Continuum shell model: the unified approach to nuclear structure and reactions

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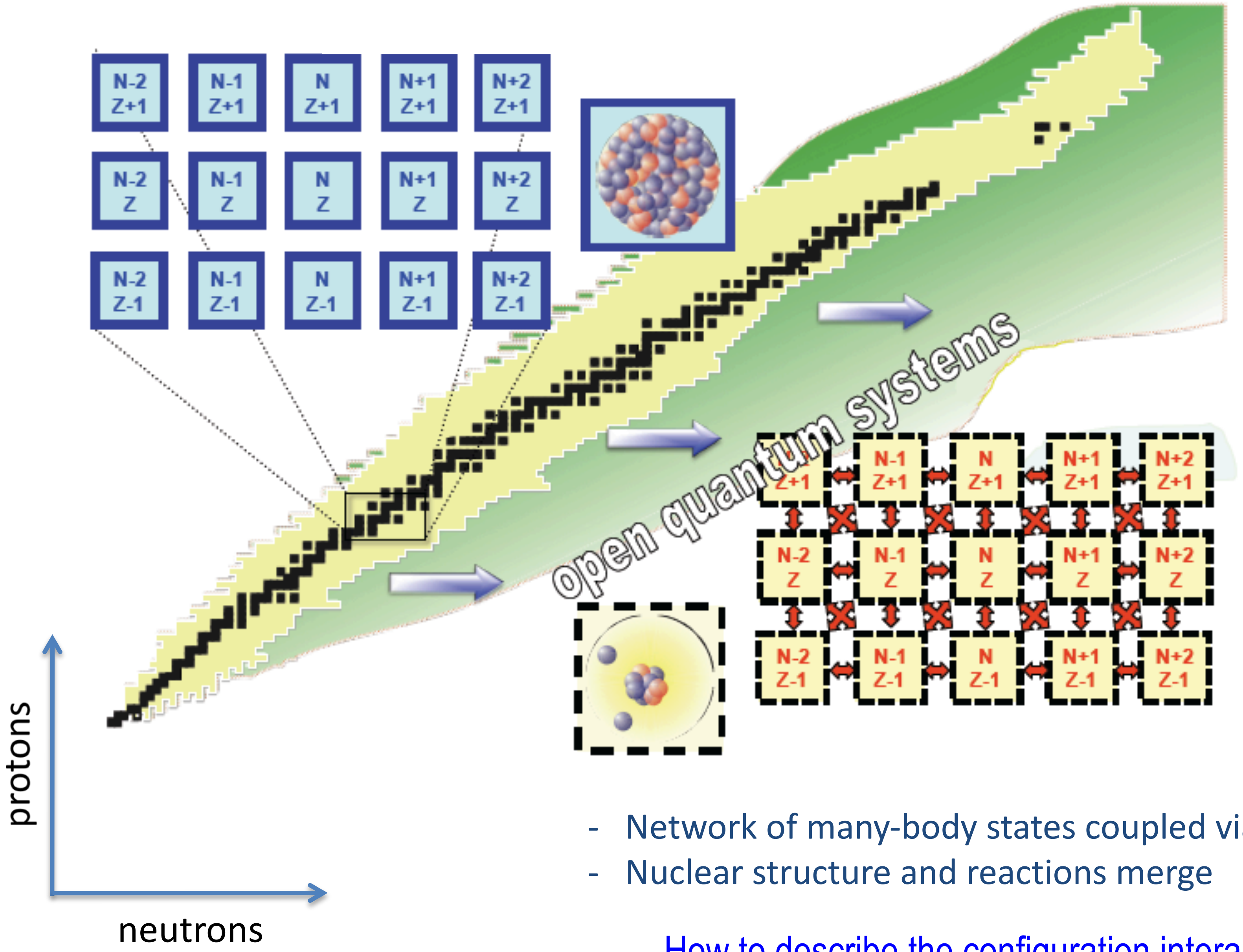
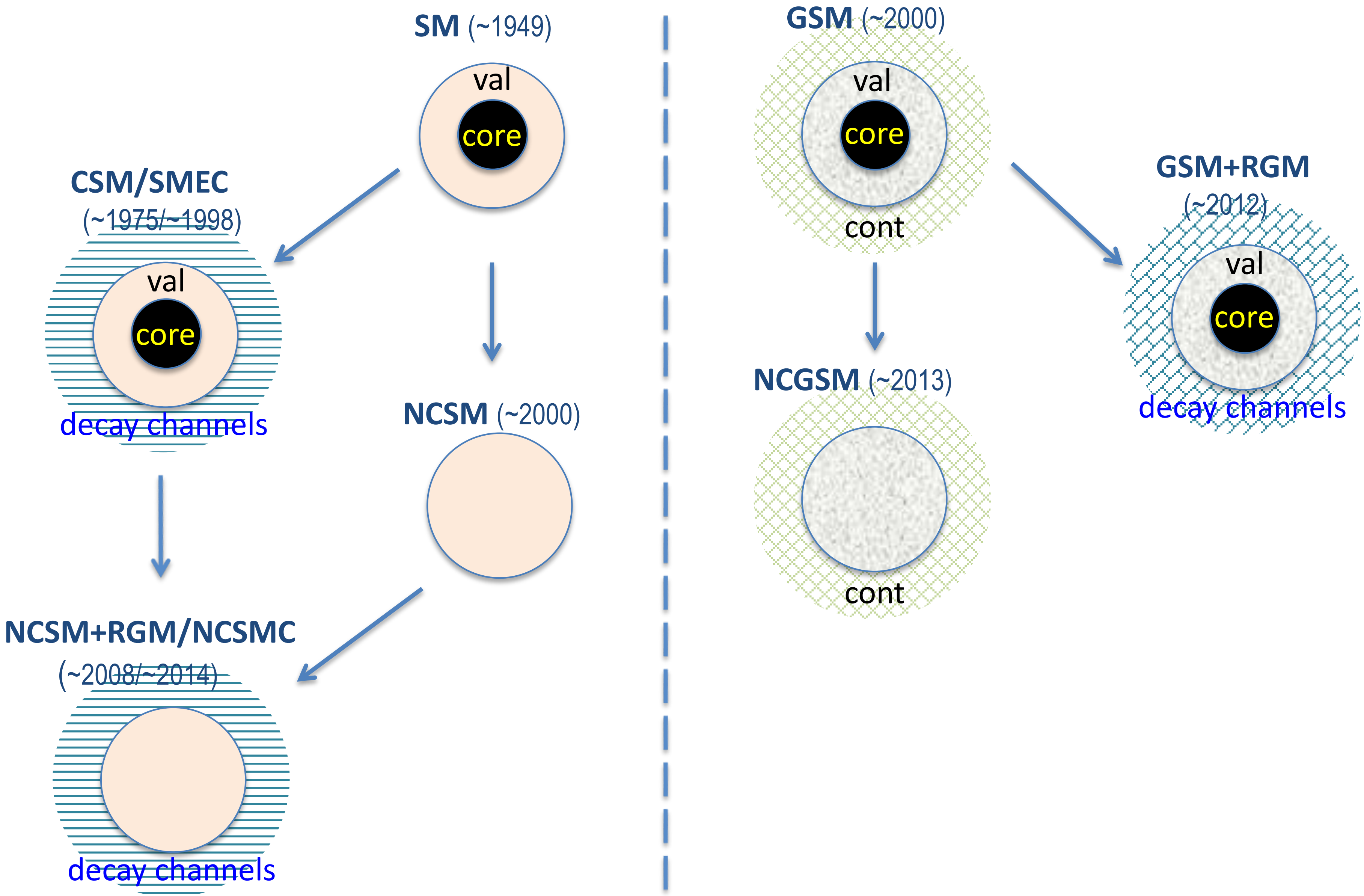


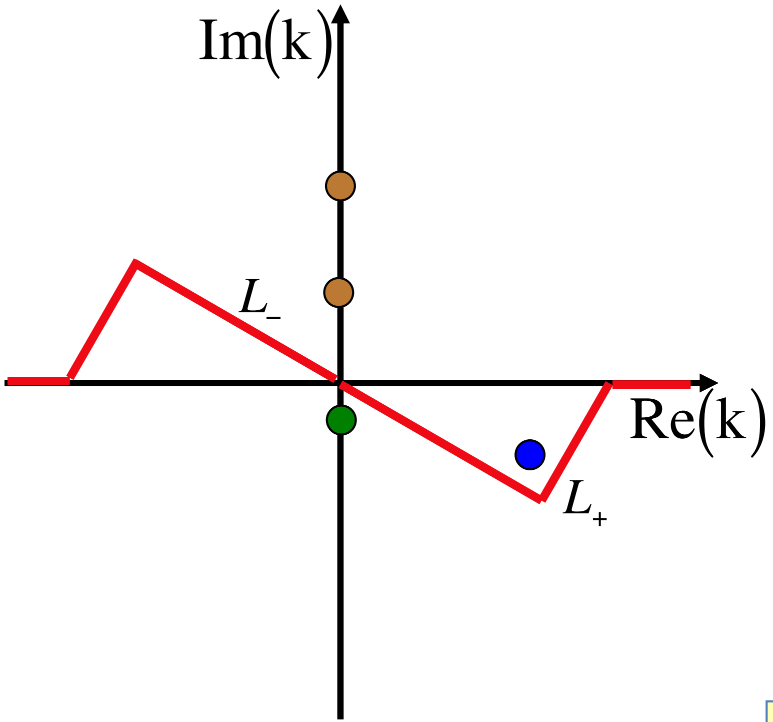
- Network of many-body states coupled via the continuum
- Nuclear structure and reactions merge

How to describe the configuration interaction in open quantum systems?

