Nuclear Level Density, Underlying Physics, and Constant Temperature Model

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June 22, 2018

In collaboration with
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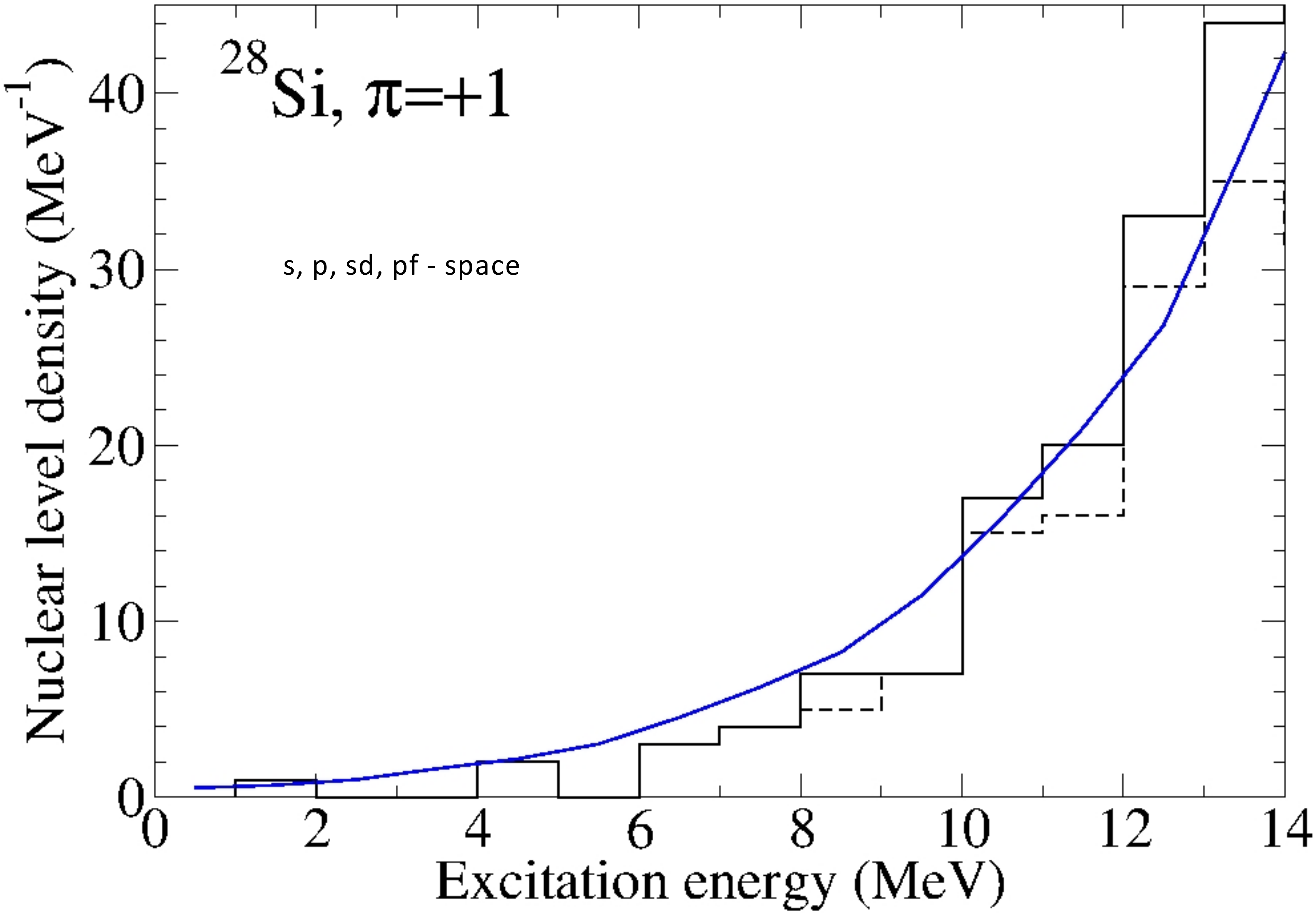
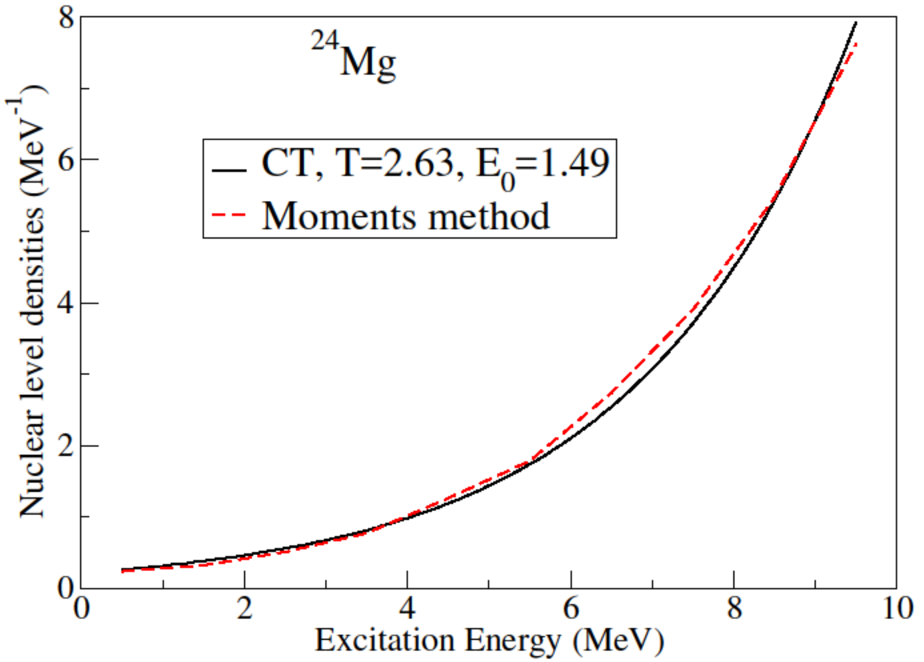
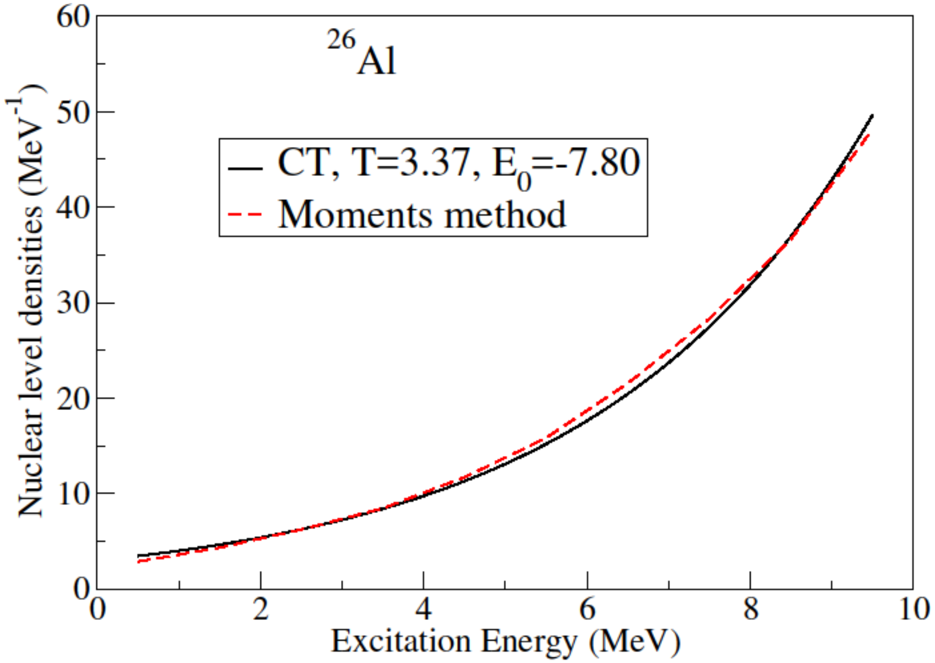
Work in progress

- "**Constant temperature model**" (CTM)
- Level density in shell model
- "**Moments method**" and related details
- **Quantum chaos** and level density
- **Thermalization** in a small mesoscopic system
- Back to CTM – **phase transition**?
- Role of small "**incoherent**" matrix elements
- Random angular momentum coupling
- CTM and "**limiting temperature**"
- Pairing and **antipairing**
- **Collective enhancement** of level density
- Projections to future



^{28}Si, π=+1
s, p, sd, pf - space
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8
10
12
14





CONSTANT TEMPERATURE PHENOMENOLOGY

LEVEL DENSITY (E) = (const) exp (E/T)

Ericson (1962)

Moretto (1975) – pairing phase transition

T – "effective constant temperature"

1/T – rate of increase of level density

How to find the level density

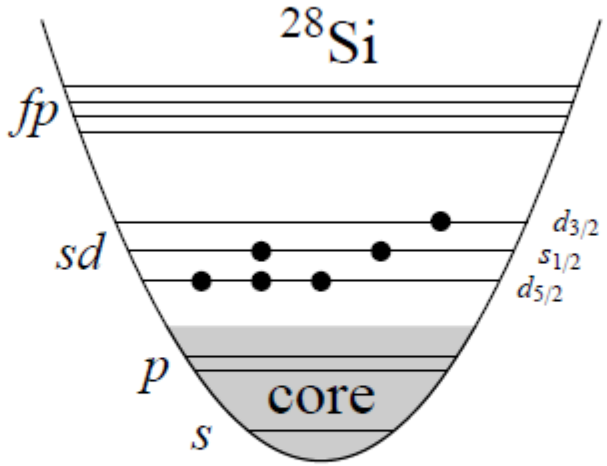
Experimentally: direct counting (low E)
neutron resonances
other resonance reactions

Theoretically: Fermi-gas phenomenology
mean-field including pairing
energy density functionals
shell model diagonalization
Monte Carlo shell model
statistical spectroscopy

Microscopic description of Nuclear Level Density

Shell model (the most successful)

- Restricted model space
  Dim(sd) $\sim 10^6$
  Dim(fp) $\sim 10^{10}$
- Need effective interaction
- Numerical diagonalization
- High accuracy: $\delta E \sim \pm 200 KeV$



How it works:

Many-body states in Shell Model: $|\alpha\rangle = \sum_{k=1}^{Dim} C_k^\alpha |k\rangle.$

Schrödinger equation: $\hat{H}|\alpha\rangle = E_\alpha|\alpha\rangle \quad \Rightarrow \quad \hat{H}\vec{C}_\alpha = E_\alpha \vec{C}_\alpha.$

Moments method

No diagonalization required

$$\rho(E,\alpha) = \sum_{\kappa} D_{\alpha\kappa} \cdot G_{\alpha\kappa}(E)$$

$$\alpha = \{n, J, T_z, \pi\}$$

Quantum numbers

Exact quantum numbers

$$\kappa = \{n_1, n_2, \dots, n_q\}$$

Partitions

$$G_{\alpha\kappa}(E) = G(E + E_{g.s.} - E_{\alpha\kappa}, \sigma_{\alpha\kappa})$$

$$G(x,\sigma) = C \cdot \begin{cases} \exp\left(-x^2/2\sigma^2\right) & , \ |x| \leq \eta \cdot \sigma \\ 0 & , \ |x| > \eta \cdot \sigma \end{cases}$$

Finite range Gaussian

$$D_{\alpha\kappa}$$

Many-body dimension

$$E_{\alpha\kappa} = \langle H \rangle_{\alpha\kappa},$$

$$\sigma_{\alpha\kappa} = \sqrt{\langle H^2 \rangle_{\alpha\kappa} - \langle H \rangle_{\alpha\kappa}^2}$$

$$\mathrm{Tr}^{(J)}[\dots] = \mathrm{Tr}^{(J_z)}[\dots]_{J_z=J} - \mathrm{Tr}^{(J_z)}[\dots]_{J_z=J+1}$$

$$\langle H \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

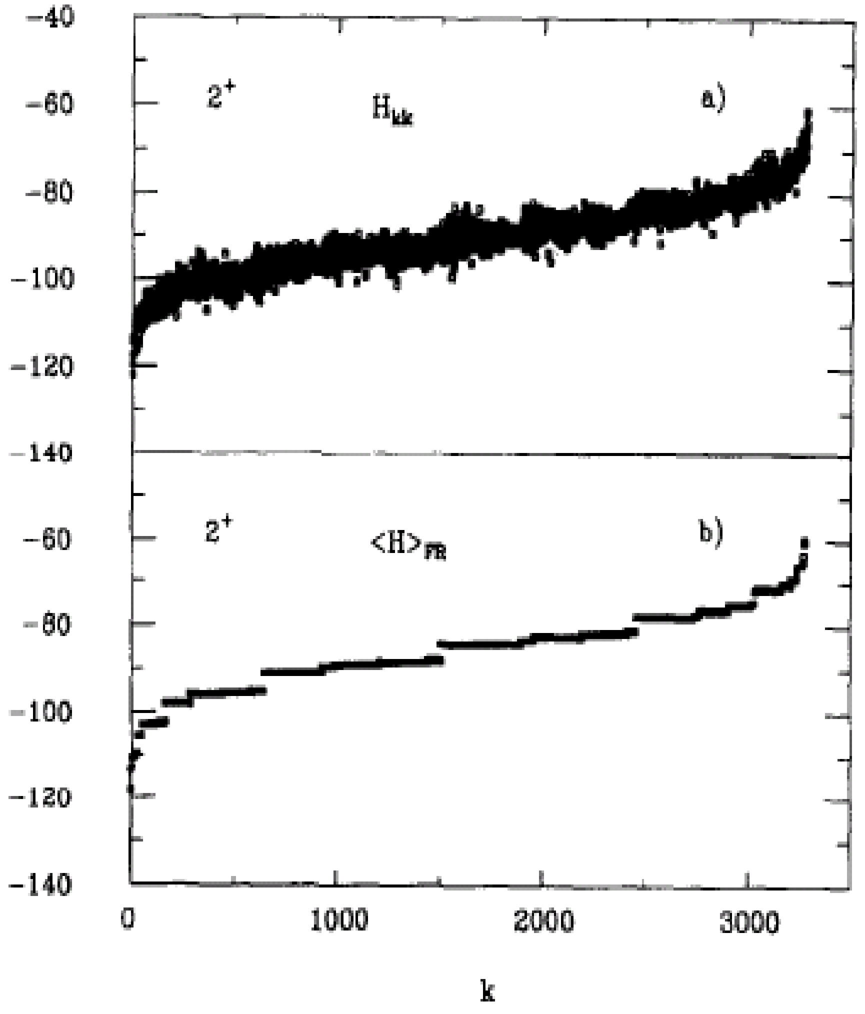
Centroids – first moment

$$\langle H^2 \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

Widths - second moment

^{28}Si

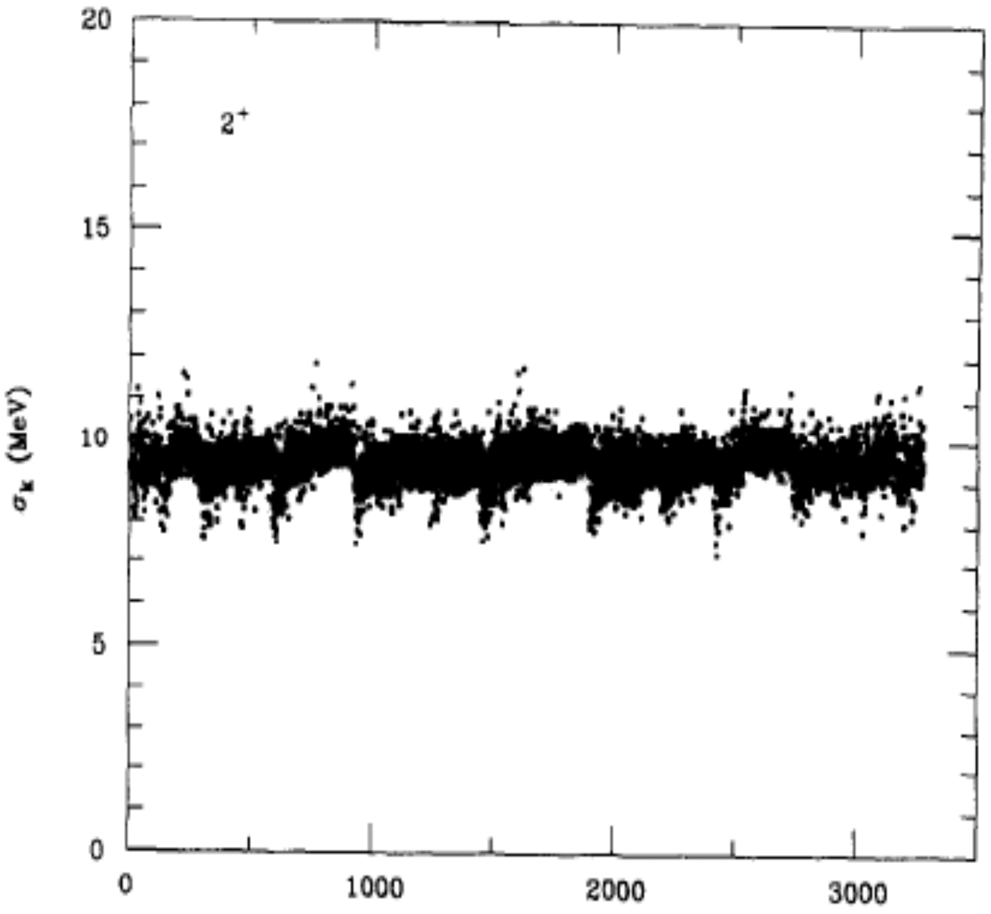
Diagonal matrix elements of the Hamiltonian in the mean-field representation



Partition structure in the shell model

(a) *All 3276 states* ; (b) *energy centroids*

^{28}Si



Energy dispersion for individual states is nearly constant

(result of **geometric chaoticity**!)

