INPP @OHIO UNIVERSITY
INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Nuclear reaction in a three body model

exact solution in P-space and exploring the story in Q-space

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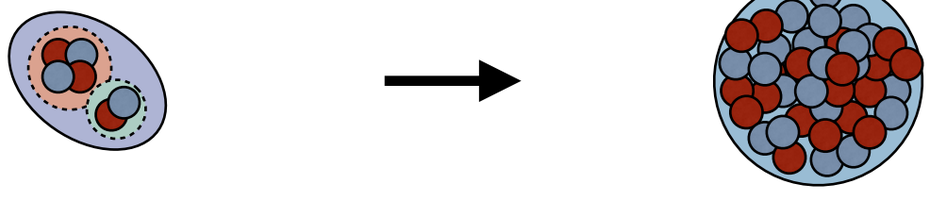
NSF

How to solve nuclear reaction?

A story between projectile and target in open quantum system

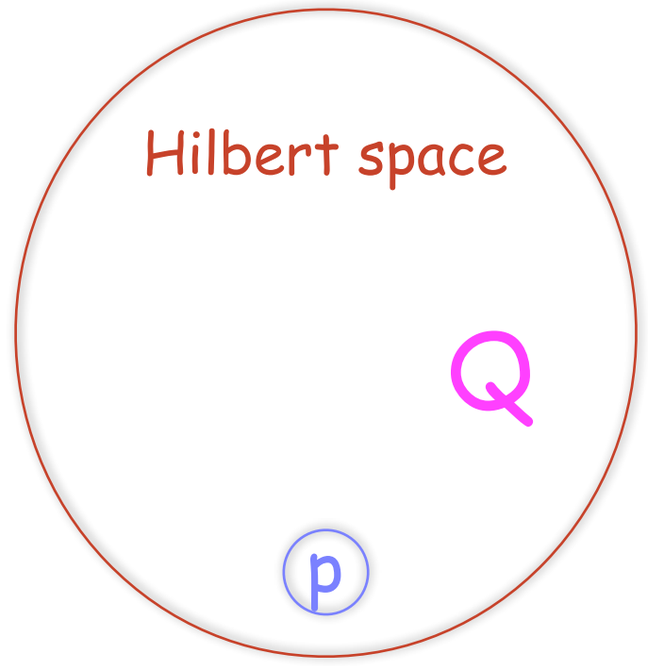


Difficult to solve due to the many body degree of freedom

Project into model space with $\mathcal{P}, \quad \mathcal{Q} = 1 - \mathcal{P}$

Solve $(E - H)\mathcal{P}\Psi = 0$ with

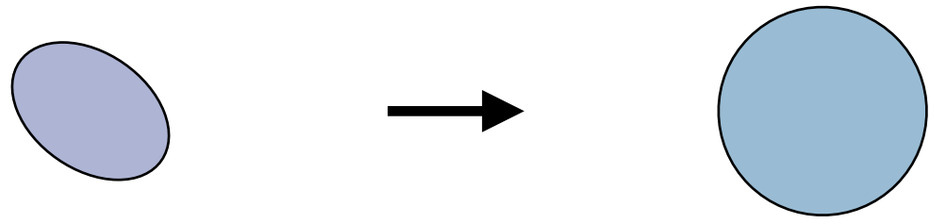
$$H = \mathcal{H}_{\mathcal{P}\mathcal{P}} + \mathcal{H}_{\mathcal{P}\mathcal{Q}} \frac{1}{E - \mathcal{H}_{\mathcal{Q}\mathcal{Q}}} \mathcal{H}_{\mathcal{Q}\mathcal{P}}$$



H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958); 19, 287(1962).

The simplest choice: two body ground state

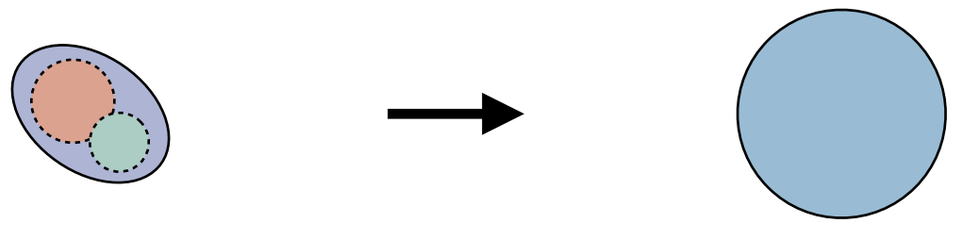
$$\mathcal{P} = |\phi_p^0 \phi_t^0\rangle\langle\phi_p^0 \phi_t^0|$$



Details in Elastic Scattering, Overall in reaction cross section

More detailed choice: three body ground state

$$\mathcal{P} = |\phi_c^0 \phi_v^0 \phi_t^0\rangle\langle\phi_c^0 \phi_v^0 \phi_t^0|$$



Details in Elastic Scattering, transfer, and breakup; Overall in reaction cross section

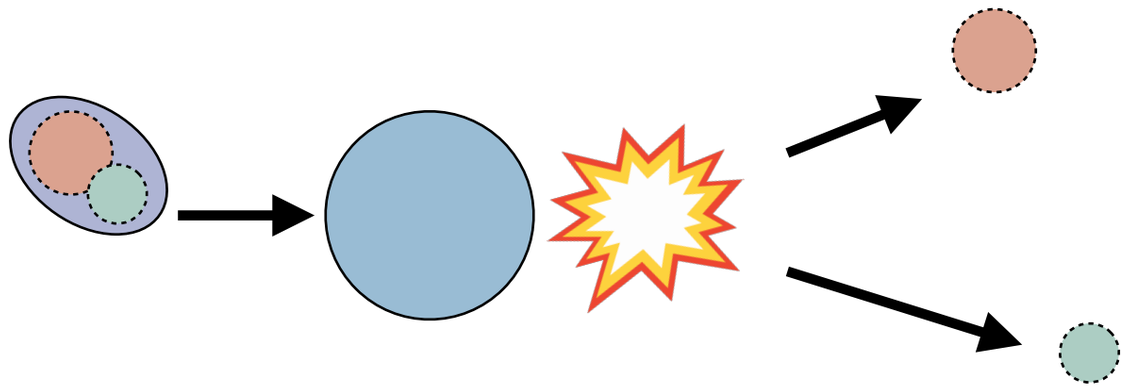
Outline

- Introduction

- Exact solution in P-space (three body model)
  - Faddeev equation in momentum space
  - Off-shell effects in three body system

- Exploring the story in Q-space
  - Nonelastic breakup: inclusive (d,p) reaction
  - continuum effects?

- Summary and perspective



Exact Solution in P-space

- Faddeev equation in momentum space: Linda's talk
- Off-shell effects in three body system

Solving Effective Three-Body Problem (I)

Solving the three body Schrodinger equation: two-body projectile with inert cores on inert target(take d+A system as an example)

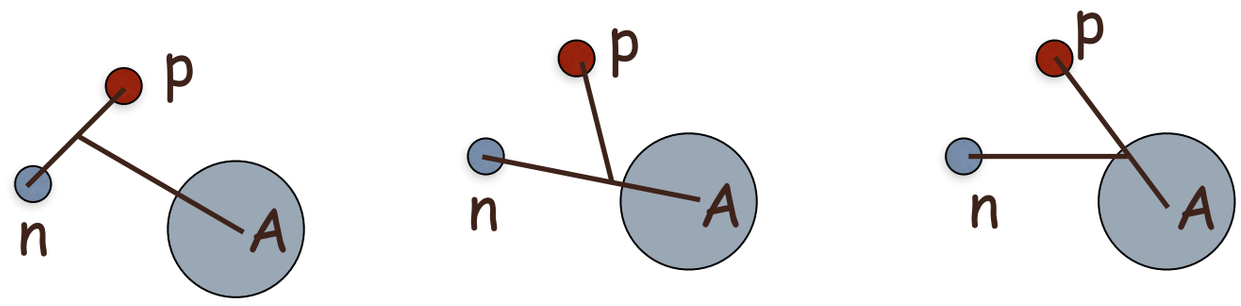
Effective (optical) potentials

$$E|\Psi\rangle = H|\Psi\rangle \qquad H = H_0 + V_{np} + U_{nA} + U_{pA}$$

V_{np} → np interaction

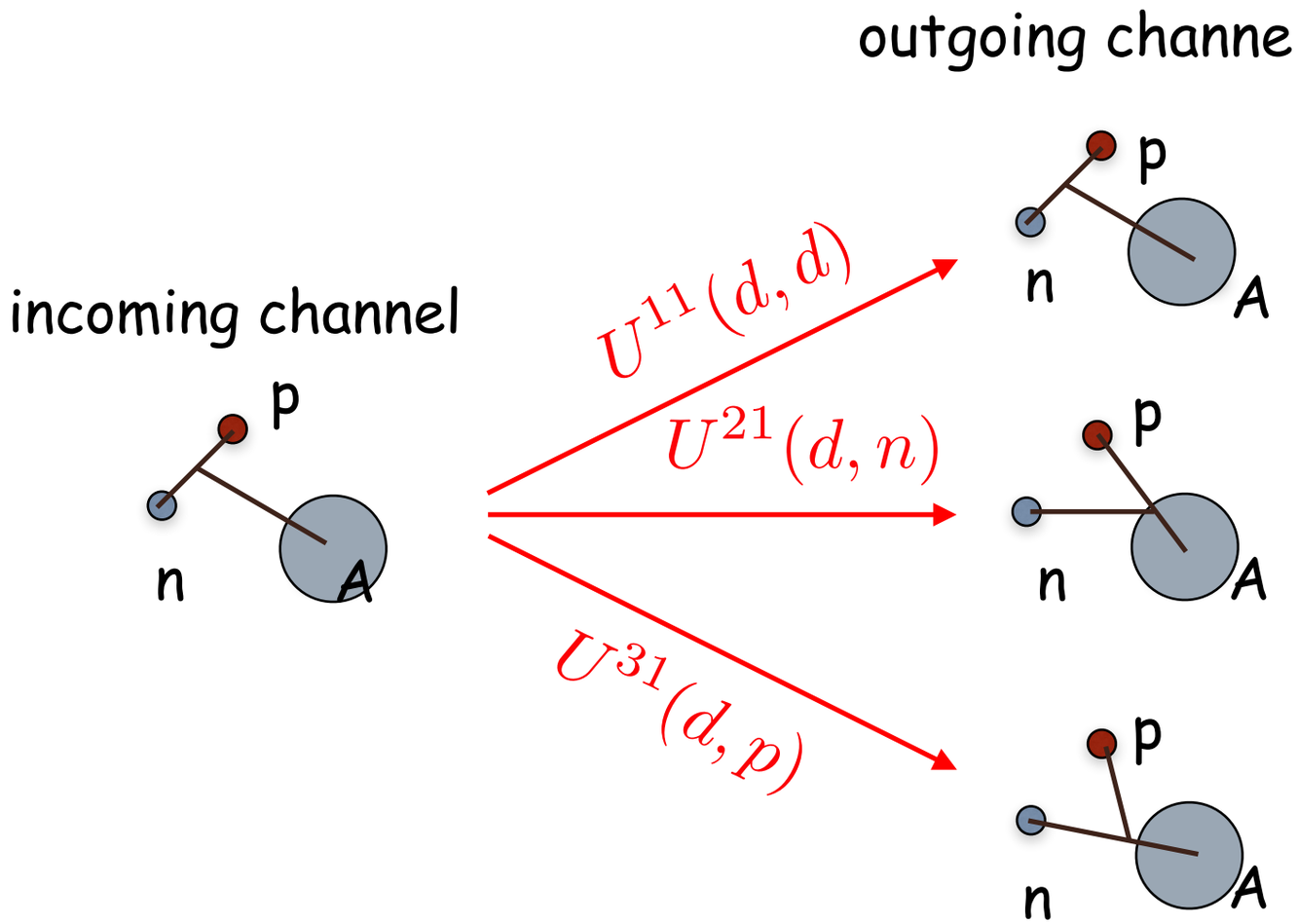
Faddeev equation: expand the three body wave function in three Jacobi system

$$|\Psi\rangle = |\psi_{np}\rangle + |\psi_{nA}\rangle + |\psi_{pA}\rangle$$



Each Jacobi coordinate specify particular boundary condition (elastic scattering/ transfer)

Solving Effective Three-Body Problem (II)



Observables:

$$\sigma_{j\leftarrow i} \propto \left|\langle \Psi_j | U^{ij} | \Psi_i \rangle\right|^2$$

Faddeev-AGS equations: [Alt et al., Nucl.Phys. B2 (1967) 167]

$$U^{ij} = \bar{\delta} G_0^{-1}(E) + \sum_k \bar{\delta}_{ik} t_k(E) G_0(E) U^{kj}$$

