Nuclear structure and reactions in EFT

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FRIB-TA Bound States to Continuum workshop, June 19, 2018

Outline

- Brief introduction to EFT
- Capture reactions in halo EFT
    - lessons learned in $^7\mathrm{Li}(n,\gamma)^8\mathrm{Li}$
    - lessons learned in $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$
- Capture reactions in lattice EFT
    - Adiabatic Projection Method
    - Coulomb
    - currents

EFT: the long and short of it

- Identify degrees of freedom

$$\mathcal{L} = c_0 \, \mathrm{O}^{(0)} + c_1 \, \mathrm{O}^{(1)} + c_2 \, \mathrm{O}^{(2)} + \cdots$$ expansion in

Hide UV ignorance- short distance

IR explicit- long distance

- Determine c_n from data (elastic, inelastic)

- EFT : ERE + currents + relativistic corrections $\left(\frac{p}{m}\right)^{2n}$

Not just Ward-Takahashi identity

expand observables in ratio of scales

power counting

Pionless EFT — $\not{\pi}$EFT

nucleon-nucleon scattering

$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

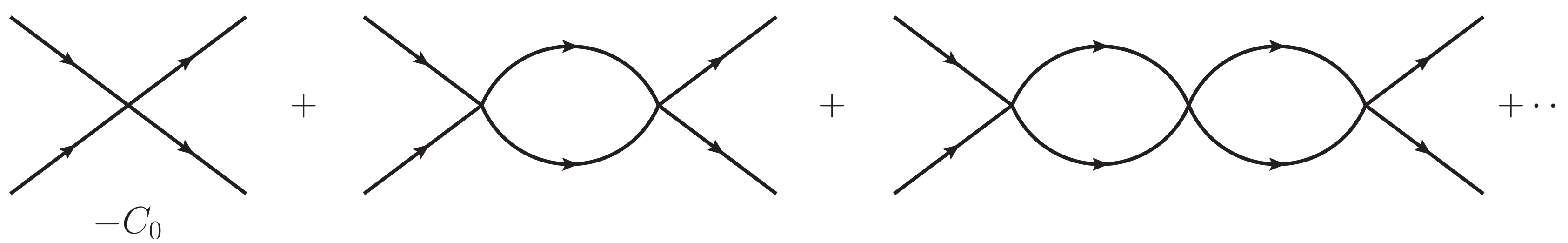
$$\approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{rp^2/2}{1/a + ip} + \cdots\right]$$, for $a \sim 1/p >> r$

Example: neutron-proton scattering

$$^1S_0 : a = -\,23.8 \text{ fm}, \quad r = 2.73 \text{ fm},$$
$$^3S_1 : a = +\,5.42 \text{ fm}, \quad r = 1.75 \text{ fm}.$$

Construct $\not{\pi}$ EFT

- Non-relativistic nucleons
- Short ranged interaction — point-like interaction

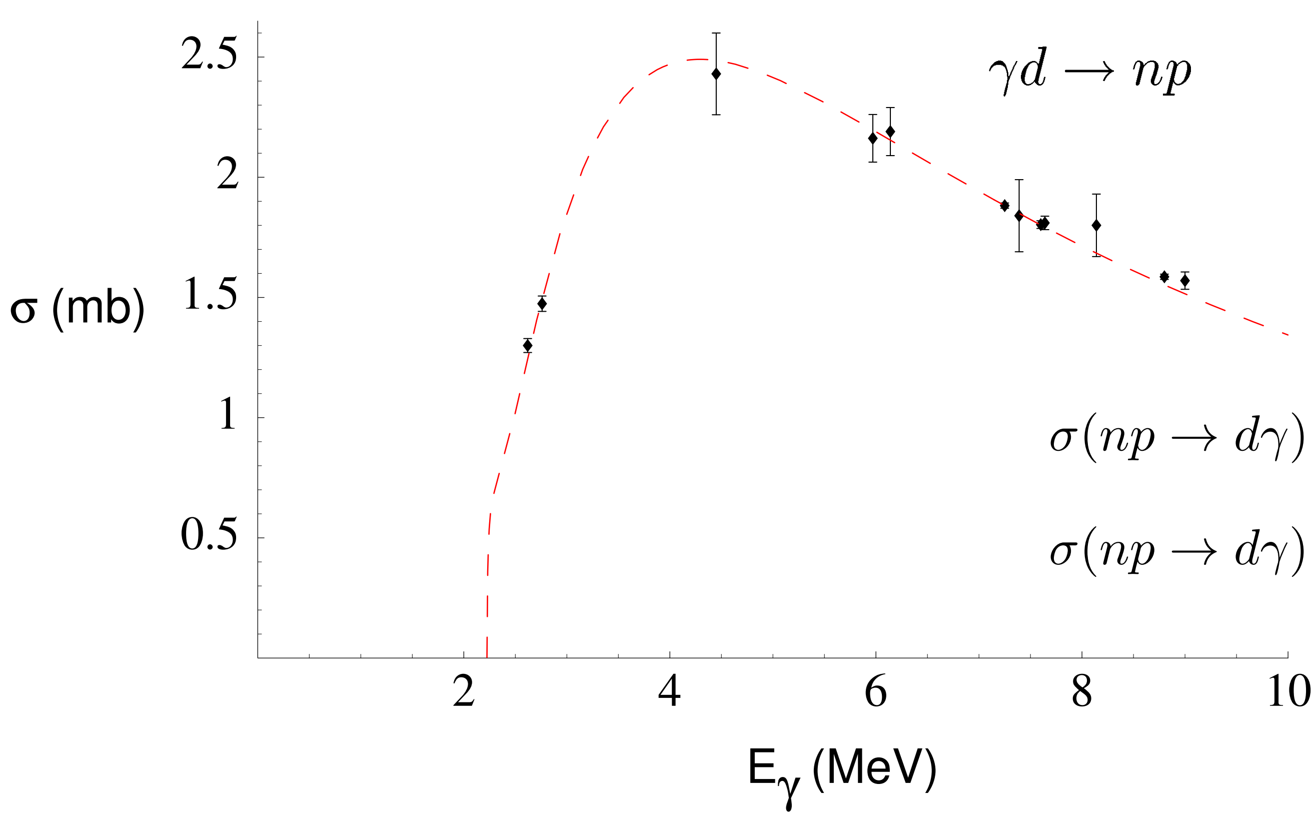


Weinberg '90
Bedaque, van Kolck '97
Kaplan, Savage, Wise '98

$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 \sim \frac{2\pi a}{\mu}$$

$$1/a \sim p \sim Q \ll 1/r \sim \Lambda \sim m_\pi$$

power-counting $C_0 \sim 1/Q$ ← single fine-tuning (rho-pion physics)



Reduce

5% → 1%

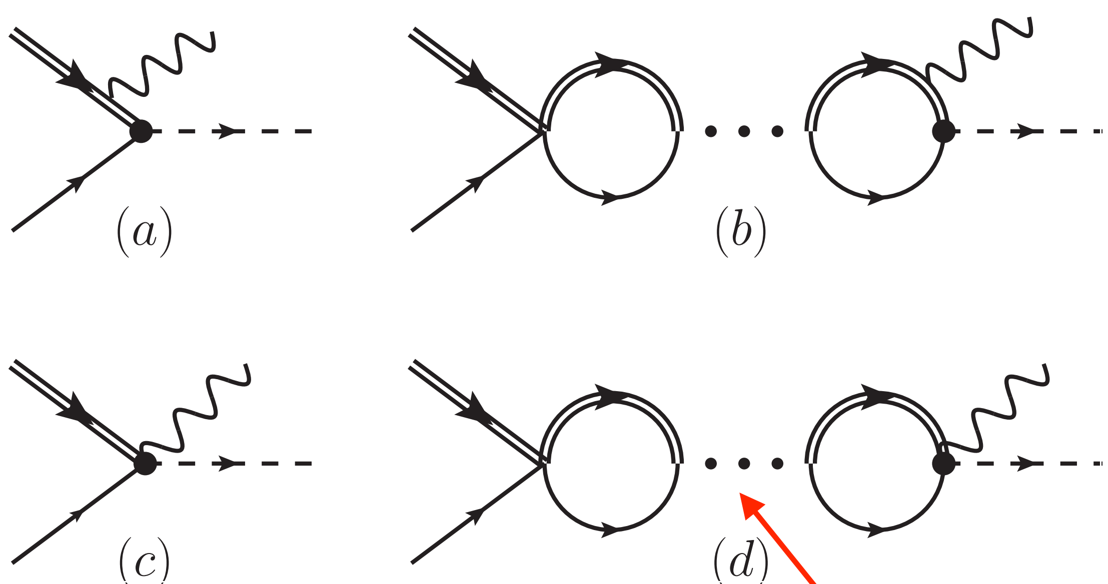
$$\sigma(np \to d\gamma)\Big|_{E=2\ \mathrm{MeV}} = 0.0218(1 + 0.6389 + 0.0135 - 0.0053 - 0.001)\ \mathrm{fm}^2$$

$$\sigma(np \to d\gamma)\Big|_{E=20\ \mathrm{keV}} = 0.1917(1 + 0.1076 + 0.0001)\ \mathrm{fm}^2$$

Rupak; NPA678, 405 (2000)

$^7\mathrm{Li}(n,\gamma)^8\mathrm{Li}$ Example

- Isospin mirror to $^7\mathrm{Li}(n,\gamma)^8\mathrm{Li} \leftrightarrow {}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$
- Inhomogeneous BBN



Whats the theory error?