Also in multiconfigurational method (hybrid of shell model and density functional)

$$\sigma_k^2 = \langle k|(H - H_{kk})^2|k\rangle = \sum_{l\neq k} H_{kl}^2 \ ,$$

Widths add in quadratures

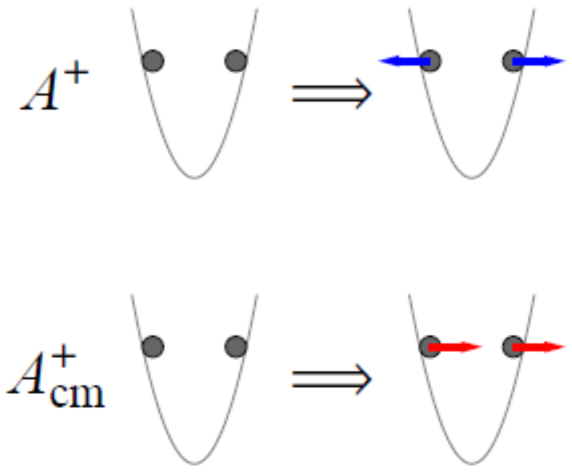
Removal of the center-of-mass spurious states

Harmonic oscillator:

$$\mathcal{N}_{spur}(K\hbar\omega) \sim \sum_{K'=1}^{K} \mathcal{N}_{pure}((K-K')\hbar\omega),$$

where K' presents how many times we act with $A^{\dagger}_{cm}$

P. Van Isacker, Phys. Rev. Lett. **89**, 262502 (2002)



Nuclear level density. Recursive method:

$$\rho_{pure}(E,J,K) = \rho(E,J,K) - \sum_{K'=1}^{K} \sum_{J_{K'}=J_{min}}^{K,\,step\ 2} \sum_{J'=|J-J_{K'}|}^{J+J_{K'}} \rho_{pure}(E,J',K-K')$$

M. Horoi and V. Zelevinsky, Phys. Rev. Lett. **98**, 262503 (2007)

$$\rho^{(0)}(E,J,0) = \rho(E,J,0)$$ $N\hbar\omega$ classification

Pure **Total** **(N=0)**

$$\rho^{(0)}(E,J,1) = \rho(E,J,1) - \sum_{J'=|J-1|}^{J+1} \rho(E,J',0)$$ **(N=1)**

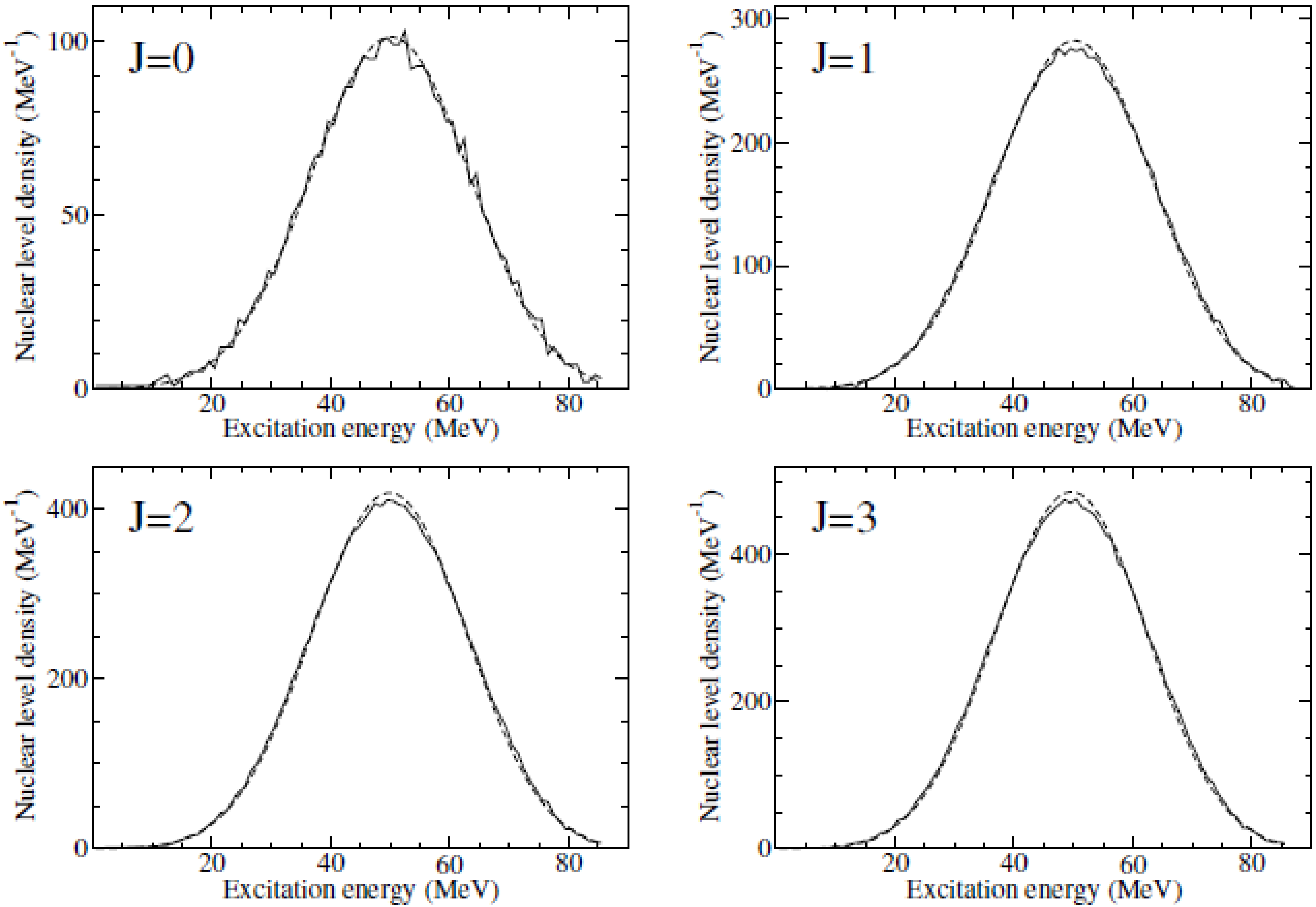
$$\rho^{(0)}(E,J,N) = \rho(E,J,N) -$$

$$- \sum_{K=1}^{N} \sum_{J_K=J_{\min}}^{N,\text{step }2} \sum_{J'=|J-J_K|}^{J+J_K} \rho^{(0)}(E,J',(N-K))$$

Recursive relation

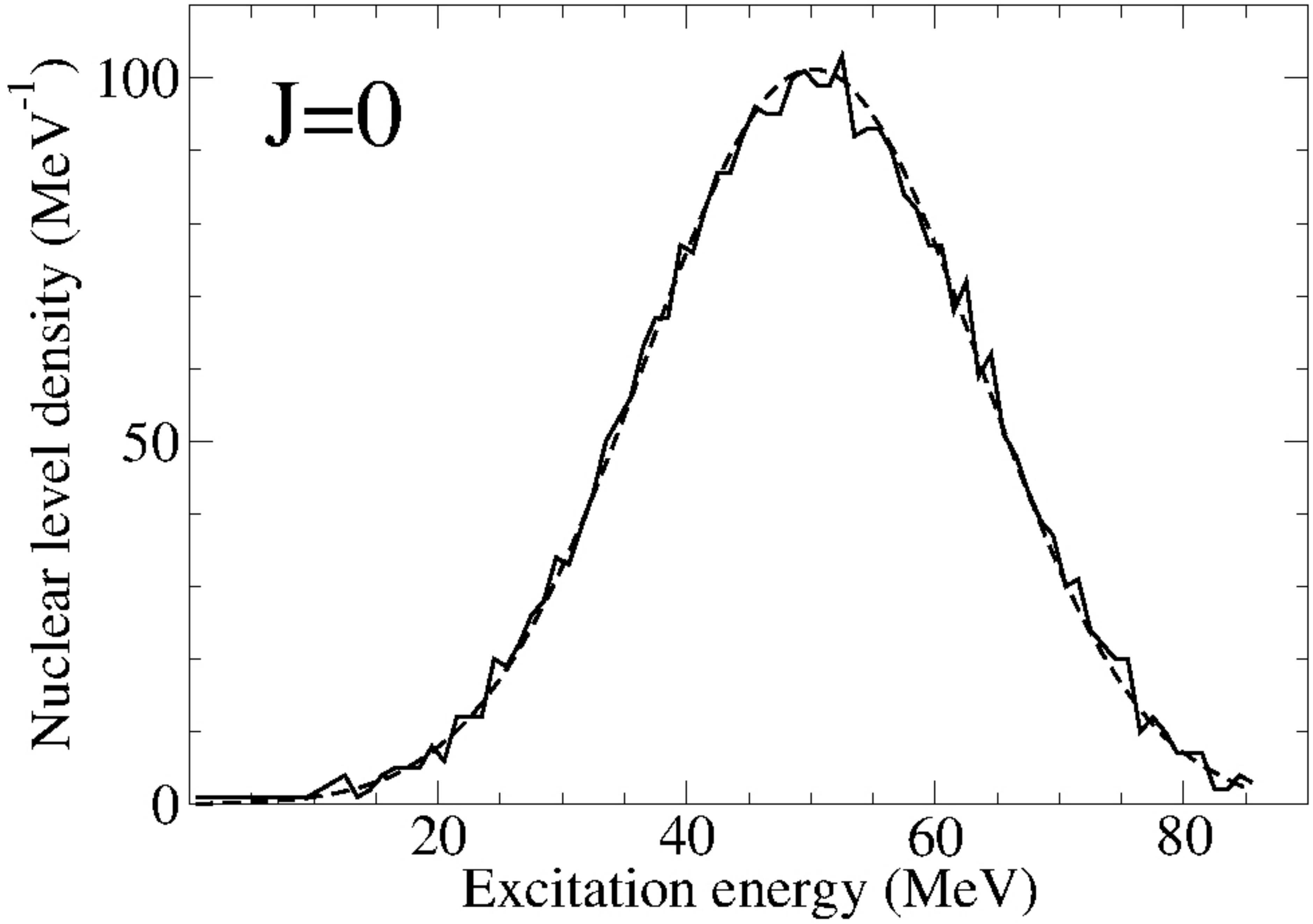
^{28}Si, parity=+1, some *J*, *sd*-shell

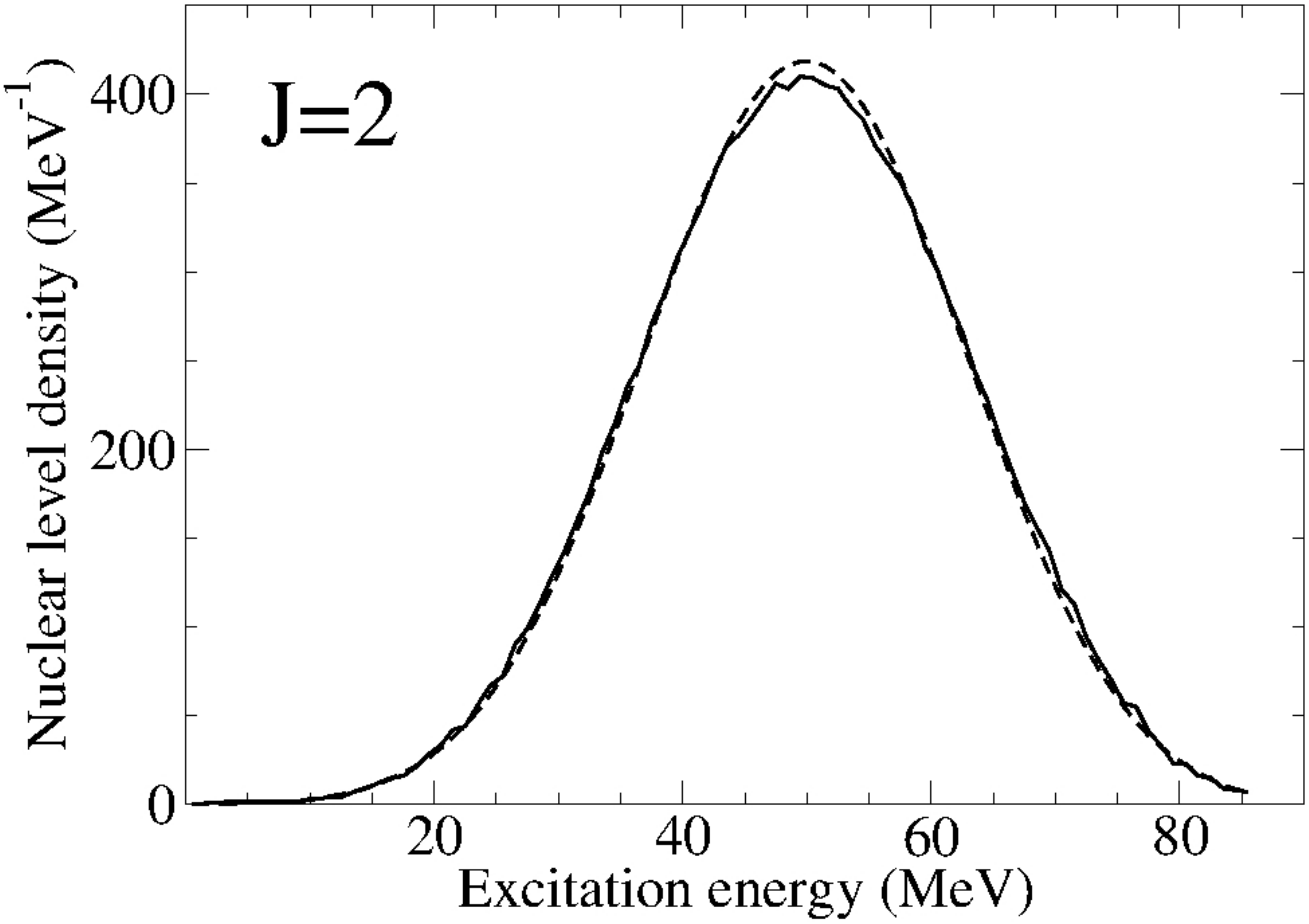
Shell Model (solid line) vs. Moments Method (dashed line).



Shell-model level density.

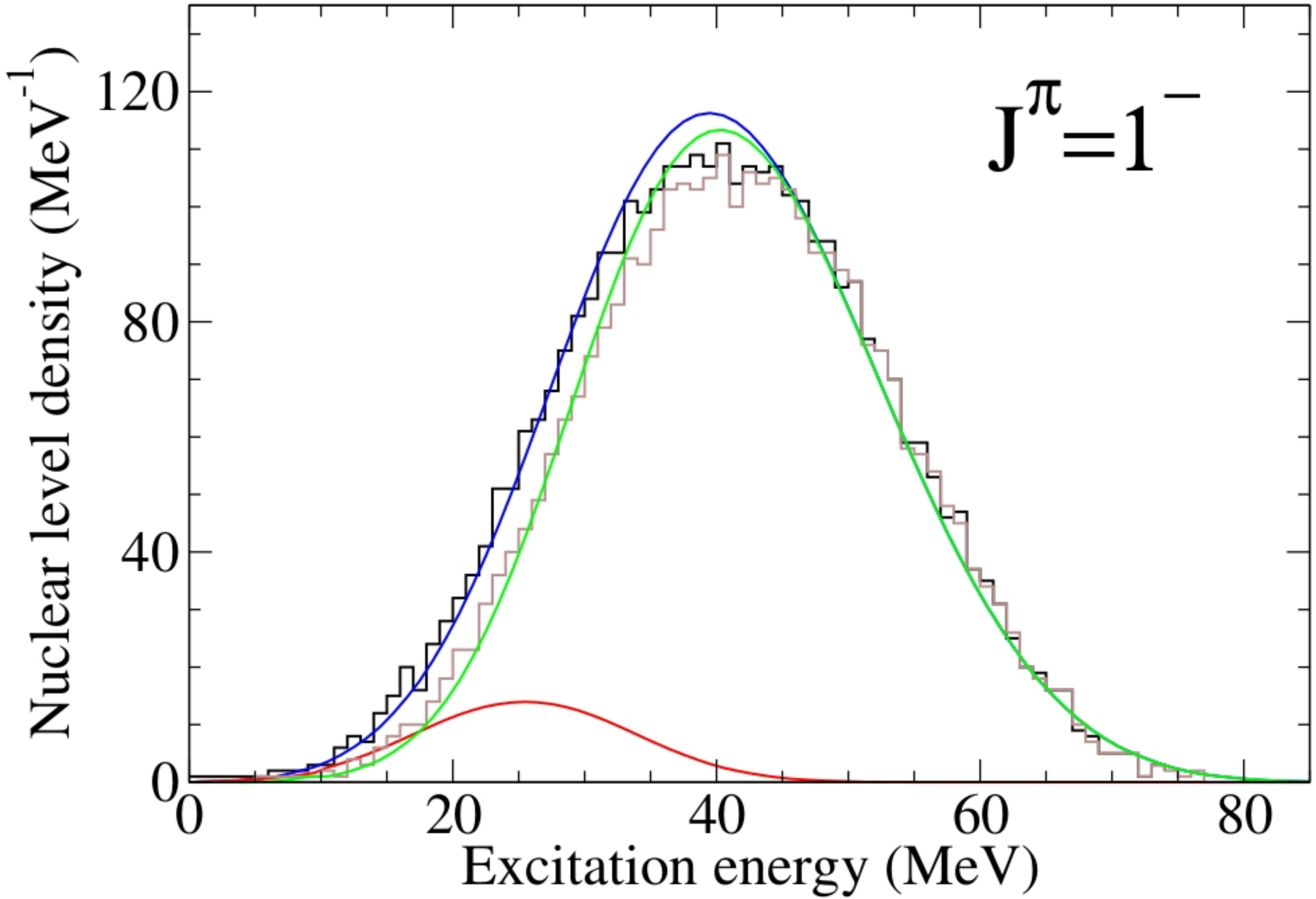
Moments method (no diagonalization)