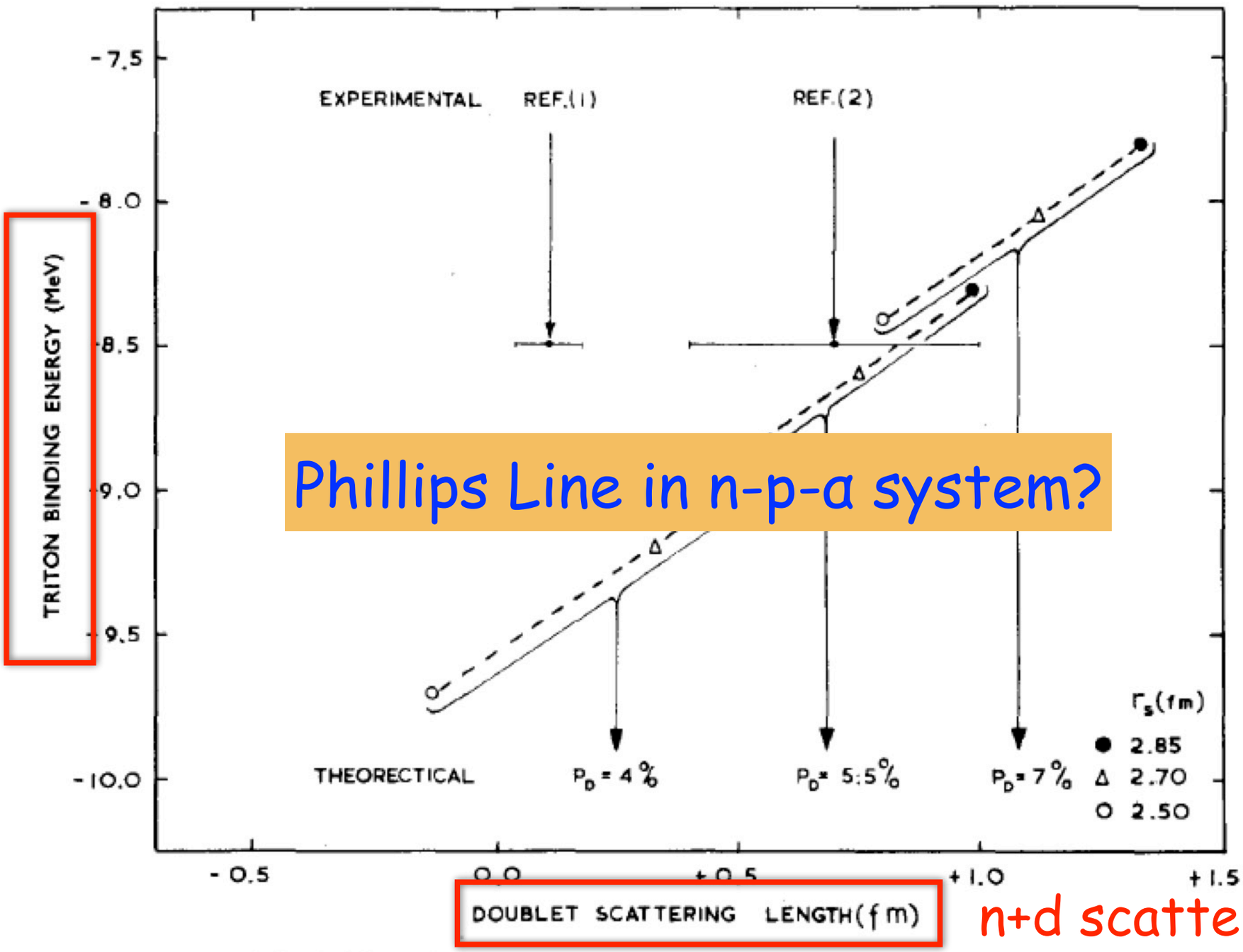
Applications to ^{6}Li bound state and d+a reactions

- Solve as effective three body problem: n+p+a
- For bound state
  - ~various type interaction for n-p and n(p)-a
  - ~solve with separable potentials (new codes)
  - ~benchmark achieve 4 significant figures
- For scattering
  - ~solve with separable potentials (new code)
  - ~consider the incident energy below and above the breakup threshold
  - ~benchmark with non-separable potentials

What is the connection ?

See details in Linda's talk

Phillips Line in n-n-p system



Phillips Line in n-p-α system?

n+d scattering

Fig. 1. The relation between ^2a and E_T for three values of r_s.

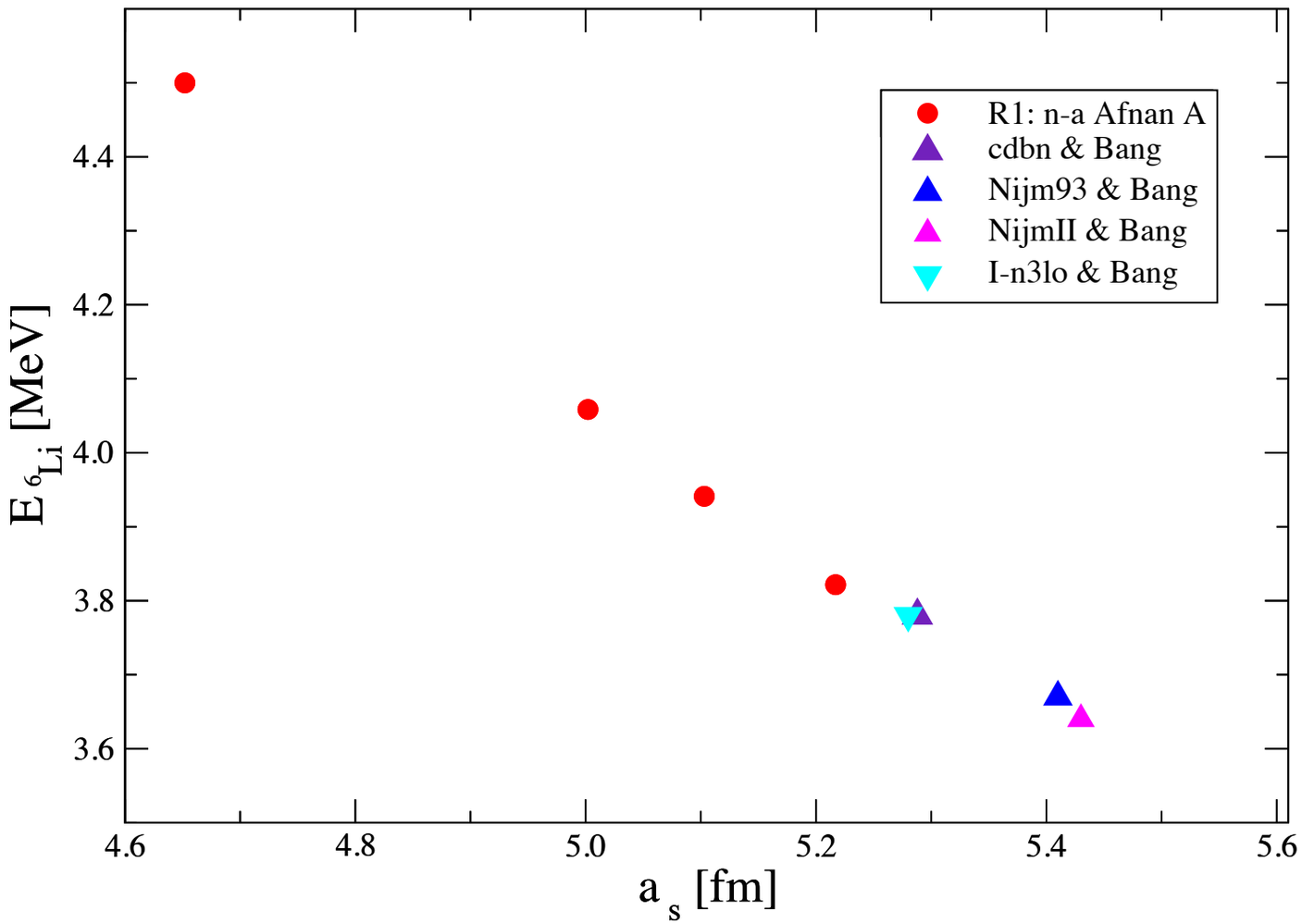
A.C.Phillips, Nucl. Phys. *A* **107** 209 (1968)

Off-shell effects in three body system

np interactions fit to the data with $\chi 2$/d.o.f.=1

(on-shell equivalent potentials)

Apply to three body calculation (fix N-a) interaction



A. Eskandarian and I. R. Afnan
Phys. Rev. C 46 2344 (1992)

effects of N-a interaction ??