wave function normalization

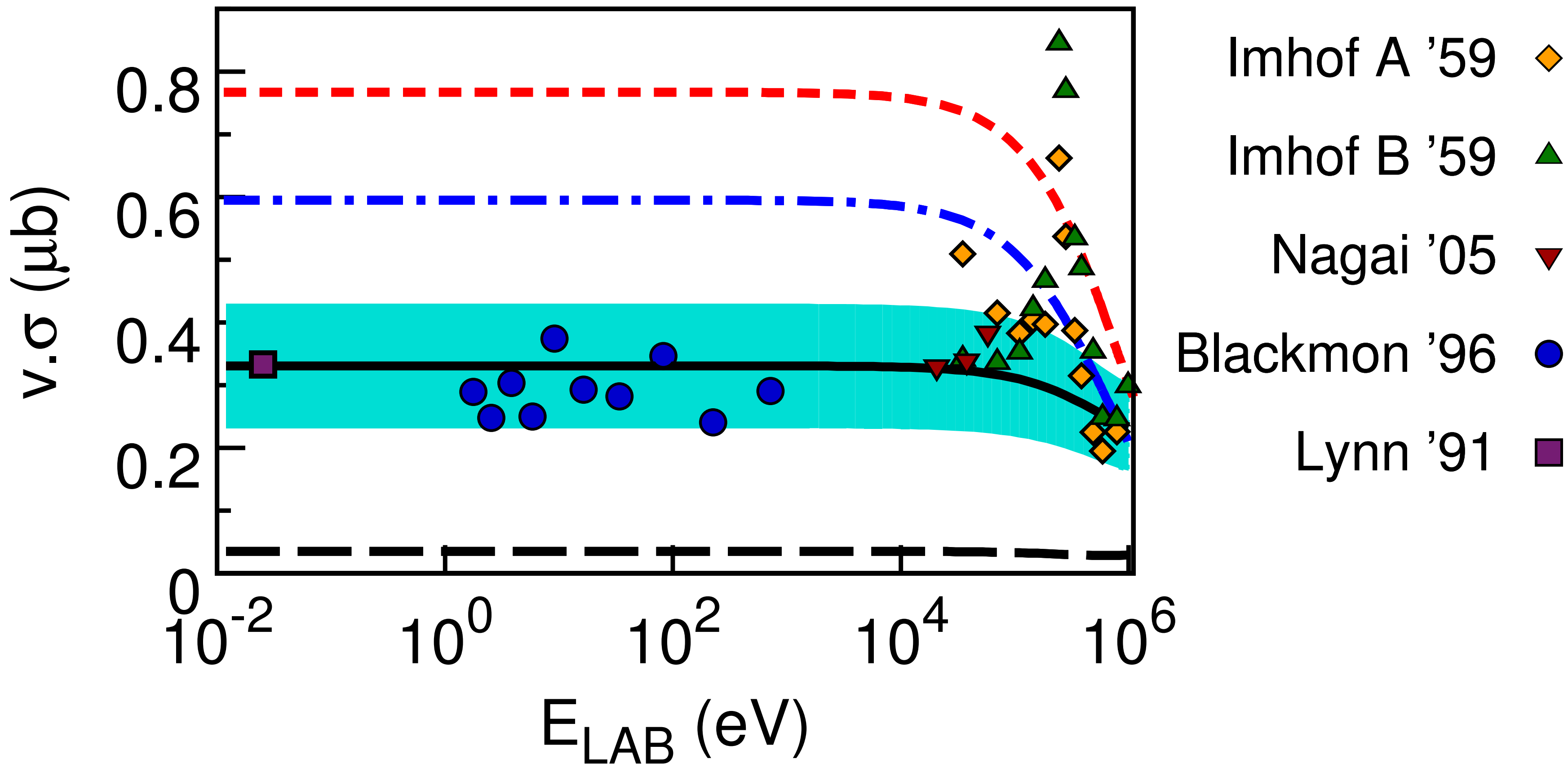
$$\sqrt{\mathcal{Z}} \propto \sqrt{\frac{1}{1+3\gamma/r_1}}$$

initial s-wave interaction

Two EFT operators for p-wave bound state at LO

Bertulani, Hammer, van Kolck (2002)
Bedaque, Hammer, van Kolck (2003)

need binding momentum and effective range at LO



Red: Tombrello (1965)
Blue: Davids-Typel (2003)
Black: EFT

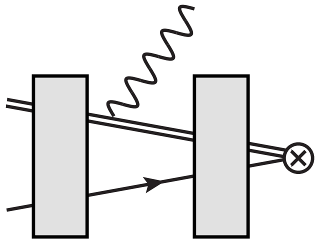
RADCAP: Bertulani
CDXS+: Typel

Rupak, Higa; PRL 106, 222501 (2011)
Fernando, Rupak, Higa; 48, 24 (2012)

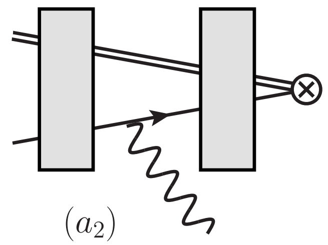
Initial state: s-wave $a_0 \sim 1/Q, r_0 \sim 1/\Lambda$
Final state: shallow p-wave (ground and excited); two operators

Lesson: Need accurate binding energy and effective range (phase shift) for p-wave

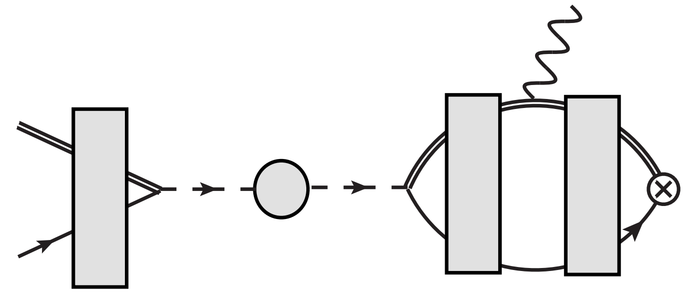
$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ Example



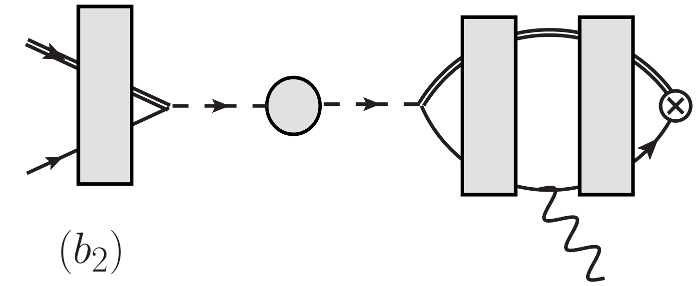
(a_1)



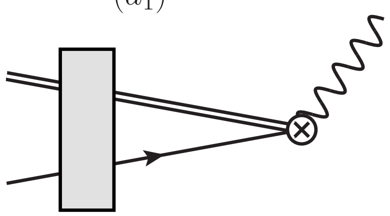
(a_2)



(b_1)

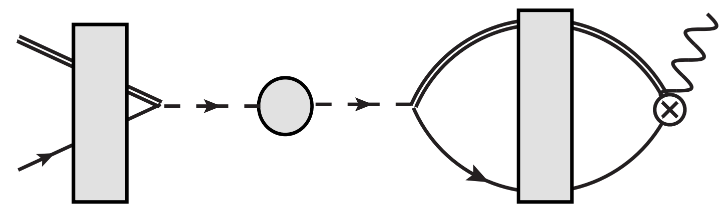


(b_2)



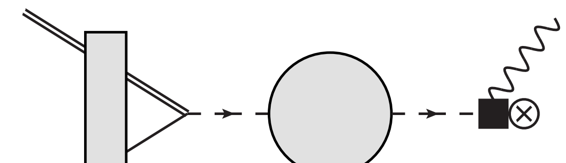
(a_3)

$\mathcal{O}(1)$



(b_3)