J=0
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
50
100
20
40
60
80

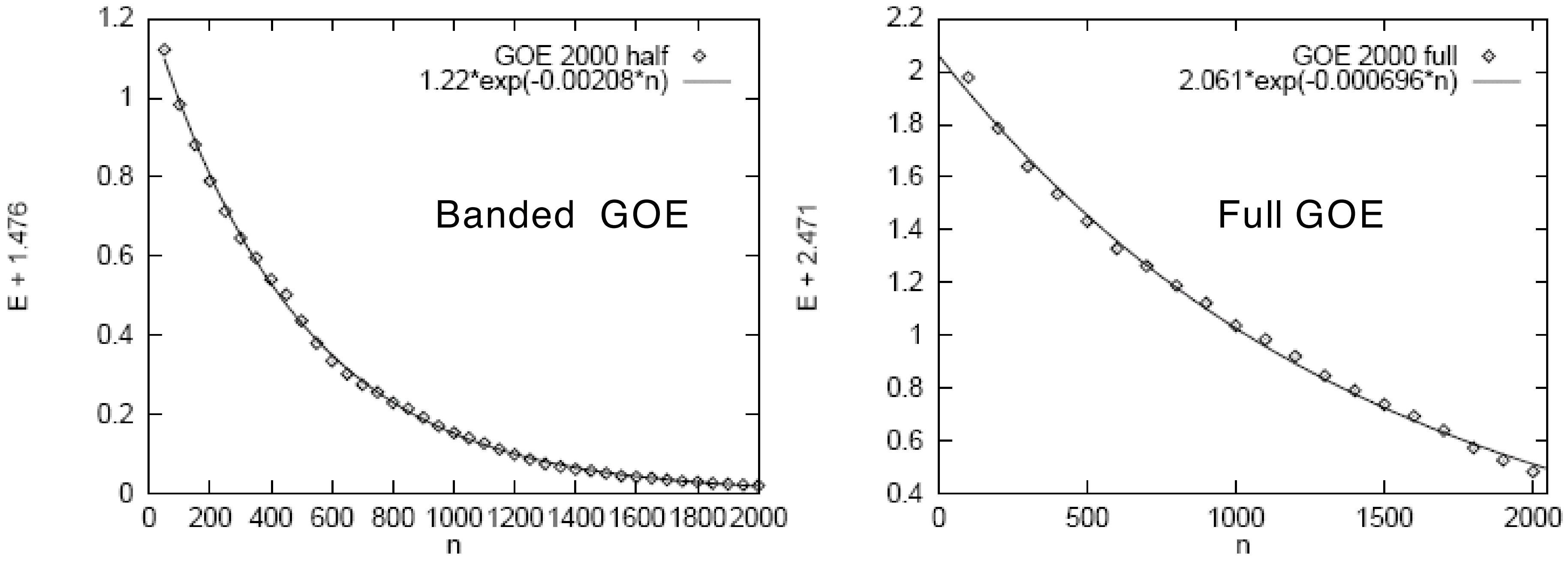


J=2
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
200
400
20
40
60
80

^{20}Ne (1ħω)



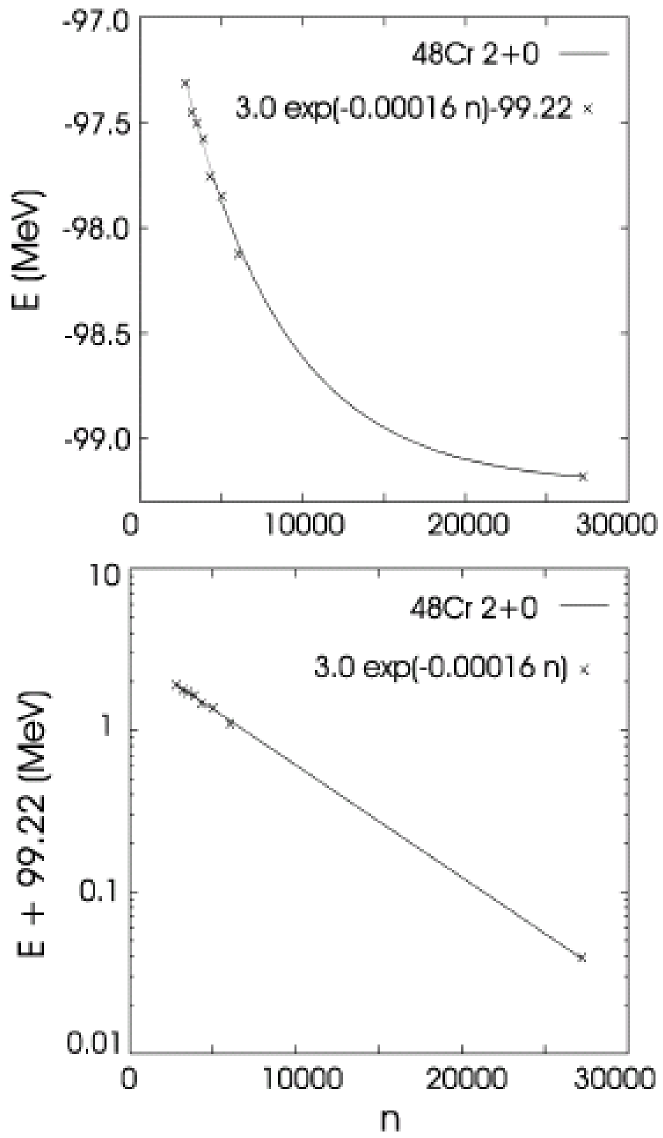
$J^{\pi}=1^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
40
80
120
0
20
40
60
80



GROUND STATE ENERGY OF RANDOM MATRICES

EXPONENTIAL CONVERGENCE

SPECIFIC PROPERTY of RANDOM MATRICES ?



REALISTIC
SHELL
MODEL

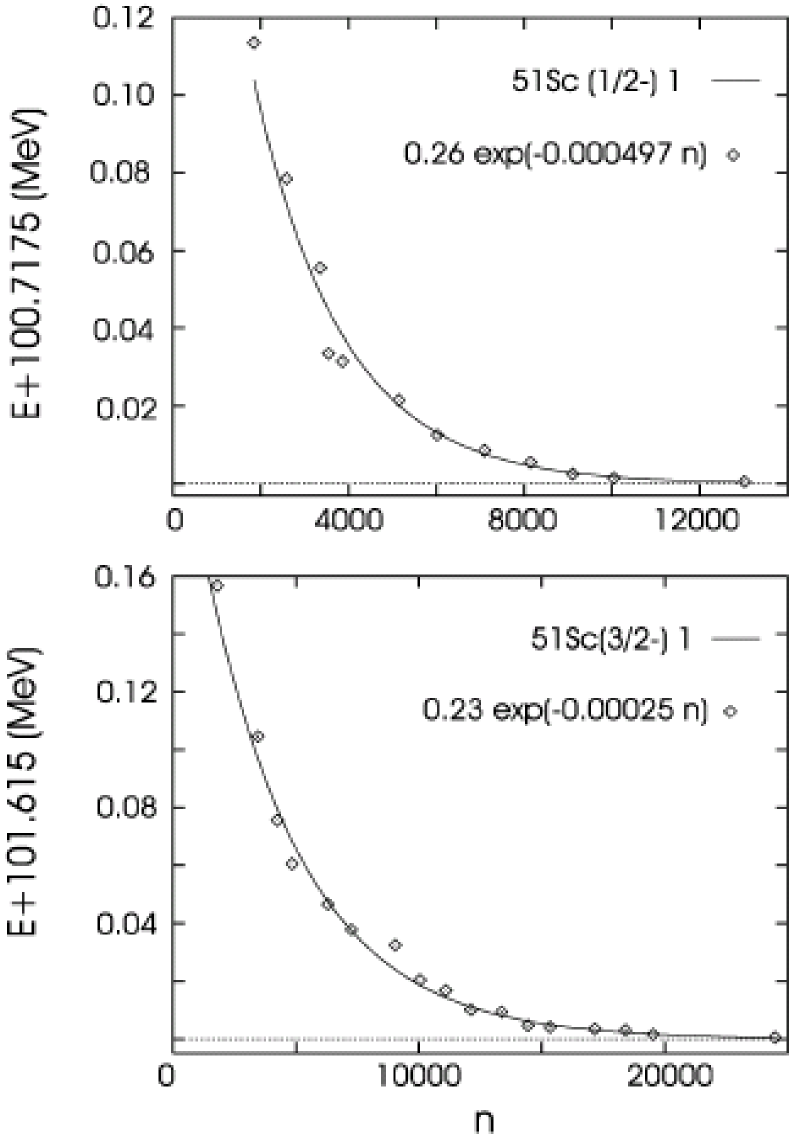
48 Cr

Excited state
J=2, T=0

EXPONENTIAL CONVERGENCE !

E(n) = E + exp(-an)

n ~ 4/N



REALISTIC
SHELL
MODEL

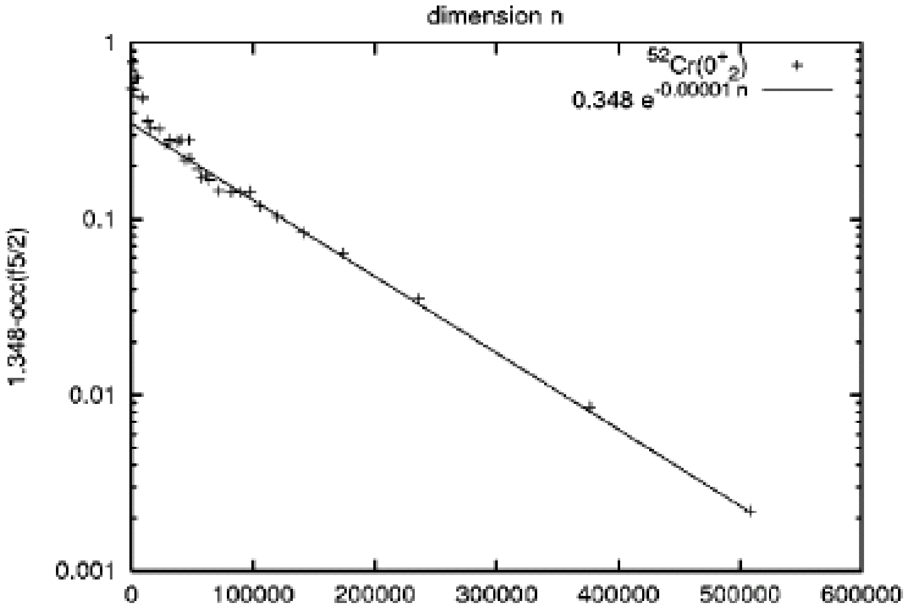
EXCITED STATES
51Sc

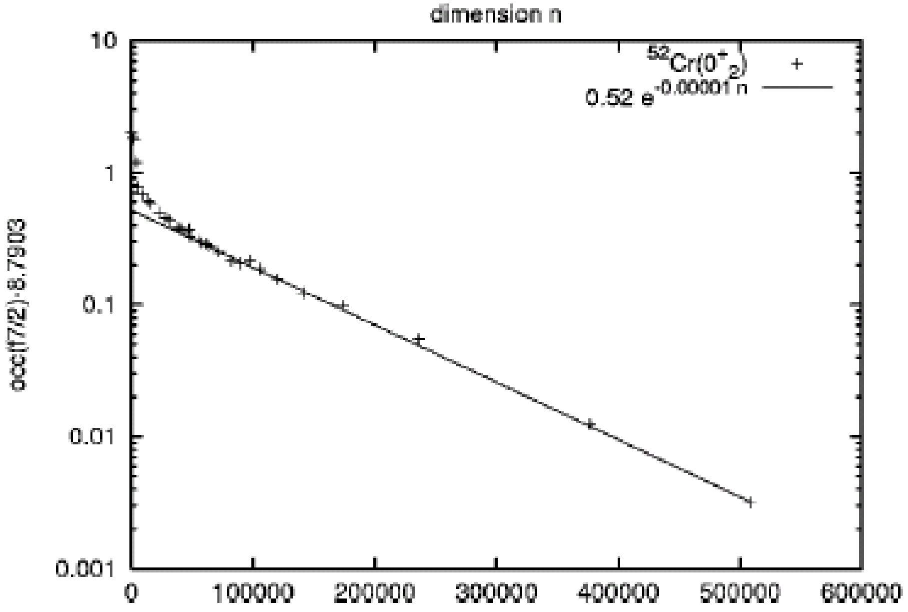
1/2-, 3/2-

Faster convergence:

$E(n) = E + \exp(-an)$

$a \sim 6/N$

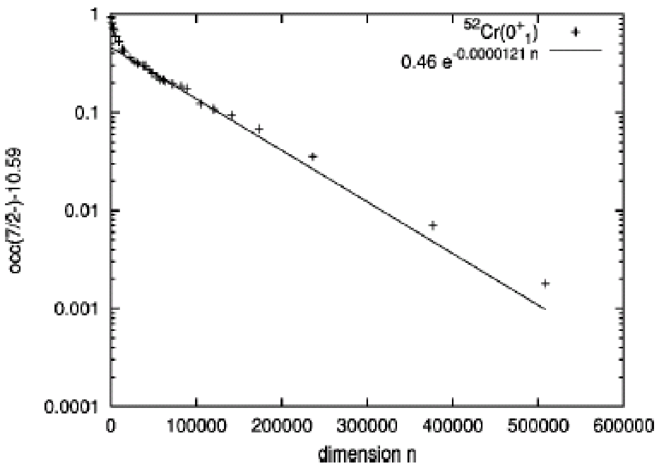




EXPONENTIAL CONVERGENCE OF SINGLE-PARTICLE OCCUPANCIES

(first excited state J=0)

^{52}Cr



Fit with $\gamma' = \gamma$

Analytical results for tridiagonal matrices

$$H = \begin{pmatrix} \epsilon_1 & V_2 & 0 & 0 & 0 & 0 \\ V_2 & \epsilon_2 & V_3 & 0 & 0 & 0 \\ 0 & V_3 & \epsilon_3 & V_4 & 0 & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & V_n \\ 0 & 0 & 0 & 0 & V_n & \epsilon_n \end{pmatrix}$$