Unitary transform N-α interaction

Start with Afnan N-α interaction

A. Eskandarian and I. R. Afnan
Phys. Rev. C 46 2344 (1992)

$$V^{n\alpha}_{ll'}(p,p') = g_l(p) C_{ll'} g_{l'}(p')$$

with

$$g_l(p) = \frac{p^l}{[p^2+\beta^2]^{l+1}}$$

perform unitary transform with

$$\tilde{H} = UHU^\dagger = K + \tilde{V}$$

where

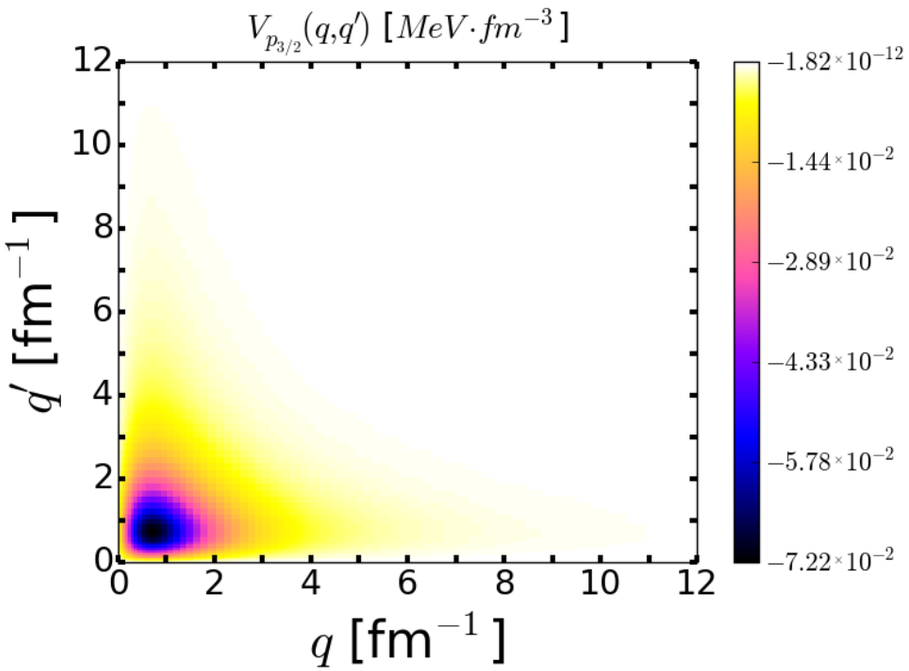
$$U = 1 - 2\,|h\rangle\langle h|$$

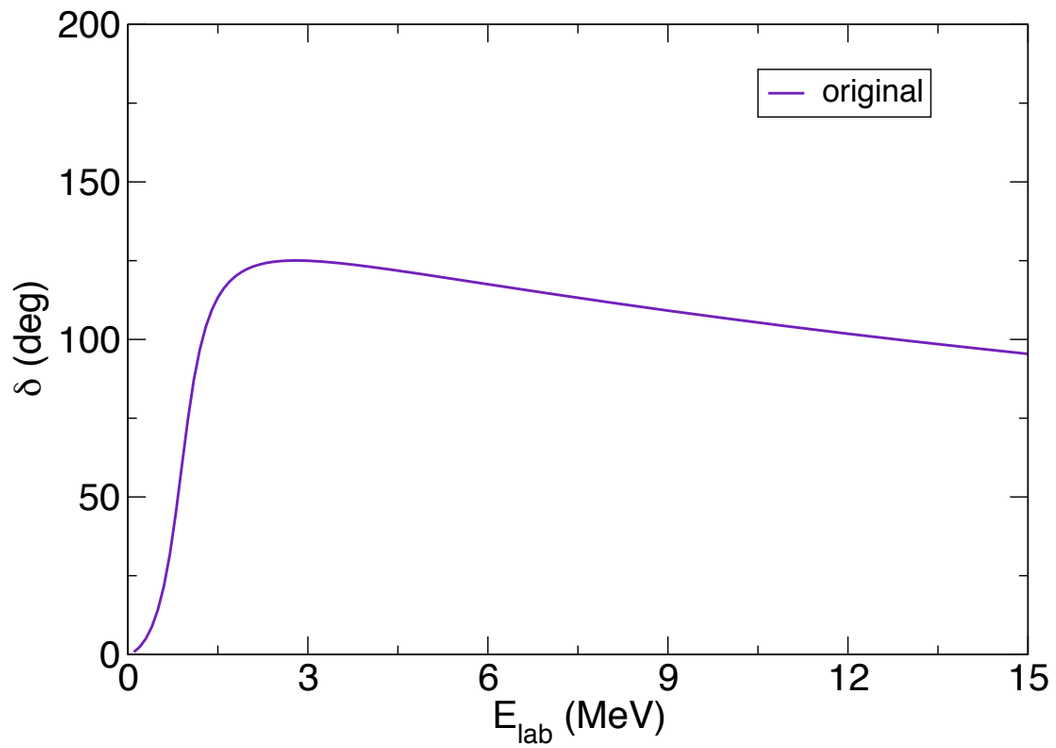
and

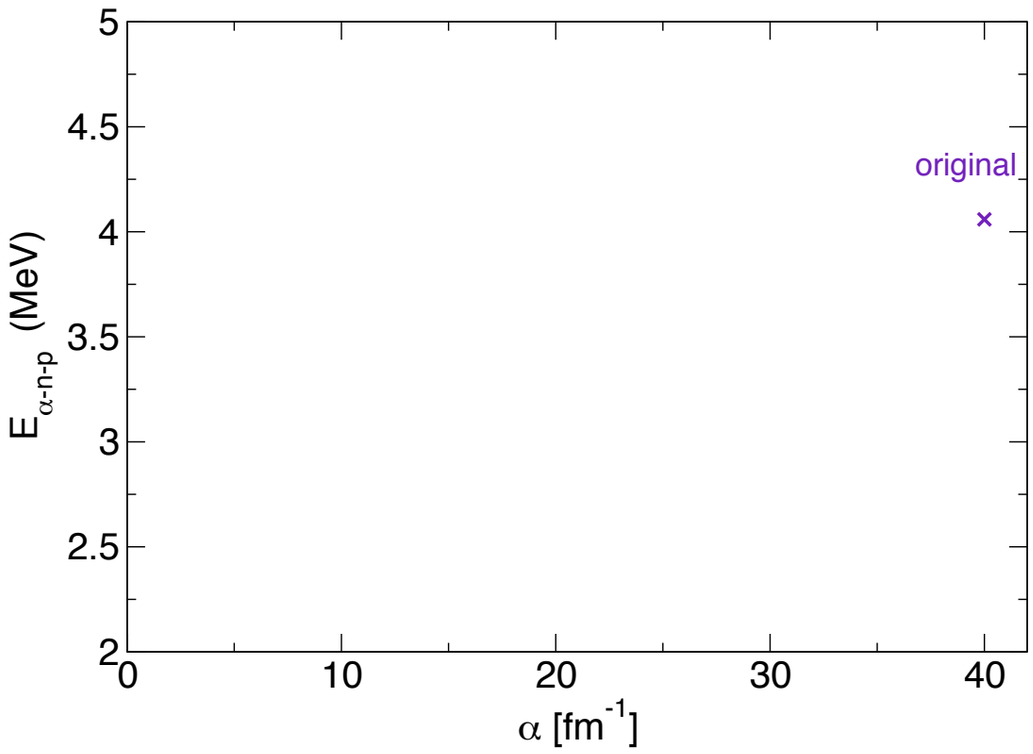
$$\langle r\,|\,h\rangle = Ne^{-\alpha r}(1-\xi r)$$

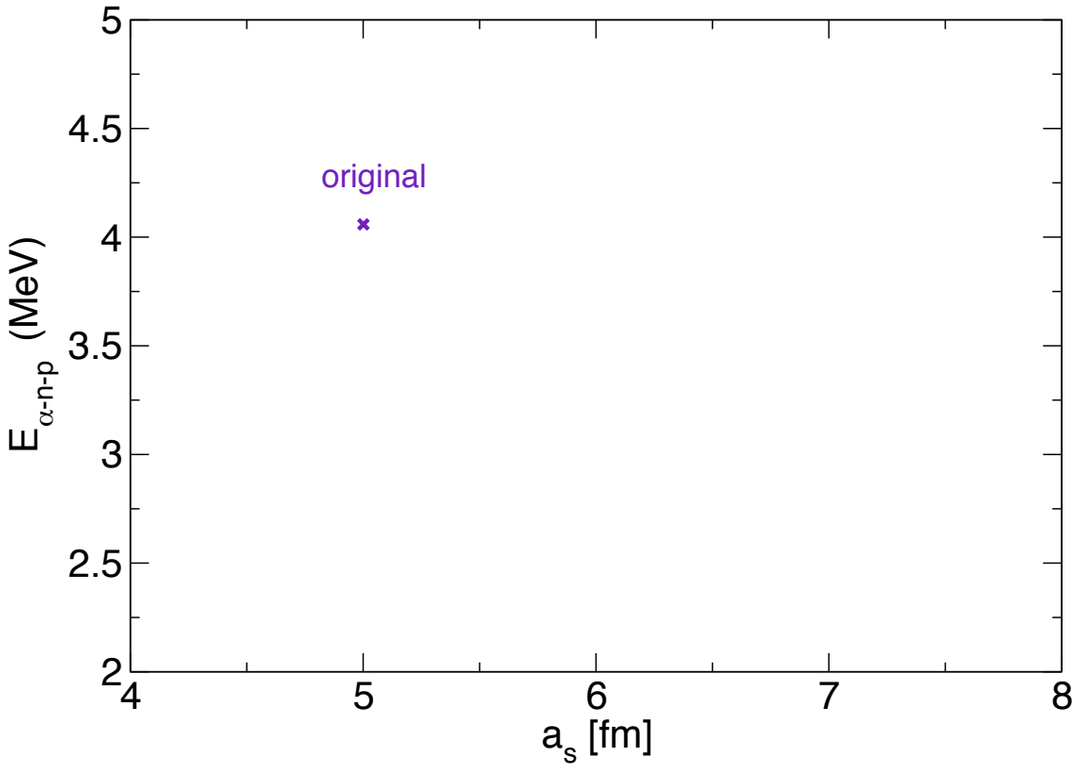
A. W. Thomas, I. R. Afnan
Phys. Lett, B 55 425 (1975)