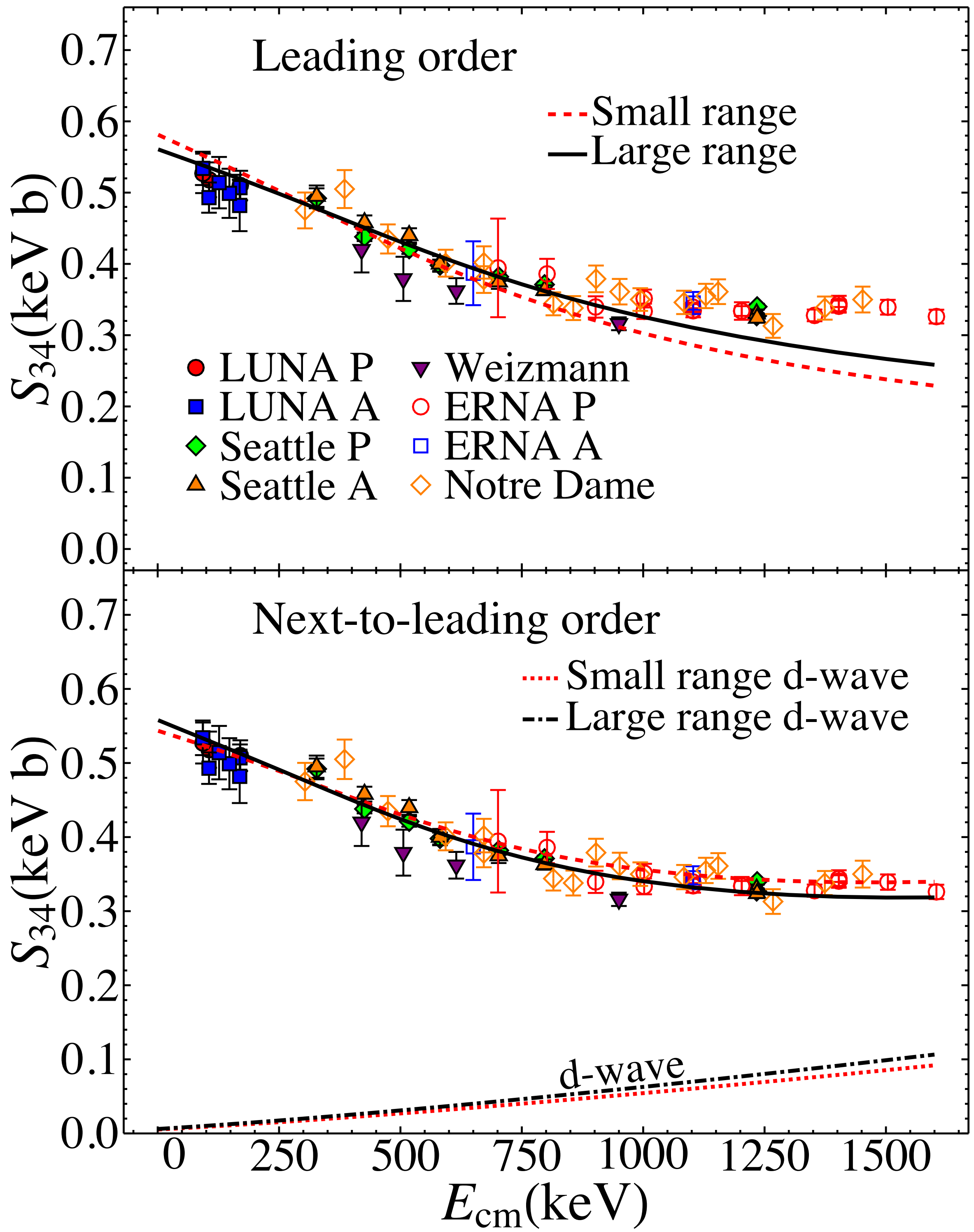
$\sim a_0(B+\mu J_0)$



$L_{E1}^{(\zeta)}$

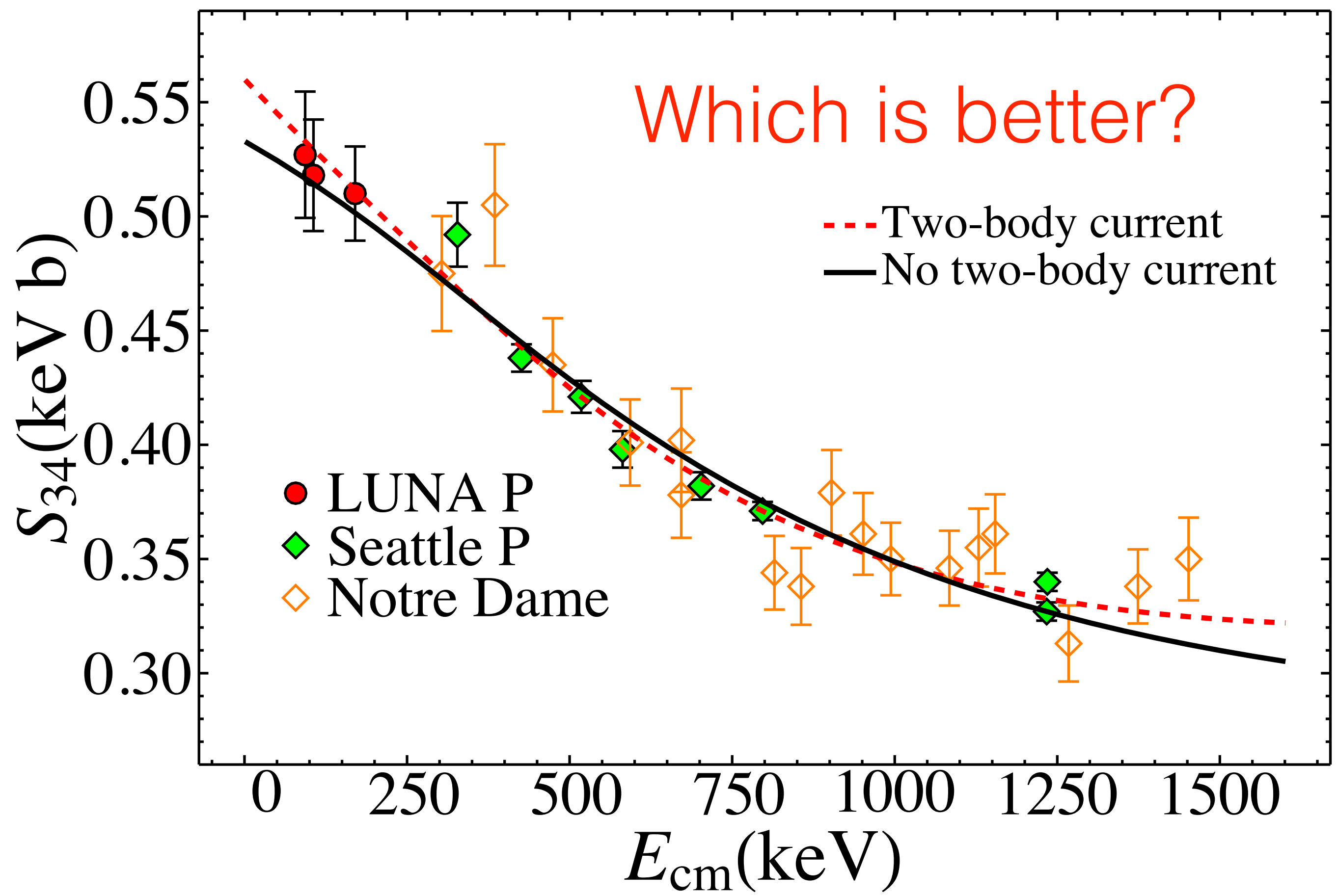
Many fine tunings:

- Initial state: s-wave
  - $a_0 \sim \Lambda^2/Q^3, r_0 \sim 1/\Lambda$
  - However $a_0(B+\mu J_0)\sim 1$, and nearly cancels set A
  - Two-body current enhanced to LO by large a_0
- Final state: shallow p-wave, ground and excited states



Fit	$S_{34}(0)$ (keV b)
Small range LO	0.582± 0.011 (fit) ± 0.194 (EFT)
Large range LO	0.561± 0.007 (fit) ± 0.187 (EFT)
Small range NLO	0.544± 0.012 (fit) ± 0.054 (EFT)
Large range NLO	0.558± 0.008 (fit) ± 0.056 (EFT)

Higa, Rupak, Vaghani; EPJA 54, 89 (2018)



$$C_{1,j}^2 = \frac{\gamma_j^2 \Gamma(2+\kappa_C/\gamma_j)}{\pi} h_j^2 \mathcal{Z}_j \propto \frac{1}{\rho_j + \#_j}$$

ground (excited) state poles when effective range about -47 MeV (-32 MeV) — near cancellations of set A and B, and two-body currents can be compensated with small change in effective range ρ_j

Bayesian analysis

posterior PDF: $$P(\boldsymbol{\theta}|D,I) = \frac{P(D|\boldsymbol{\theta},I)P(\boldsymbol{\theta}|I)}{P(D|I)}$$

prior PDF: $$P(\boldsymbol{\theta}|I)$$ ← EFT expectations/bias

likelihood PDF: $$P(D|\boldsymbol{\theta},I) \propto \exp(-\frac{1}{2}\chi^2), \quad \chi^2 = \sum_{i=1}^{N} \frac{[D_i - \mu_i(\boldsymbol{\theta})]^2}{\sigma_i^2}$$

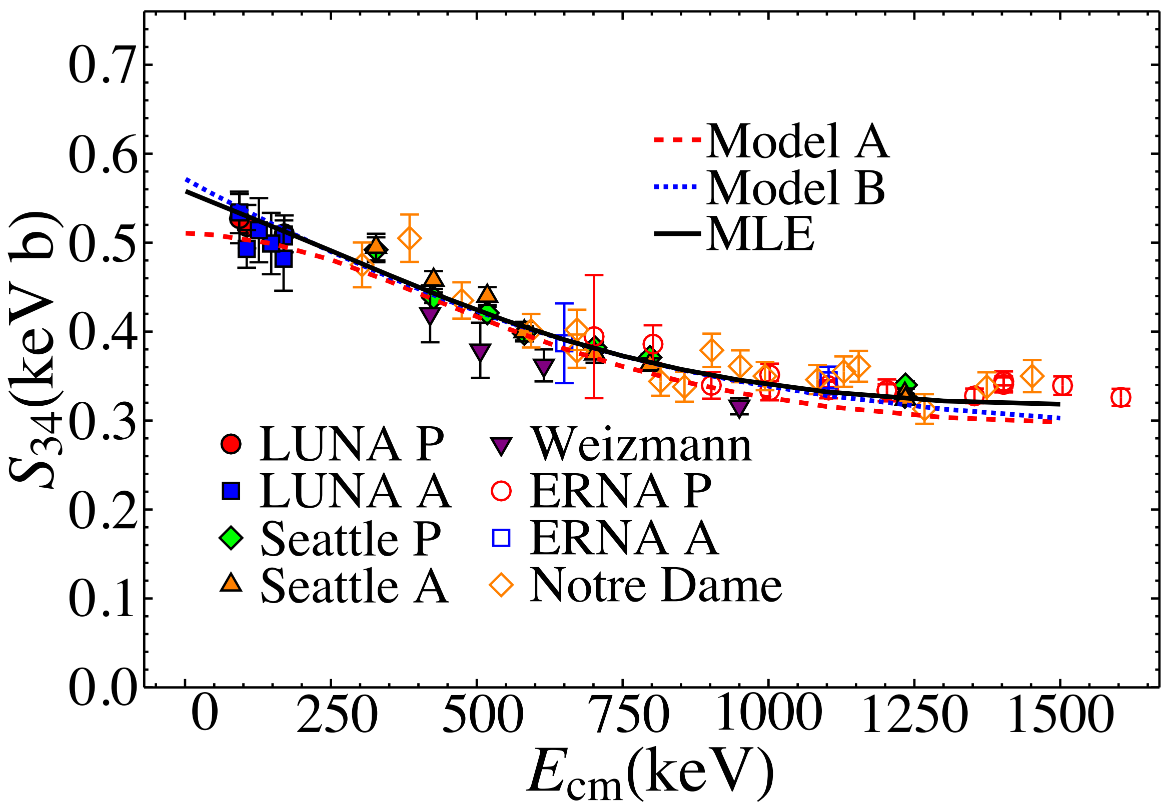
evidence: $$P(D|I) = \int [d\boldsymbol{\theta}] P(D|\boldsymbol{\theta},I)P(\boldsymbol{\theta}|I)$$