Evolution of paradigms



Gamow Shell Model



$$H \rightarrow [H]_{ij} = [H]_{ji}$$

Complex-symmetric eigenvalue problem for hermitian Hamiltonian

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|dk = 1\,;\ \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

bound states resonances

non-resonant continuum

$$|SD_i\rangle = |u_{i_1}\ldots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

↓

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311

No identification of reaction channels

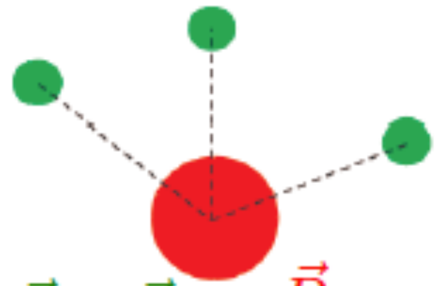
→ GSM in this representation is a tool *par excellence* for nuclear structure studies

- Center of mass treatment: Cluster Orbital Shell Model relative coordinates

Y. Suzuki, K. Ikeda, PRC 38 (1998) 410

$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2\mu} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{p_i p_j}{A_c}\right)$$

"Recoil" term coming from the expression of H in the COSM coordinates. No spurious states

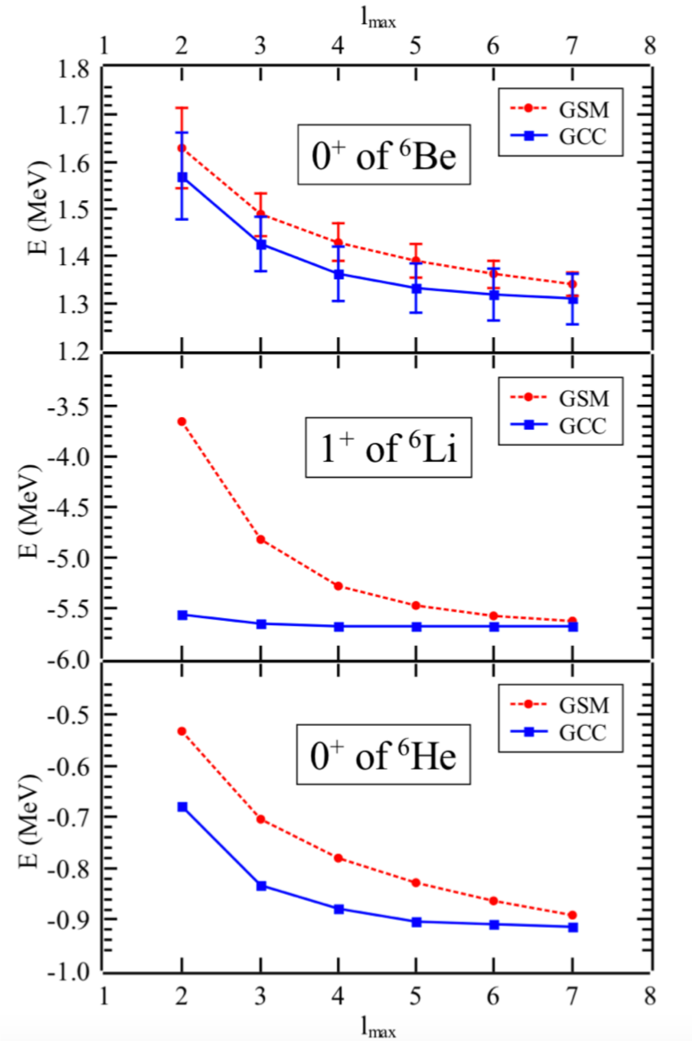


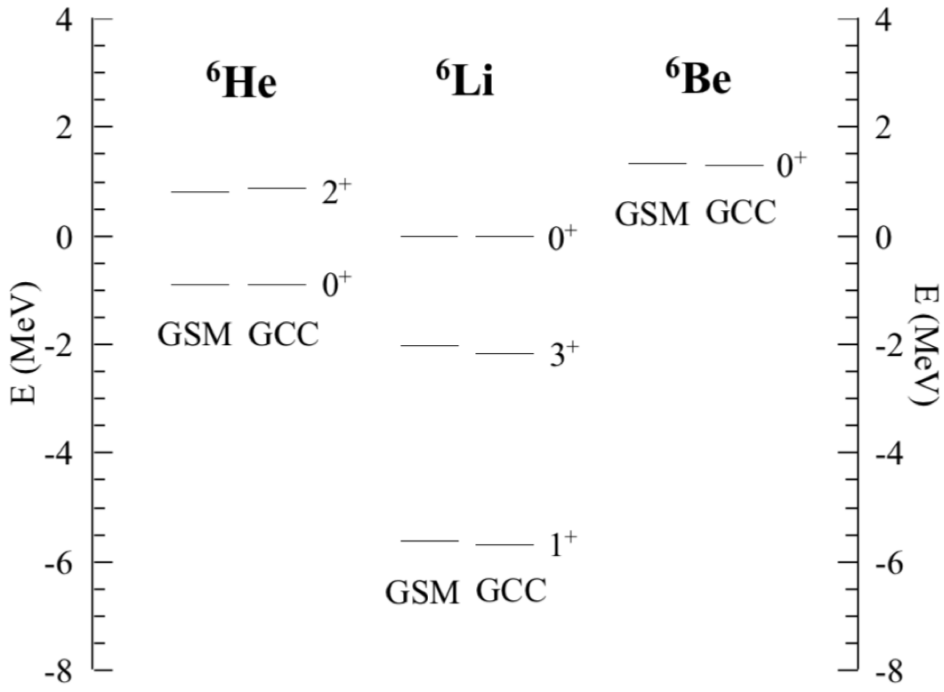
$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

- Center of mass treatment: Cluster Orbital Shell Model relative coordinates

Y. Suzuki, K. Ikeda, PRC 38 (1998) 410

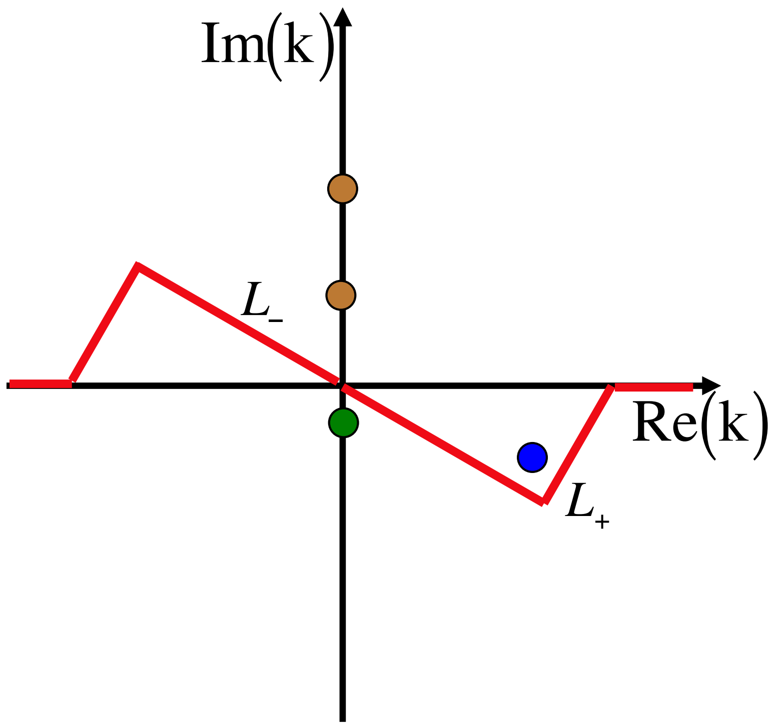
Jacobi vs COSM coordinates





S.M. Wang et al, PRC 96, 044307 (2017)

Coupled channel formulation of the Gamow shell model



$$|\Psi\rangle = \sum_c \int_0^\infty dr \frac{u_c(r)}{r} r^2 \hat{A} |CS\rangle_c$$

⇓ GSM channel state

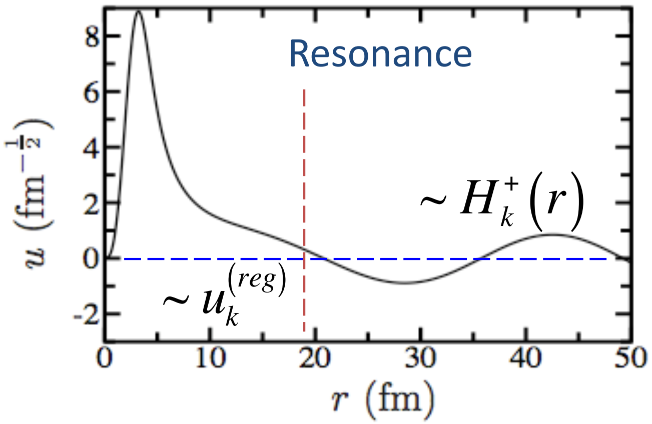
Channel basis: $\{c\} = \{A_T, J_T; a_P, \ell_P, J_{\text{int}}, J_P\}$

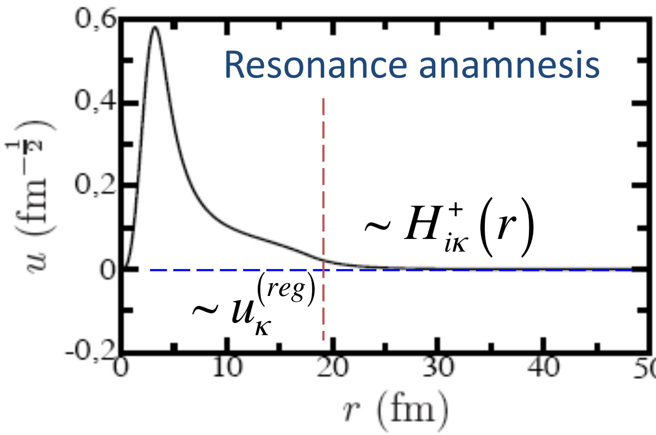
$$\hat{A}|CS\rangle_c \equiv |(c,r)\rangle = \hat{A}\left[|\Psi_T^{J_T}\rangle \otimes |r,\ell_P,J_{\text{int}},J_P\rangle\right]_{M_A}^{J_A}$$

Y. Jaganathen et al, PRC 88, 044318 (2014)
K. Fossez et al., PRC 91, 034609 (2015)

- Entrance and exit reaction channels defined
  → Unification of nuclear structure and reactions
- Scattering wave functions $|\Psi_{\text{GSM}}(A-p) \otimes \Phi_{\text{proj}}(p)\rangle$ are the many-body states
- Antisymmetry handled exactly
- Core arbitrary