Recurrence relation for determinants

$$D_n(E) = (\epsilon_n - E)D_{n-1}(E) - V_n^2 D_{n-2}(E)$$

Convergence is determined by

$$\lambda_n^2 = V_n^2/(\epsilon_n \epsilon_{n-1})$$

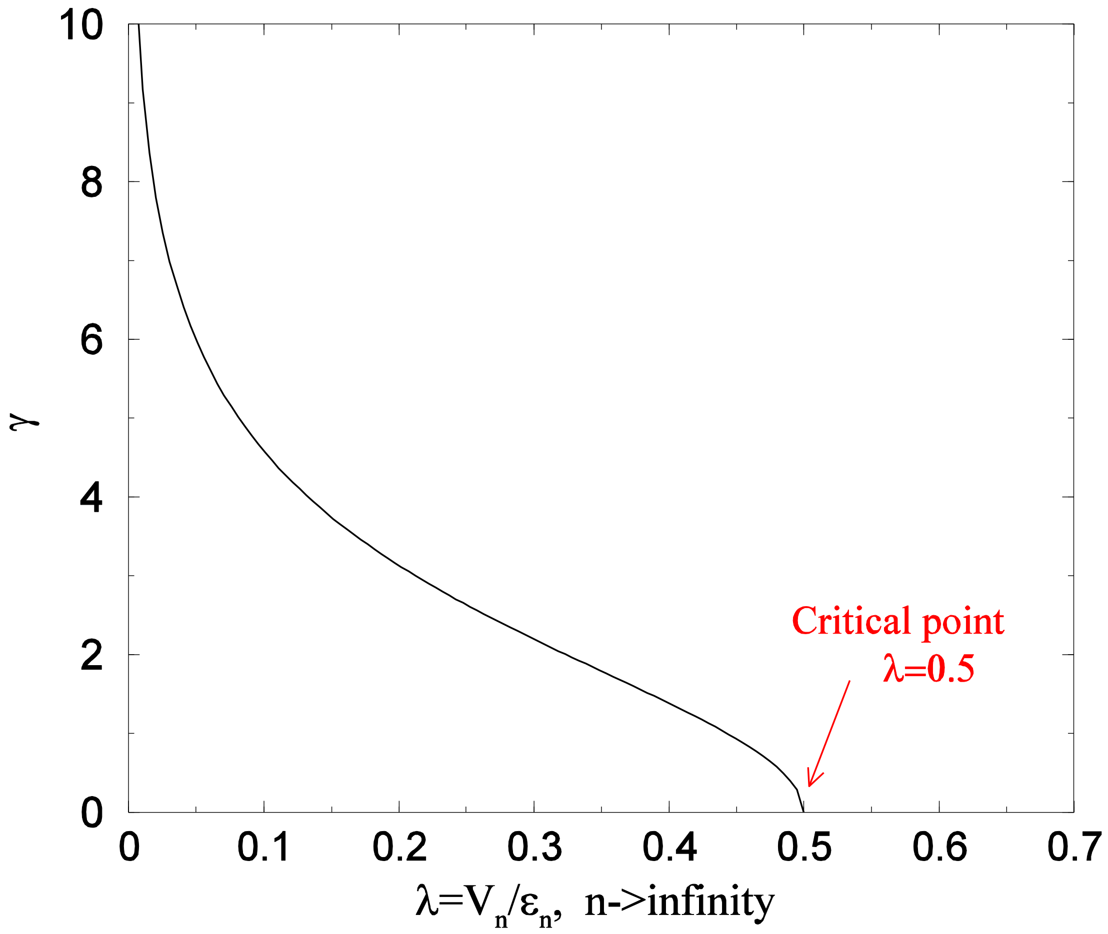
Assume existence of the limit

$$\lambda_n \Rightarrow \lambda$$

at

$$n \Rightarrow \infty$$

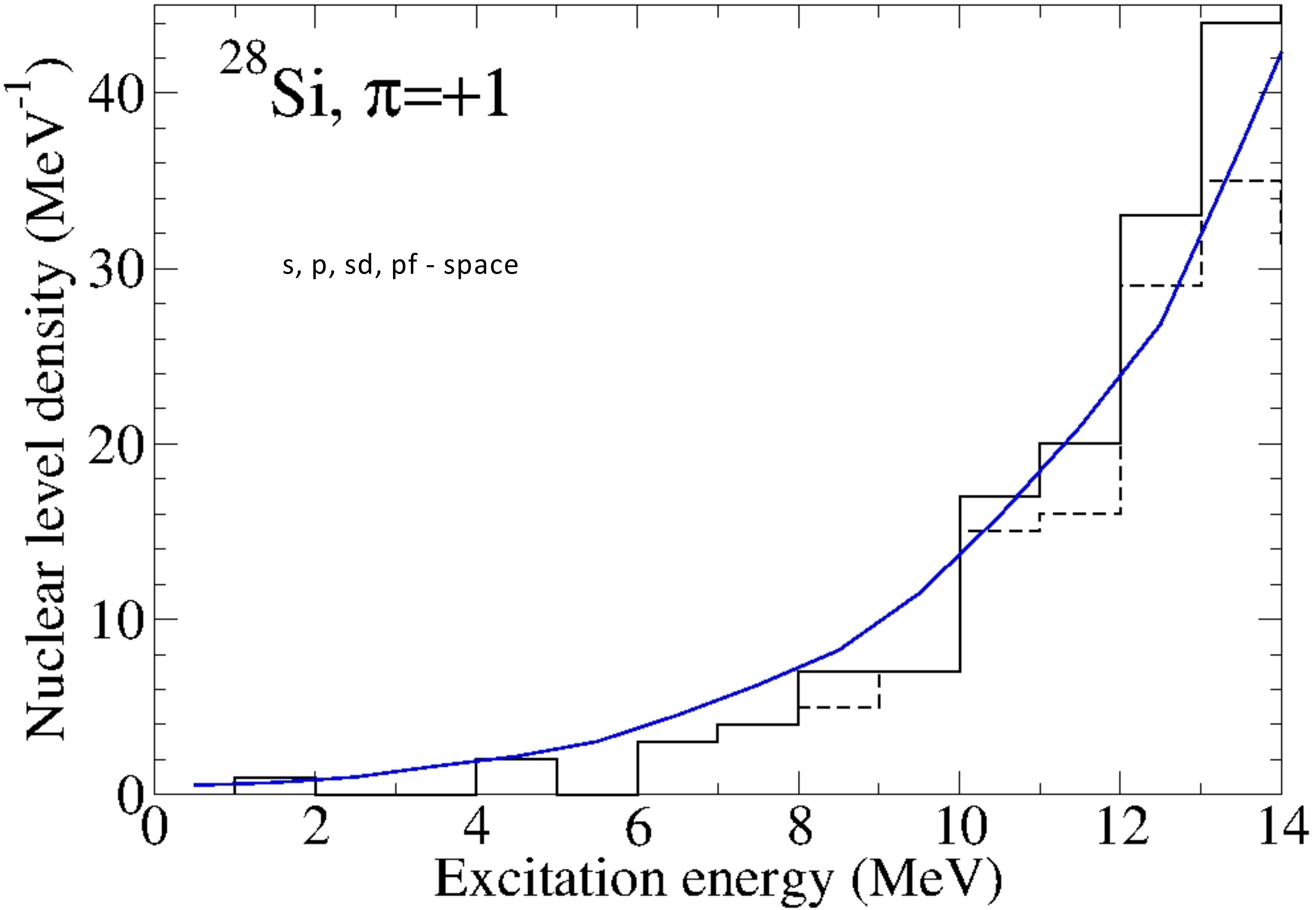
New method for shell-model level density /B.A. Brown, 2018/



CONVERGENCE REGIMES

- $\lambda = 0$ → *Fast convergence*
- $0 < \lambda < 1/2$ → *Exponential convergence*
- $\lambda = 1/2$ → Power law
- $\lambda > 1/2$ → *Divergence*

Exponential convergence $\sim \exp(-\gamma n)$ → $\gamma = -\ln\left\{\frac{1}{2\lambda^2}\left(1 - 2\lambda^2 - \sqrt{1 - 4\lambda^2}\right)\right\}$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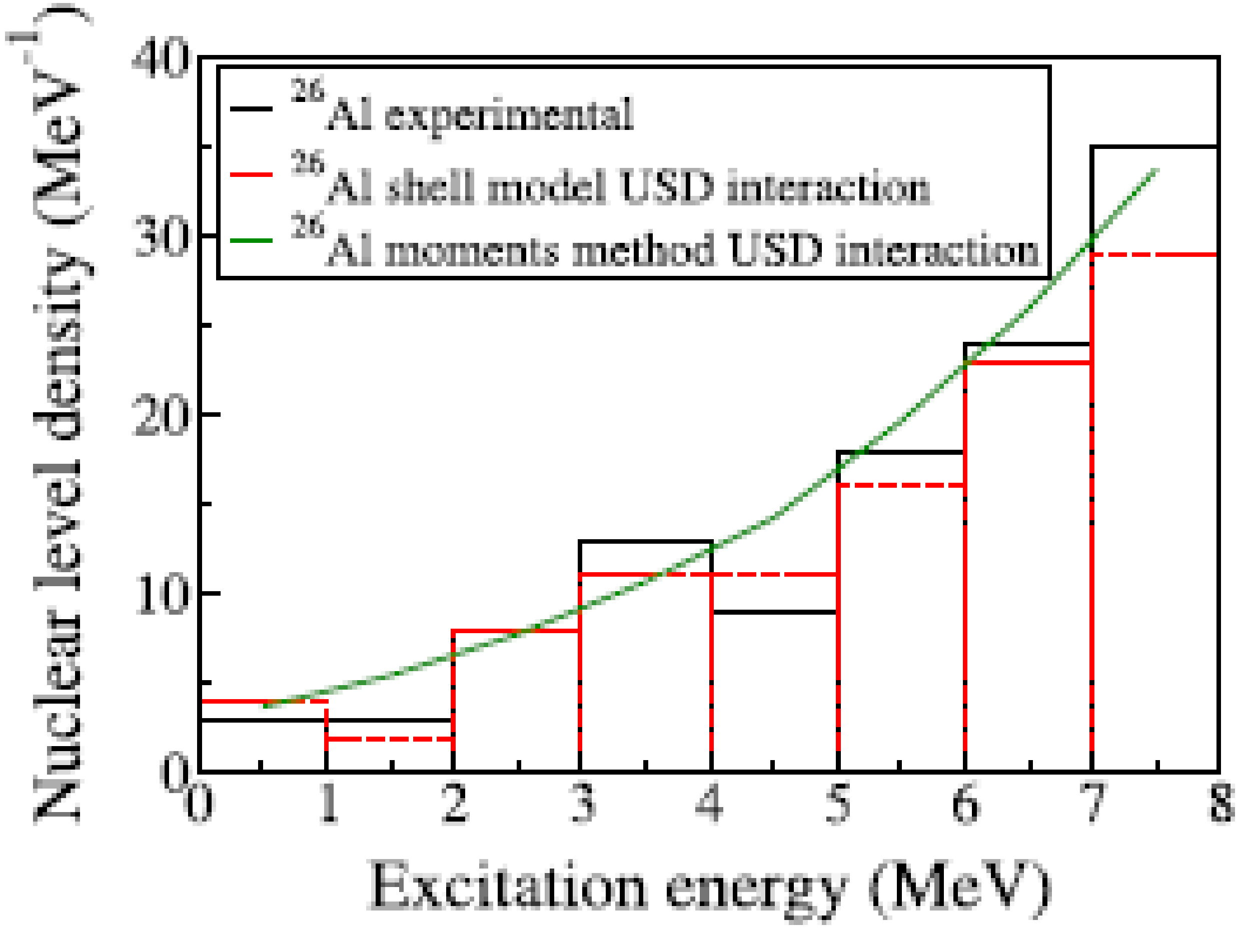
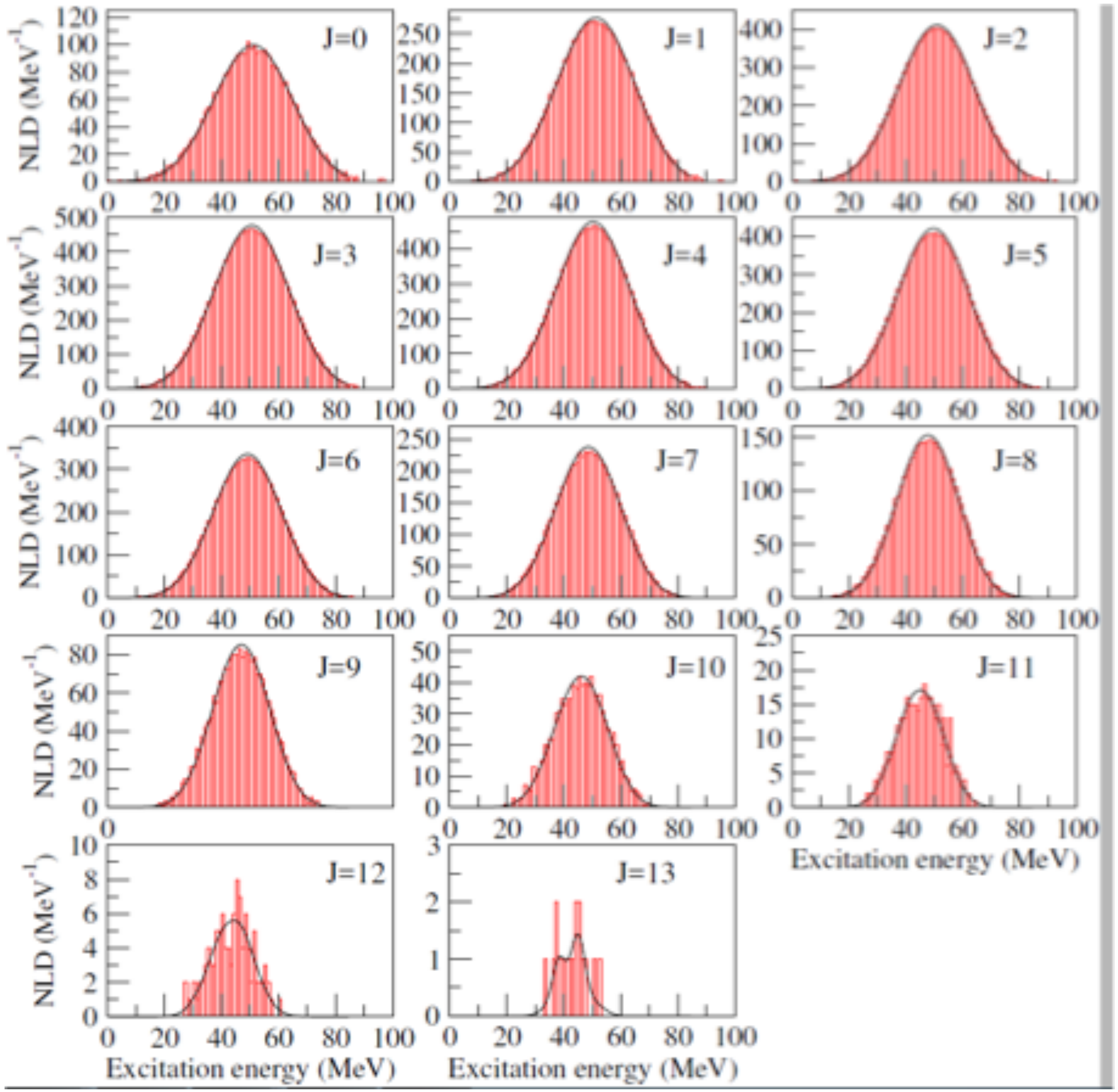


S. Karampagia, V.Z.
Nucl. Phys. A962 (2017)

J = 0 – 7, positive parity level density

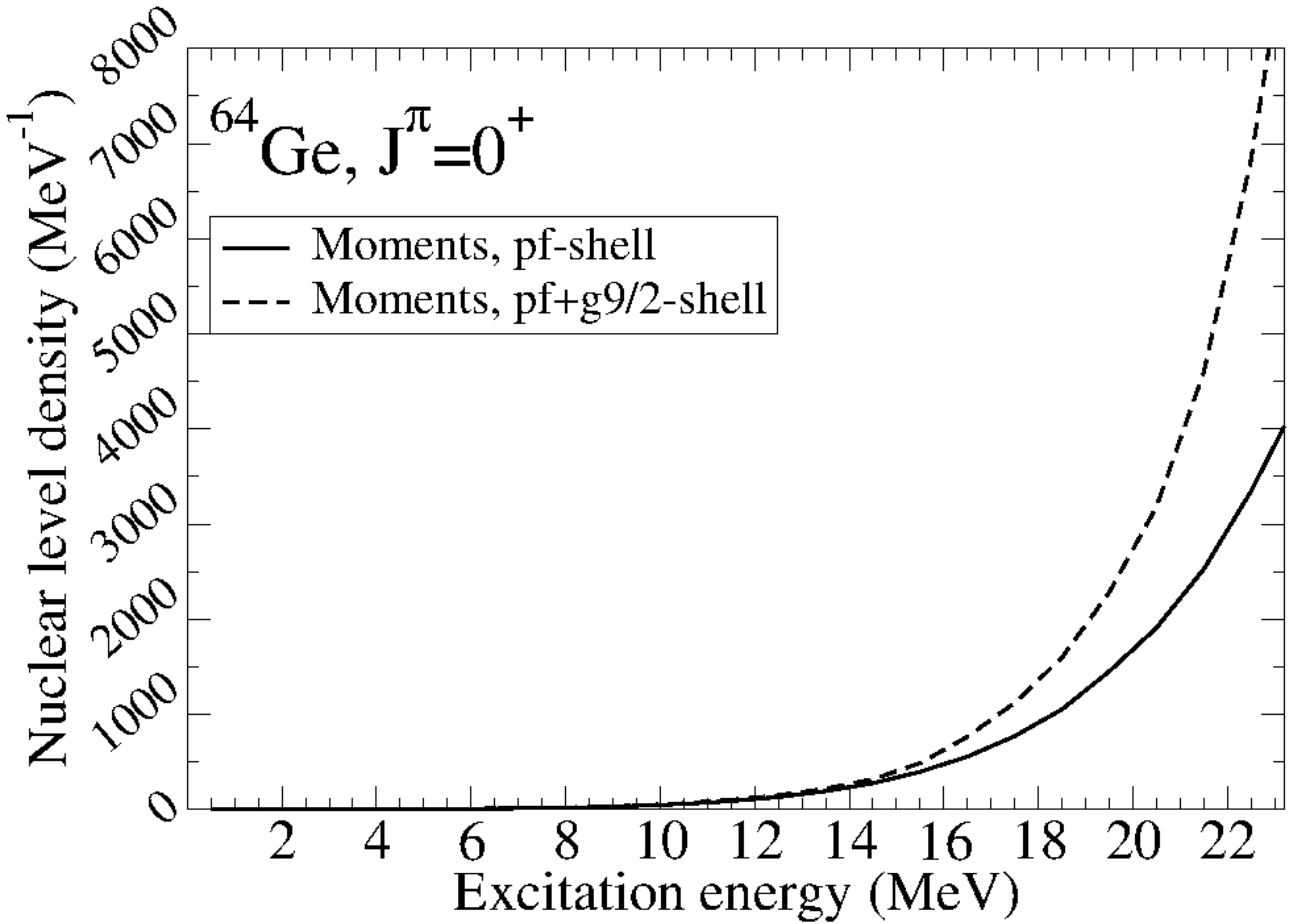


Generic shape (Gaussian)

Level density for different classes of states in 28Si

Full agreement between exact shell model and moments method

Problems: truncated orbital space, only positive parity in sd-model, …



R.Sen’kov, V.Z.
PRC 93 (2016)

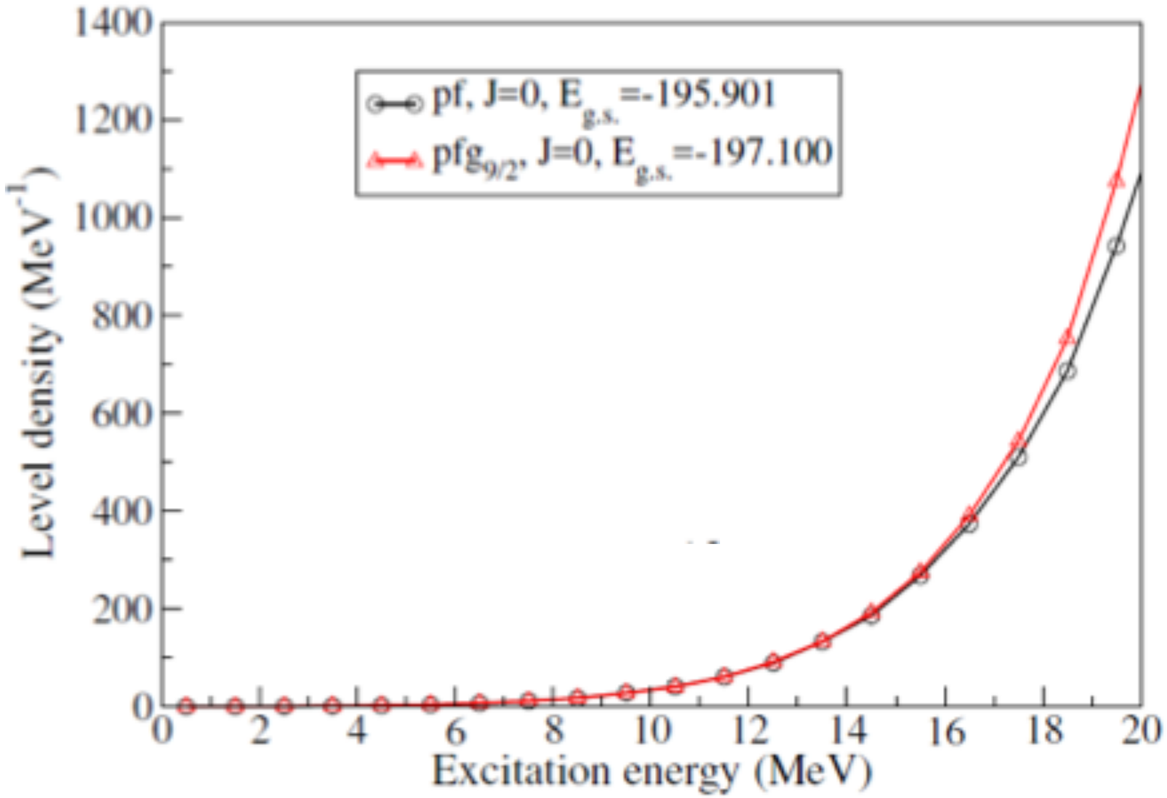


FIG. 3: Comparison of the level density of ^{56}Fe calculated in the pf model space (black line with circles) versus the one calculated in the $pf + g_{9/2}$ model space (red line with triangles), using the $E_{g.s.}$=-197.100, which minimizes the difference of the low-lying level densities between the two model spaces.