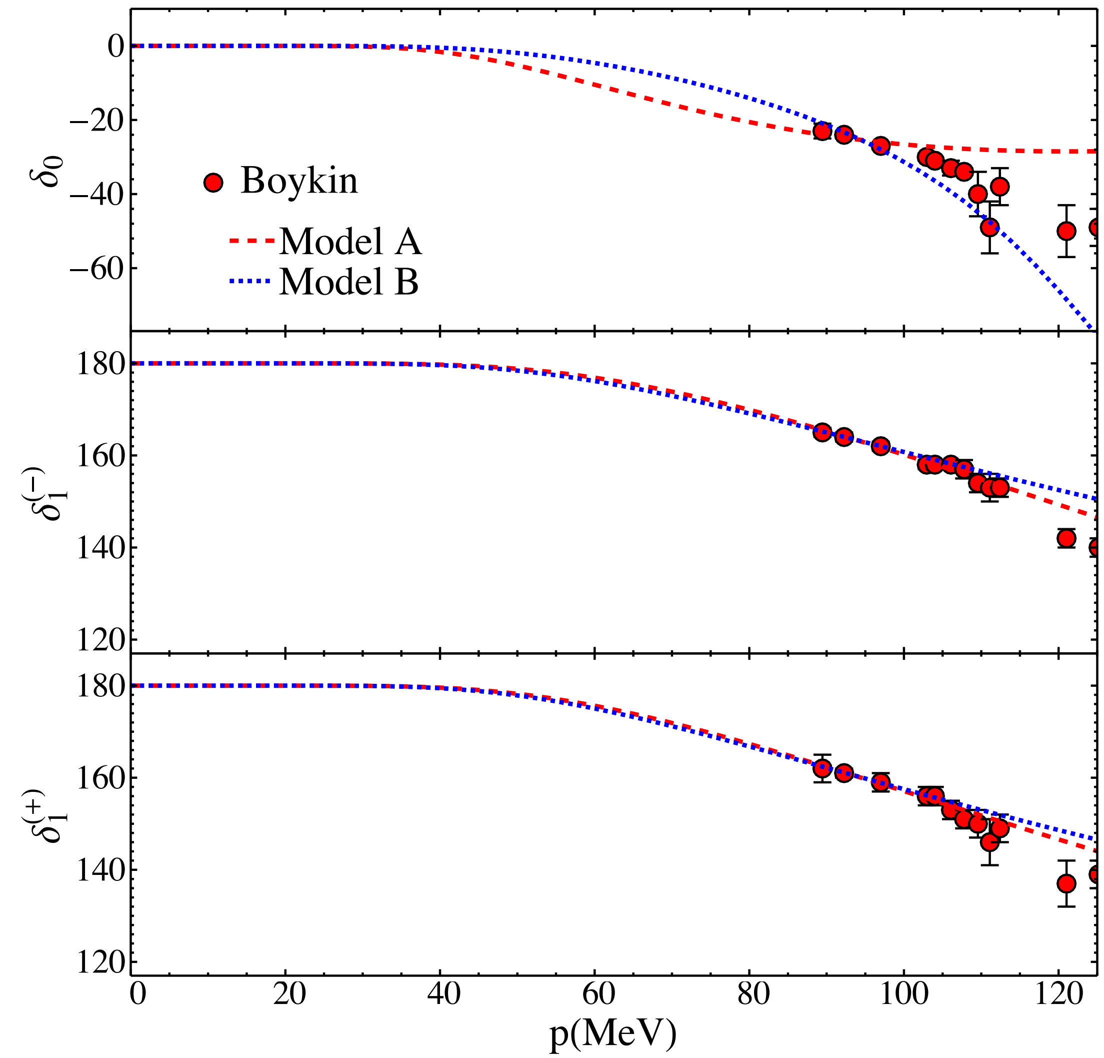
— harder to calculate
— not needed for parameter estimation
— needed to compare models

Models: $$\frac{P(M_1|D,I)}{P(M_2|D,I)} = \frac{P(M_1|I)}{P(M_2|I)} \times \frac{P(D|M_1,I)}{P(D|M_2,I)}$$

↖ bias goes here

Preliminary Results





Both models equally likely without phase shift input, model A much more likely with phase shift input.

$$\ln(Z_A) \approx -63 \pm 1$$

$$\ln(Z_B) \approx -67 \pm 1$$

Evidence calculated using Nested Sampling

Skilling; Baysian Analysis 4, 833 (2006)

Elastic scattering of $^3\mathrm{He} + \alpha$ with SONIK

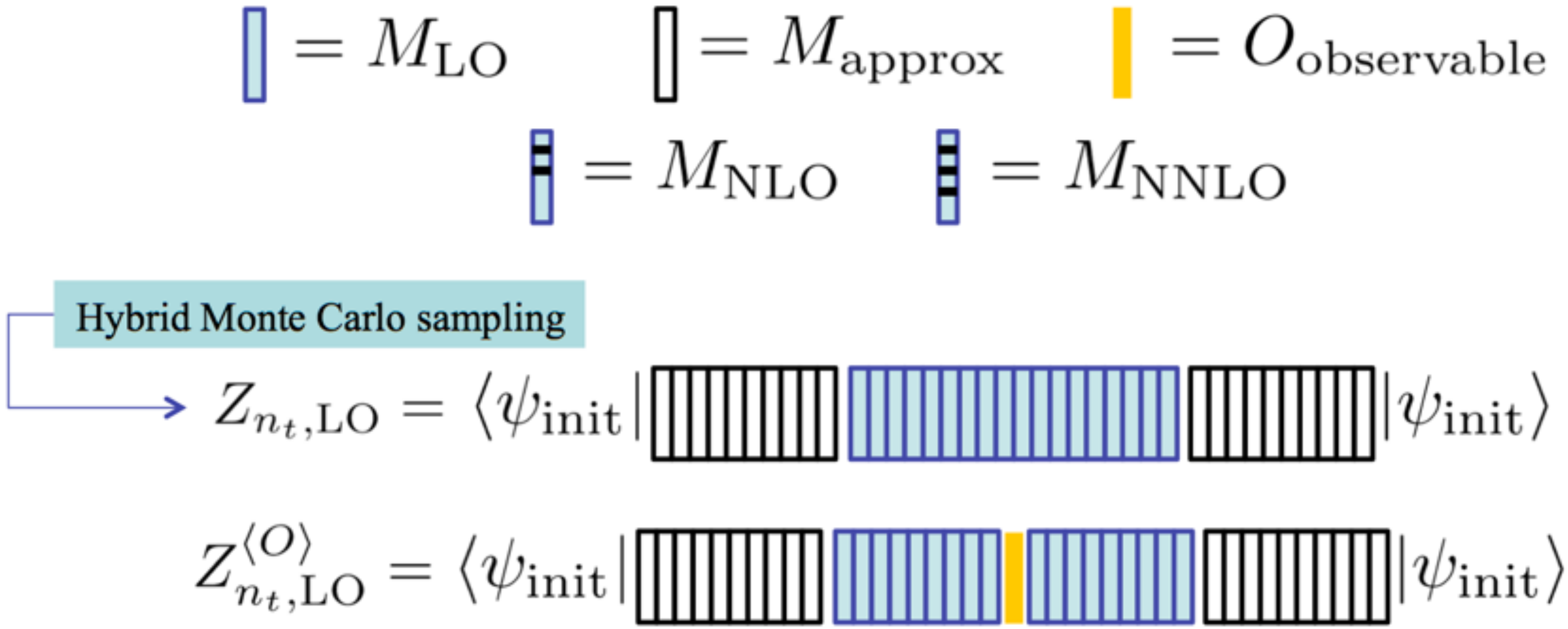
~ 500 keV to 3 MeV

Spokespersons: Connolly, Davids, Greife

Reactions in lattice EFT

- Consider: $a(b,\gamma)c$; $a(b,c)d$
- Need effective “cluster” Hamiltonian — acts in cluster coordinates, spins, etc.
- Reactions using cluster Hamiltonian — traditional methods, continuum (halo/cluster) EFT, lattice EFT

Schematic of lattice Monte Carlo calculation

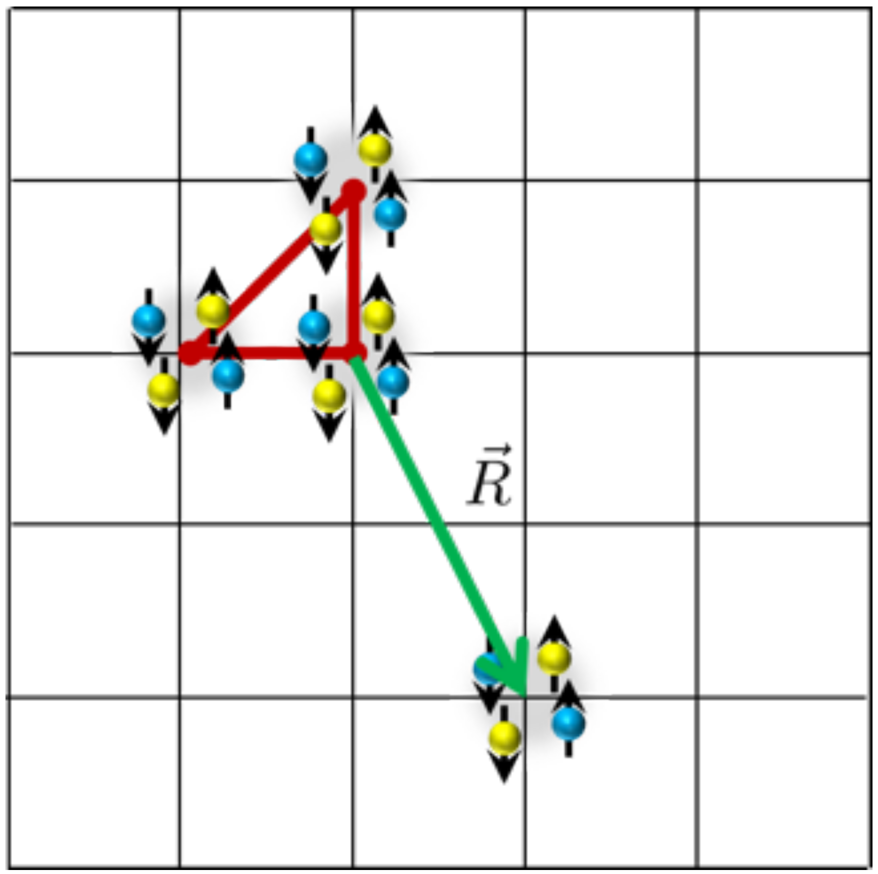


$$e^{-E_{0,\mathrm{LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

$$\langle O \rangle_{0,\mathrm{LO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{LO}} / Z_{n_t,\mathrm{LO}}$$