Original Afnan N-α interaction

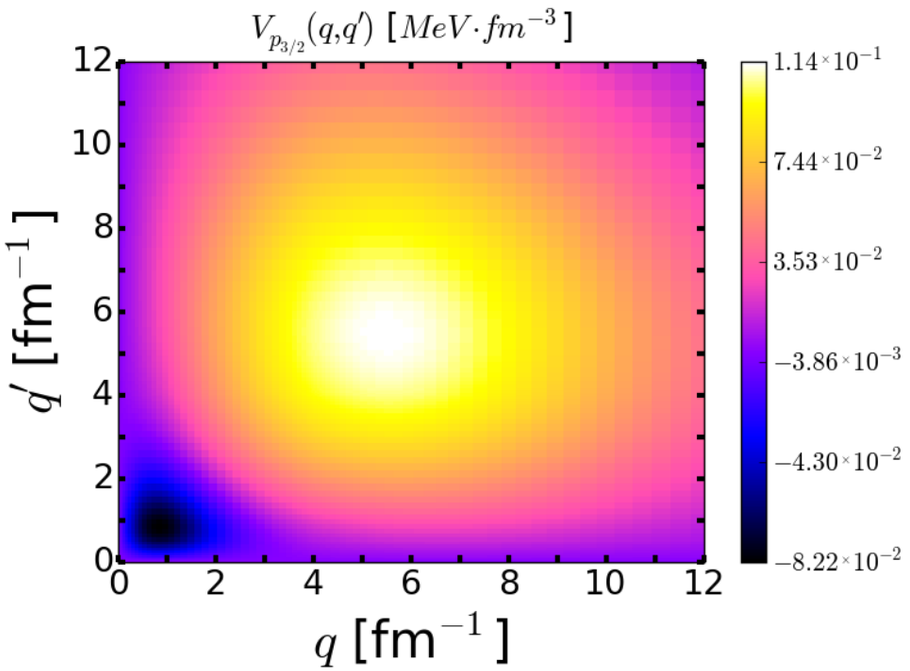


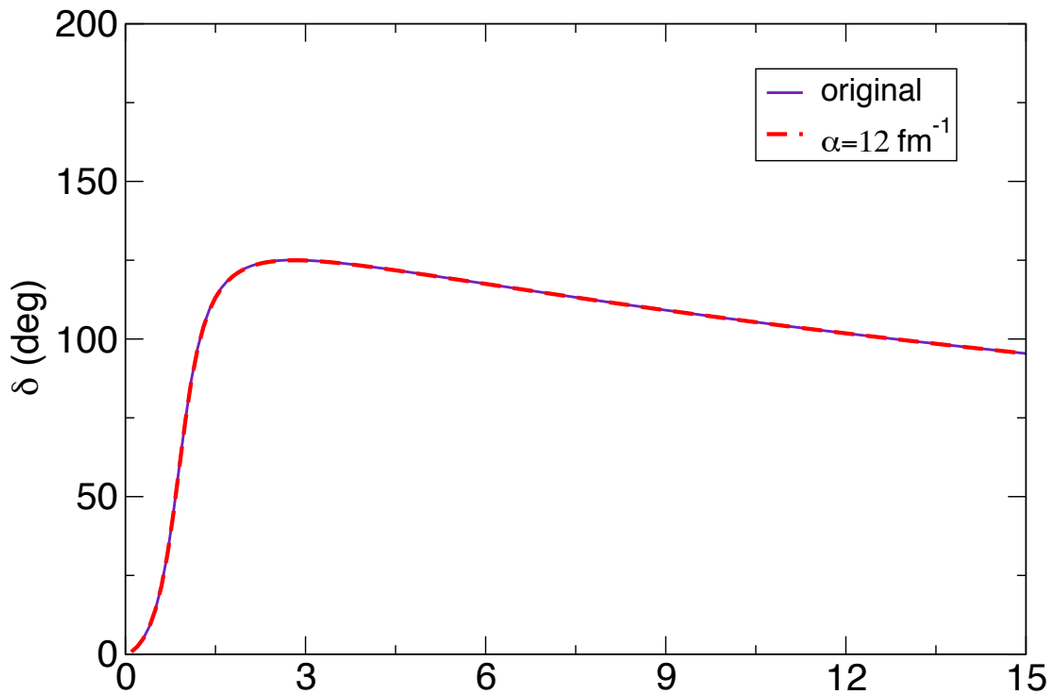


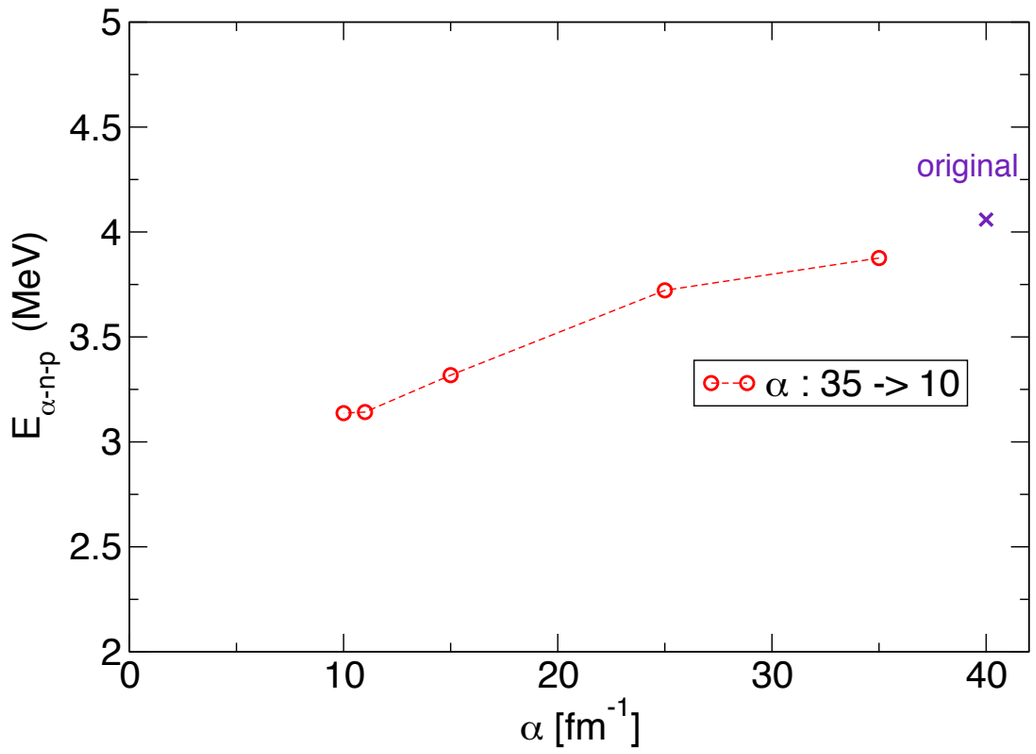


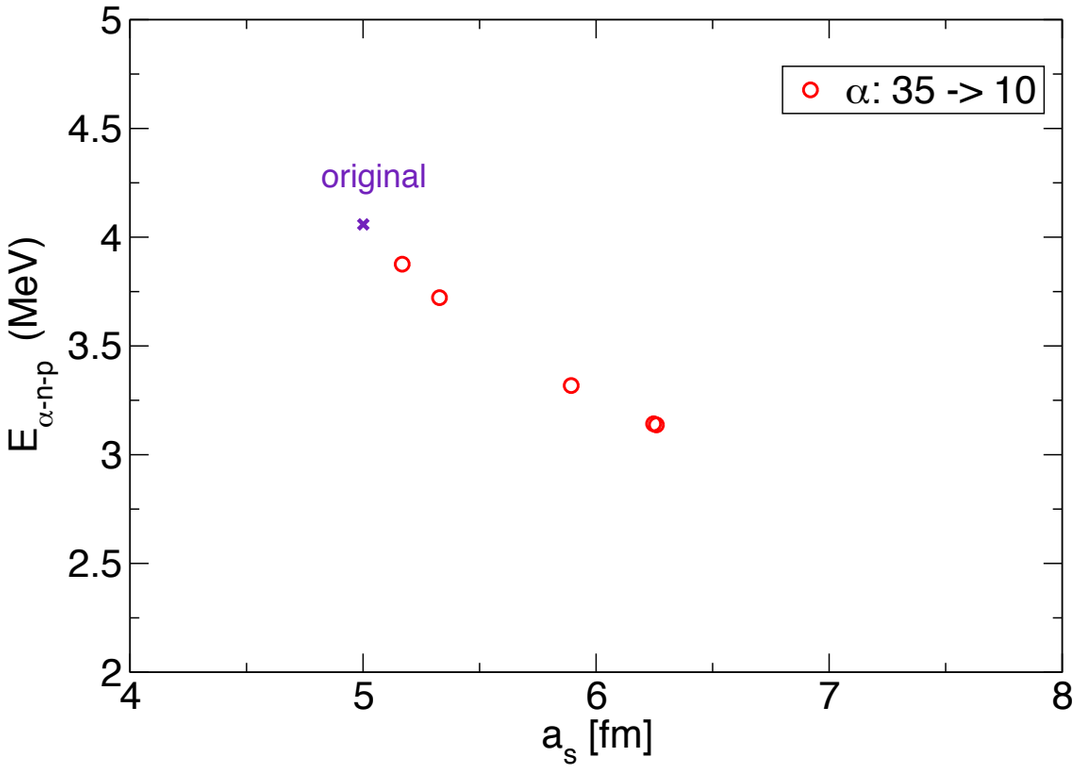


$\alpha = 12\ fm^{-1}$

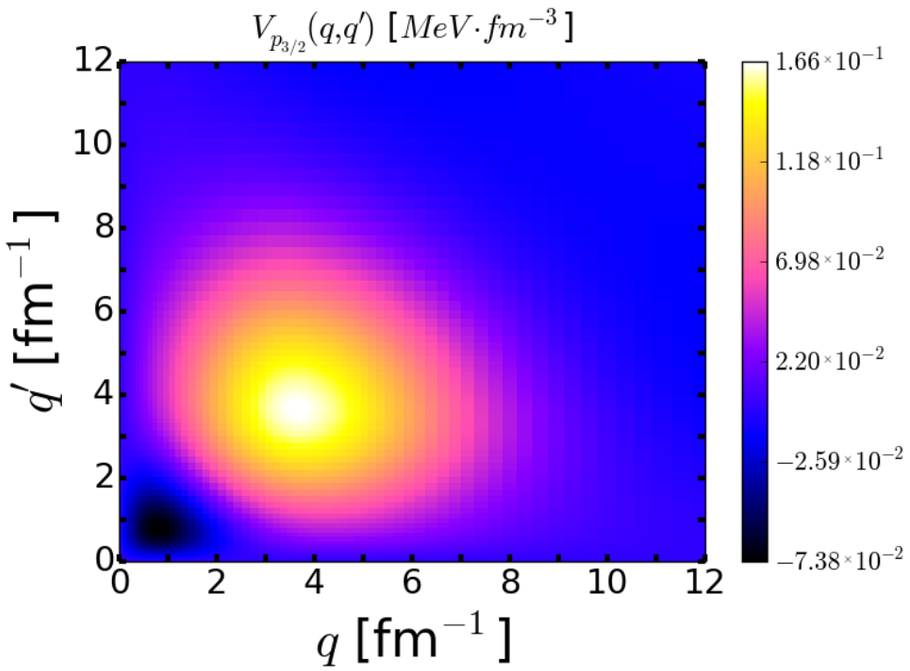

$V_{p_{3/2}}(q,q')$ [$MeV \cdot fm^{-3}$]
q' [fm^{-1}]
q [fm^{-1}]
1.14×10^{-1}
7.44×10^{-2}
3.53×10^{-2}
-3.86×10^{-3}
-4.30×10^{-2}
-8.22×10^{-2}

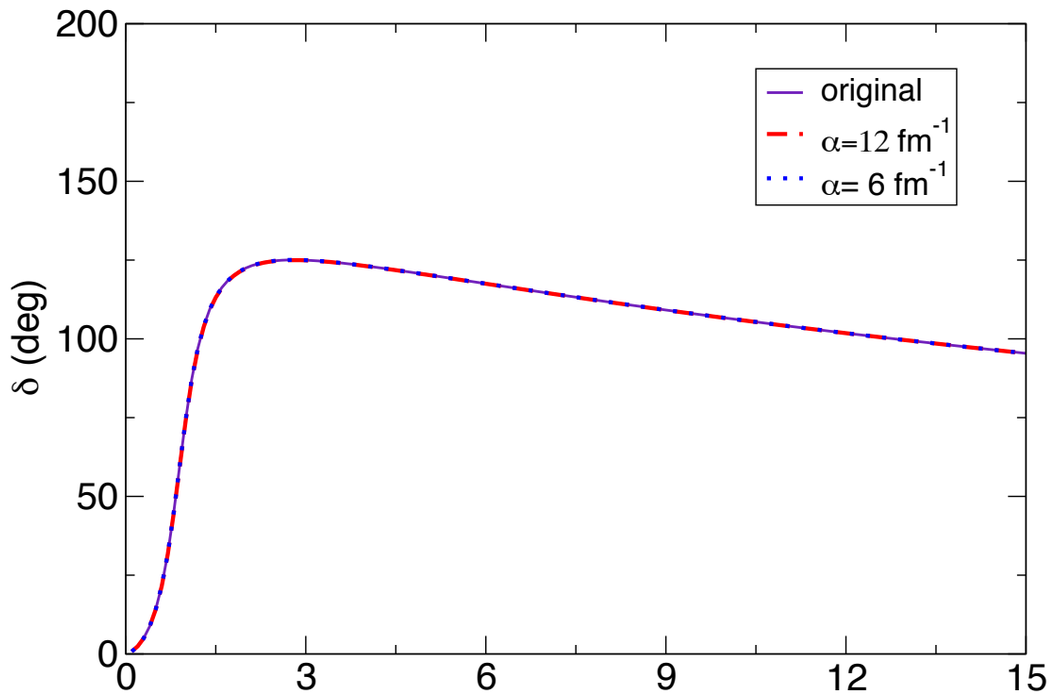

original
α=12 fm^{-1}
δ (deg)
E_{lab} (MeV)

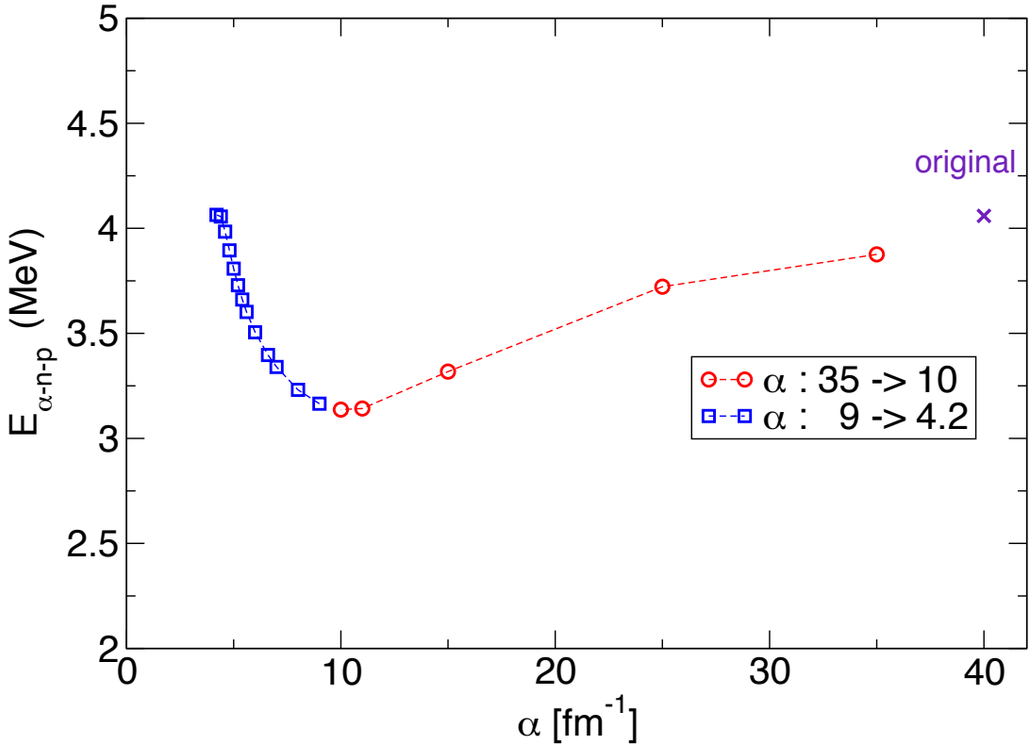

original
α : 35 -> 10
$E_{α\text{-}n\text{-}p}$ (MeV)
α [fm^{-1}]

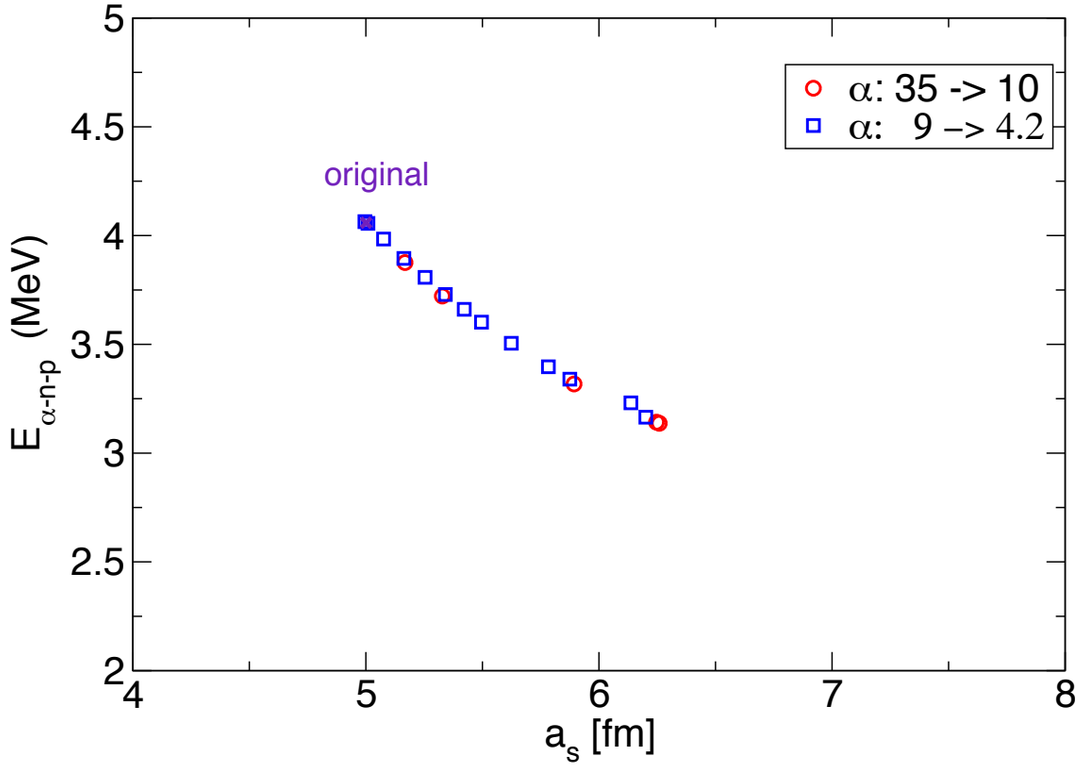

α: 35 -> 10
original
$E_{α\text{-}n\text{-}p}$ (MeV)
a_s [fm]