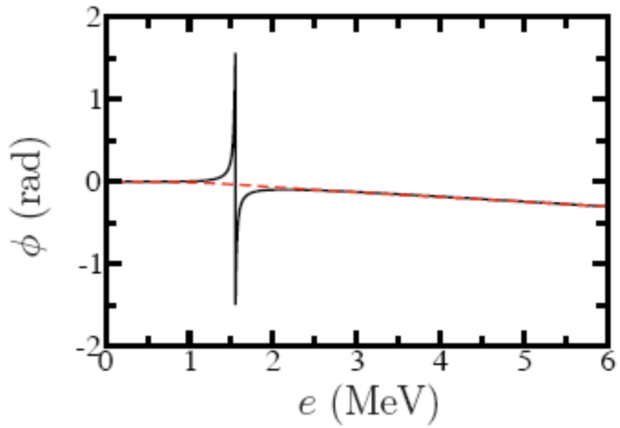
L^2 basis for s.p. resonances





$$k \rightarrow \kappa = \sqrt{\mathcal{R}\left((k)^2\right)}$$

$$\mathcal{W}\left(u_k^{(reg)}, H_{i\kappa}^+\right)(r)\Big|_{r=R} = 0$$



Bound states and resonance anamneses form together a discrete subset $\{|\tilde{u}_n\rangle\}$ of the complete set of basis states in Hilbert space

$$\hat{h} \rightarrow \hat{\hat{h}} = \sum_n |\tilde{u}_n\rangle \tilde{e}_n \langle \tilde{u}_n| + \hat{p}\hat{h}\hat{p}$$

$$\tilde{e}_n = \begin{cases} e_n \\ e_n^{(res)} = \hbar^2\kappa^2/2\mu \end{cases} \qquad \hat{p} = 1 - \sum_n |\tilde{u}_n\rangle\langle\tilde{u}_n|$$

Discrete states $\quad \tilde{e}_n : \left(\tilde{e}_n - \hat{\hat{h}}\right)|\tilde{u}_n\rangle = 0$

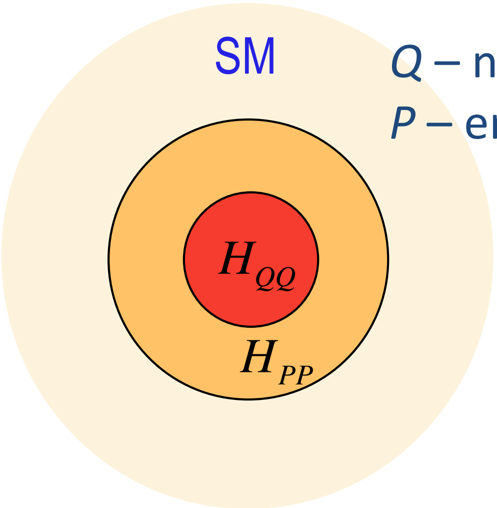
Scattering states $\quad \{\tilde{e}\} : \left(\tilde{e} - \hat{p}\hat{h}\hat{p}\right)|\tilde{u}\rangle = 0$

$$\sum_n |\tilde{u}_n\rangle\langle\tilde{u}_n| + \int_{R^+} |\tilde{u}_k\rangle\langle\tilde{u}_k| = 1 \quad ; \quad \langle\tilde{u}_i|\tilde{u}_j\rangle = \delta_{ij}$$

$$|SD_i\rangle = |\tilde{u}_{i_1} \ldots \tilde{u}_{i_A}\rangle \quad ; \quad \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

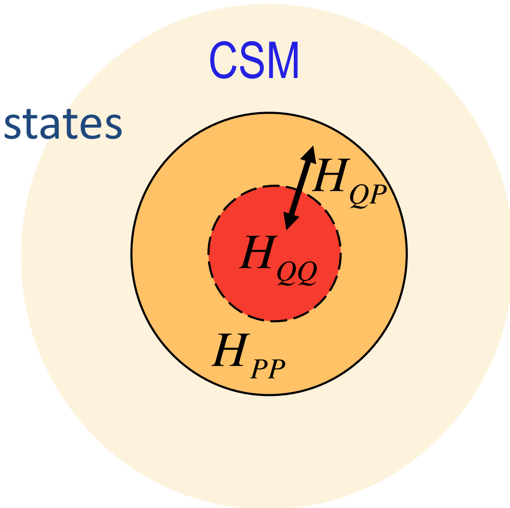
J.B. Faes, M.P., Nucl. Phys. A 800 (2008) 21

Shell Model Embedded in the Continuum (SMEC)



Q – nucleus, localized states

P – environment, scattering states



$$H_{QQ} \rightarrow \mathcal{H}_{QQ}^{eff}(E) = H'_{QQ}(E) - \frac{i}{2} V(E) V^T(E)$$

closed quantum system

$$= \underbrace{H_{QQ}^{(SM)} + u_{QQ}(E)}_{\text{hermitian}} \underbrace{- \frac{i}{2} w_{QQ}(E)}_{\text{anti-hermitian}}$$

open quantum system

C. Mahaux, H.A. Weidenmüller, Shell Model Approach to Nuclear Reactions (1969)

H.W.Bartz et al, Nucl. Phys. A275 (1977) 111

R.J. Philpott, Nucl. Phys. A289 (1977) 109

K. Bennaceur et al, Nucl. Phys. A651 (1999) 289

J. Rotureau et al, Nucl. Phys. A767 (2006) 13

$$H\Psi = E\Psi$$

$$H_{QQ}\Phi_i = E_i\Phi_i \qquad (E - H_{PP})\omega_i^+ = H_{PQ}\Phi_i \qquad (E - H_{PP})\xi = 0$$

$$\omega_i^+ = G_P^+ H_{PQ}\Phi_i$$

Discrete states : $\langle \Phi_i | H_{QQ} + \underbrace{H_{QP} G_P^+(E) H_{PQ} | \Phi_j \rangle}_{\omega_j^+} = E_{ij}\delta_{ij} + \langle \underbrace{w_i}_{\langle \Phi_i | H_{QP}} | \omega_j \rangle \delta_{E_iE}\delta_{E_jE}$

$$|\Psi_k\rangle = \sum_i c_k^i |\Phi_i\rangle \ , \quad E_k(E) = E$$

Scattering solutions : $|\Psi\rangle = |\xi\rangle + \left(Q + G_P^+(E) H_{PQ}\right) \dfrac{1}{E - \mathcal{H}_{QQ}^{eff}(E)} H_{QP} |\xi\rangle$

non-resonant part — resonant part

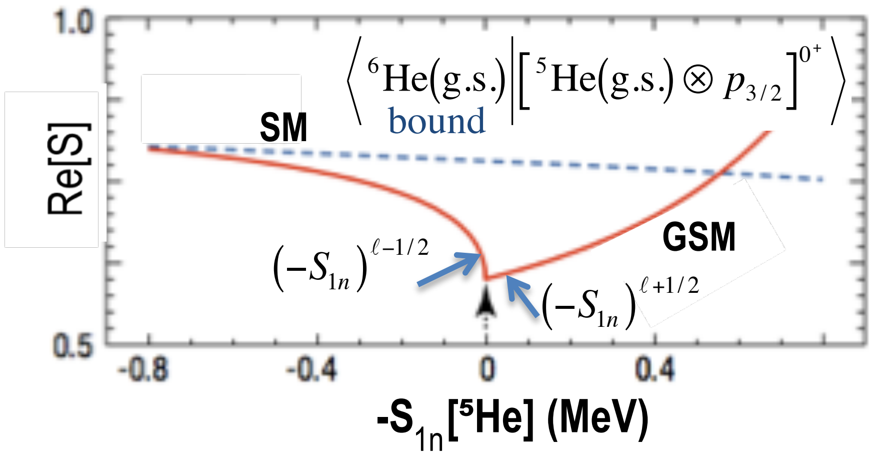
- Shell model and reaction theory reconciled
- Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations

Complex-symmetric eigenvalue problem in Continuum Shell Model (GSM/SMEC)

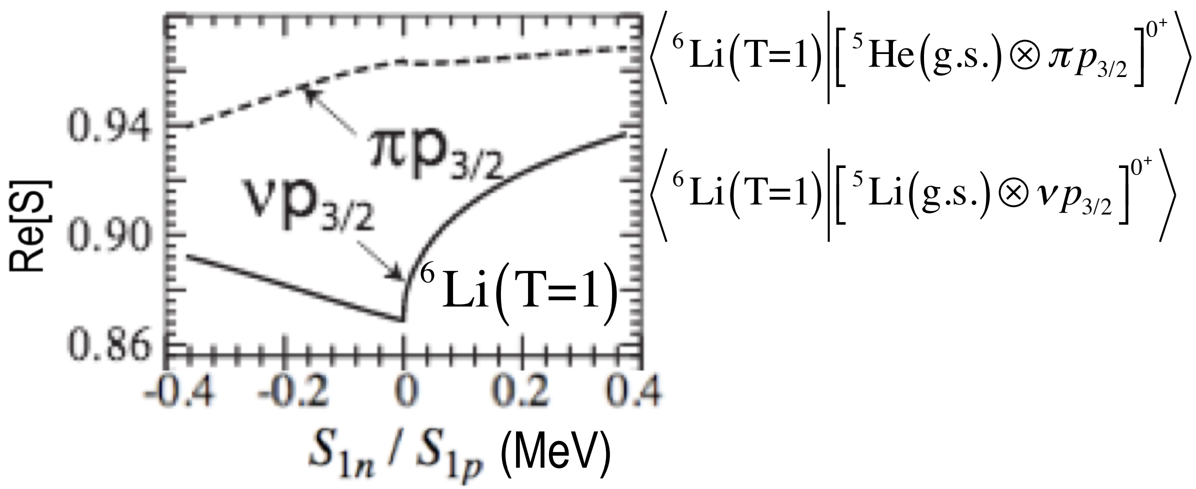
Coupling to the environment of scattering states and decay channels does not reduce to the adjustment of (hermitian) Hamiltonian and leads to new (collective) phenomena

- resonance trapping and super-radiance phenomenon
- modification of spectral fluctuations
- multichannel coupling effects in reaction cross-sections and shell occupancies
- anti-odd-even staggering of separation energies in odd-Z isotopic chains
- clustering
- exceptional points
- violation of orthogonal invariance and channel equivalence
- matter (charge) distribution (pairing anti-halo effect)
- ….