CLOSED MESOSCOPIC SYSTEM

at high level density

Two languages: *individual wave functions*
thermal excitation

* Mutually exclusive ?
* Complementary ?
* Equivalent ?

Answer depends on thermometer

CHAOS versus THERMALIZATION

L. BOLTZMANN - *Stosszahlansatz* = MOLECULAR CHAOS

N. BOHR - *Compound nucleus* = MANY-BODY CHAOS

N. S. KRYLOV - *Foundations of statistical mechanics*

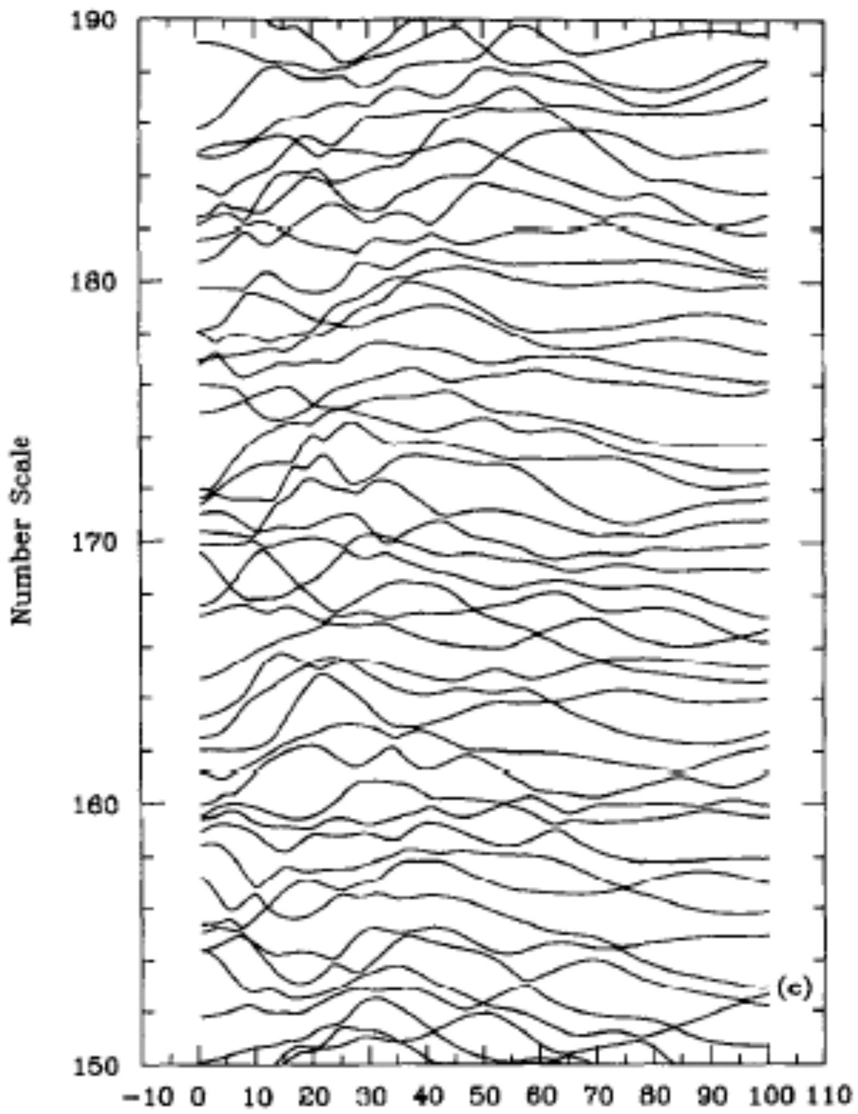
L. Van HOVE - *Quantum ergodicity*

L. D. LANDAU and E. M. LIFSHITZ - "*Statistical Physics*"

Average over the equilibrium ensemble should coincide with the expectation value in a generic individual eigenstate of the same energy – the results of measurements in a closed system do not depend on exact microscopic conditions or phase relationships if the eigenstates at the same energy have similar macroscopic properties

TOOL: MANY-BODY QUANTUM CHAOS

LEVEL DYNAMICS



WAY to CHAOS:

MULTIPLE

AVOIDED

CROSSINGS

as a function

of interaction strength

(shell model of 24Mg
as a typical example)

Fraction (%) of realistic strength

From turbulent to laminar level dynamics

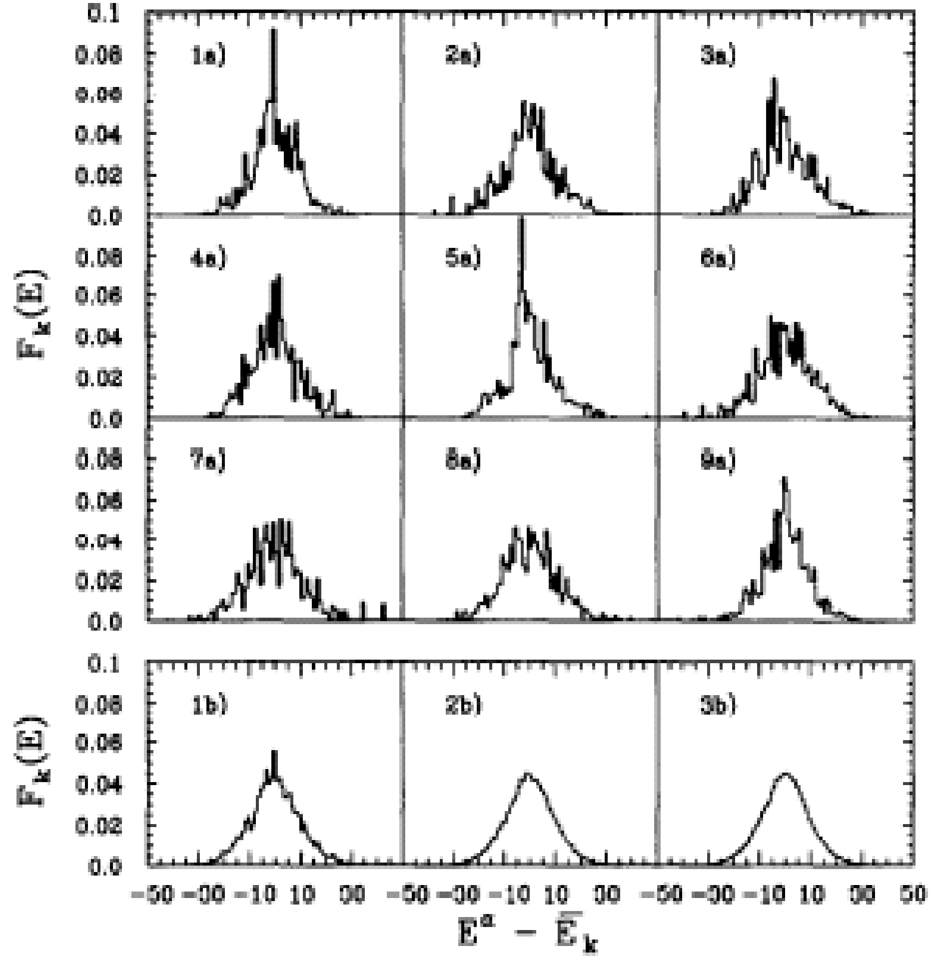
Chaos due to particle interactions at high level density

INSIDE CHAOS

I. Percival, J. Phys. **B6** (1973) L229

- *DISORDERED* wave functions
- Any *SIMPLE* operator has matrix elements of the same order of magnitude between any two of these eigenfunctions
- All typical wave functions of roughly the same energy *LOOK ROUGHLY THE SAME* being spread over the large region of configuration space

Random matrix canonical ensembles – only as mathematical limit



9 INDIVIDUAL STATES

AVERAGE OVER

10, 100, 400 STATES

STRENGTH FUNCTION

$F_k(E) = \sum_\alpha (C_k^\alpha)^2 \delta(E - E_\alpha)$

Local density of states in condensed matter physics

FAMILY OF ENTROPIES FOR A MESOSCOPIC SYSTEM

- **THERMODYNAMIC** (*Boltzmann*)

$\rho(E) \propto \exp(S_{\rm th})$

- **QUASIPARTICLE** (*Landau Fermi-liquid*)

$S^{\alpha}_{\rm s.p.} = -\sum_i \{ n_i^{\alpha} \ln(n_i^{\alpha})$
$+ (1 - n_i^{\alpha}) \ln(1 - n_i^{\alpha}) \}$

- **INFORMATION** (*Shannon*)

$|\alpha\rangle = \sum_k C_k^{\alpha} |k\rangle, \quad S^{\alpha}_{\rm inf} = -\sum_k \{ |C_k^{\alpha}|^2 \ln |C_k^{\alpha}|^2 \}$

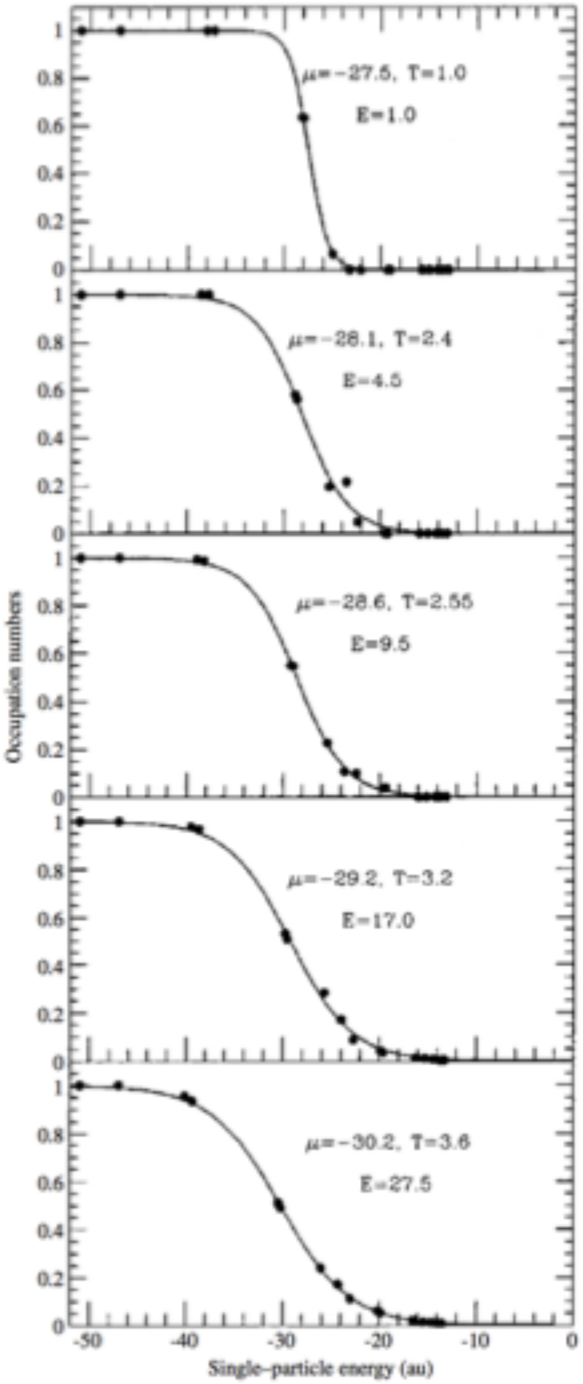
$$\langle n_i \rangle_E = [e^{(\epsilon_i - \mu)/T_{\rm s.p.}} + 1]^{-1}$$

Temperature T(E)

$$T_{\rm th} = \left(\frac{dS_{\rm th}}{dE} \right)^{-1}$$

$$T_{\rm inf} = \left(\frac{d\bar{S}_{\rm inf}}{dE} \right)^{-1}$$

T(s.p.) and T(inf) =
for individual states !



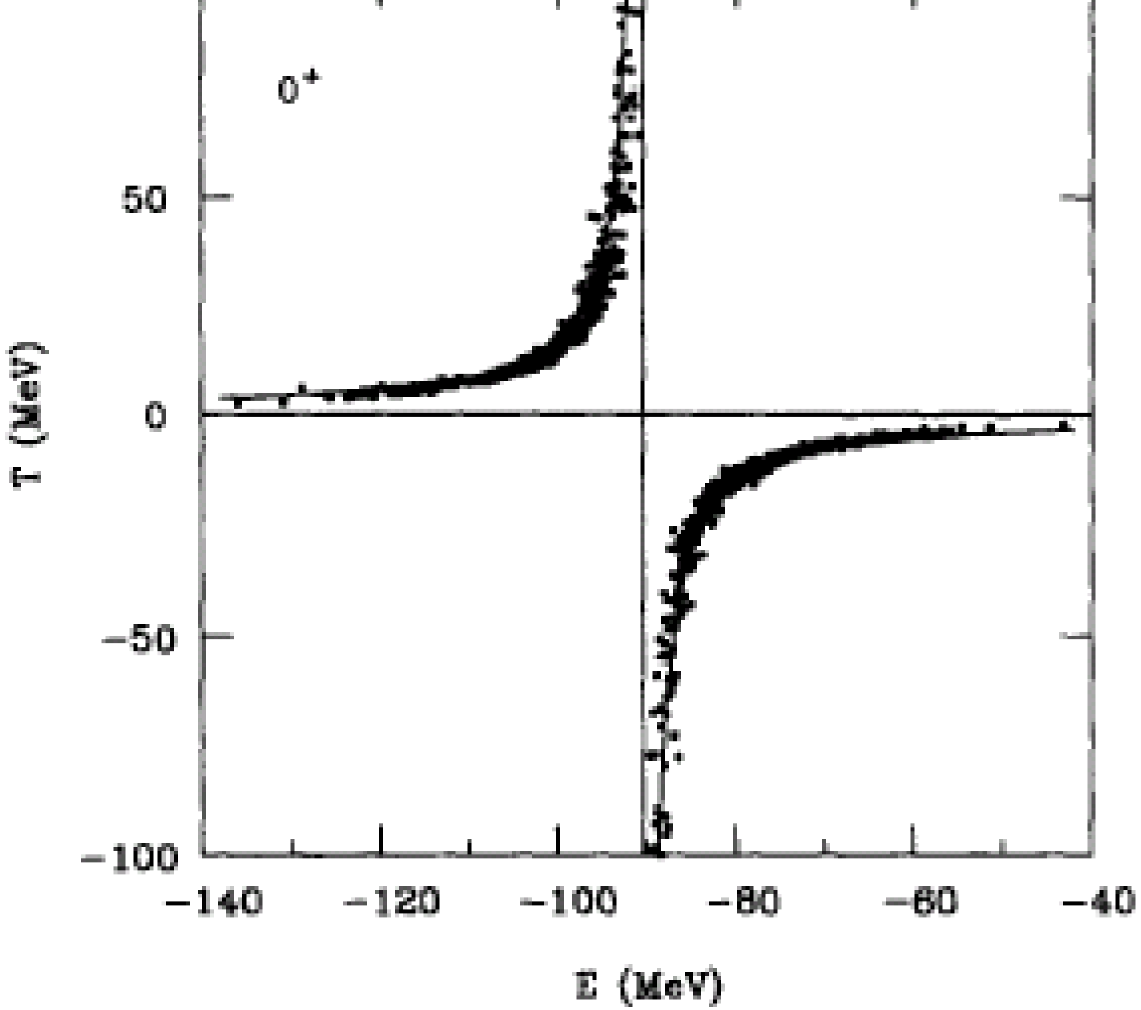
Occupation numbers in multicharged ions Au25+

(recombination as analog of neutron resonances in nuclei)

$$n_s^\alpha = \langle \alpha | \hat{n}_s | \alpha \rangle = \sum_k \left| C_k^\alpha \right|^2 \langle k | \hat{n}_s | k \rangle$$

/G. Gribakin, A. Gribakina, V. Flambaum/

Average over individual states is equivalent to a thermal ensemble



Gaussian level density

CENTROID E_0

WIDTH σ_E

Microcanonical temperature

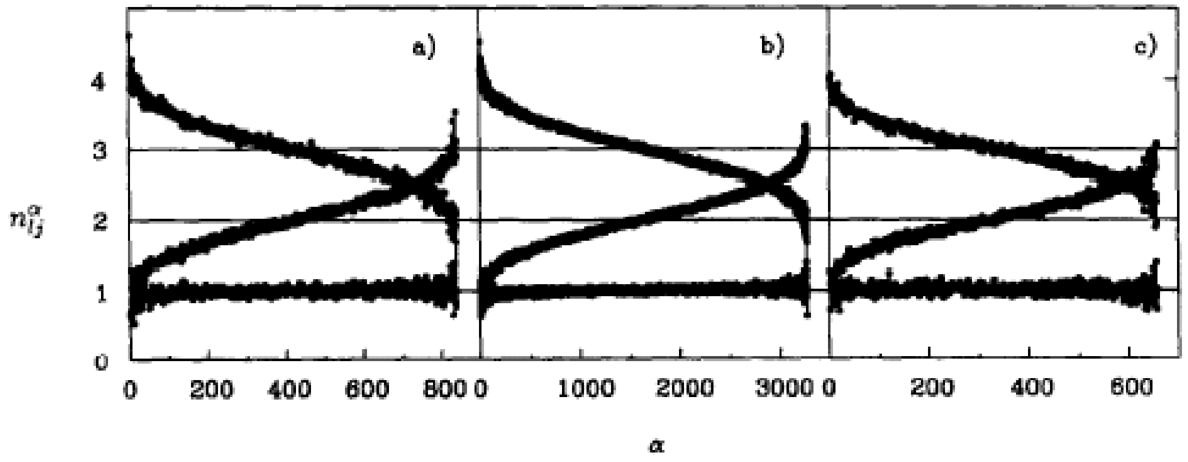
$$T_{\mathrm{th}} = \sigma_E^2/(E_0 - E)$$