Courtesy Dean Lee

Adiabatic Projection Method



$$\text{Initial state } |\vec{R}\rangle$$

$$\text{Evolved state } |\vec{R}\rangle_\tau = e^{-\tau H}|\vec{R}\rangle$$

$$_\tau\langle\vec{R}'|H|\vec{R}\rangle_\tau$$

Energy measurements in cluster basis.
Divide by norm matrix $[N_\tau]_{\vec{R},\vec{R}'} =_\tau \langle\vec{R}|\vec{R}'\rangle_\tau$

microscopic Hamiltonian: $L^{3(A-1)}$
cluster Hamiltonian: L^3

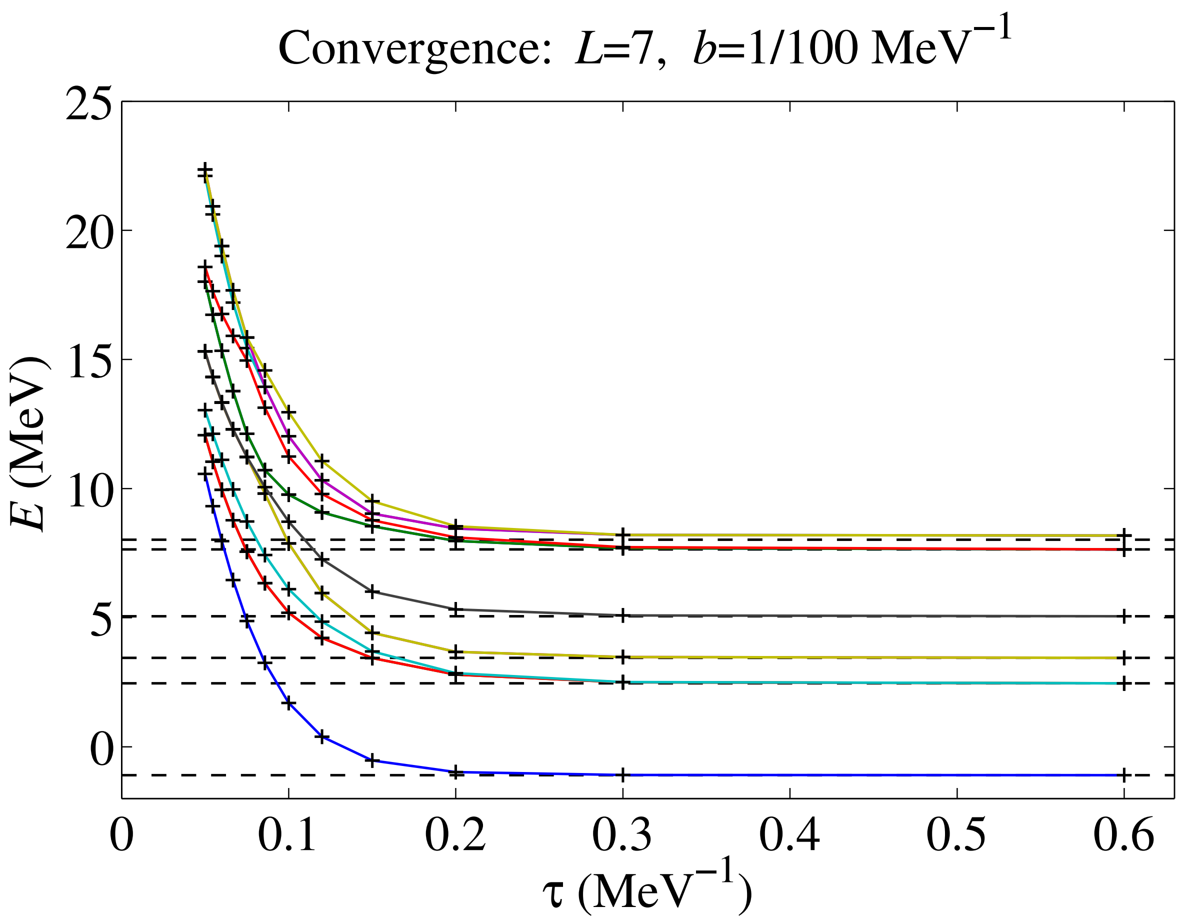
smaller in practice

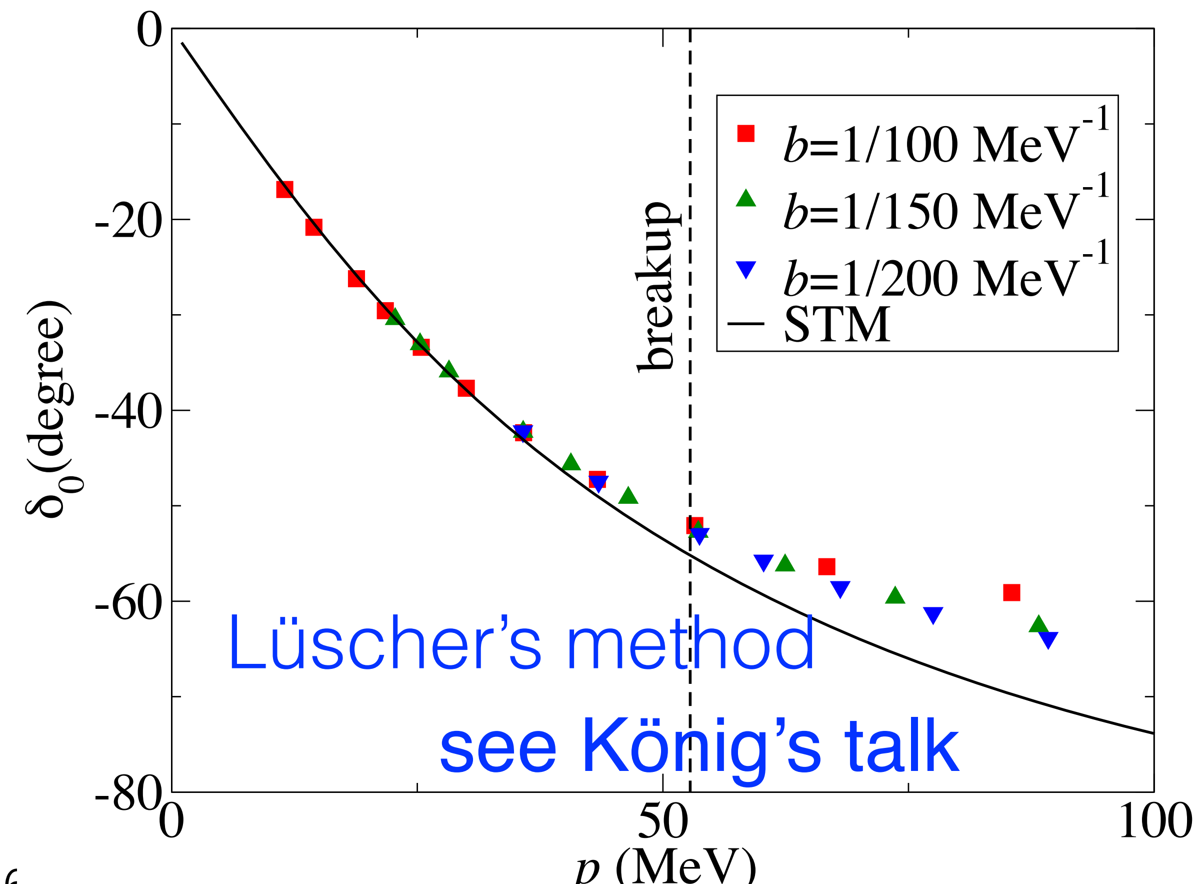
Proof of concept: quartet neutron-deuteron

$$\mathcal{L}_I = -g(\psi_\uparrow^\dagger \psi_\uparrow)(\psi_\downarrow^\dagger \psi_\downarrow)$$

$$T(p) = h(p) + \int dq K(p,q) T(q)$$

$$T(p) = \frac{2\pi}{\mu} \frac{1}{p \cot \delta - ip}$$





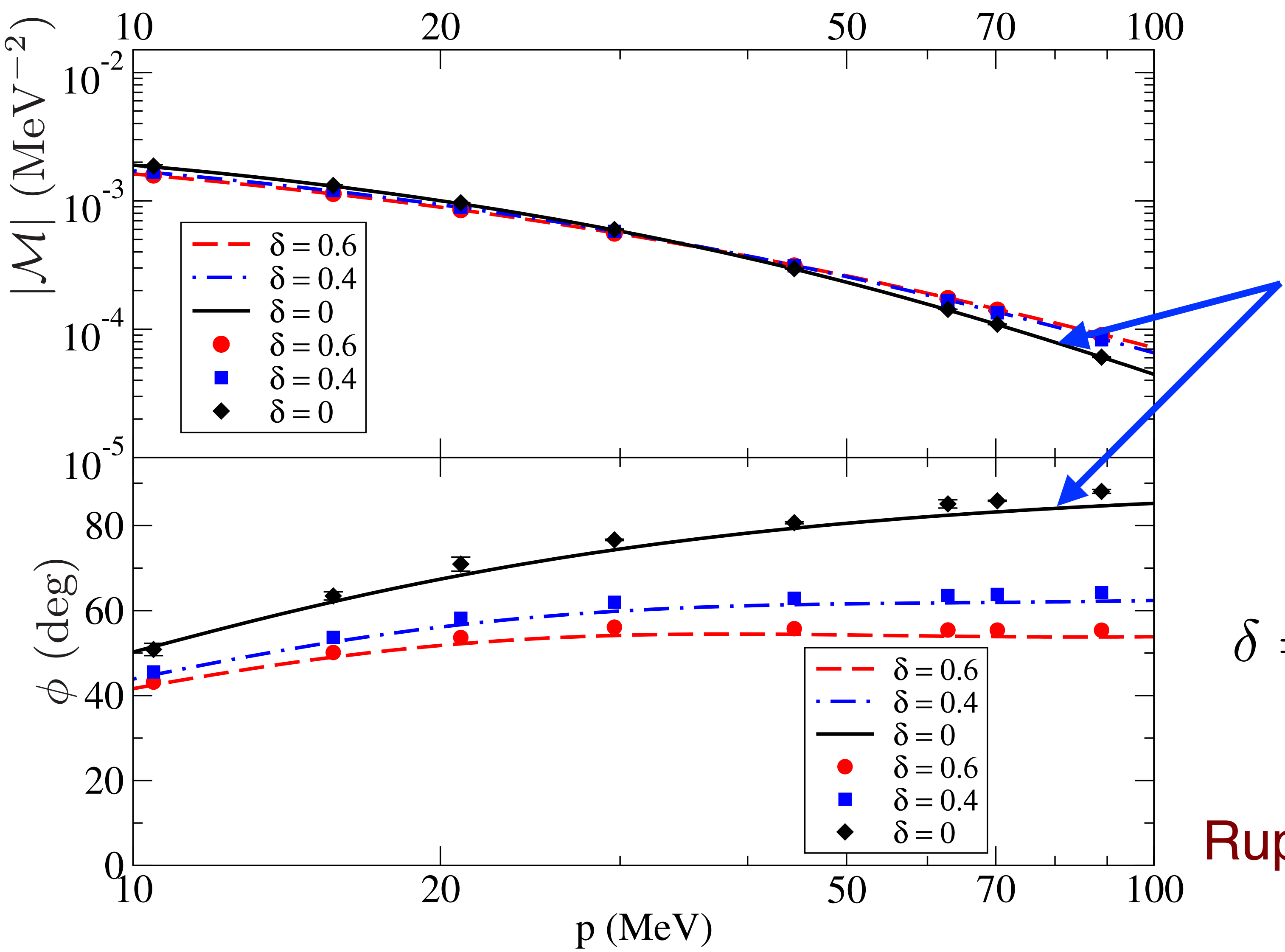
Rupak, Lee; PRL 111, 032502 (2013)
Pine, Lee, Rupak; EPJA 49, 151 (2013)

