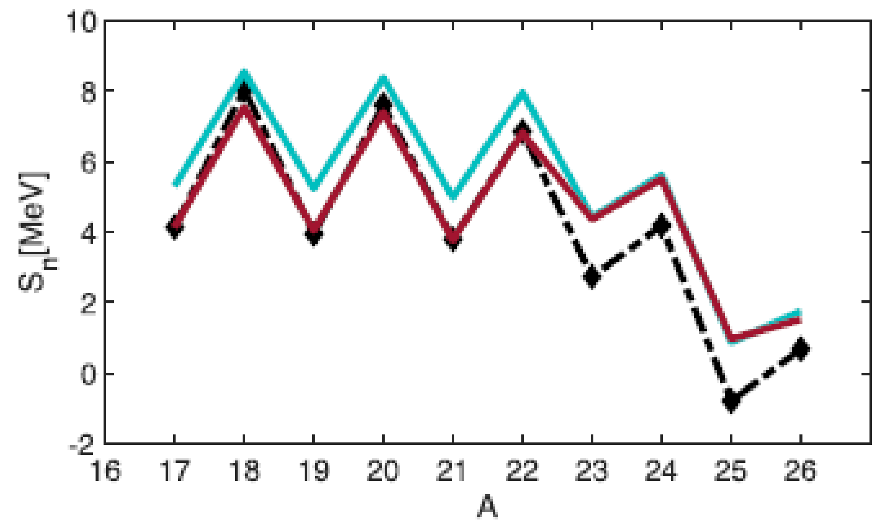
$0d_{5/2}$ **-5.31-0.00i (MeV)**

WS SPE's

Expt. SPE's

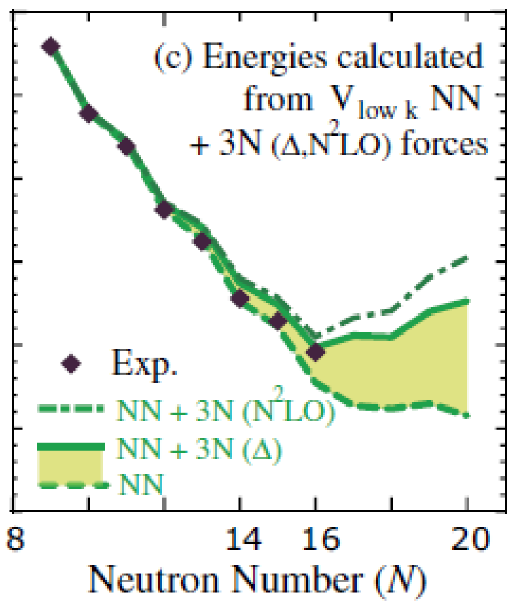
[Extracted from ^{17}O,
by Michel *et al*., PRC 67, 054311 (2003)]