839 states (28 Si) J=0

EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution (**dots**)

From thermodynamic entropy defined by level density (**lines**) d5/2, d3/2, s1/2



28 Si

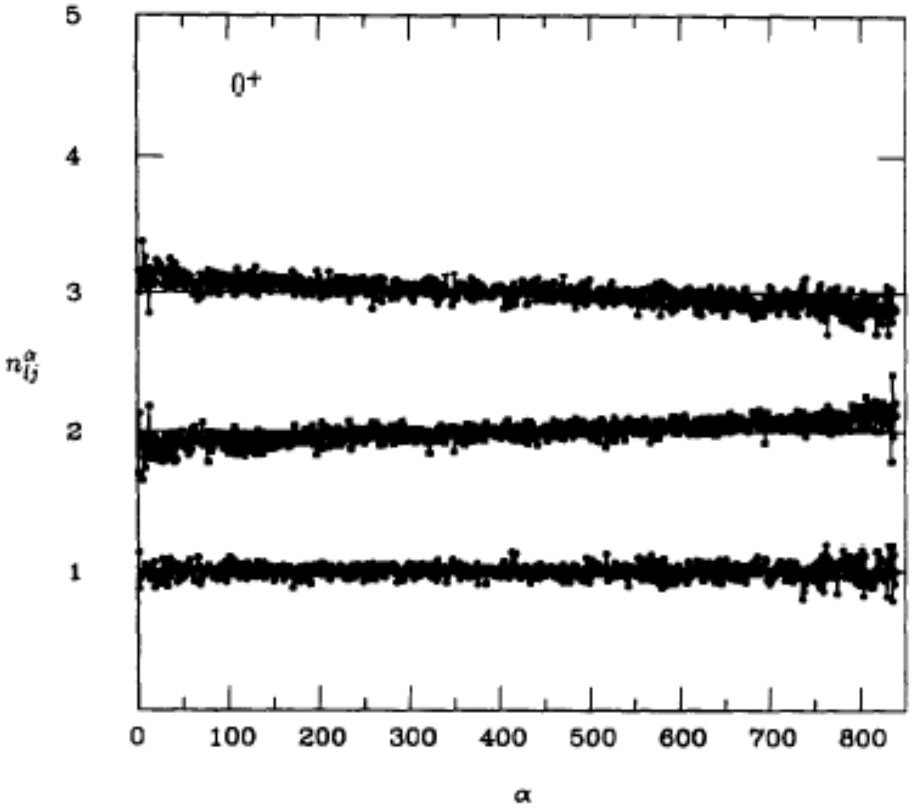
J=0 *J=2* *J=9*

Single – particle occupation numbers

Thermodynamic behavior
identical in all symmetry classes

FERMI-LIQUID PICTURE

J=0



Artificially strong interaction (factor of 10)

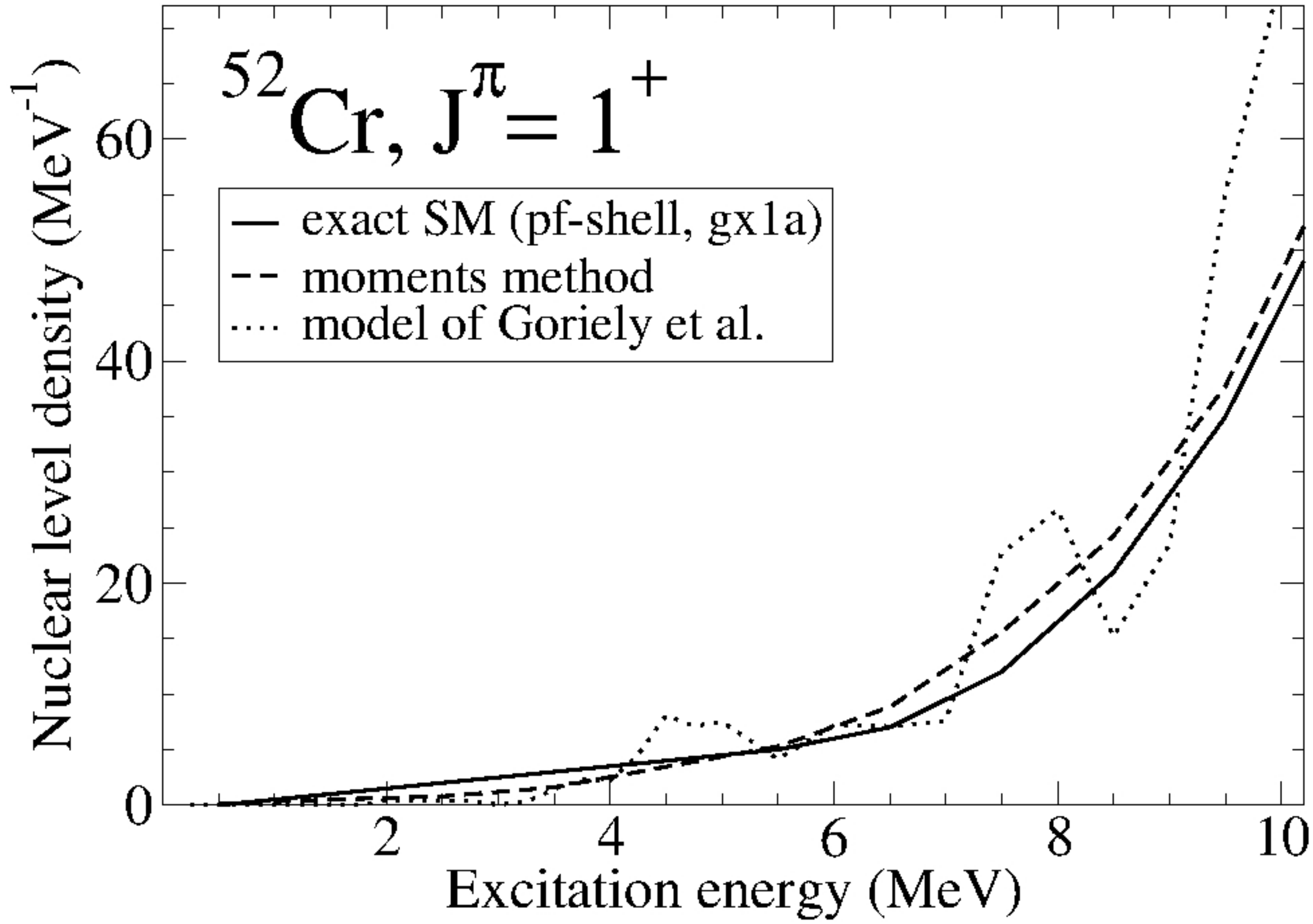
Single-particle thermometer cannot resolve spectral evolution

MEAN FIELD COMBINATORICS

S. Goriely et al. Phys. Rev. C 78, 064307 (2008)
C 79, 024612 (2009)

http://www.astro.ulb.ac.be/pmwiki/Brusslin/Level

Hartree – Fock – Bogoliubov plus
Collective enhancement with certain phonons



^{52}Cr, $J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
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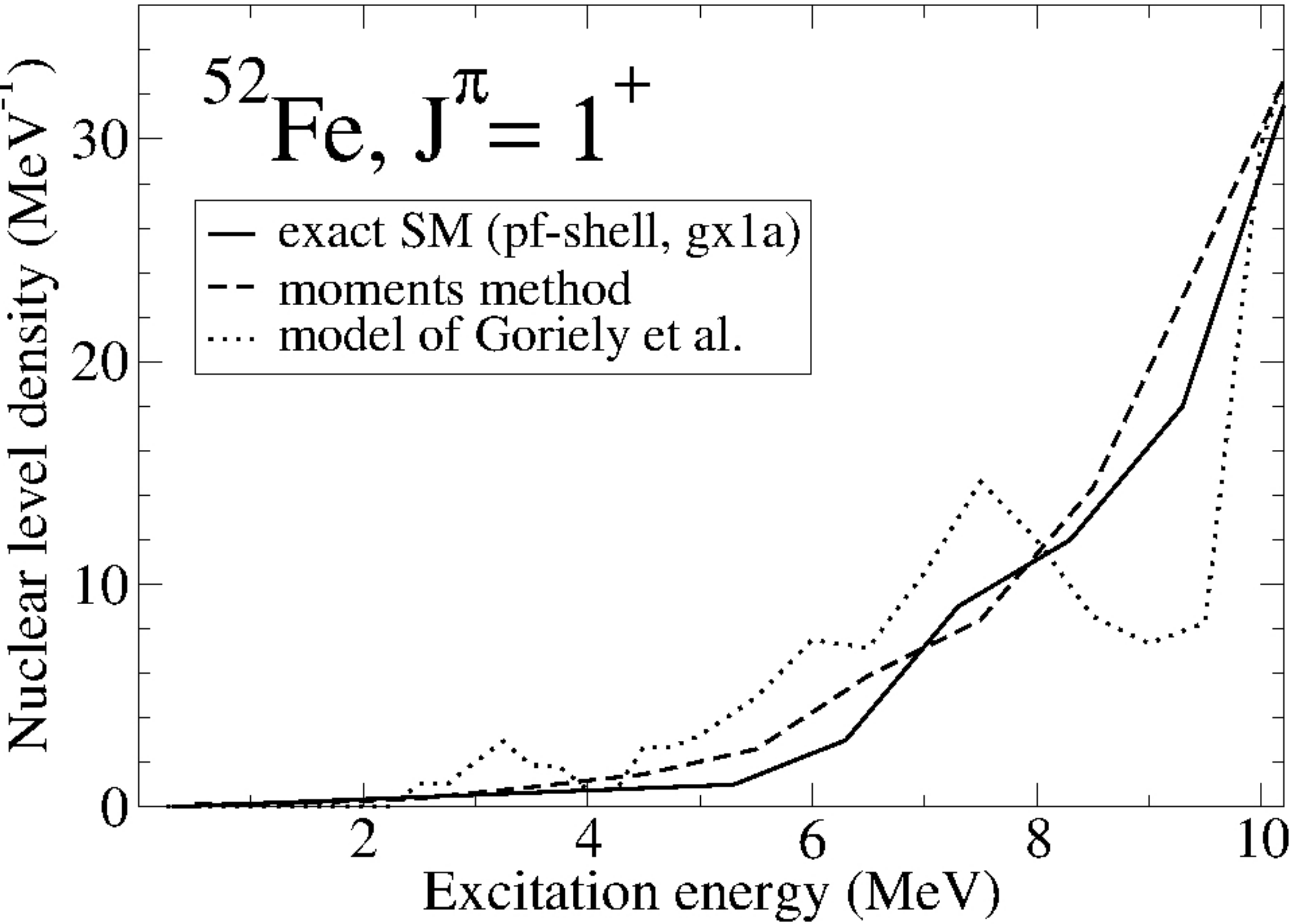

$^{52}Fe, J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
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Nuclear level density (MeV^{-1})
Excitation energy (MeV)
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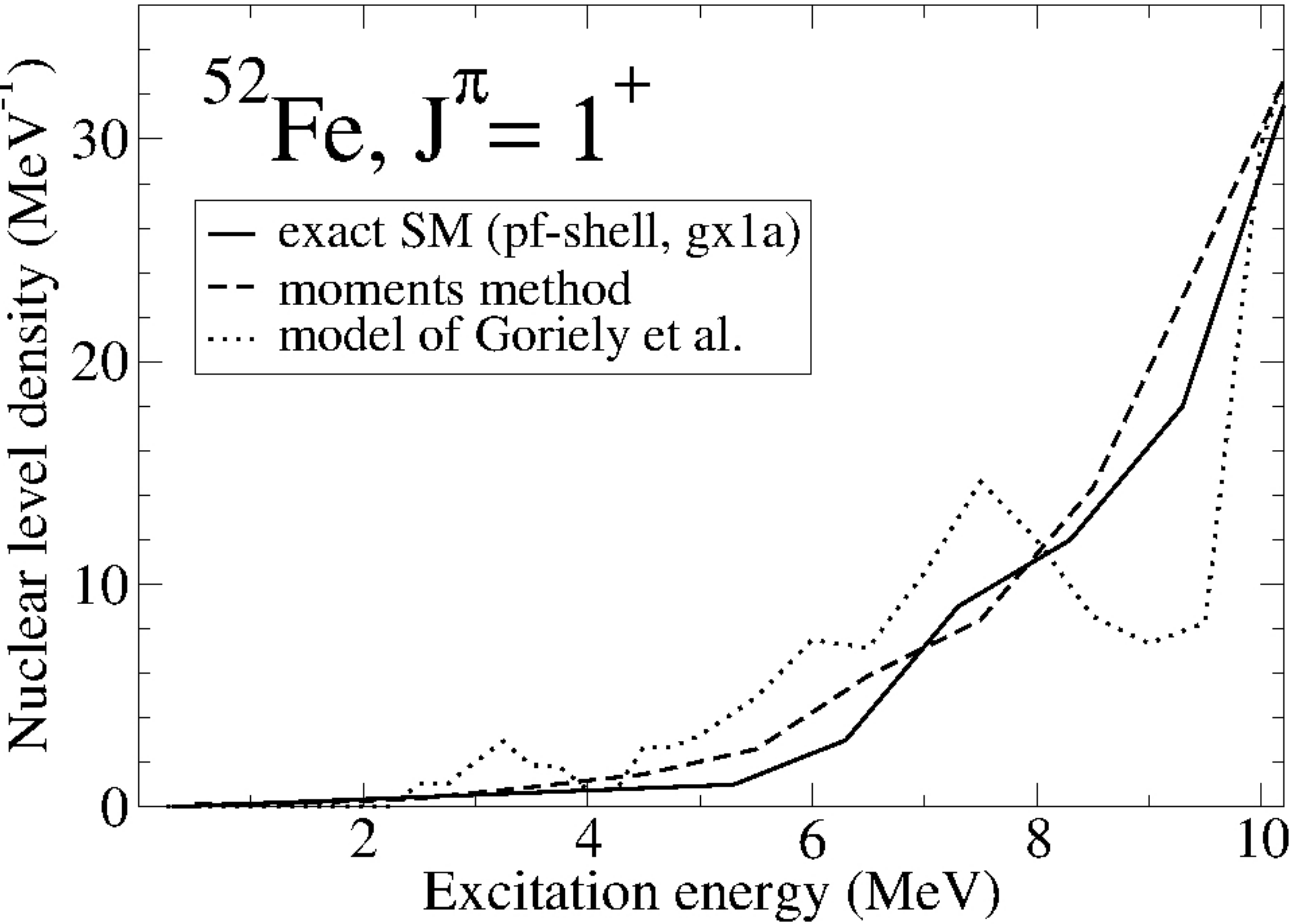