Configuration mixing in weakly bound/unbound states



- Analogy with the Wigner threshold phenomenon for reaction cross-sections
- The interference phenomenon between resonant states and non-resonant continuum in the vicinity of the particle emission threshold

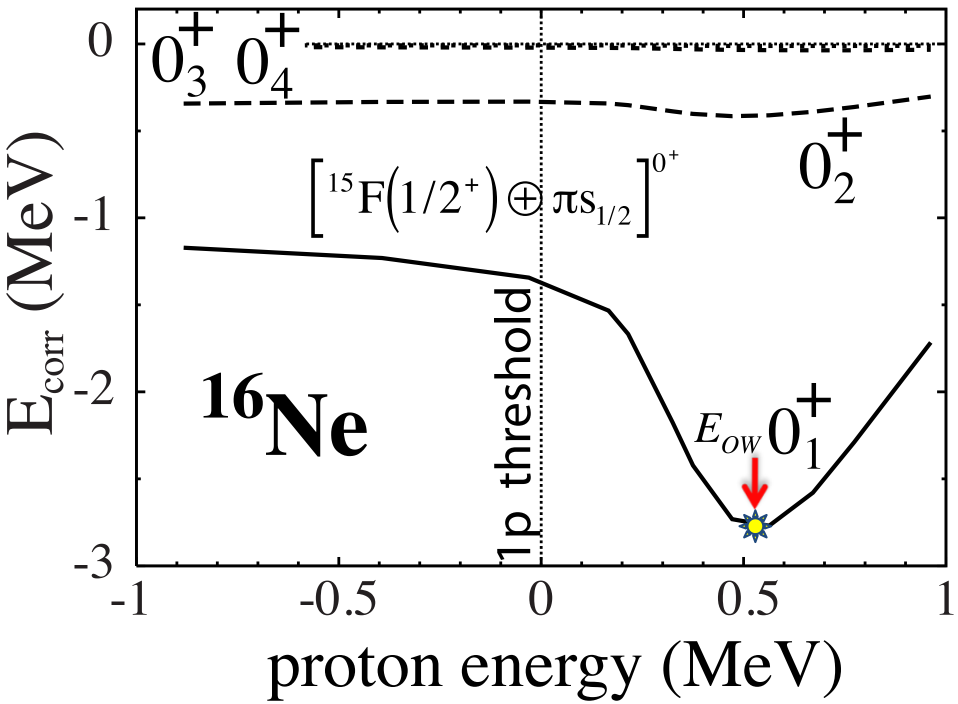


Near-threshold configuration mixing acts differently at the proton and neutron drip lines

N. Michel et al., PRC(R) 75, 031301 (2007)

Continuum coupling correlation energy

$$E_{corr;i}(E) = \mathrm{Re}\left\{\left\langle \Psi_i^A \middle| \mathcal{H}_{QQ}^{eff}(E) - H_{QQ} \middle| \Psi_i^A \right\rangle\right\}$$



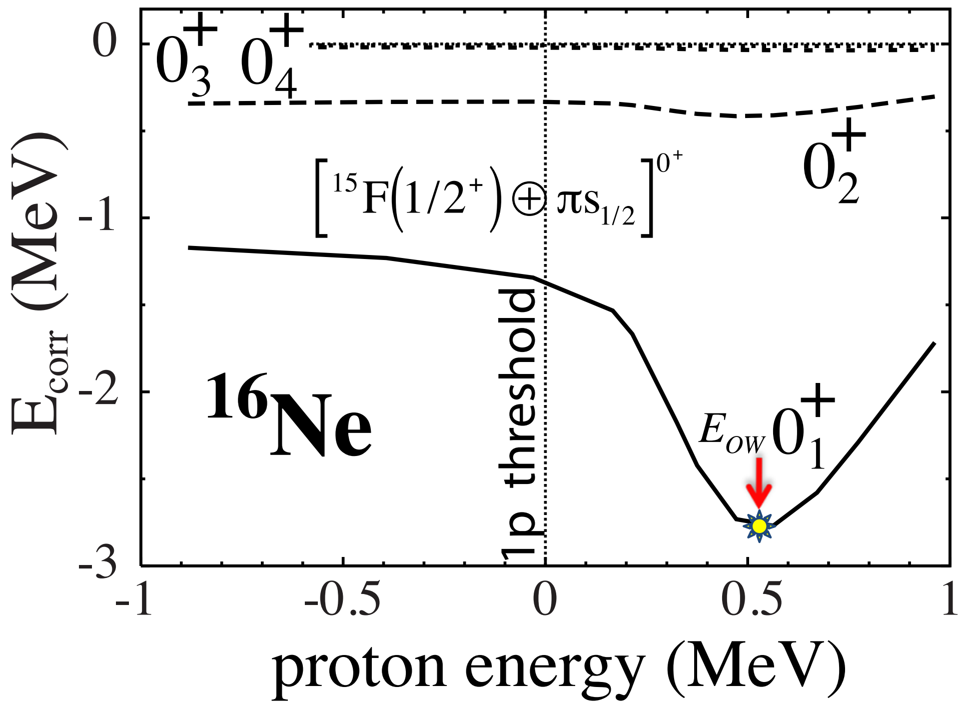
Okolowicz et al., Prog. Theor. Phys. Suppl. 196 (2012) 230
Fortschr. Phys. 61 (2013) 66

- Interaction through the continuum leads to the formation of the collective eigenstate ('aligned state') which couples strongly to the decay channel and carries many of its characteristics
- Aligned state is a superposition of SM eigenstates having the same quantum numbers
- Point of the strongest collectivity (centroid of the 'opportunity energy window') is determined by an interplay between the competing forces of repulsion (Coulomb and centrifugal int.) and attraction (continuum coupling)

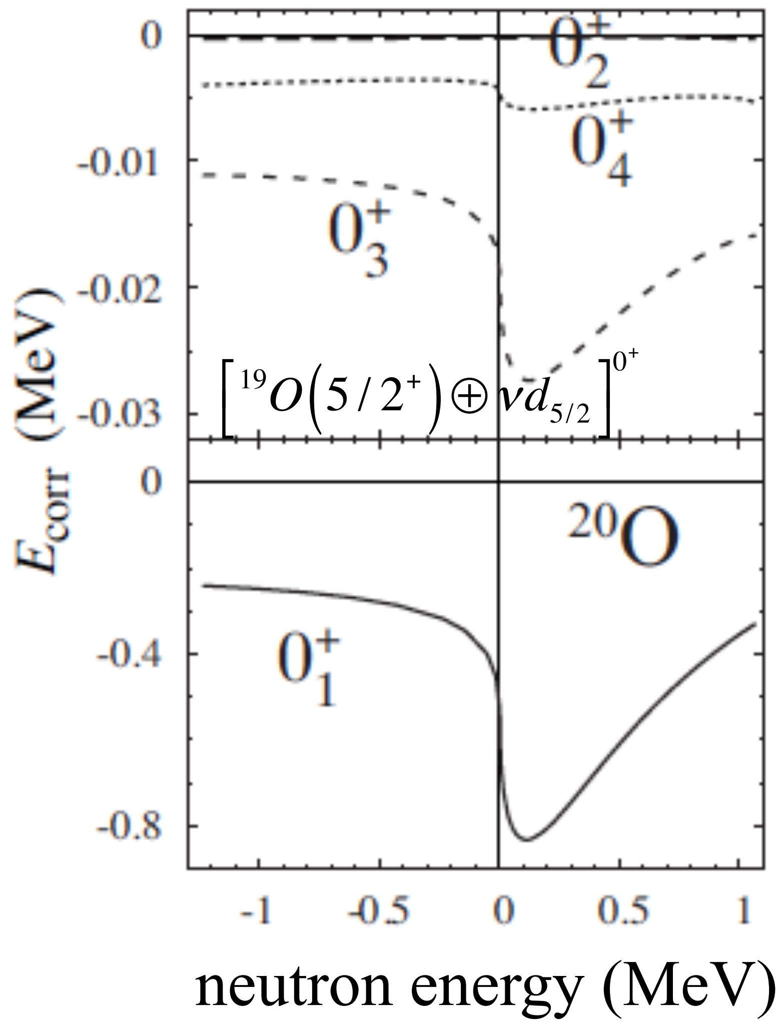
→ Emergence of new energy scale related to the external configuration mixing via decay channel(s)

Continuum coupling correlation energy

$$E_{corr;i}(E) = \mathrm{Re}\left\{\left\langle \Psi_i^A \middle| \mathcal{H}_{QQ}^{eff}(E) - H_{QQ} \middle| \Psi_i^A \right\rangle\right\}$$

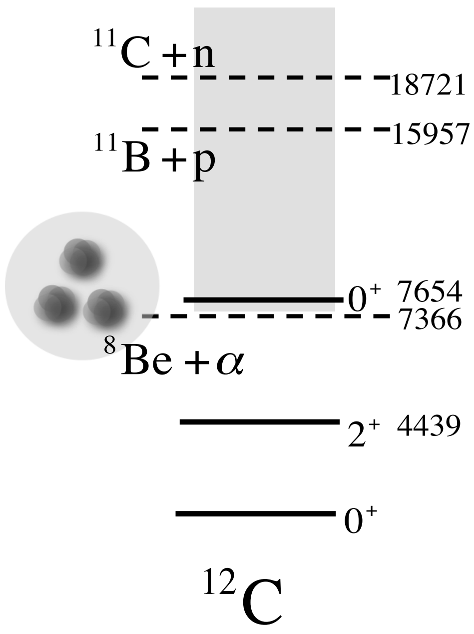


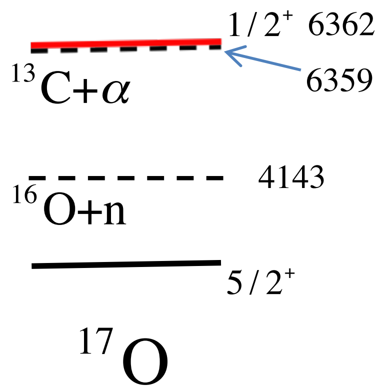
J. Okolowicz et al., Prog. Theor. Phys. Suppl. 196 (2012) 230
Fortschr. Phys. 61 (2013) 66
J. Okolowicz et al., APP B45, 331 (2014)



→ In contrast to charged particle case, the strong (multi)neutron correlations exist also in heavy nuclei

This generic phenomenon in open quantum systems explains why so many states, both on and off the nucleosynthesis path, exist 'fortuitously' close to open channels