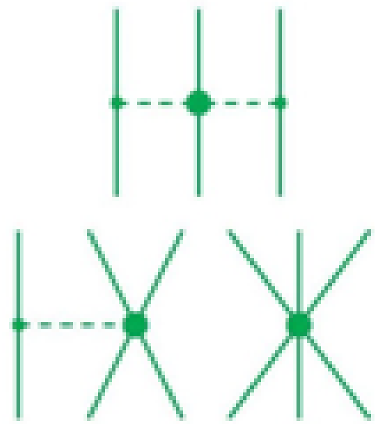


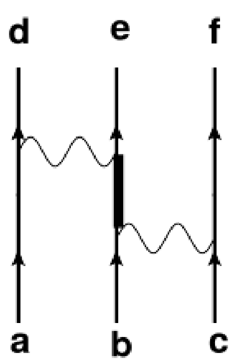


3NFs are important for binding energy calculations

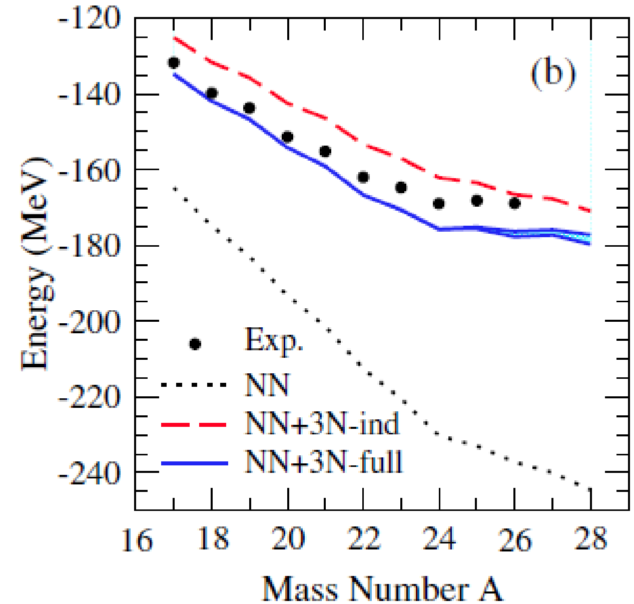
Otsuka *et al.*, PRL 105, 032501 (2010): SM, N^{3}LO, N^{2}LO 3NF

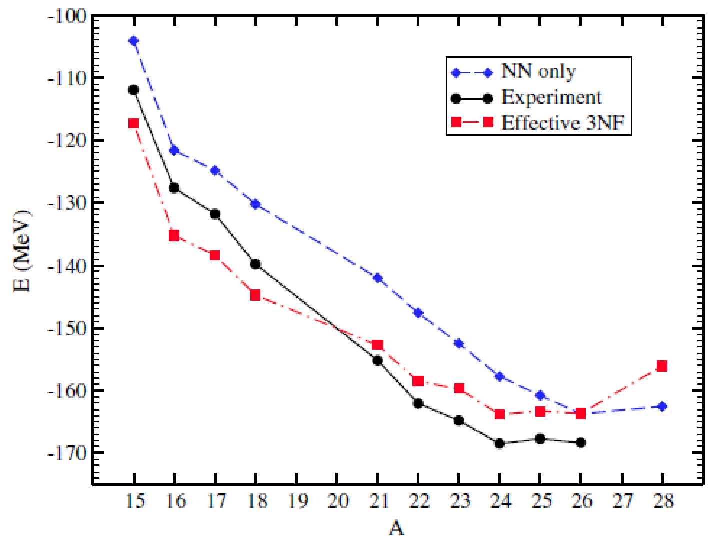






Bogner *et al.*, PRL 113, 142501 (2014): In-medium SRG, N^{3}LO, 3NF=induced / initial (N^{2}LO)