$p(n,\gamma)d$ in lattice EFT

Use retarded Green's function to evaluate $\langle\psi_B|O_{\rm EM}|\psi_i\rangle$

$$\mathcal{M}(\epsilon) = \left(\frac{p^2}{M} - E - i\epsilon\right)\sum_{\boldsymbol{x},\boldsymbol{y}}\psi_B^*(\boldsymbol{y})\langle\boldsymbol{y}|\frac{1}{E-\hat{H}_s+i\epsilon}|\boldsymbol{x}\rangle e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

cluster Hamiltonian goes here

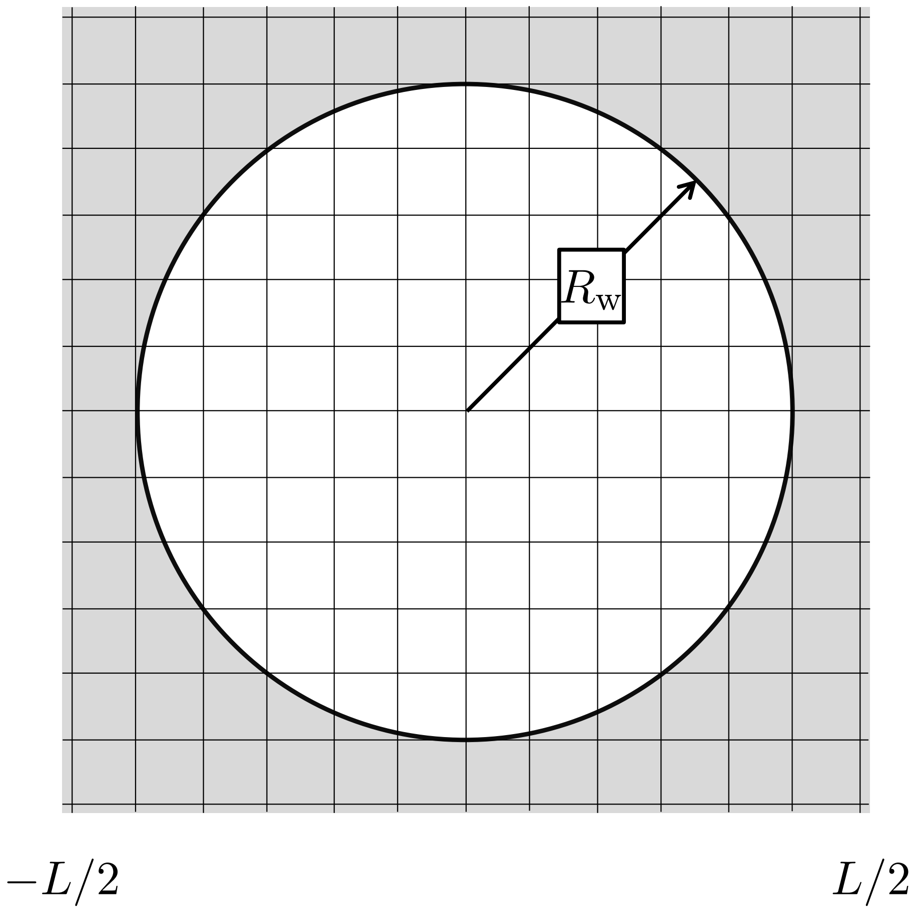


$\delta = \epsilon M/p^2$

Rupak, Lee; PRL 111, 032502 (2013)

Coulomb interaction

- implement with spherical hard-wall



$$\psi_{\text{short}}(r) \propto j_0(kr) \cot \delta_s - n_0(kr),$$
$$\psi_{\text{Coulomb}}(r) \propto F_0(kr) \cot \delta_{sc} + G_0(kr)$$

— the lattice provides the momentum, which fixes the phase shift

We adjust wall size from free theory

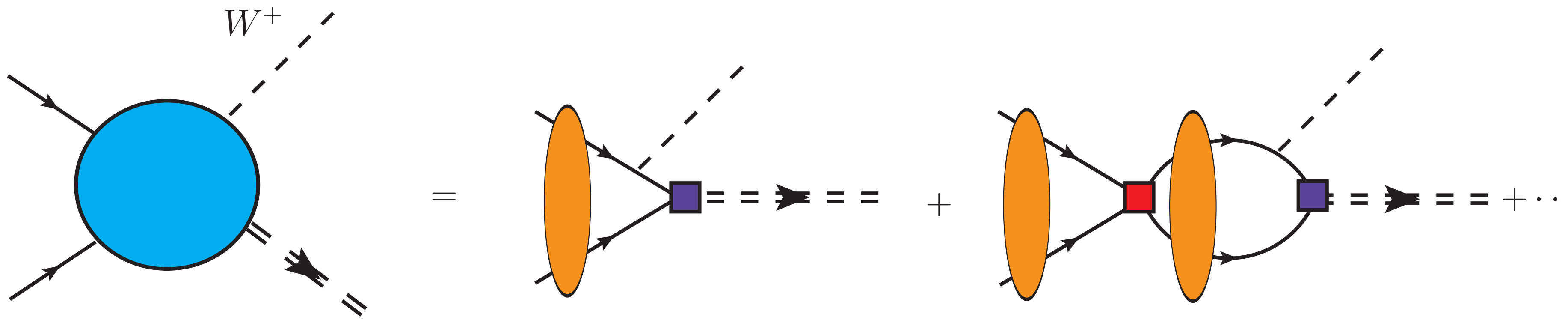
$$j_0(k_0 R_w) = 0$$

IR scale setting

Borasoy et al. 2007

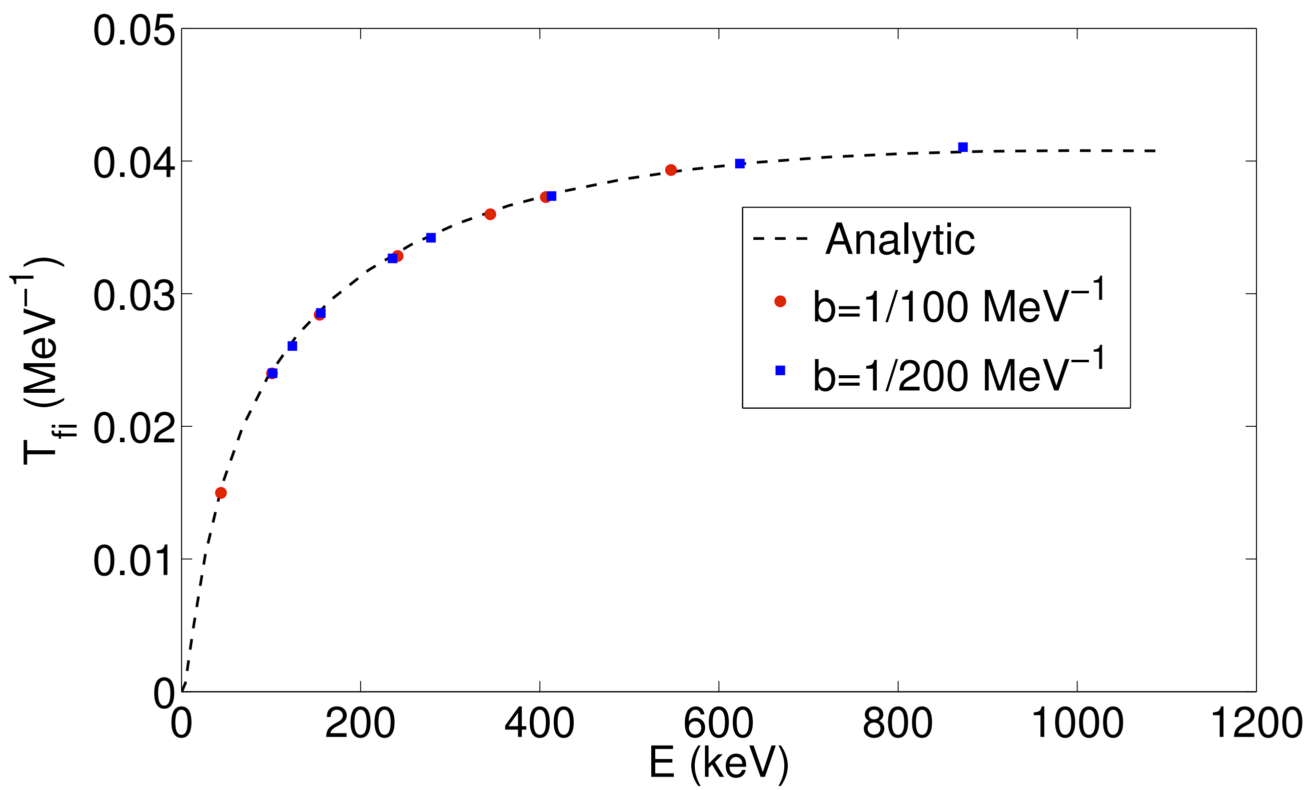
Carlson et al. 1984

proton-proton fusion in lattice EFT



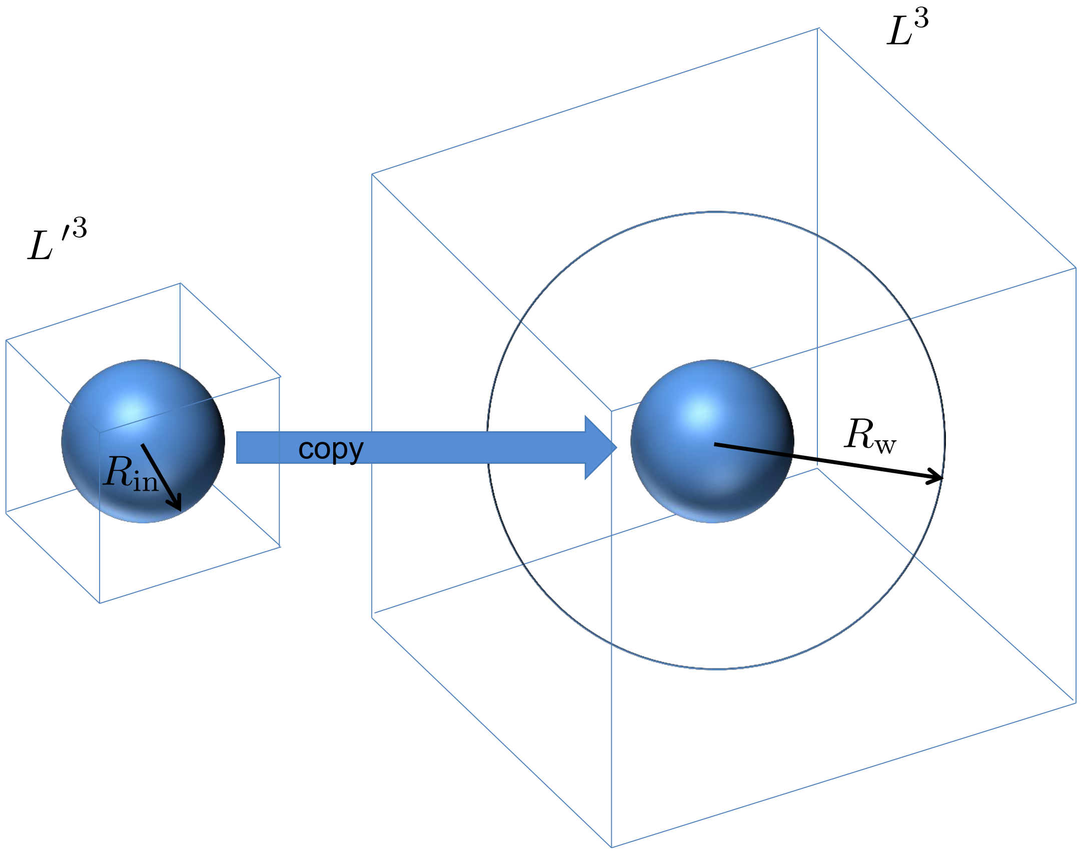
electro-weak operator similar to $np \to d\gamma$, get incoming Coulomb wave function correct