$\alpha = 6\ fm^{-1}$

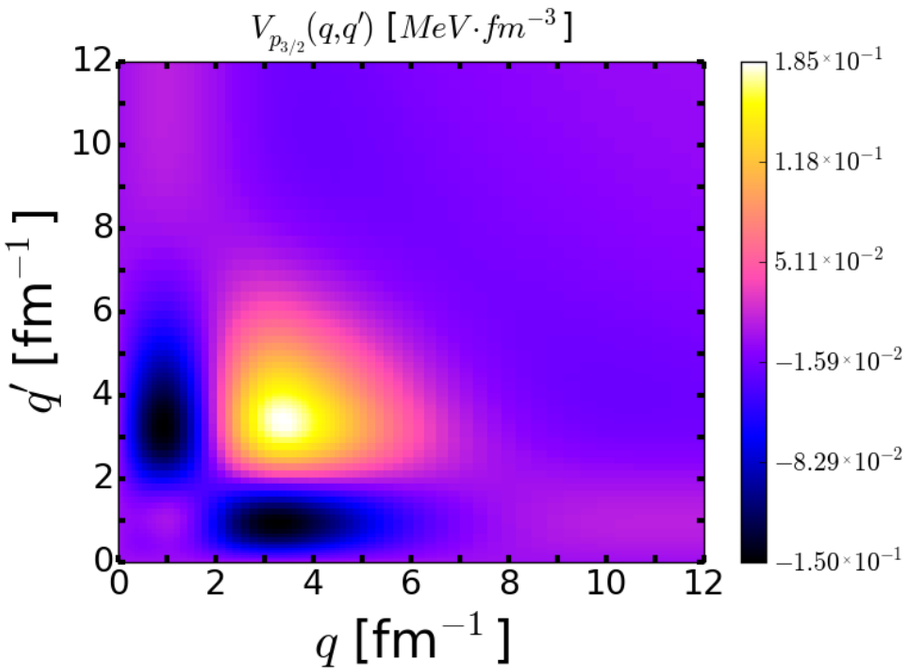

$V_{p_{3/2}}(q,q')$ [$MeV \cdot fm^{-3}$]
q' [fm^{-1}]
q [fm^{-1}]
0
2
4
6
8
10
12
1.66×10^{-1}
1.18×10^{-1}
6.98×10^{-2}
2.20×10^{-2}
-2.59×10^{-2}
-7.38×10^{-2}

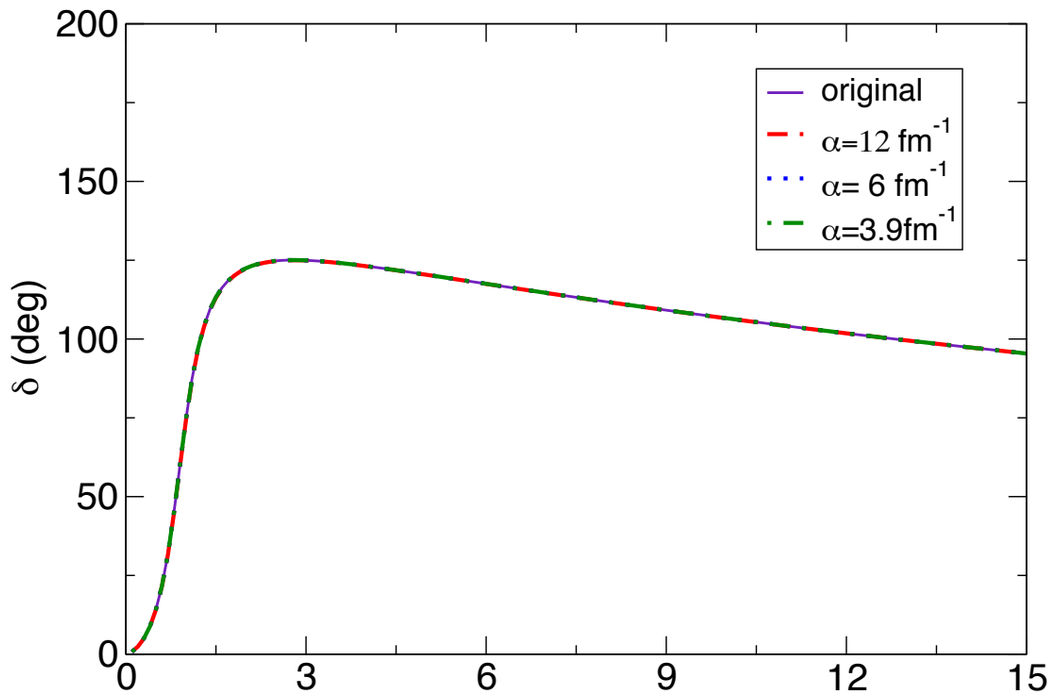

original
α=12 fm^{-1}
α= 6 fm^{-1}
δ (deg)
E_{lab} (MeV)
0
50
100
150
200
0
3
6
9
12
15

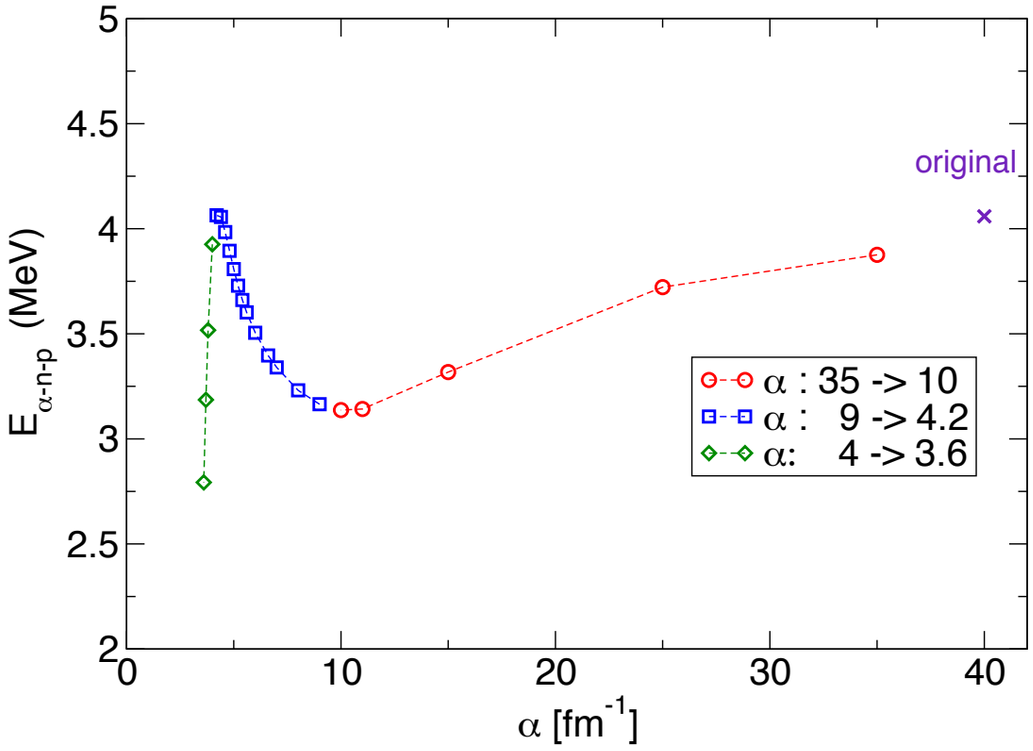

original
α : 35 -> 10
α : 9 -> 4.2
$E_{\alpha\text{-n-p}}$ (MeV)
α [fm^{-1}]
2
2.5
3
3.5
4
4.5
5
0
10
20
30
40

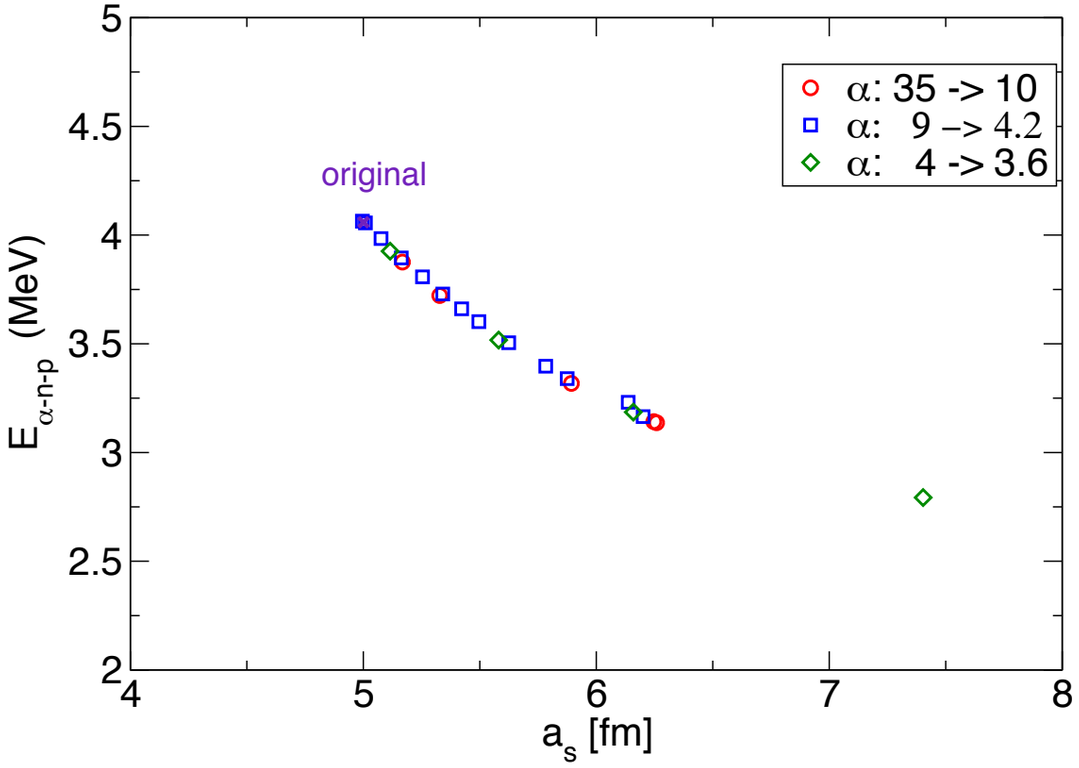

original
α: 35 -> 10
α: 9 –> 4.2
$E_{\alpha\text{-n-p}}$ (MeV)
a_s [fm]
2
2.5
3
3.5
4
4.5
5
4
5
6
7
8

$\alpha = 3.9\ fm^{-1}$

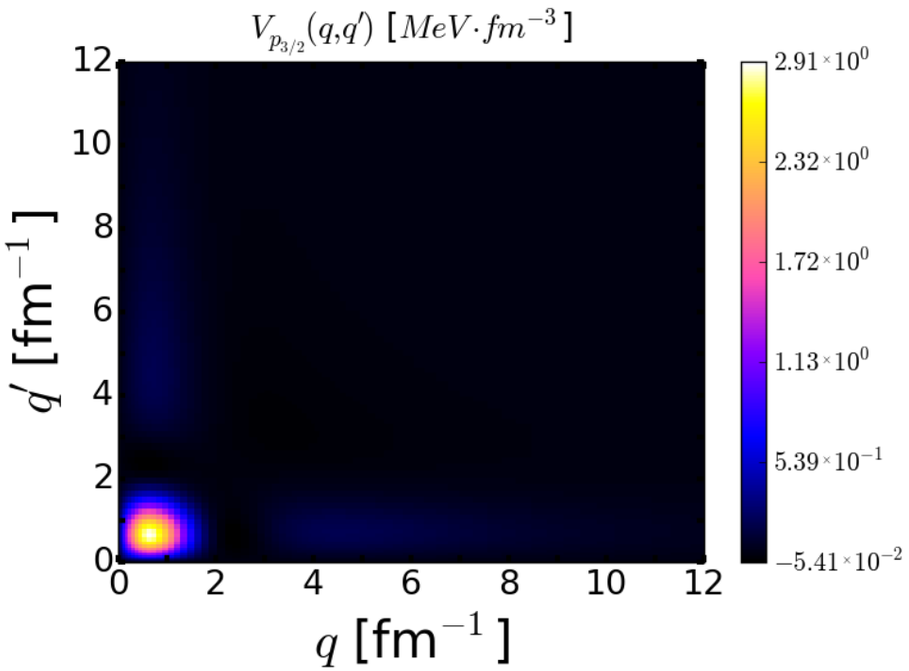

$V_{p_{3/2}}(q,q')$ [$MeV \cdot fm^{-3}$]
q' [fm^{-1}]
q [fm^{-1}]
1.85×10^{-1}
1.18×10^{-1}
5.11×10^{-2}
-1.59×10^{-2}
-8.29×10^{-2}
-1.50×10^{-1}