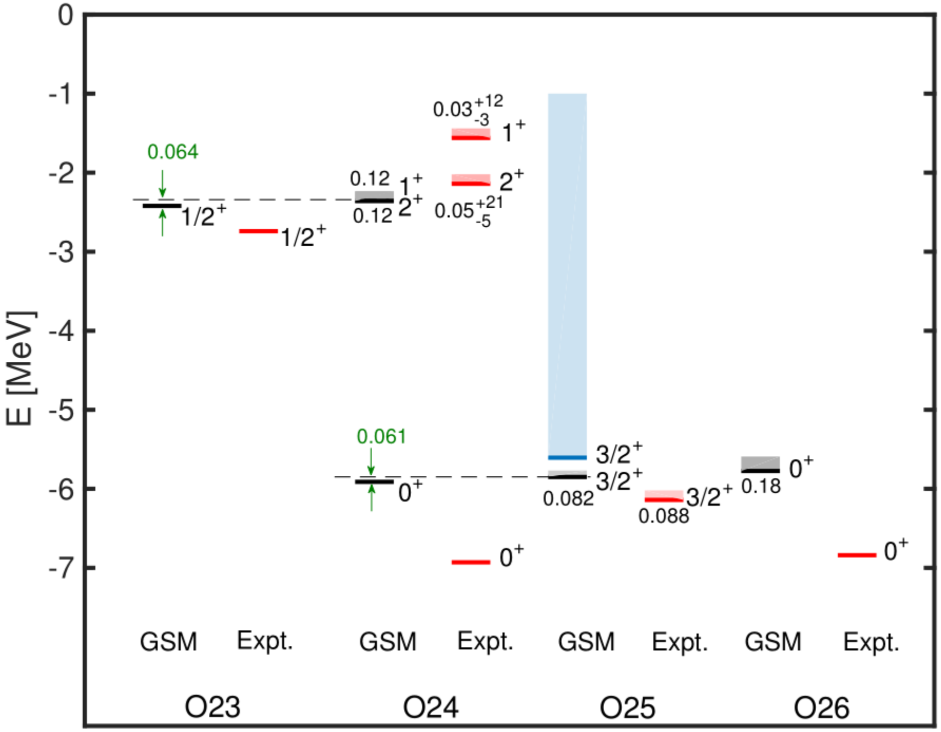




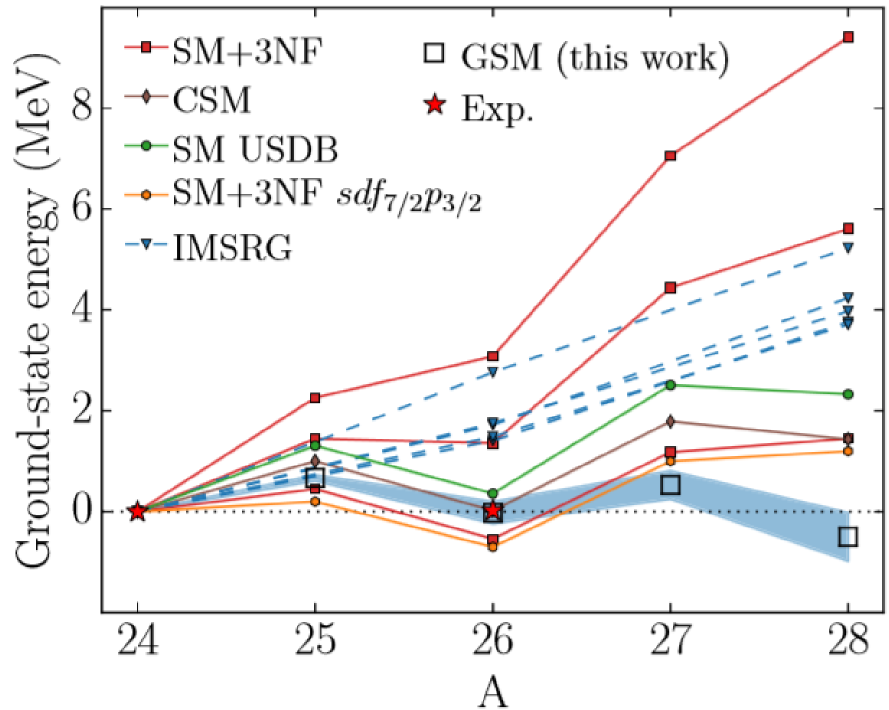
G. Hagen *et al.*, PRL 108, 242501 (2012): Continuum CC, N^{3}LO, N^{2}LO 3NF

3NF effects

^{22}O core (N=14 closed shells)



K. Fossez, J. Rotureau, N. Michel, W. Zarewicz, PRC 96, 024308 (2017)



Summary

Realistic nuclear forces (CD Bonn)

↓

Renormalization by $V_{low\text{-}k}$

↓

Many-body solutions by CGSM → **Full Q-box folded diagrams in nondegenerate complex-*k* space, which includes contributions from core polarization and excluded space.**

- ✓ **Successfully applied to excitation spectra of weakly-bound or unbound oxygen isotopes.**

Physics Letters B 769 (2017) 227–232

ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Resonance and continuum Gamow shell model with realistic nuclear forces

CrossMark

Z.H. Sun, Q. Wu, Z.H. Zhao, B.S. Hu, S.J. Dai, F.R. Xu *

Group members

School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Thank you for your attention

Connecting Bound States to the Continuum
Facility for Rare Isotope Beams (FRIB)
June 11-22, 2018