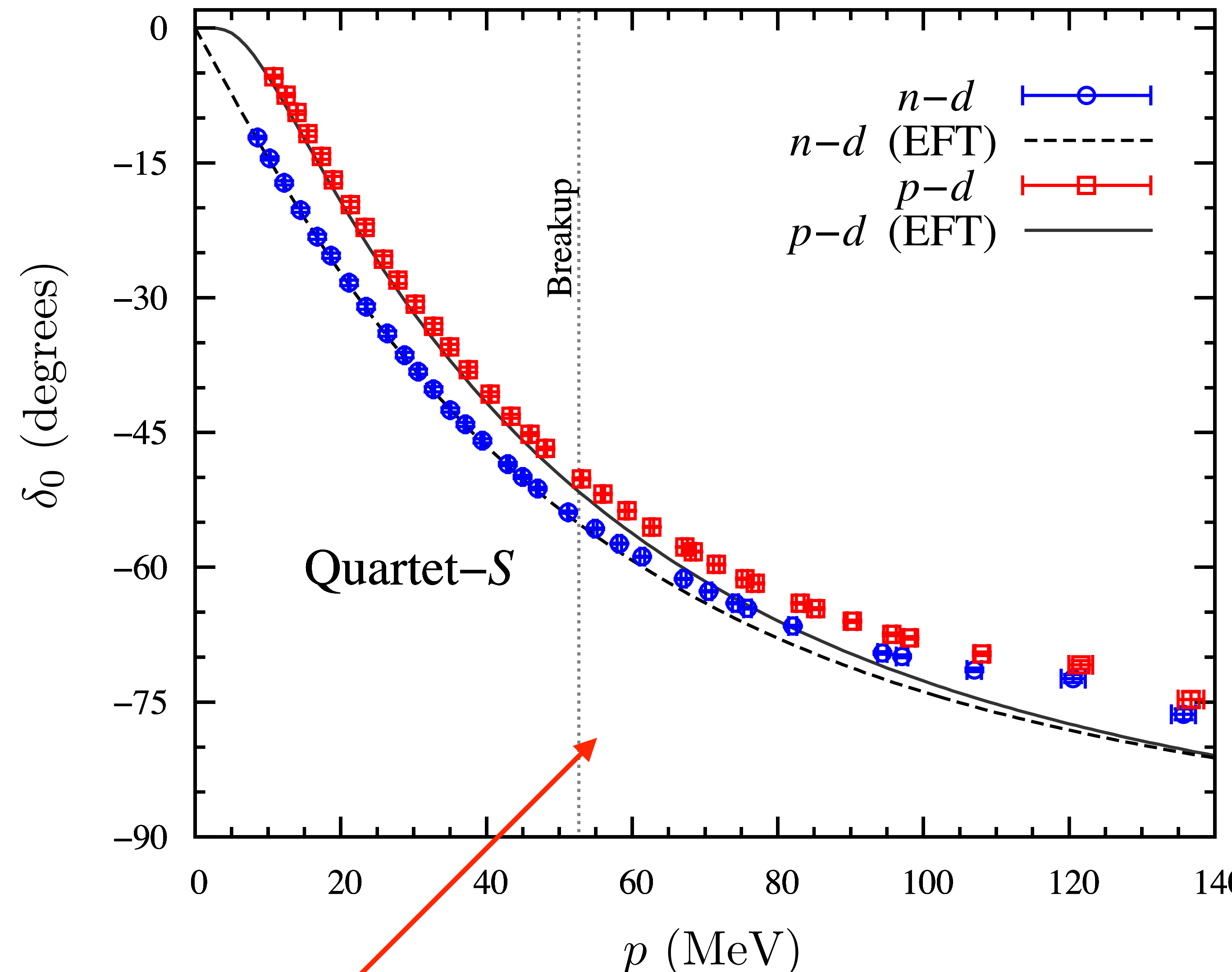
$$\Lambda(p) = \sqrt{\frac{\gamma^2}{8\pi C_\eta^2}} |T_{fi}(p)|$$



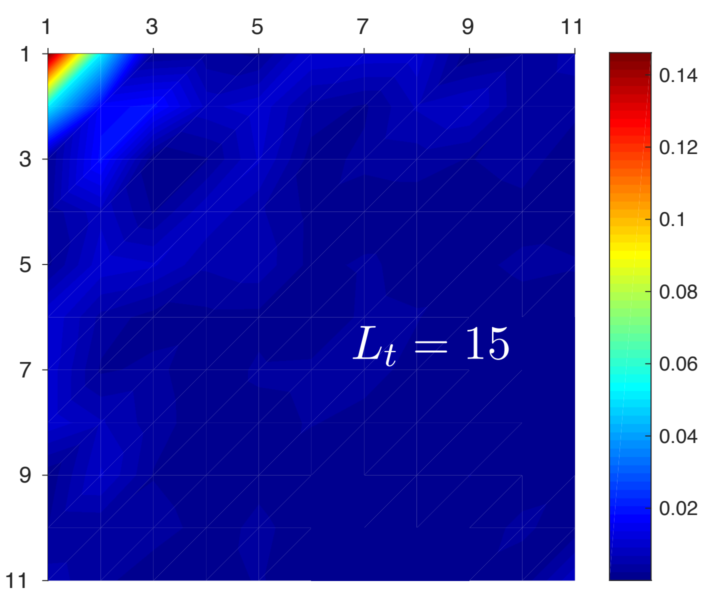
$\Lambda_{EFT}(0) \approx 2.51$ Kong, Ravndal; NPA 656, 421 (1999)

$\text{Lattice fit}: \Lambda(0) \approx 2.49 \pm 0.02$ Rupak, Ravi; PLB 741, 301 (2015)





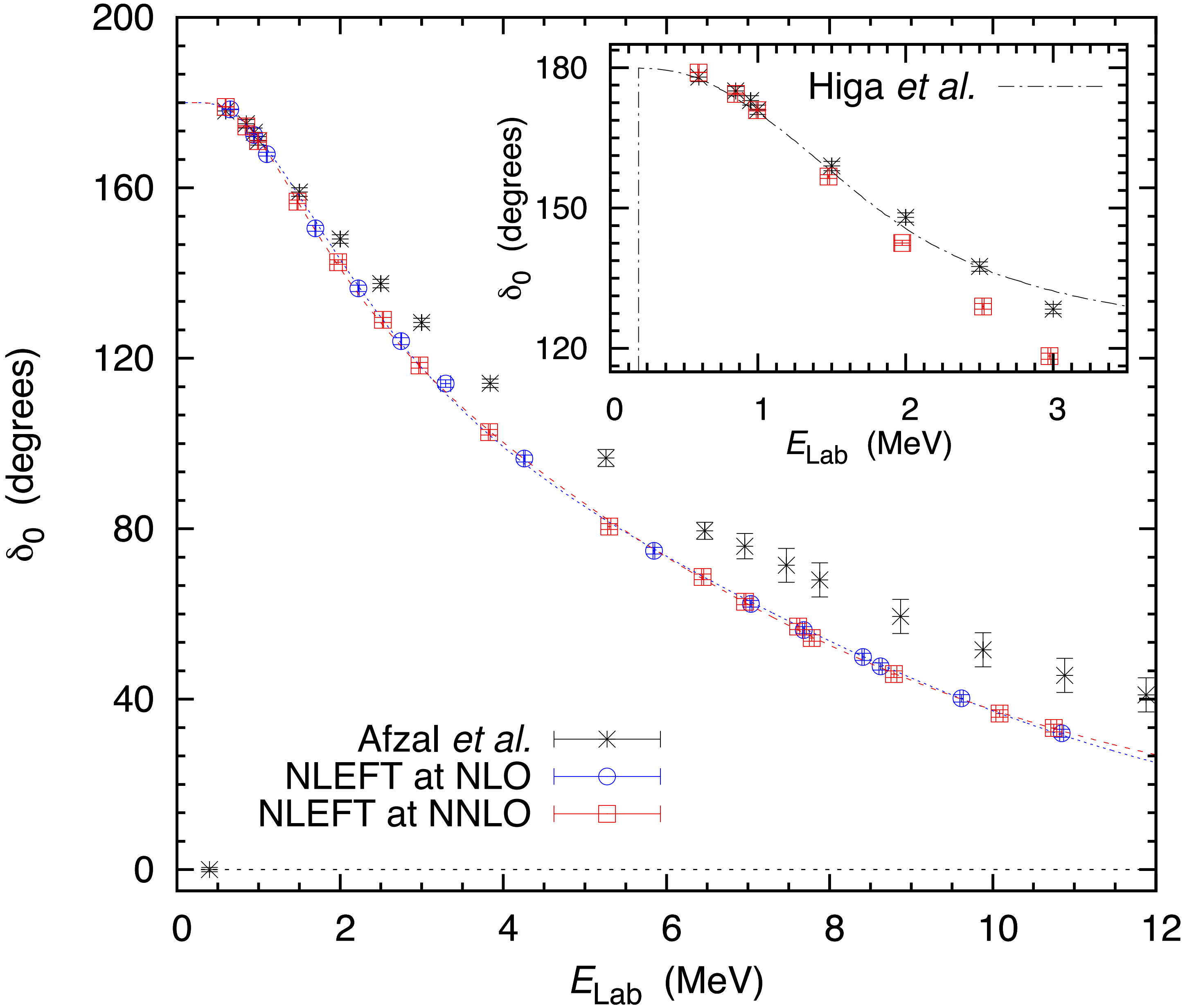
fermion-dimer



matching 15 to 82

Elhatisari, Lee; Meißner, Rupak; EPJA 52, 174 (2016)

alpha-alpha scattering



microscopic adiabatic Hamiltonian calculated to 16 fm, then extended to 120 fm box

Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner;
Nature 528, 111 (2015)

pinhole algorithm

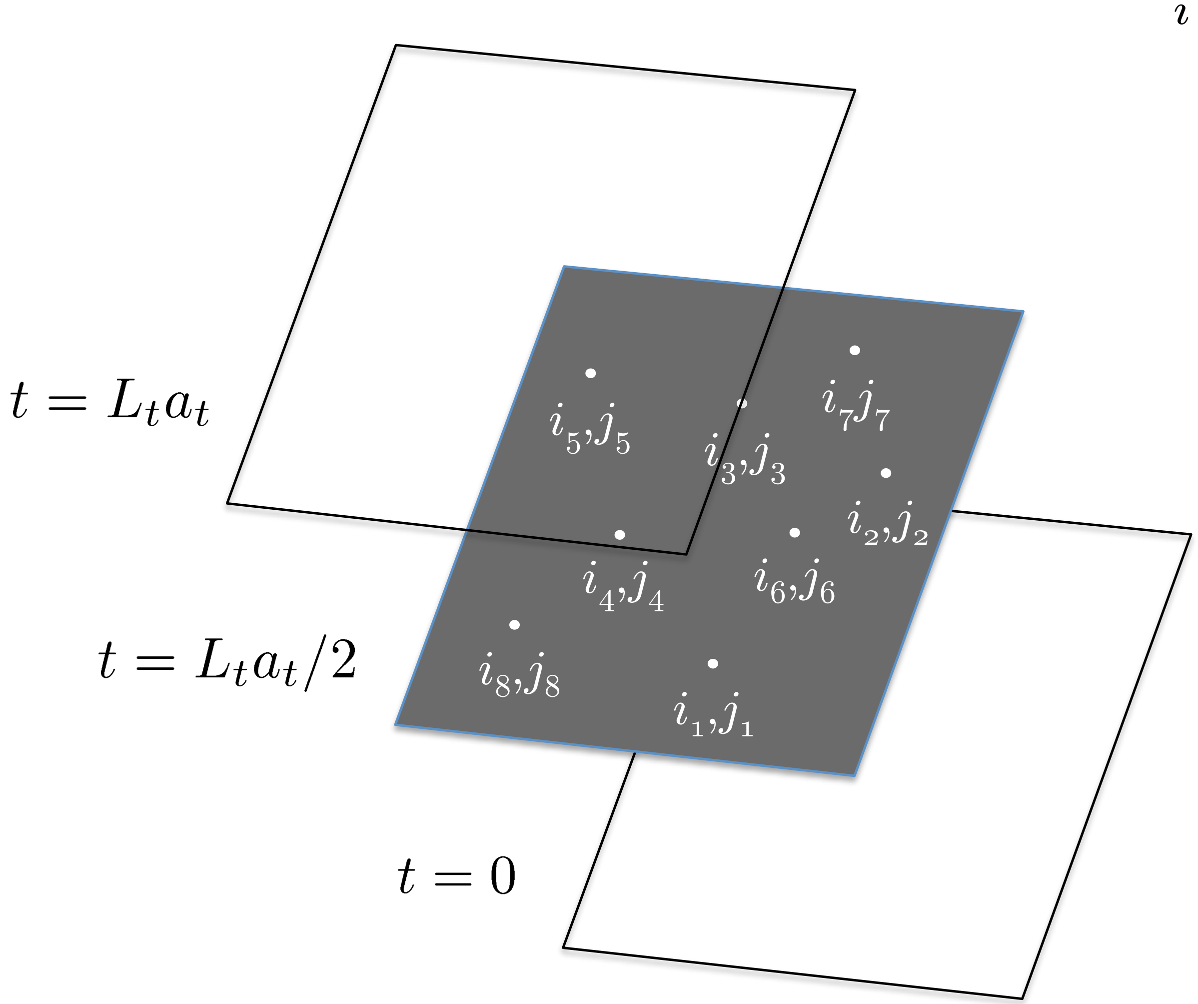
Calculates charge distribution in CM coordinates
Importance sampling according to magnitude for

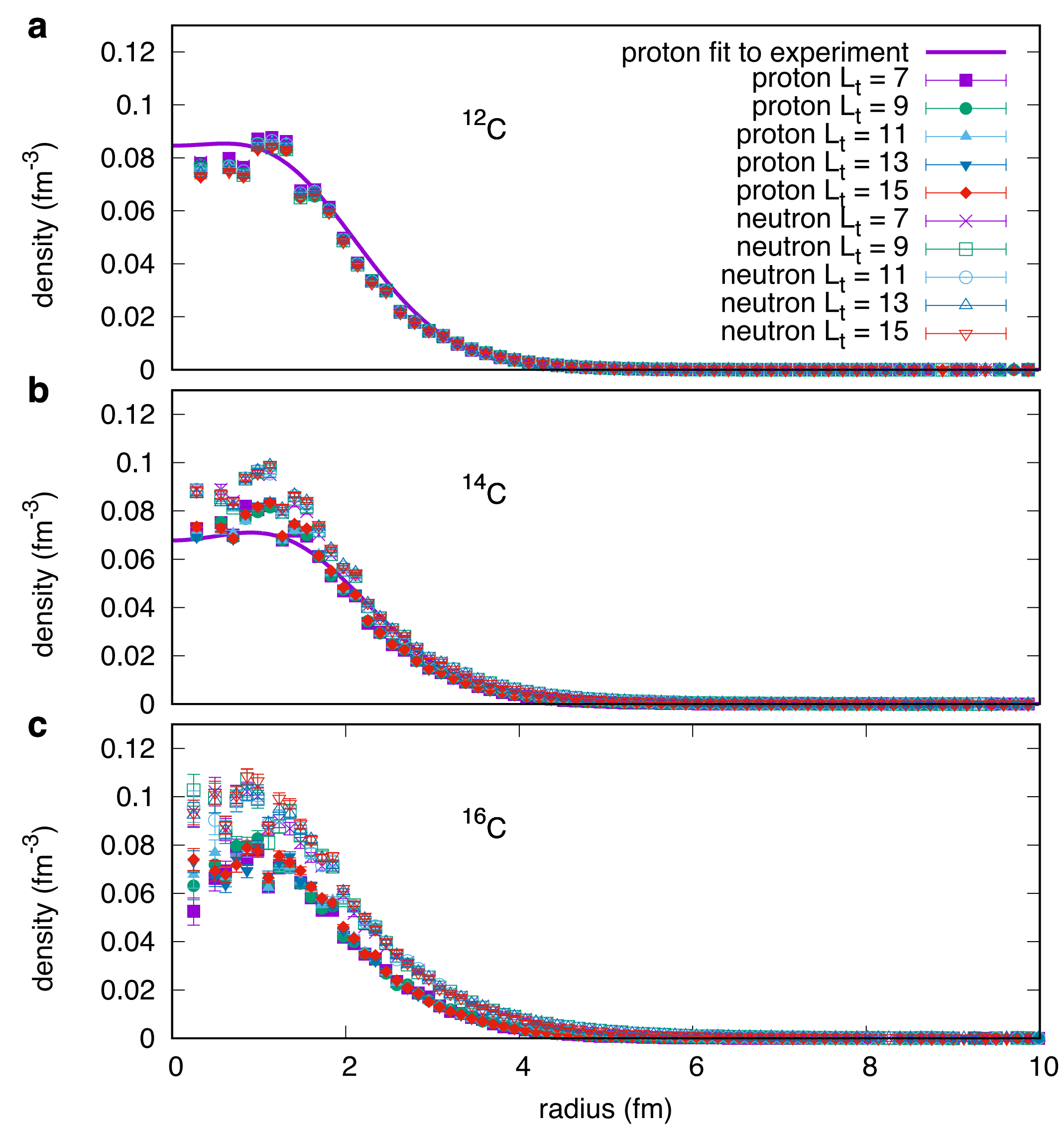
$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A) =: \rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

$$\langle\Psi_f|M_*^{L_t'}M^{L_t/2}\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A)M^{L_t/2}M_*^{L_t'}|\Psi_i\rangle$$

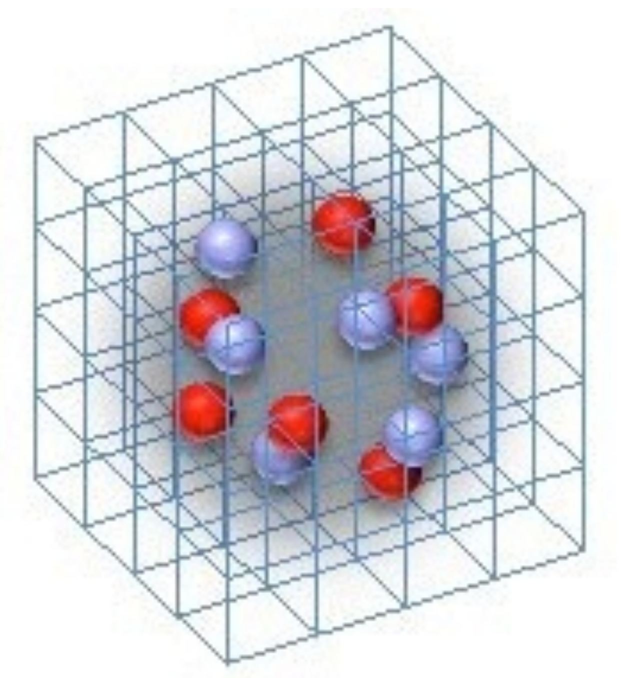
CM location minimizes the rms radius $\sum_i |\vec{\boldsymbol{R}}_{\mathrm{CM}} - \vec{\boldsymbol{r}}_i|^2$

Elhatisari et al.;
PRL 119, 222505 (2017)





Nuclear Lattice EFT Collaboration



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Summary

- Halo/cluster EFT for nuclear astrophysics
- Adiabatic Projection Method for two cluster Hamiltonian
- Reactions with or without Coulomb in lattice EFT
- Pinhole algorithm
  - Magnetic moments
  - $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, etc.