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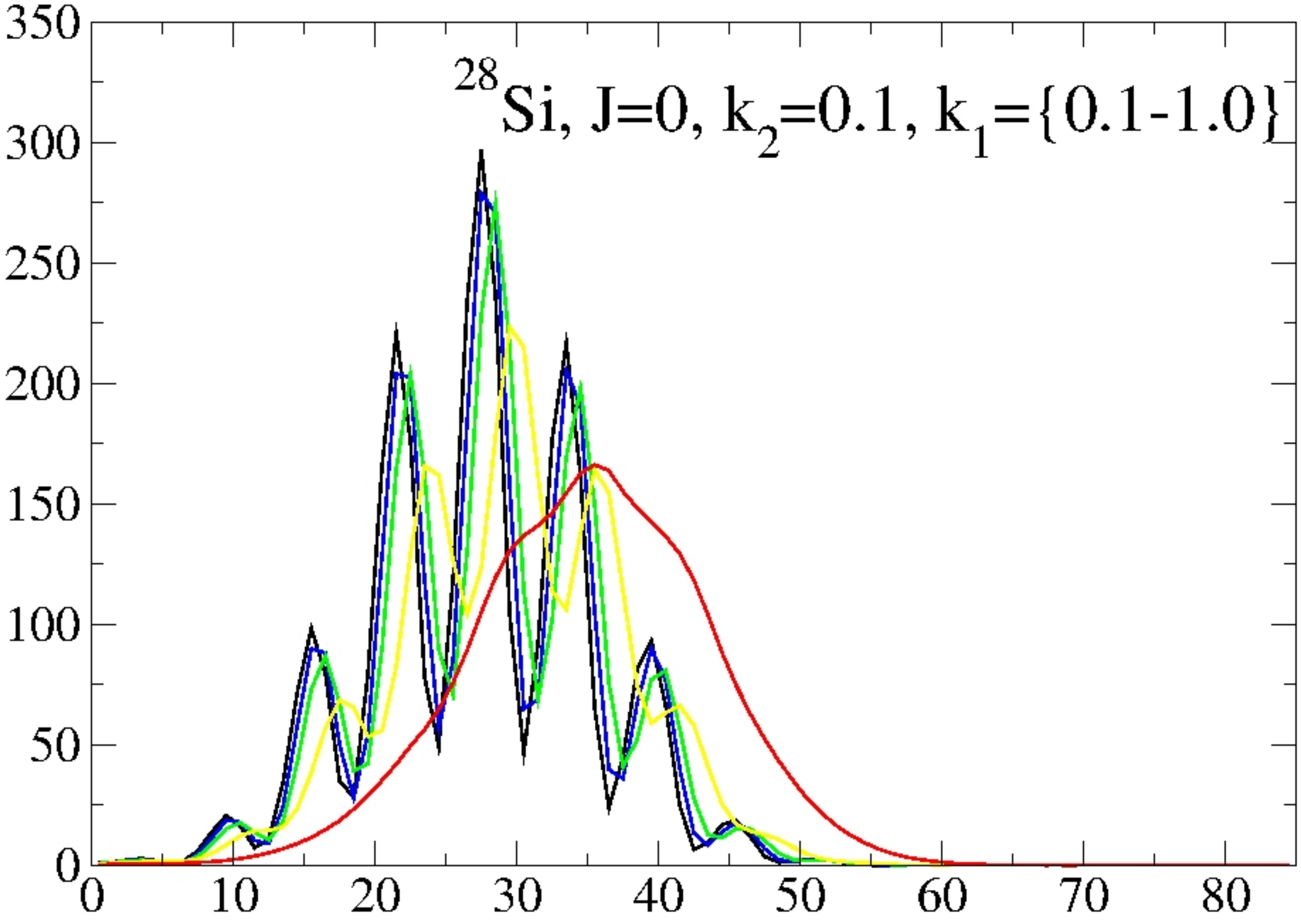
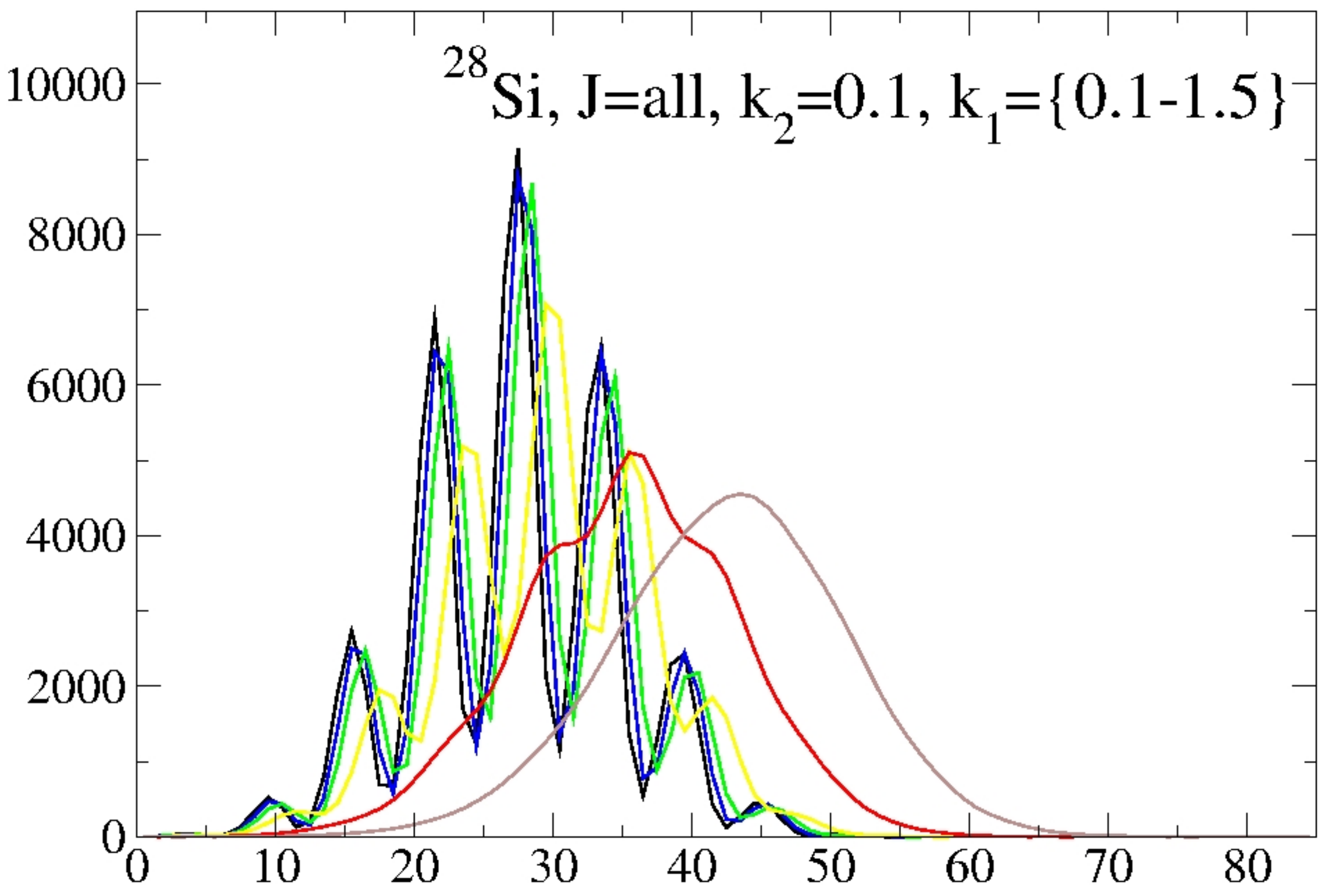
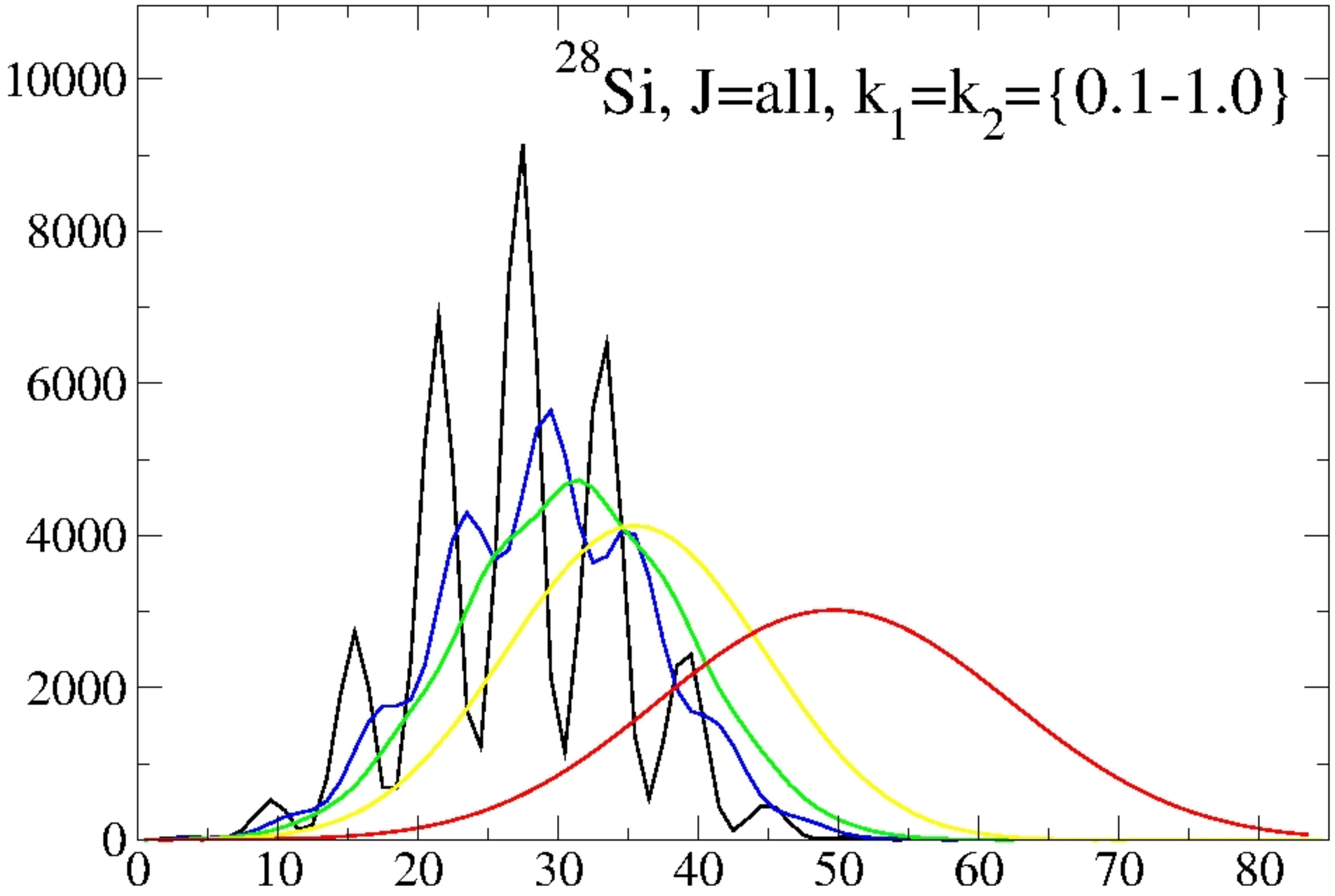


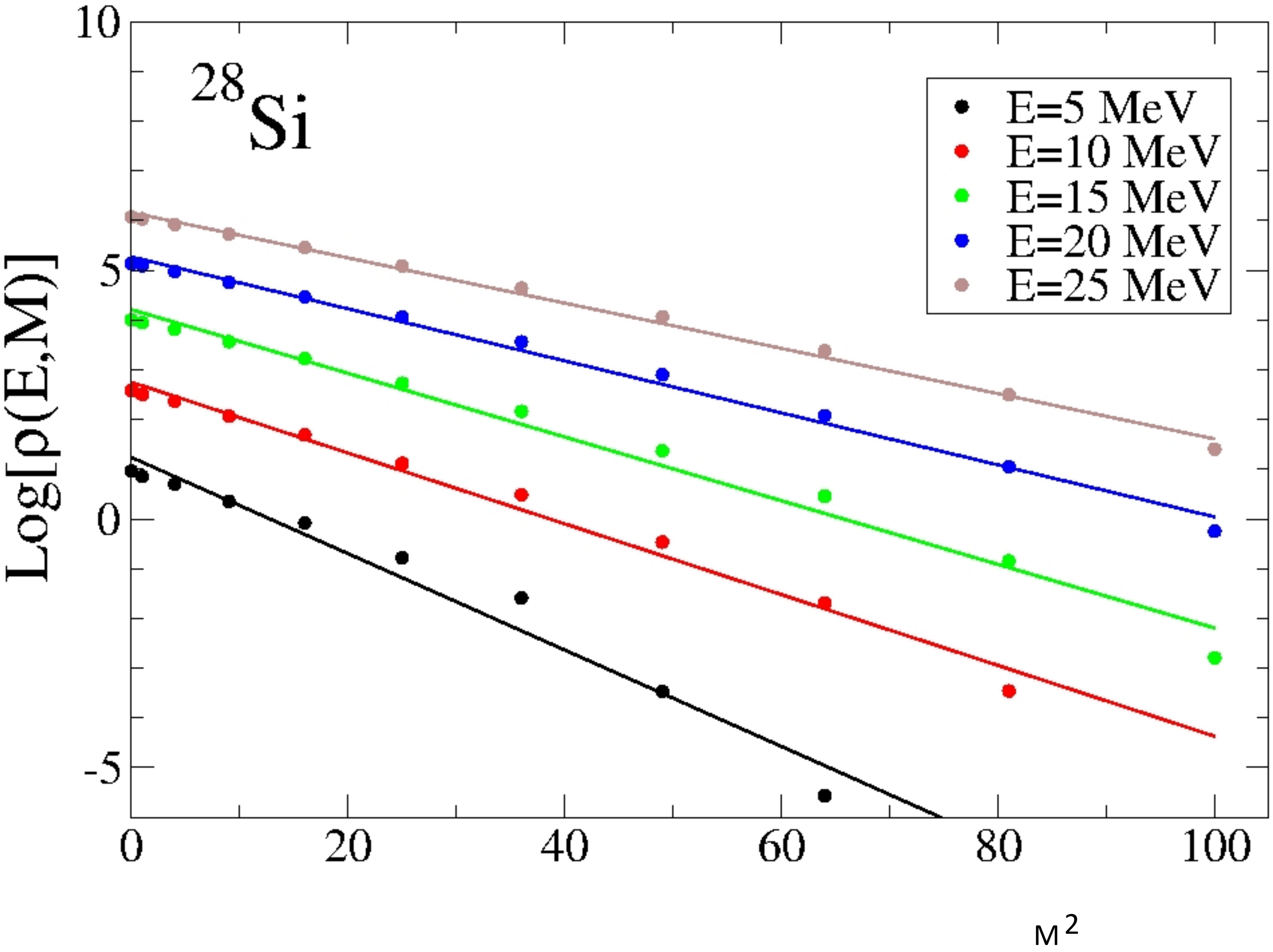
^{28}Si, J=0, k_2=0.1, k_1={0.1-1.0}
0
50
100
150
200
250
300
350
0
10
20
30
40
50
60
70
80



^{28}Si, J=all, k_2=0.1, k_1={0.1-1.5}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80



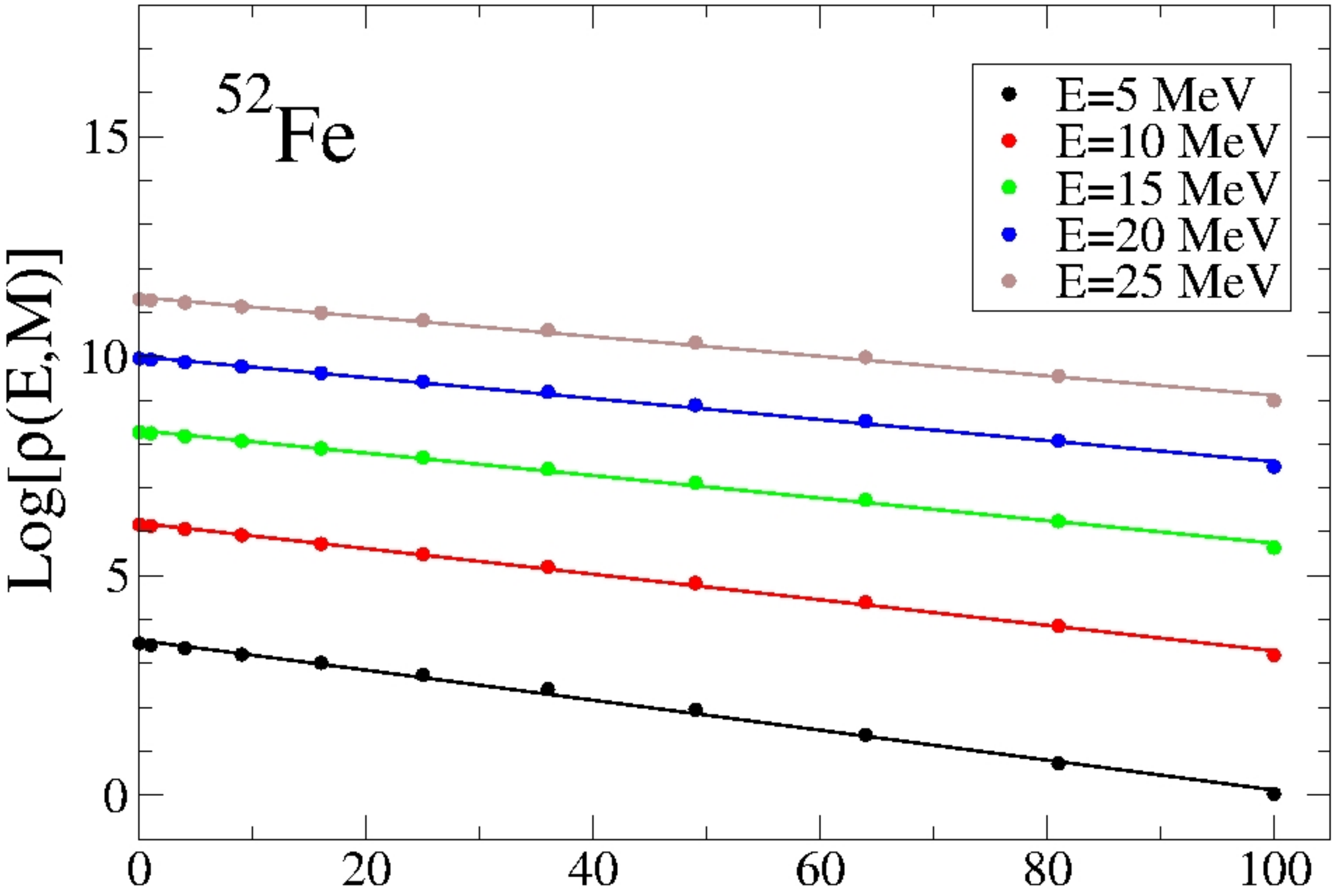
^{28}Si, J=all, k_1=k_2={0.1-1.0}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80



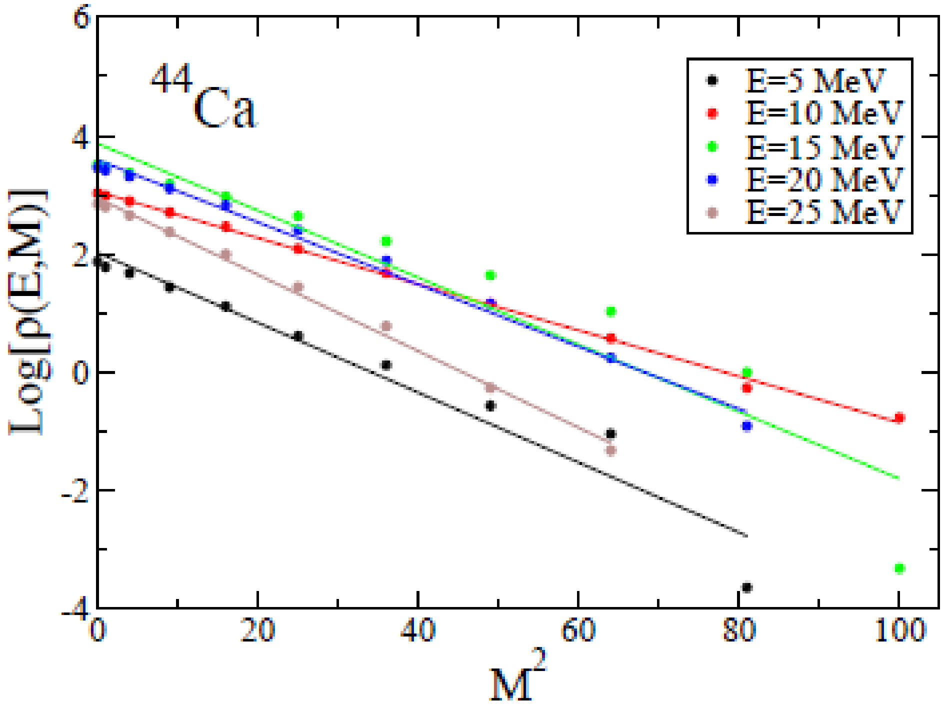
"Spin cut-off" parameter

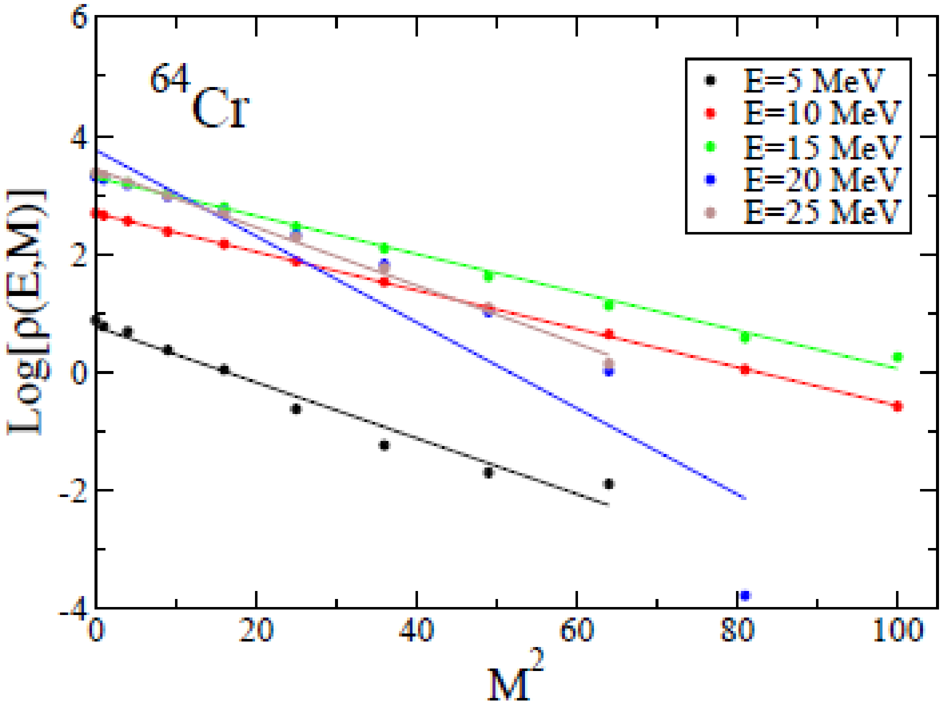
$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$