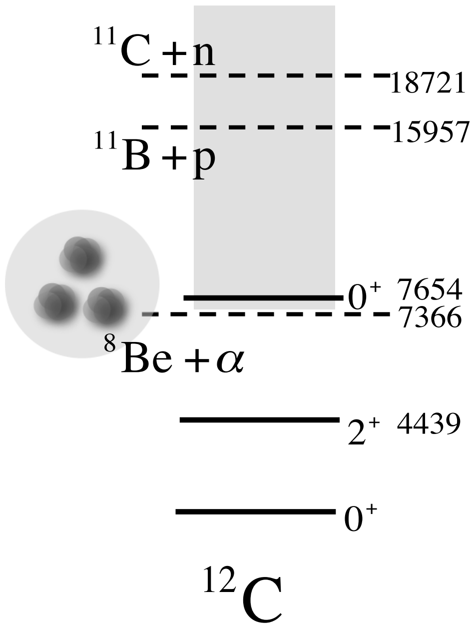


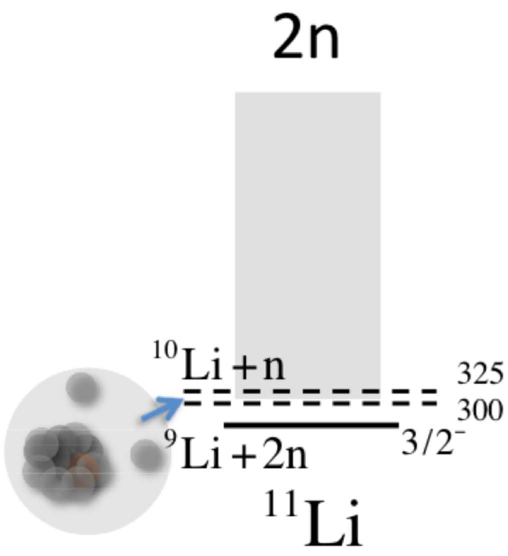


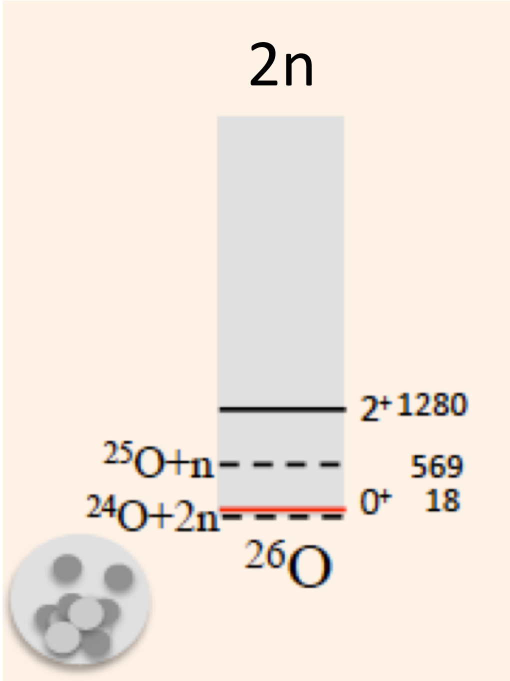
Γ_γ branch of 0^+_2 decay to particle-bound state(s) of ^{12}C forms a seed for the synthesis of heavier elements

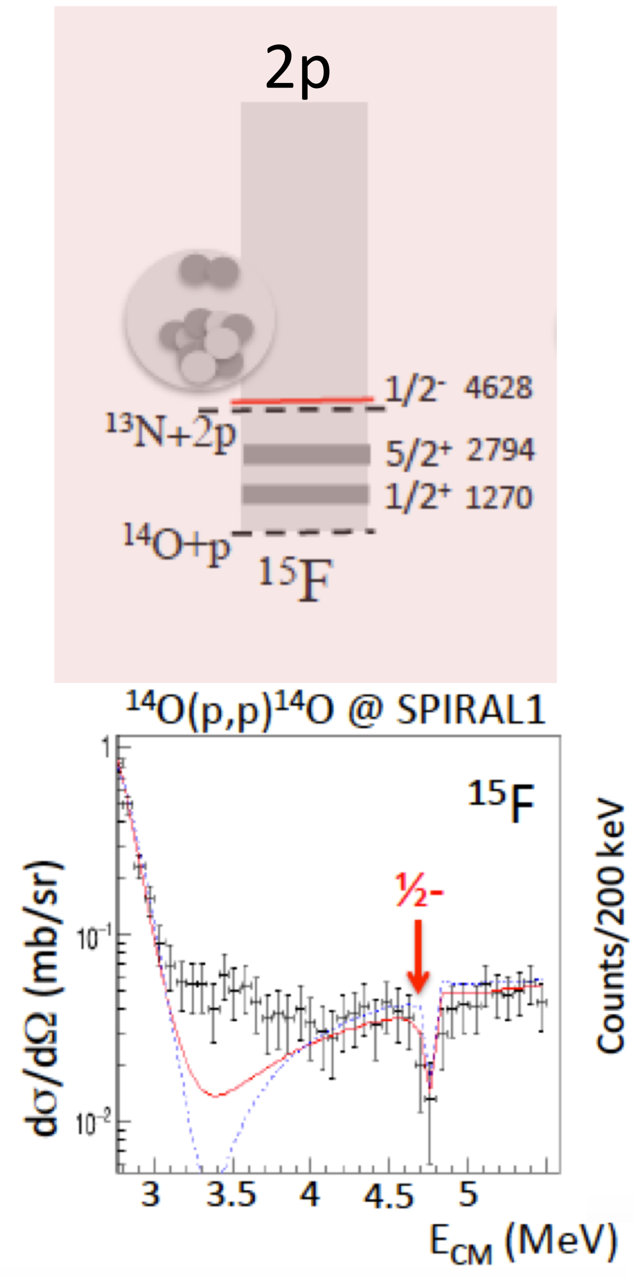
½+ resonance lying <3 keV above $^{13}C+\alpha$ threshold enables slow neutron-capture process

This generic phenomenon in open quantum systems explains why so many states, both on and off the nucleosynthesis path, exist 'fortuitously' close to open channels