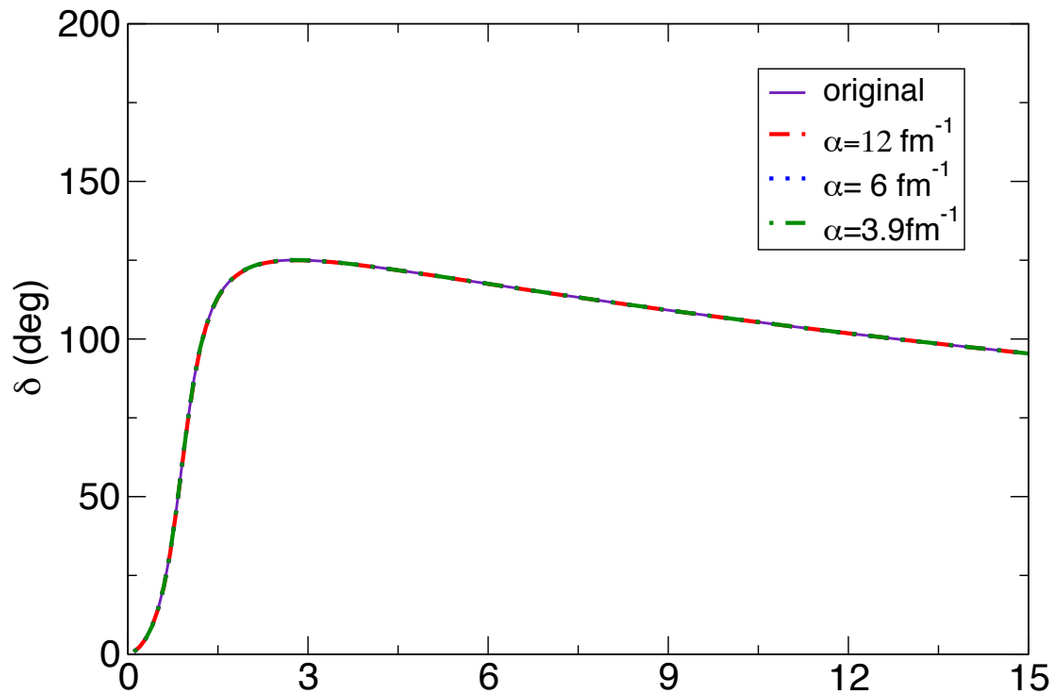

original
α=12 fm^{-1}
α= 6 fm^{-1}
α=3.9fm^{-1}
δ (deg)
E$_{lab}$ (MeV)

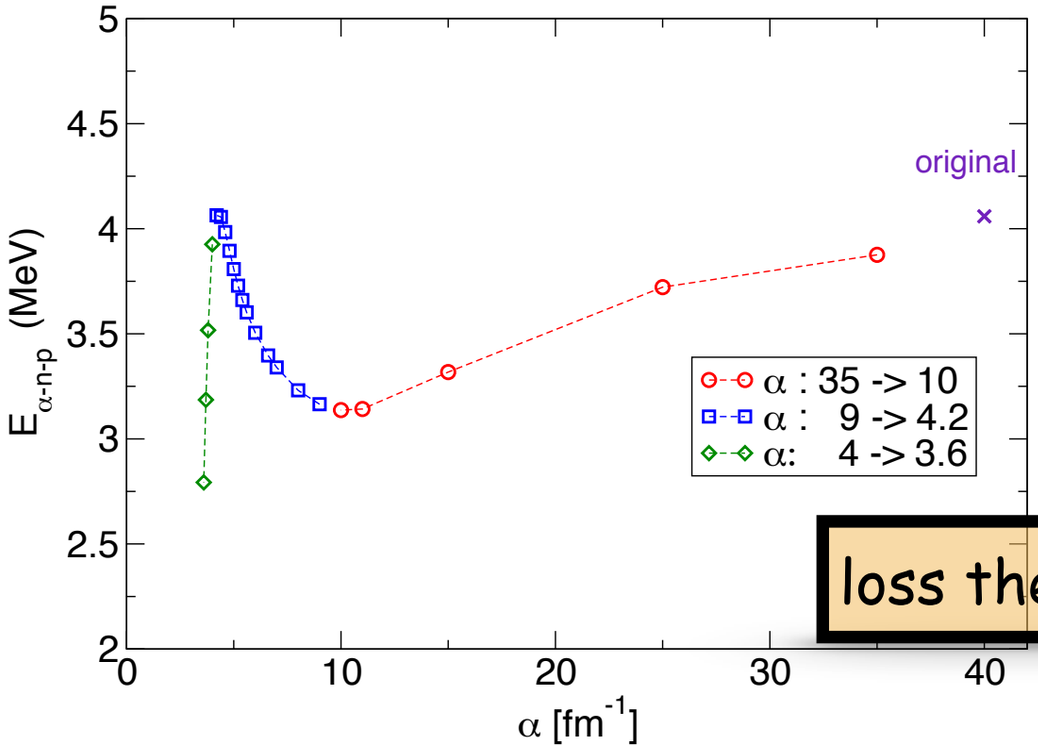

original
α : 35 -> 10
α : 9 -> 4.2
α: 4 -> 3.6
E$_{α\text{-}n\text{-}p}$ (MeV)
α [fm^{-1}]

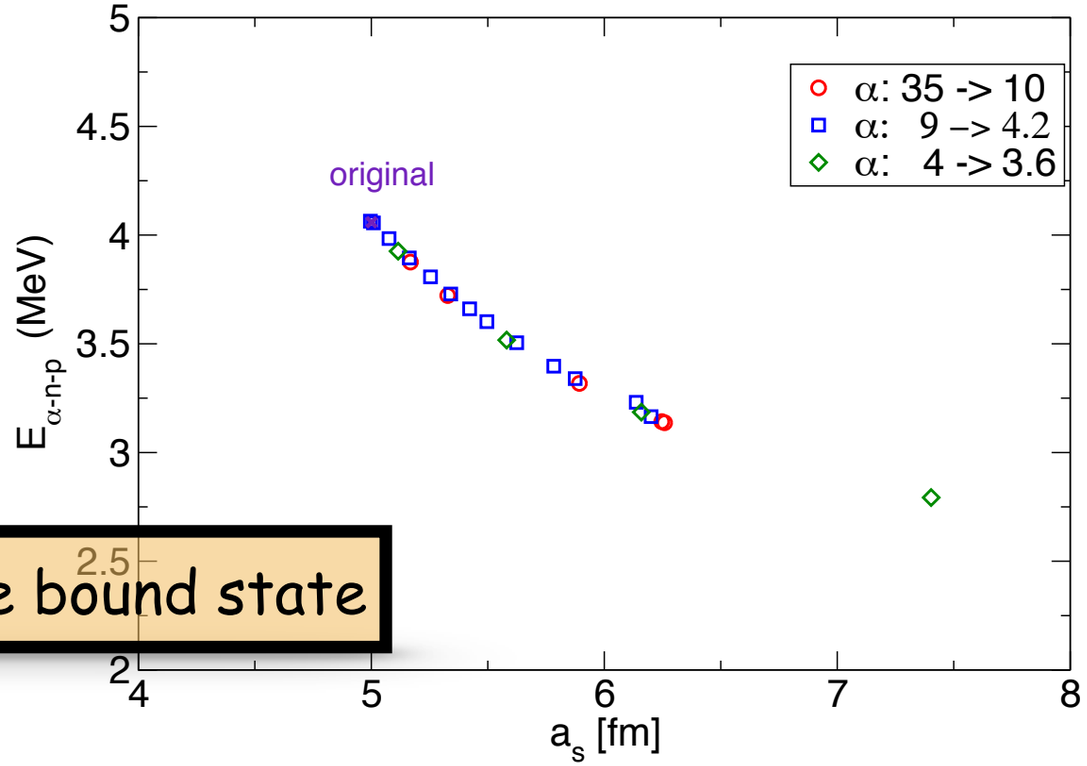

original
α: 35 -> 10
α: 9 –> 4.2
α: 4 -> 3.6
E$_{α\text{-}n\text{-}p}$ (MeV)
a$_s$ [fm]

$\alpha = 2\ fm^{-1}$



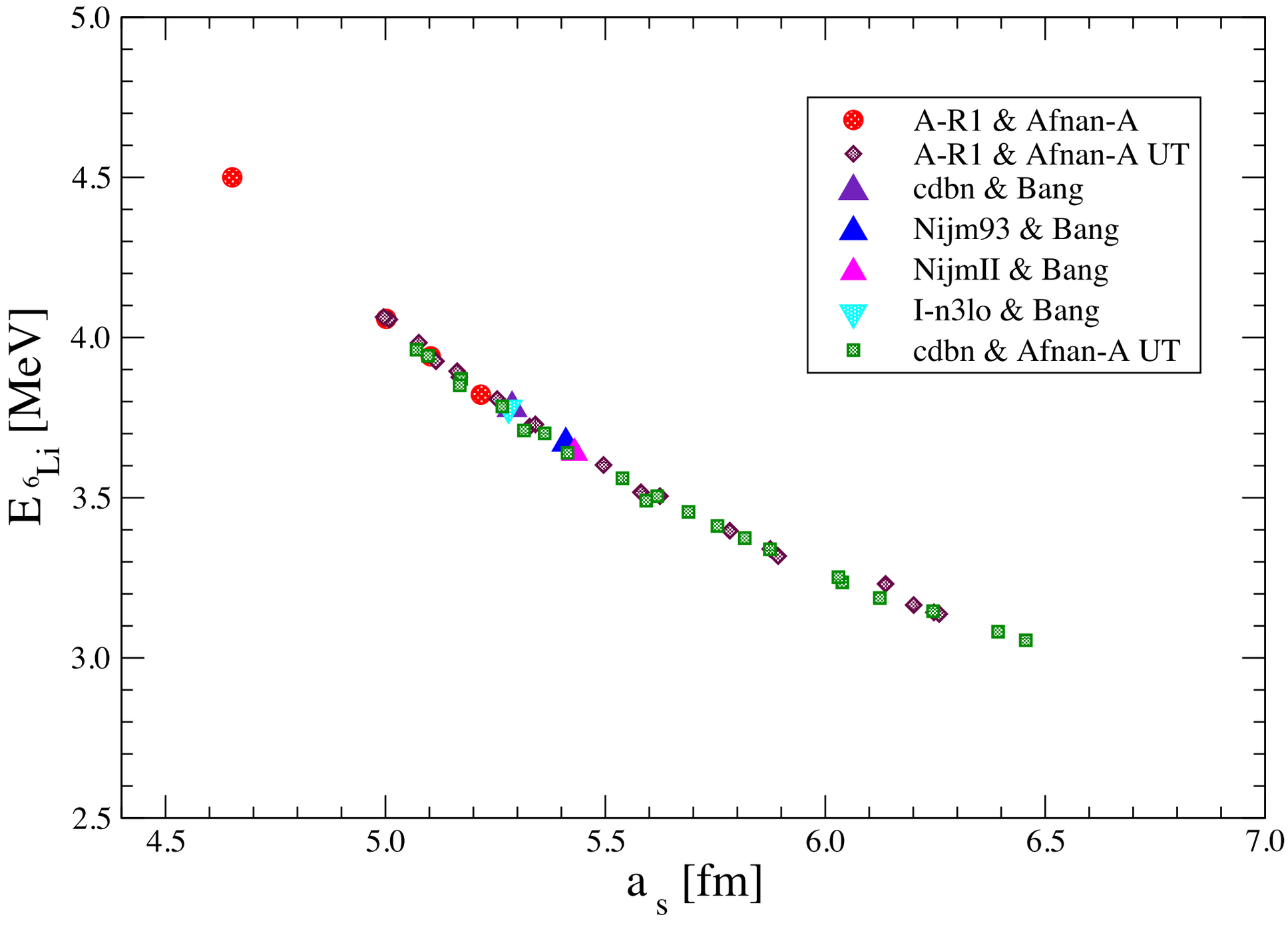




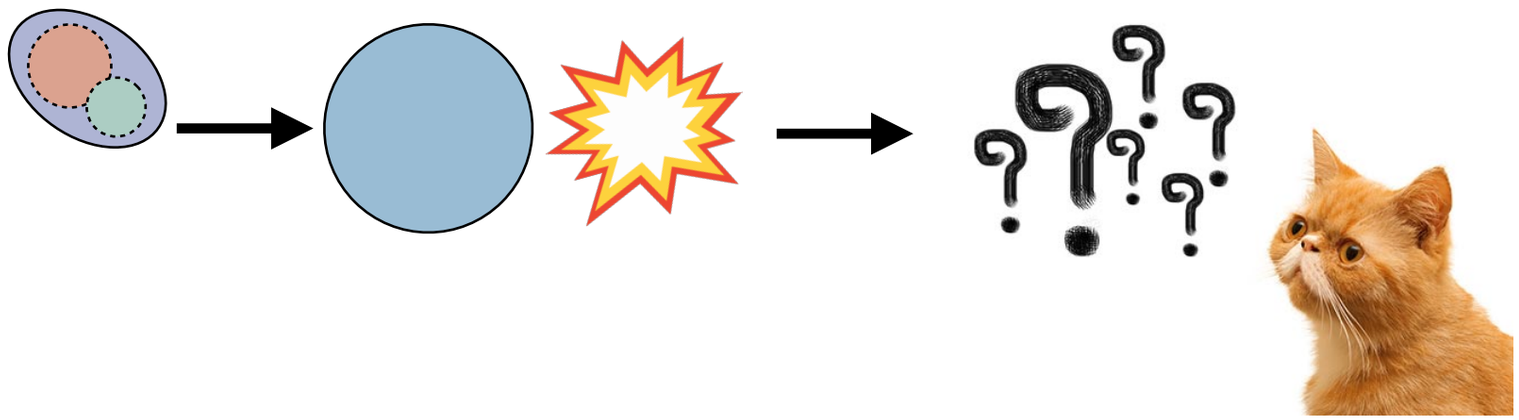


loss the bound state

Phillips Line in n-p-α system !!



All results in one plot!