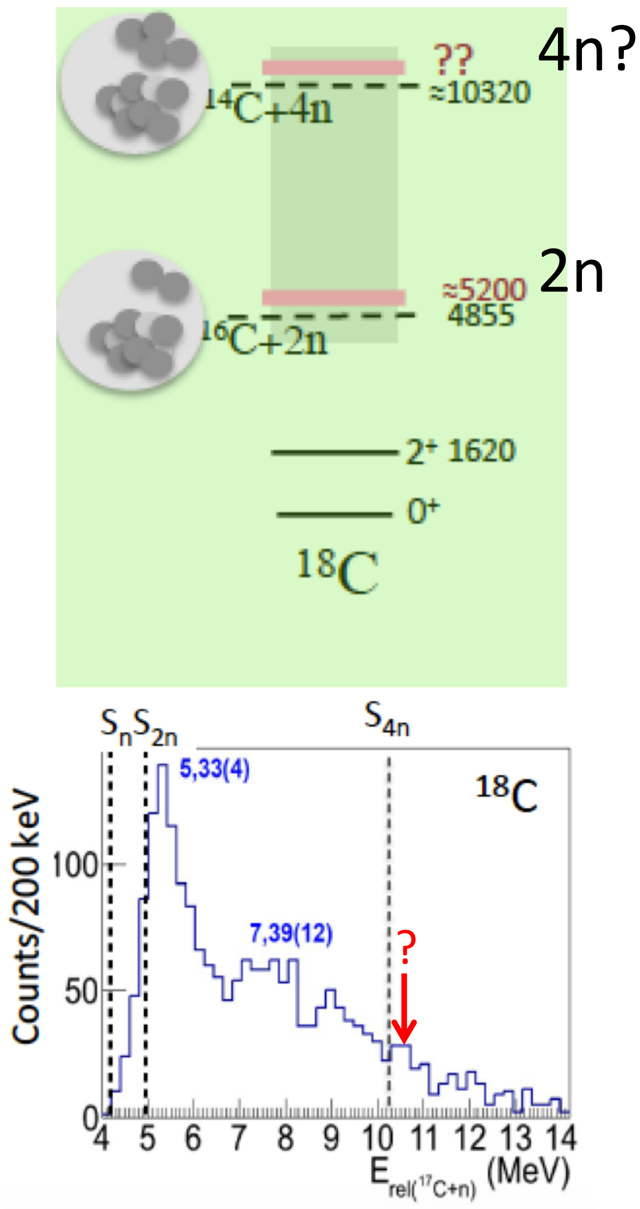




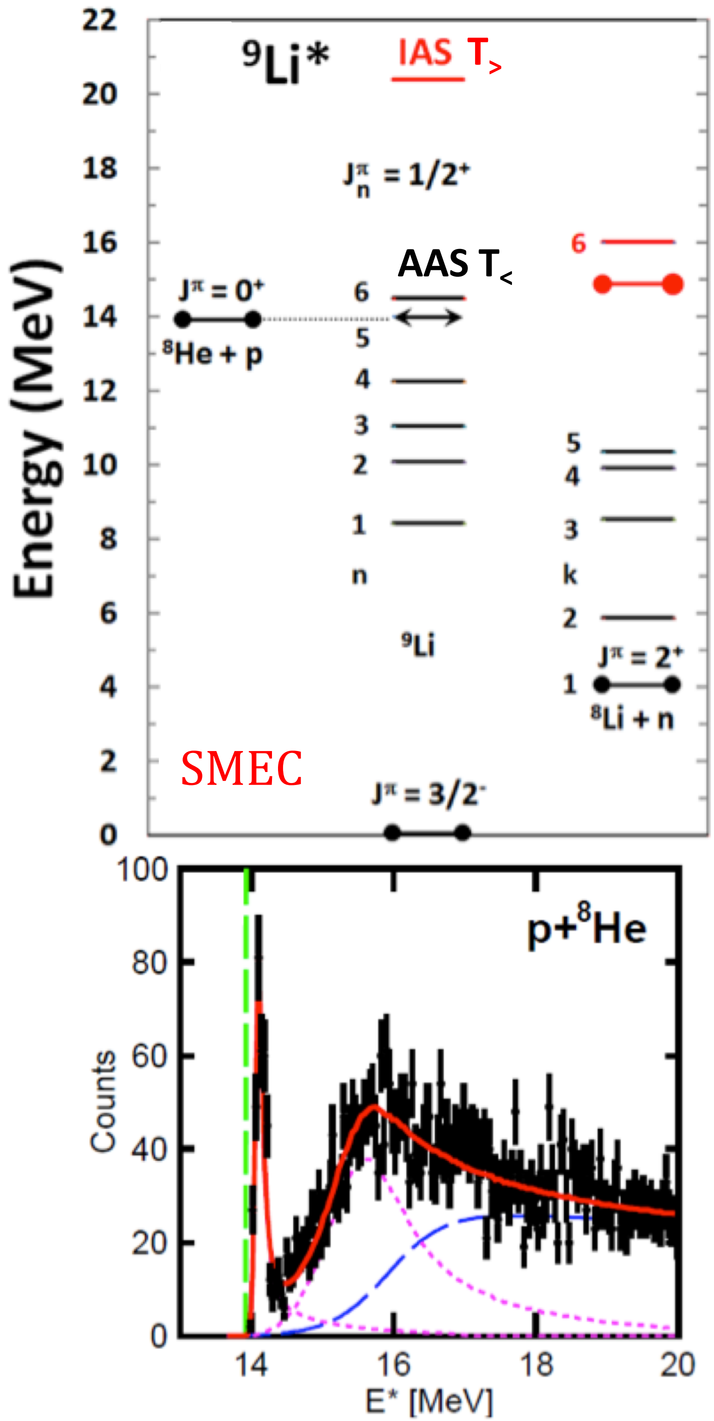




F. De Grancey et al, PLB 758, 26 (2016)

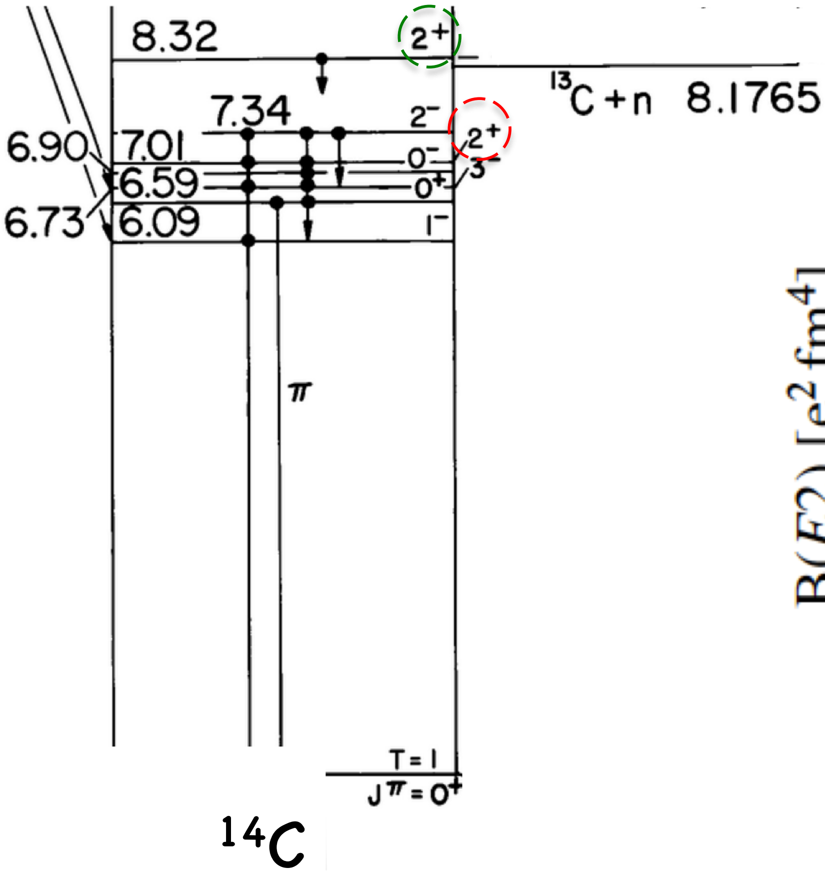


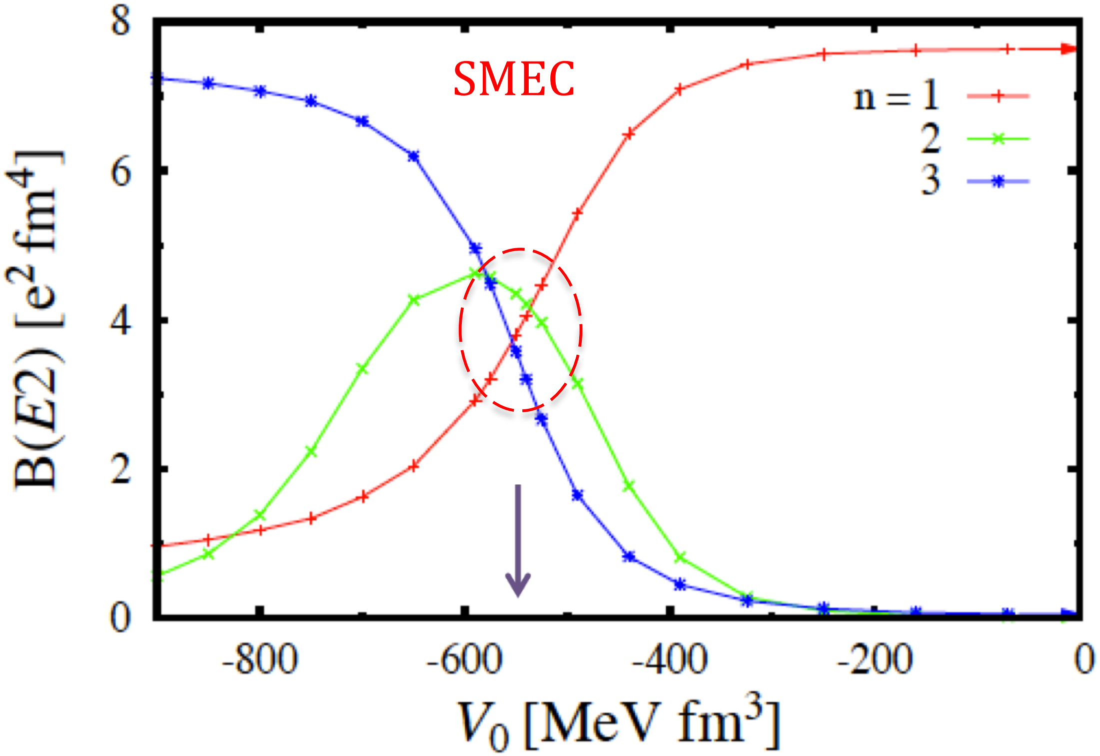
Courtesy of O. Sorlin



Exp: R.J. Charity et al, PRC 78, 054307 (2008)
Th: J. Okolowicz et al., PRC 97, 044303 (2018)

Near-threshold collectivization of electromagnetic transtions





- Strong collectivization of the B(E2) in ^{14}C from the near-threshold resonance 2^+_2 to the ground state 0^+_1
- Another example: strong collective B(E1) transition between halo state $1/2^-_1$ (S_n=181 keV) and the ground state $1/2^+_1$ in ^{11}Be

Unified desciption of structure and reactions in the Gamow Shell Model

p+^{18}Ne excitation function

Y. Jaganathen, et al., PRC 89 (2014) 034624

^{18}Ne	EXP	GSM	GSM-CC
0+	0.00	0.00	
2+	1.89	1.56	

S_p=3.921 MeV
S_n=19.237 MeV

^{19}Na	EXP	GSM	GSM-CC
5/2+	0.32	0.28	0.29
3/2+	0.44	0.25	0.27
1/2+	1.07	1.08	1.13

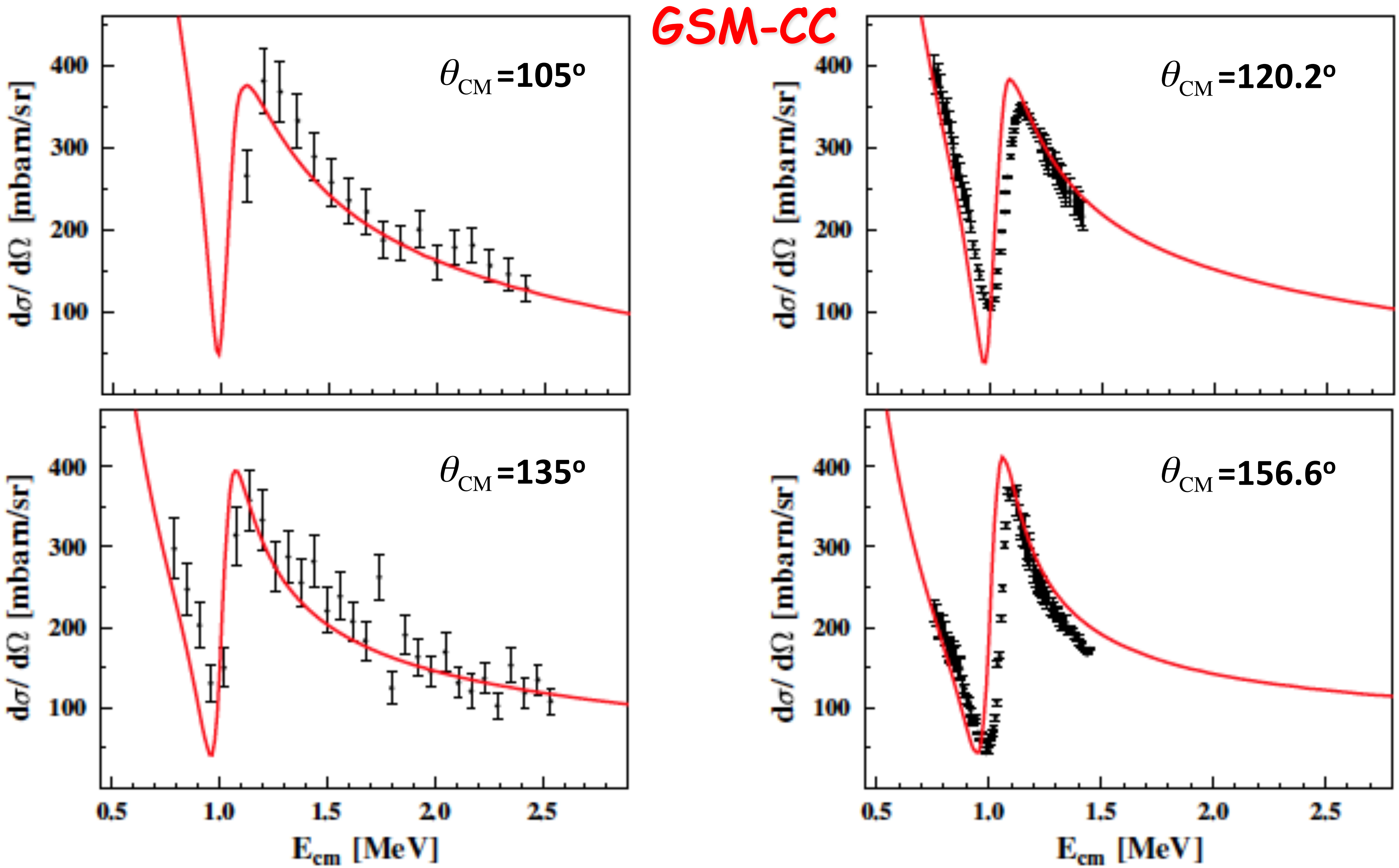
S_p=-0.32 MeV
S_n=20.18 MeV

Interaction: FHT finite-range interaction: $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

H. Furutani, H. Horiuchi, R. Tamagaki, PTP 60 (1978) 307; 62 (1979) 981

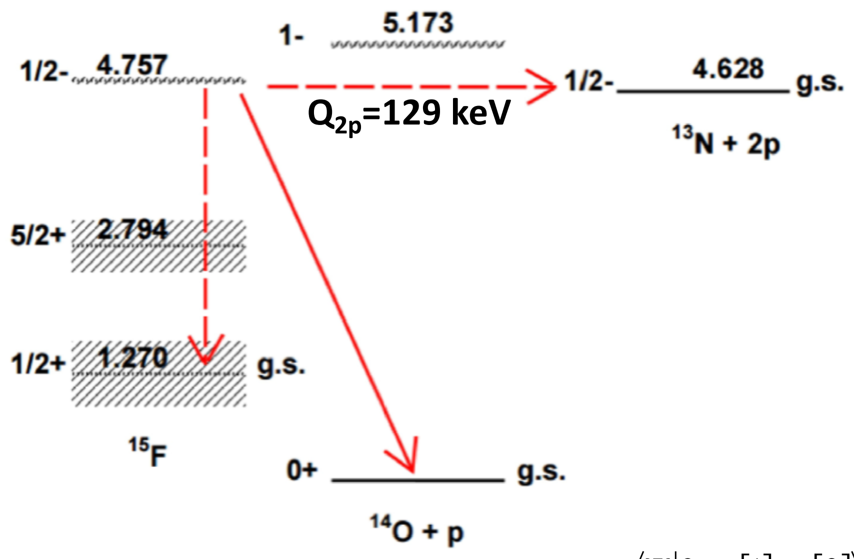
GSM and GSM-CC results (almost) identical → Scattering states J=$0^+,1^+,2^+$,... and higher lying (bound) states in ^{18}Ne are less important for the completeness of channel basis

p+^{18}Ne excitation function at different angles



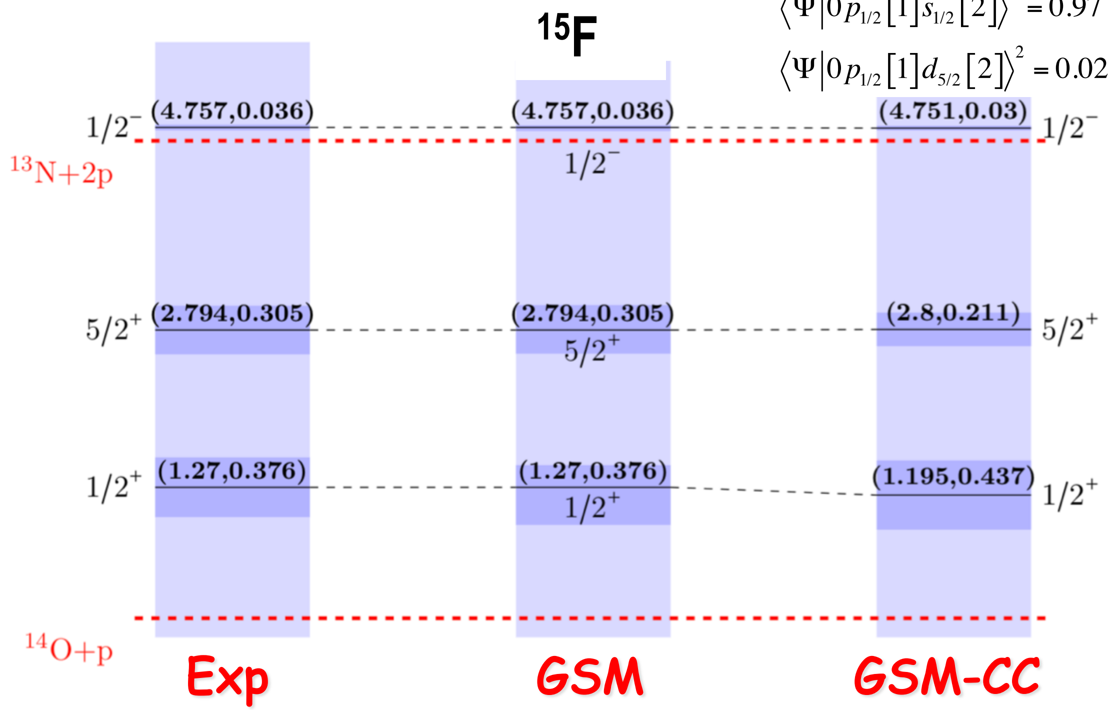
Exp: F. de Oliveira Santos et al., Eur. Phys. J. A24, 237 (2005)
B. Skorodumov et al., Phys. Atom. Nucl. 69, 1979 (2006)
C. Angulo et al., PRC 67, 014308 (2003)

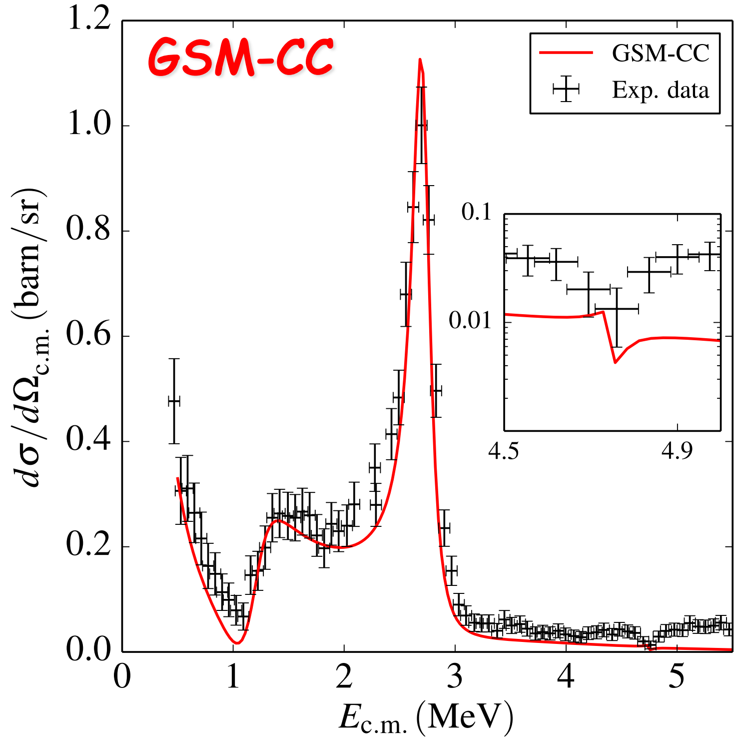
p+^{14}O excitation function and spectroscopy of ^{15}F



$$\langle \Psi | 0p_{1/2}[1] s_{1/2}[2] \rangle^2 = 0.97$$

$$\langle \Psi | 0p_{1/2}[1] d_{5/2}[2] \rangle^2 = 0.02$$

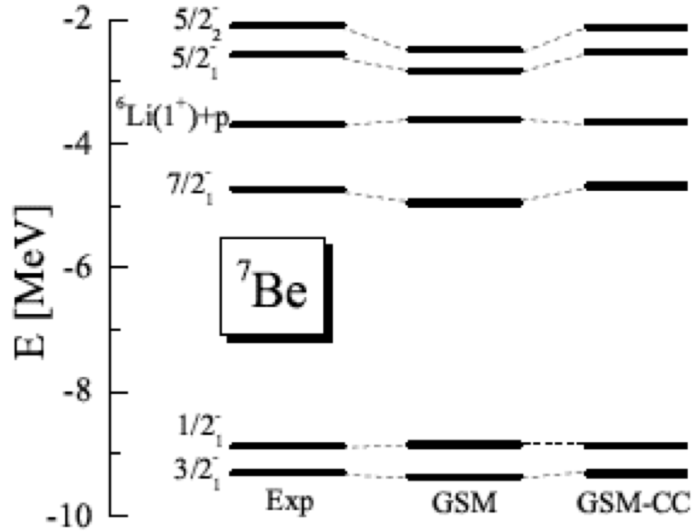


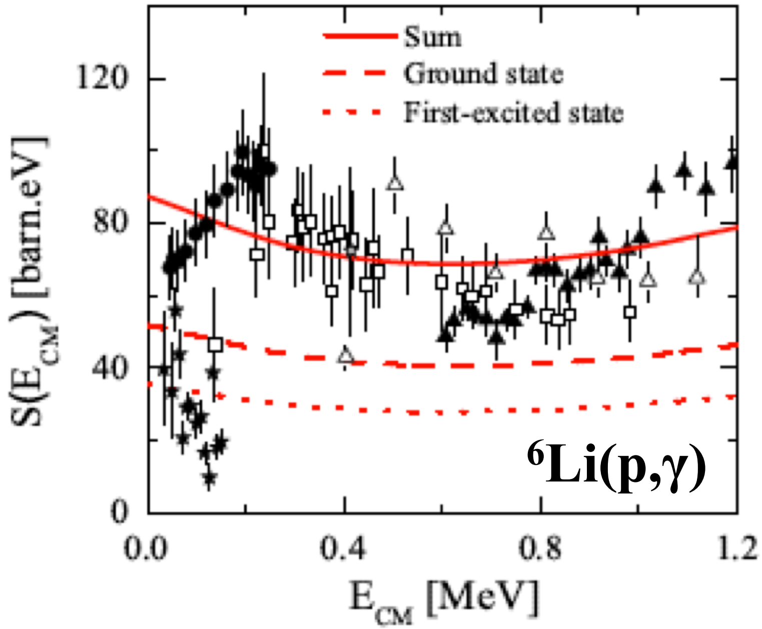


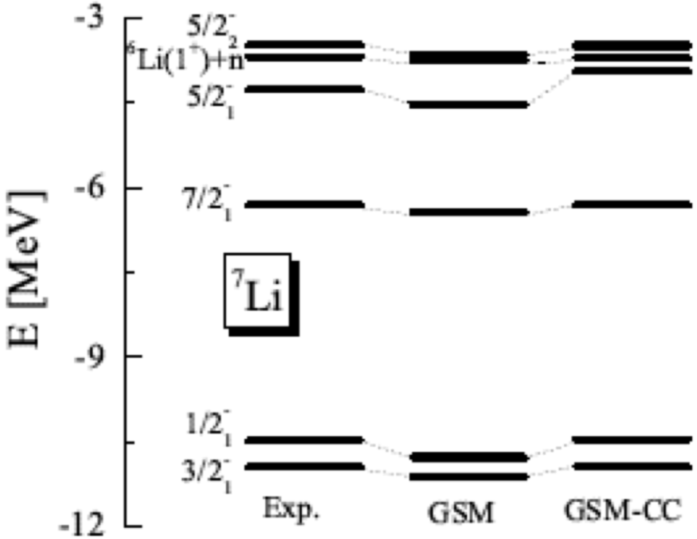
F. De Grancey, A. Mercenne, et al, PLB 758, 26 (2016)