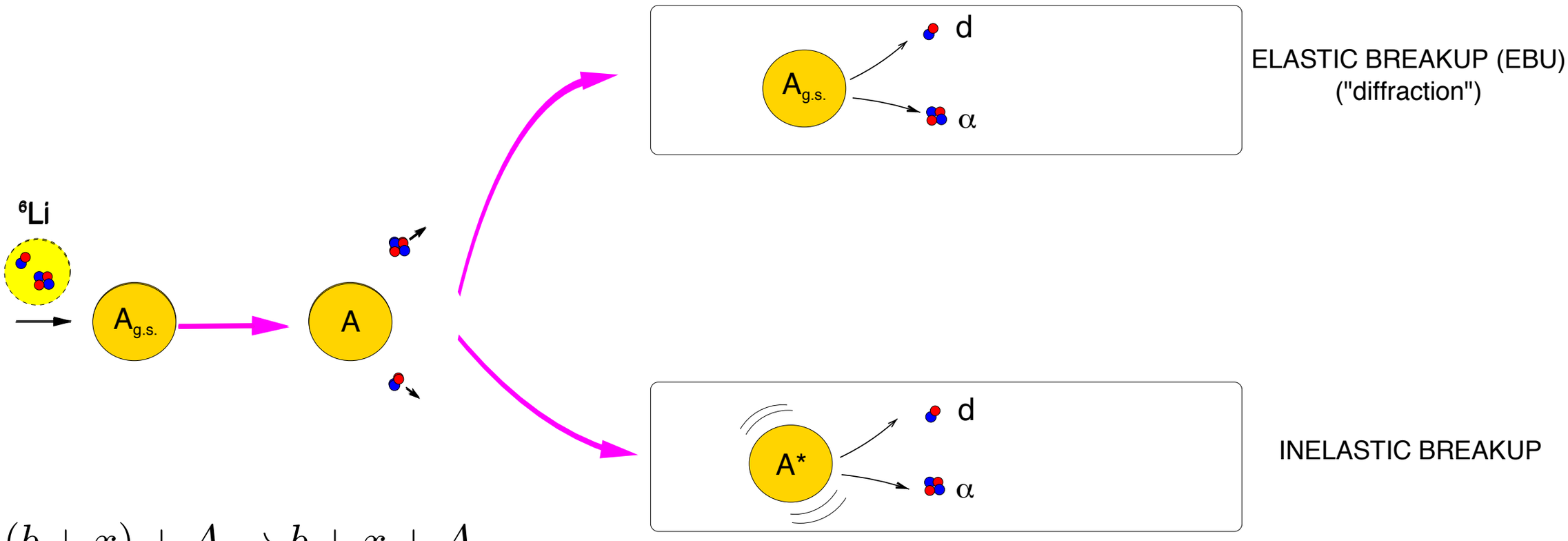


Exploring The Story in Q-space

- Nonelastic breakup: inclusive (d,p) reaction
- continuum effects?

From experimental point of view

Exclusive breakup

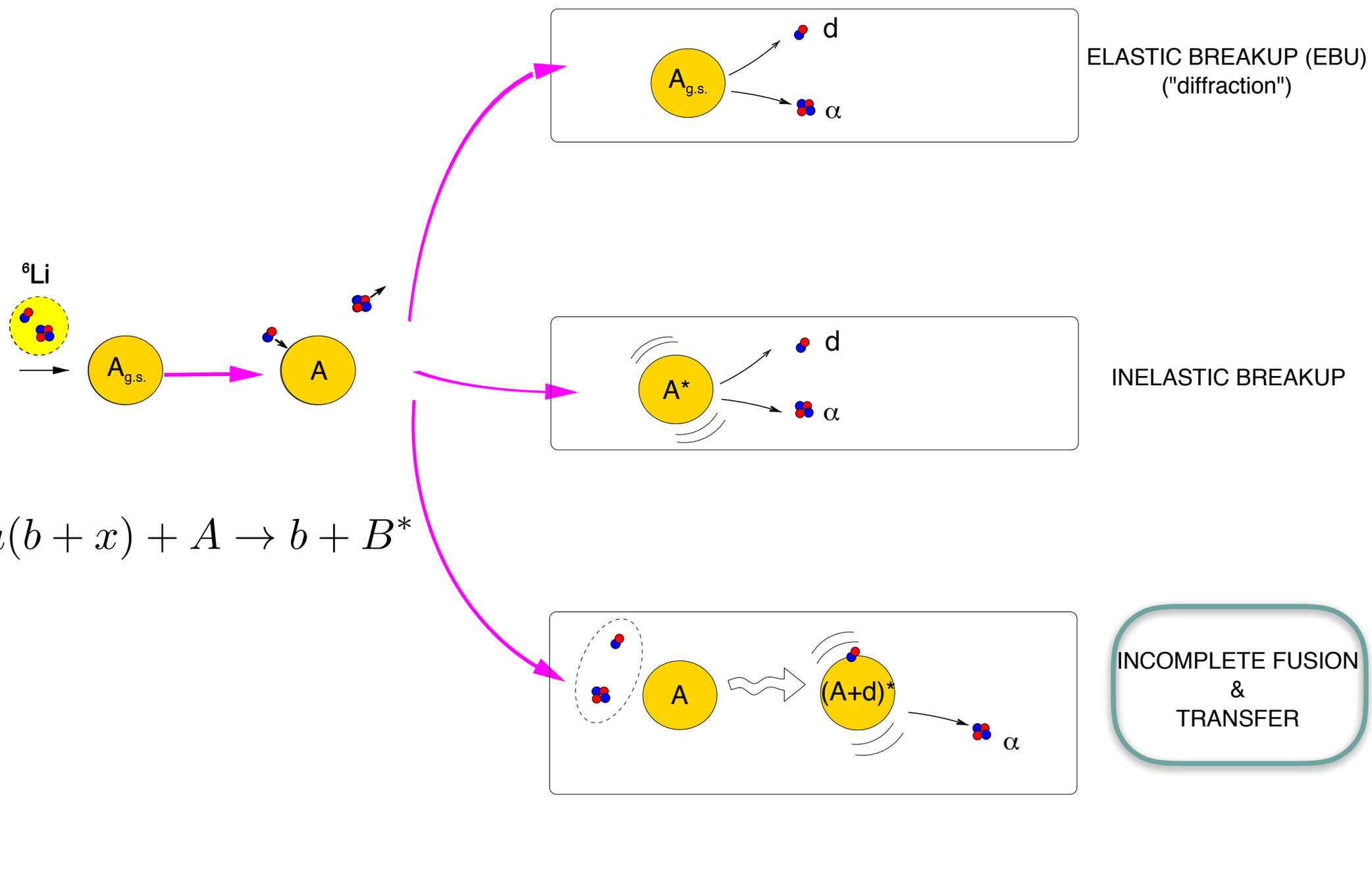


$$a(b+x)+A \rightarrow b+x+A$$

From experimental point of view

Inclusive breakup

CDCC/Faddeev 😀

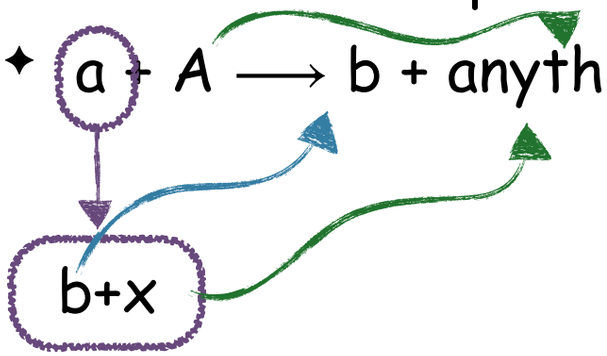


$$a(b+x)+A \rightarrow b+B^*$$

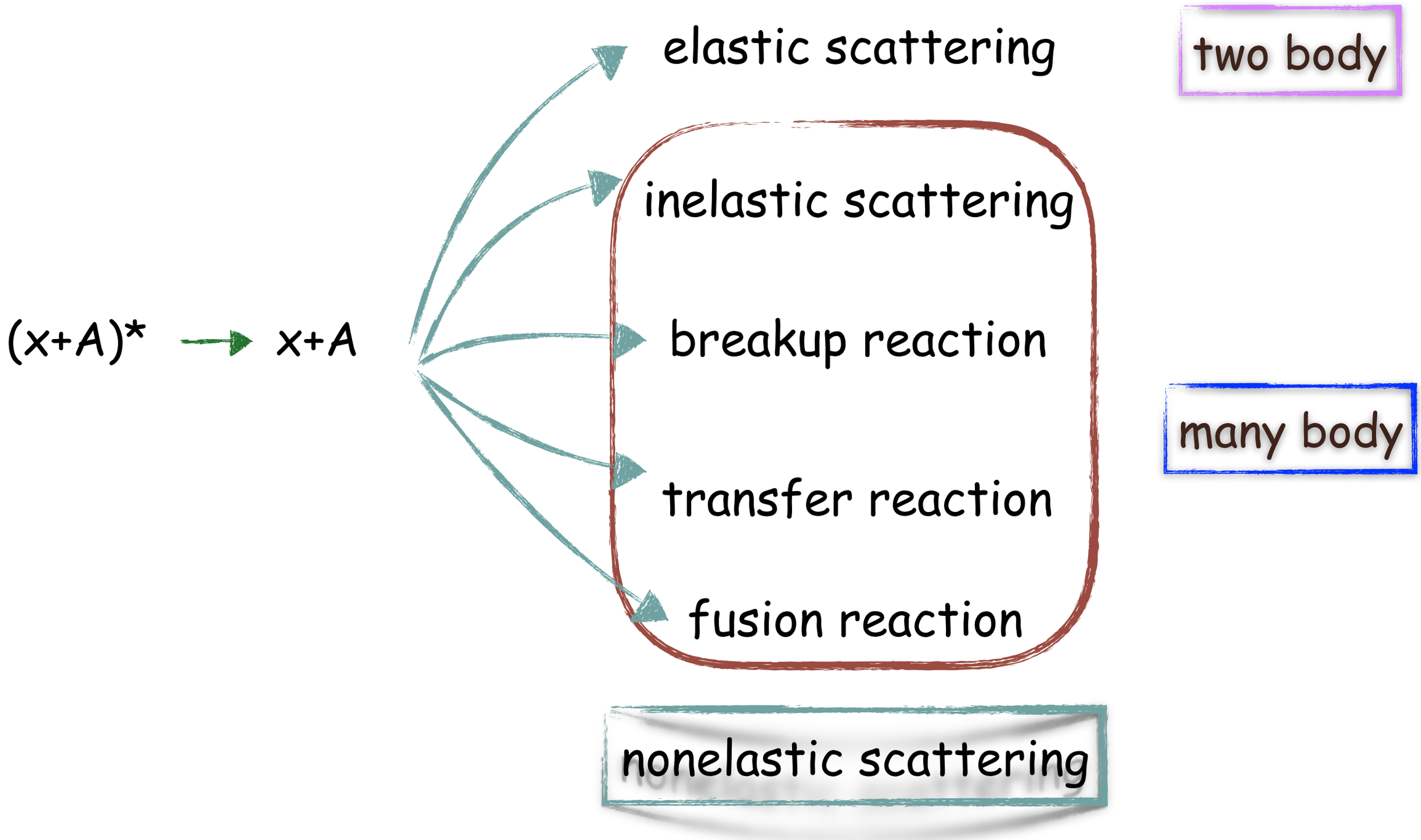
🤔

The Ichimura, Austern, Vincent Model

- Inclusive breakup :
  - a + A ⟶ b + anything → (x+A)*



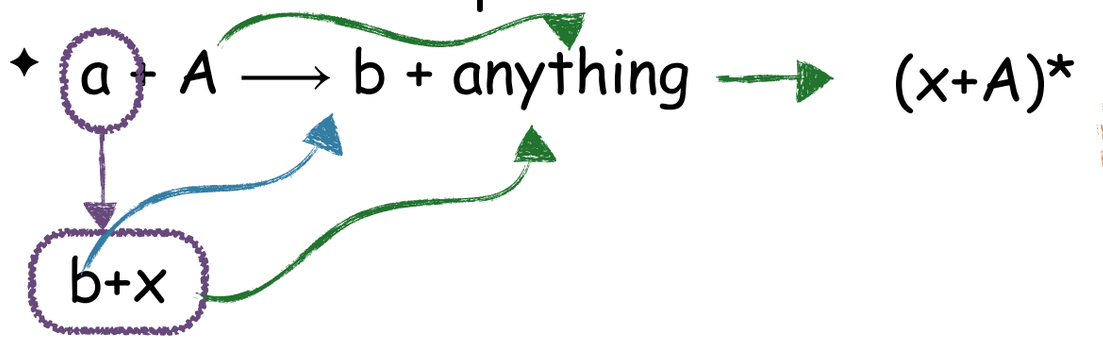
The Ichimura, Austern, Vincent Model

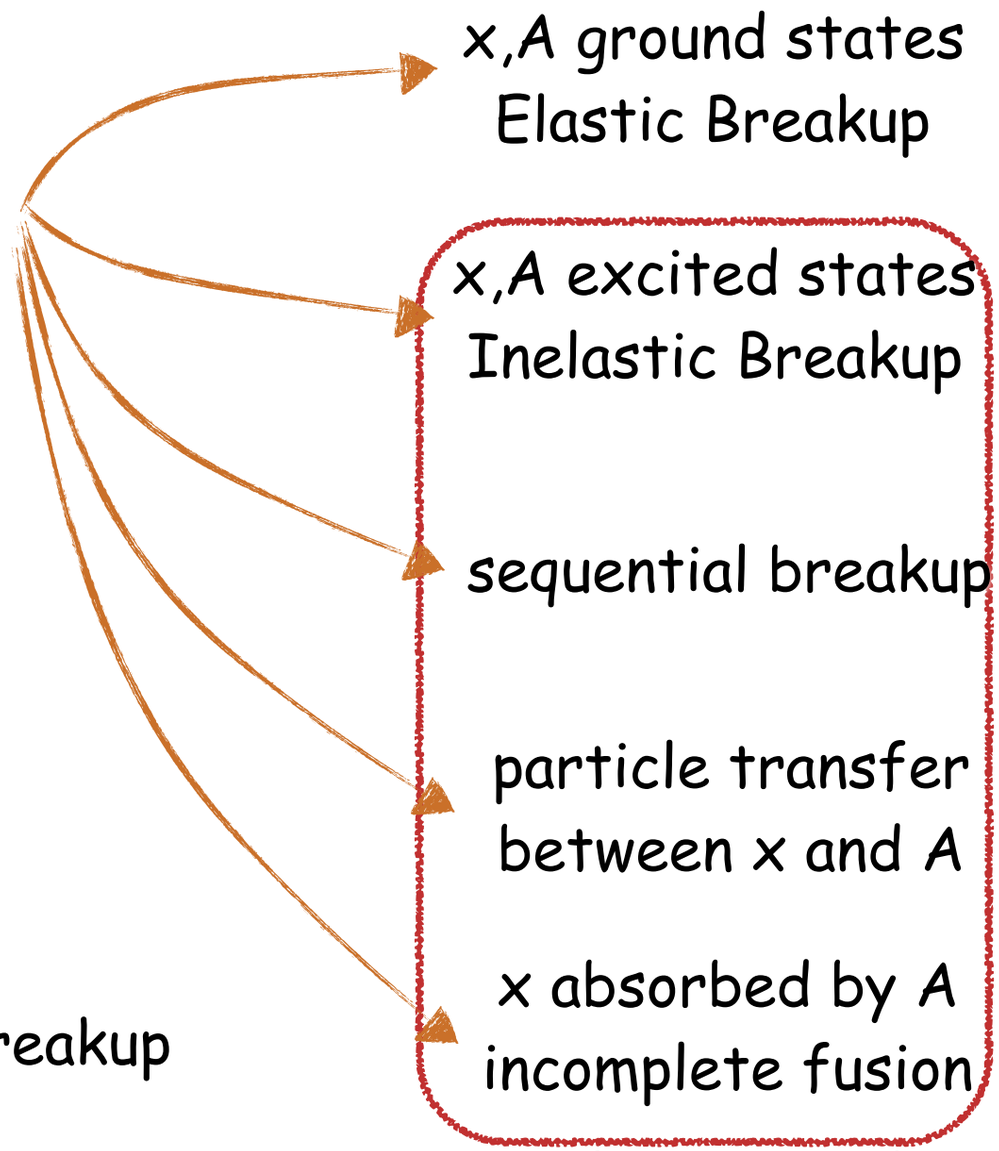


The Ichimura, Austern, Vincent Model

three body

- Inclusive breakup :





- Inclusive differential cross sections :

$$\sigma_b^{TBU} = \sigma_b^{EBU} + \sigma_b^{NEB}$$

- Post form expression for nonelastic breakup

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E_b \mathrm{d}\Omega_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\left\langle \varphi_x^0(\vec{r}_x,\vec{k}_b)\middle|W_x\middle|\varphi_x^0(\vec{r}_x,\vec{k}_b)\right\rangle$$

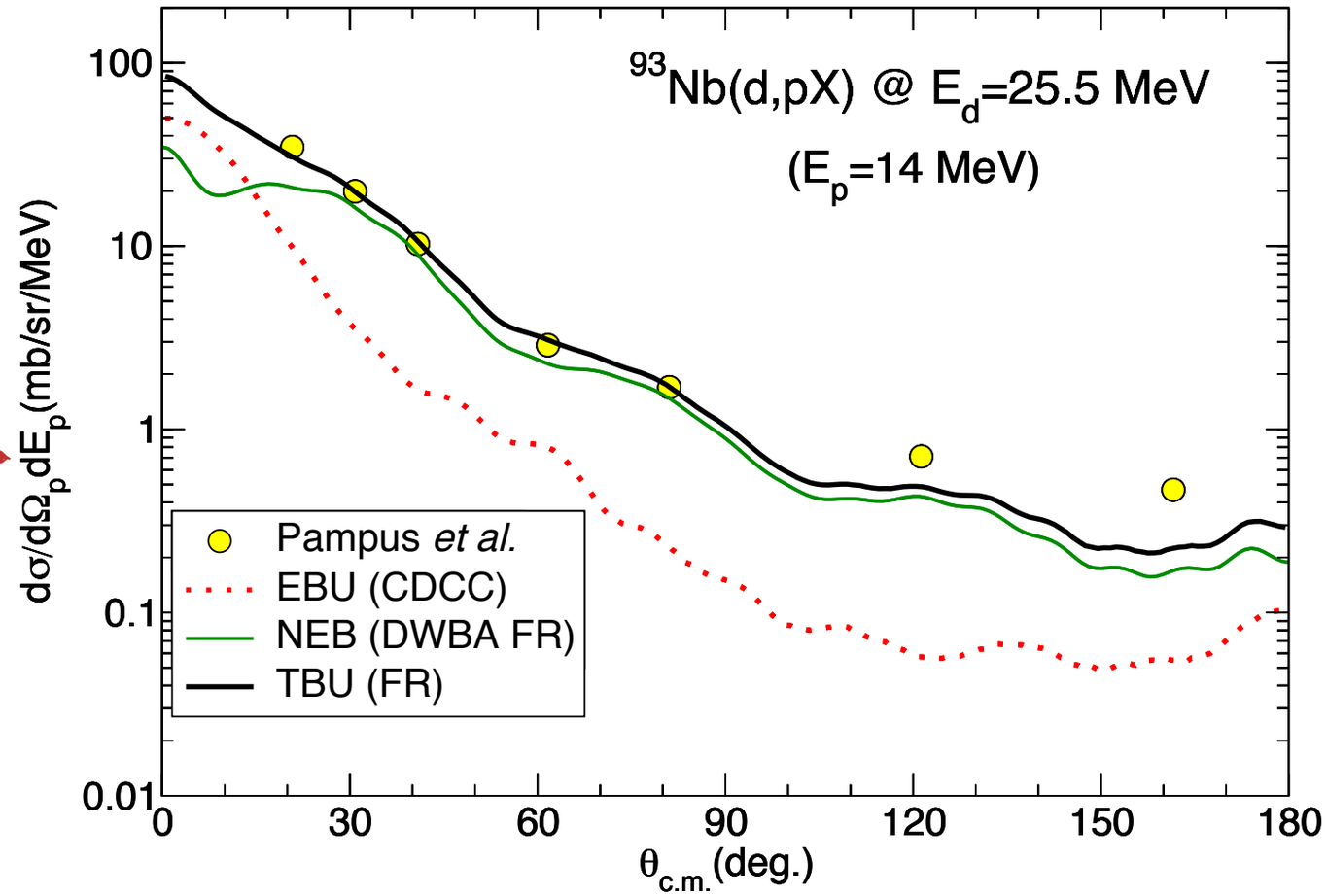
imaginary part of U_x

many body

Deuteron Breakup

- d ⇒ (n + p), S_p=2.224 MeV
- only proton is detected
- EBU : CDCC (FRESCO)
- NEB : IAV model
  - DWBA $\Psi^{3b} \simeq \chi_a \varphi_a \Phi_A$
  - Exact Finite Range
- TBU=EBU+NEB 😃

J. Pampus et al, Nucl. Phys. A311, 141 (1978).



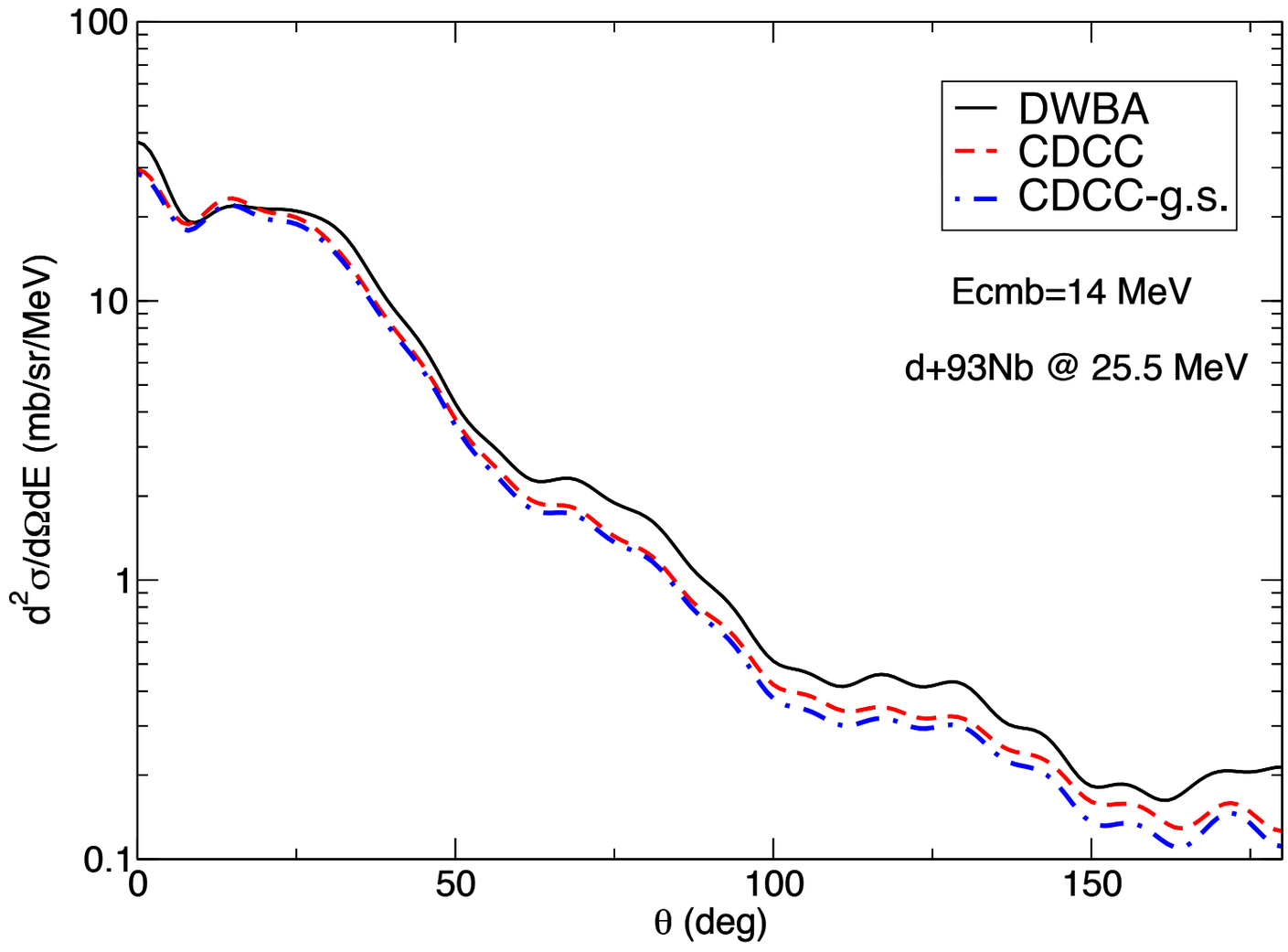
Continuum effects?

$$(E_x - K_x - U_x)\varphi_x^0(\vec{r}_x, \vec{k}_b) = (\chi_b^{(-)}(\vec{r}_b, \vec{k}_b) \,|\, V_{post} \,|\, \Psi^{3b}\rangle$$

$\Psi^{3b} \sim \phi_a \chi_a$ DWBA

$\Psi^{3b} \sim \sum_{n\alpha} \phi_a^{n\alpha} \chi_a^{n\alpha}$ CDCC

Go beyond DWBA



No breakup fusion!

Summary and perspective

- Solve the nuclear reaction in three body model
  - P-space
    - solve the three body problem with Faddeev equations
    - discover the low energy universality behavior for n+p+a
  - Q-space
    - apply IAV model to inclusive breakup problem
    - discuss the breakup effects

perspective

- Connecting to the bound state (many body technique)
- Berggren basis solve the three body scattering problem