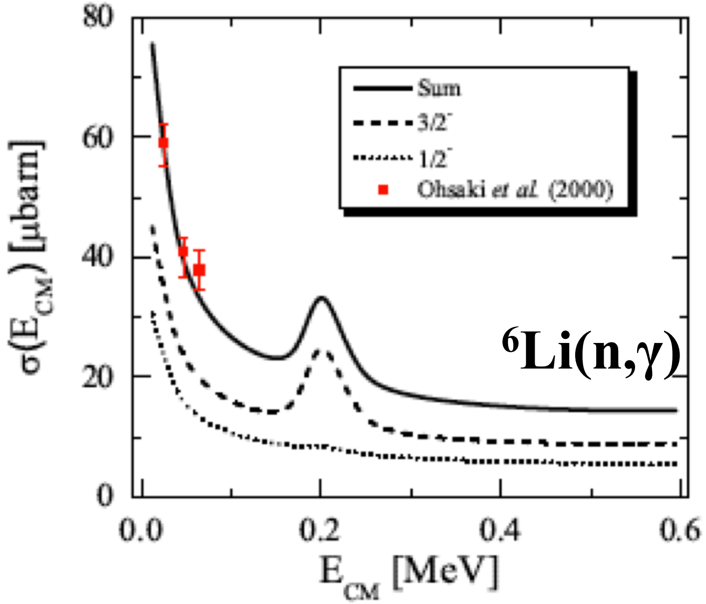
Scattering states are important for the completeness of channel basis

Mirror radiative capture cross sections





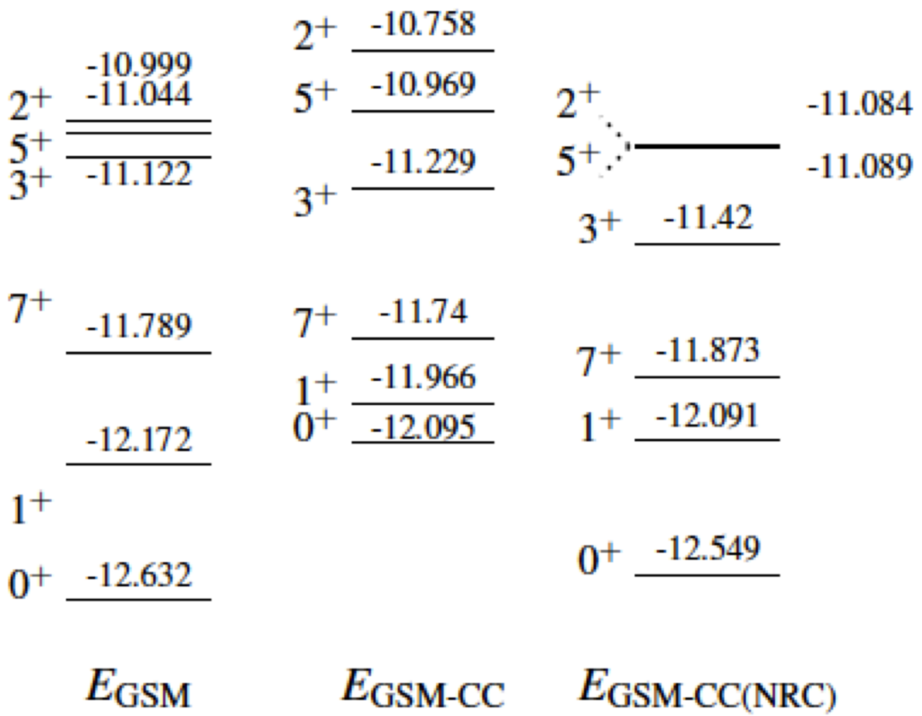




G. Dong, et al., J. Phys. G: Nucl. Part Phys. 44. 045201 (2017)

Role of the non-resonant reaction channels $\left[\left| J^{\pi}_{scat}(\text{A-1}) \right\rangle \otimes \left| j^{\pi'}(\text{n}) \right\rangle \right]^{J^{\pi}(\text{A})}$

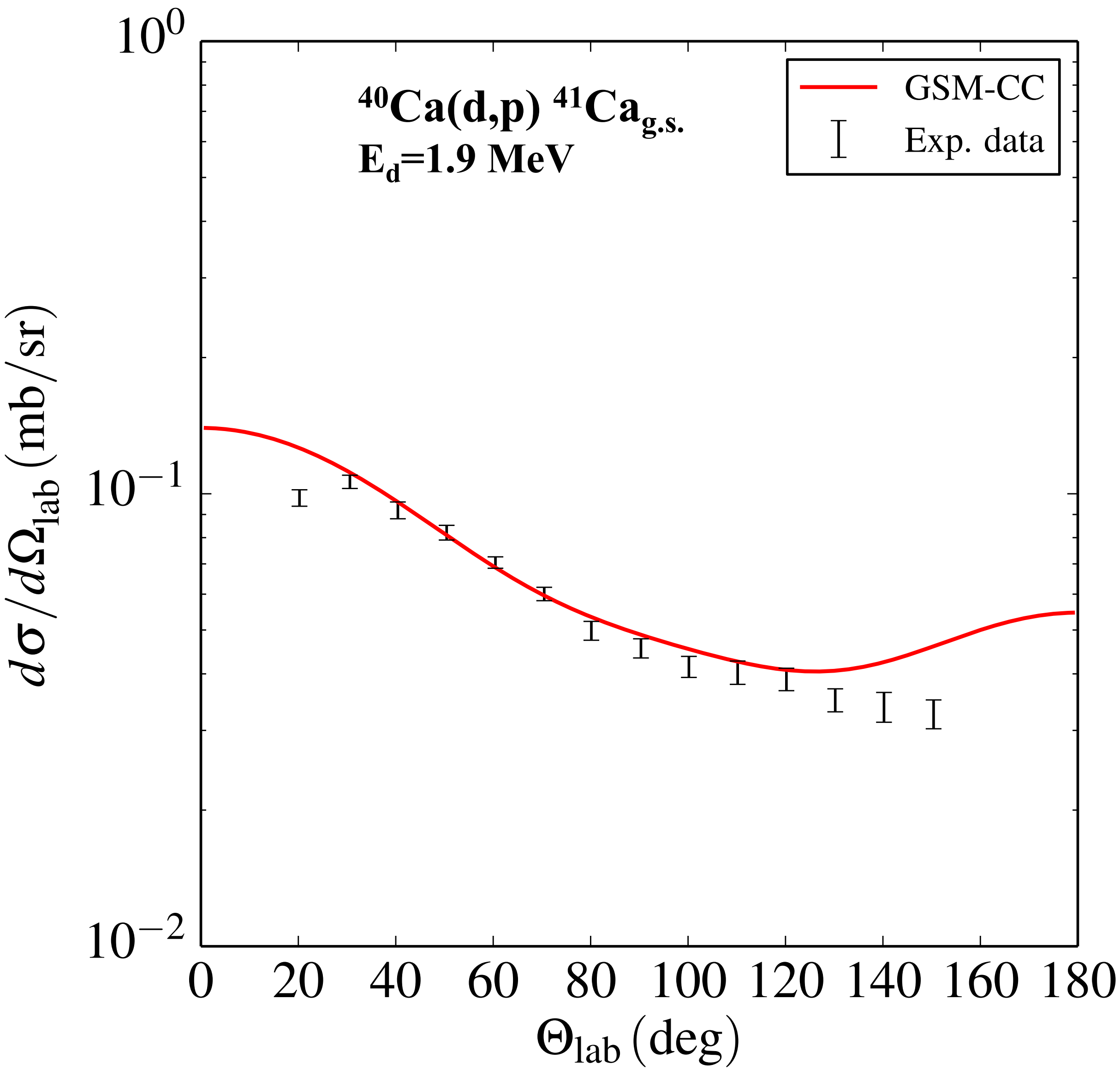
^{42}Sc



Channels:
^{40}Ca+d
^{41}Ca+p
^{41}Sc+n

Non-resonant channels built of continuum states:
1/2+, 3/2+, 5/2+, 7/2+, 9/2+
1/2−, 3/2−, 5/2−, 7/2−
in ^{41}Sc and ^{41}Ca

^{40}Ca(d,p) transfer reaction



Exp: I. Fodor et al., Nucl. Phys.. 73, 155 (1965)

Th: A. Mercenne, et al., (2018), in preparation

Outlook

- Shell model treatment of weakly bound/unbound states → unification of nuclear structure and reactions
  - Crucial role of non-resonant reaction channels

- Collectivization of nuclear wave functions due to:
  - internal mixing by interactions: rotational and vibrational states
  - external mixing via the decay(s) channels: coherent enhancement/suppression of radiation, multi-channel effects in shell occupancies and reaction cross-sections, …
  - interplay of internal and external mixing: near-threshold cluster/correlated states, breaking of the mirror/isospin symmetry, near-threshold collectivization of electromagnetic transitions, exceptional points, anti-odd-even effect in binding energies, …

- Future challenges:
  - how effective NN interactions are modified in weakly-bound/unbound states
  - γ-selection rules for in- and out- band transitions in the resonance bands
  - new kinds of multi-nucleon correlations and clustering in the vicinity of particle emission thresholds
  - effects of exceptional points in nuclear spectroscopy and reactions
  - … …