Markovian
random process
of angular momentum
coupling



$$\frac{\rho(E,M)}{\rho(E)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-M^2/2\sigma^2}$$

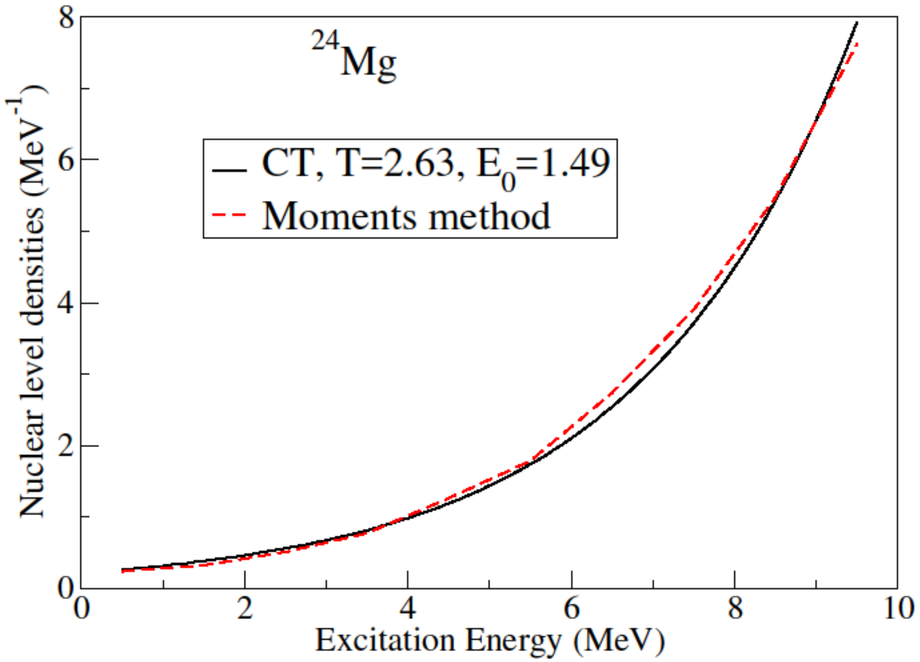


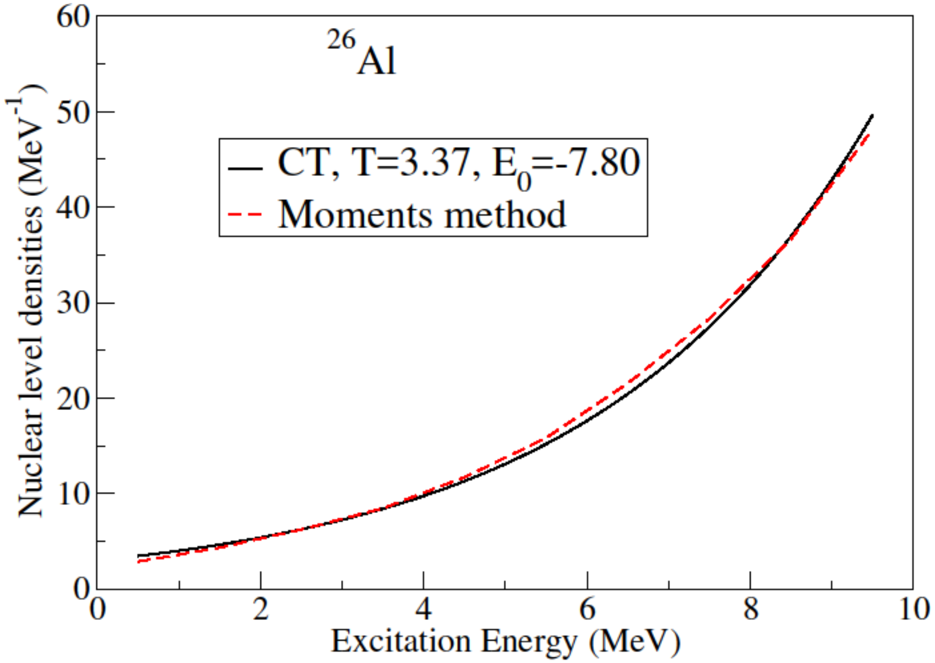


4 valence neutrons

4 proton holes

Space – only T=2,
Two-body interaction through T=1 channel





CONSTANT TEMPERATURE PHENOMENOLOGY

Level density (const) exp(E/T)

$$T_{t-d} = \left(\frac{\partial S}{\partial E}\right)^{-1} = T\left(1 - e^{-E/T}\right)$$

Partition function = **Trace{exp[-H/T(t-d)]} diverges** at **T > T(t-d)**

Cumulative level number

$N(E) = \exp(S)$,

Entropy **$S(E)= \ln(N)$**

Thermodynamic temperature

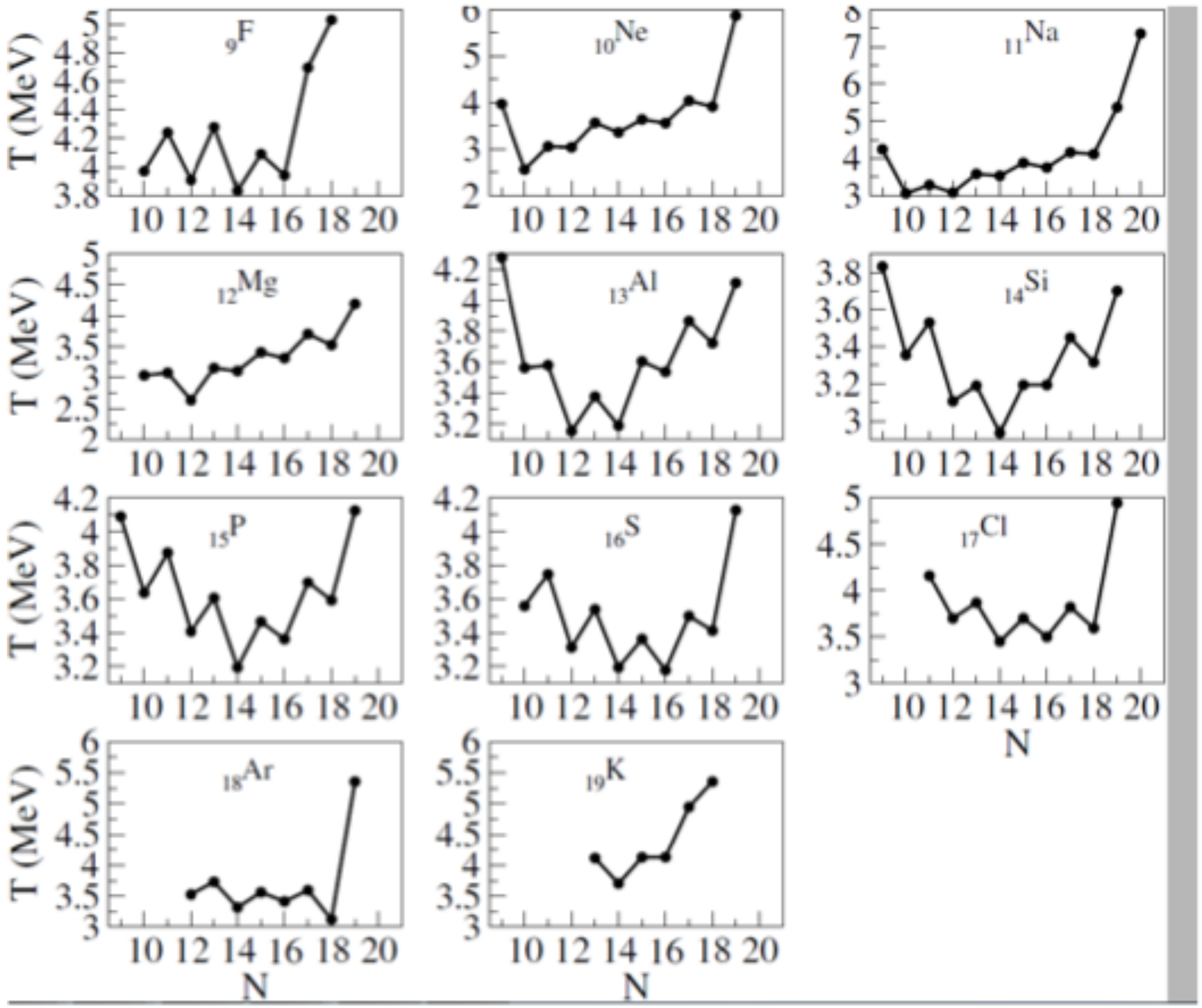
$T(t\text{-}d) = dS/dE = T[1 – \exp(- E/T)]$

Parameter T is ***limiting temperature***

(***Hagedorn temperature*** in particle physics)

Pairing phase transition? (Moretto) - Chaotization

1/T – rate of increase of the level density



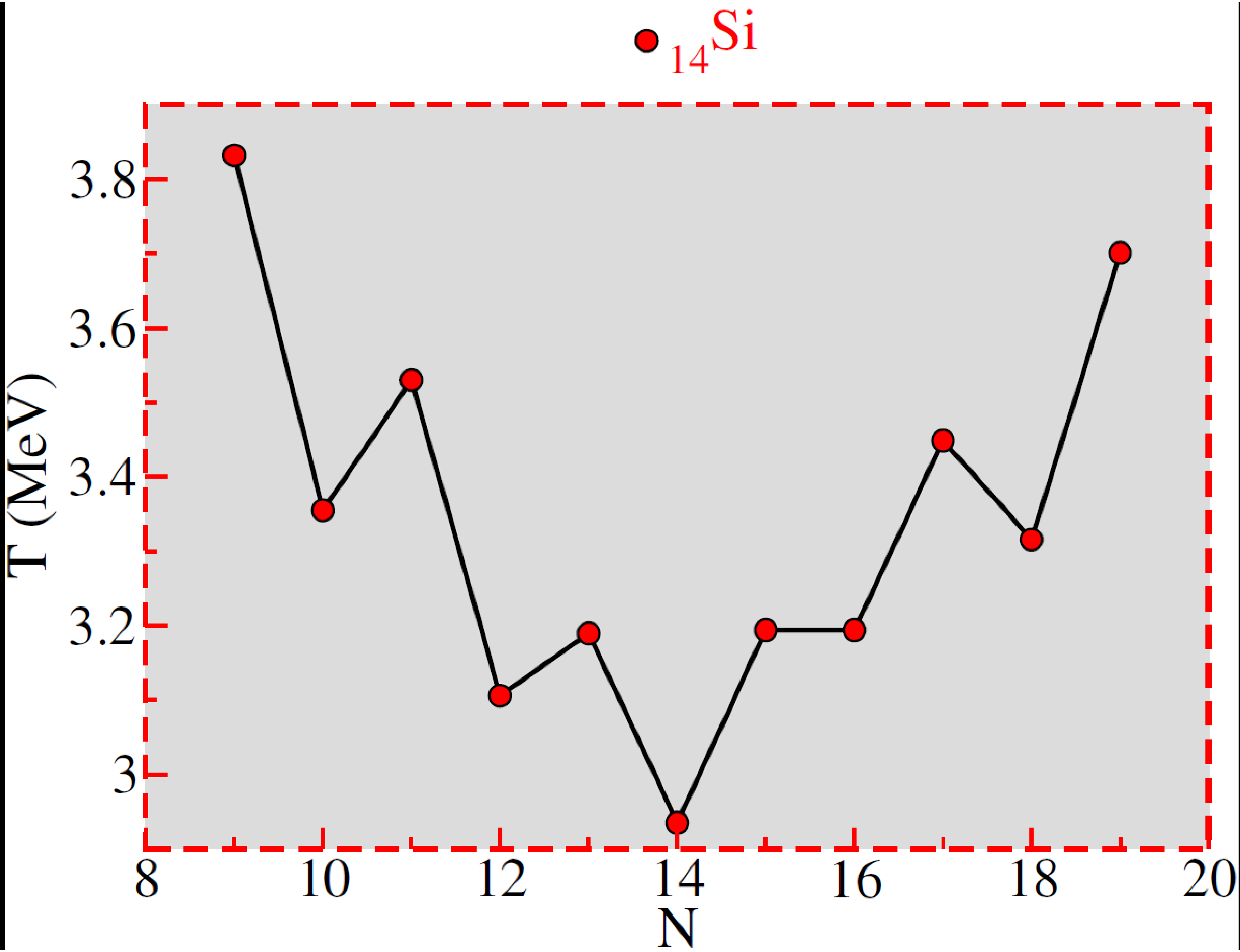
Effective temperature **T**

for **(sd)** – nuclei,

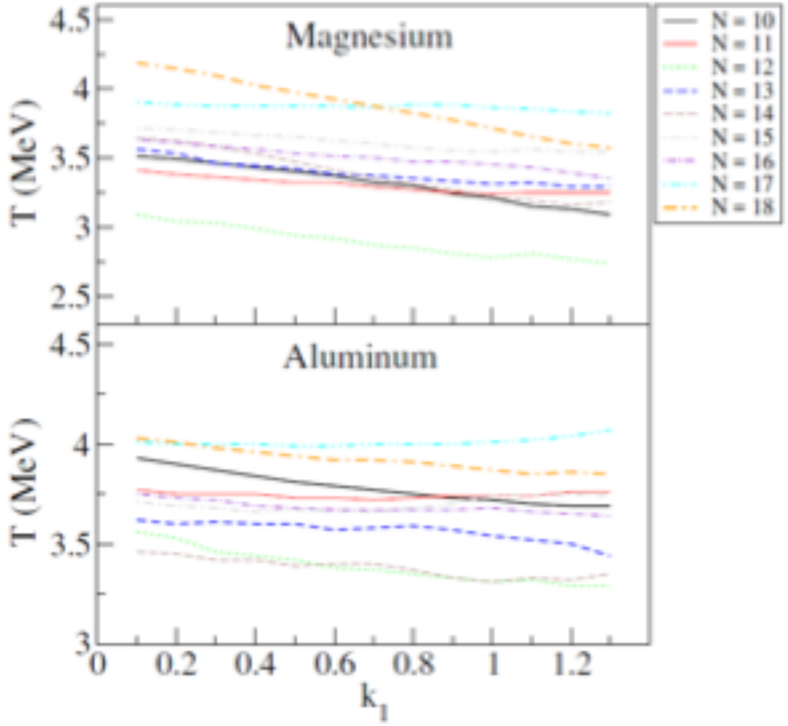
tabulated for all

classes of spin

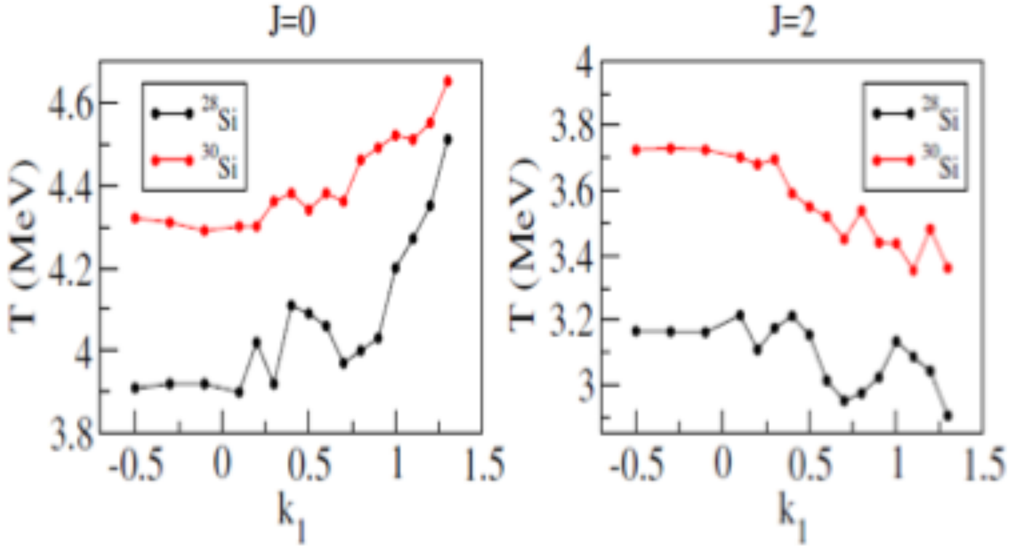
(ADNDT, 2018)



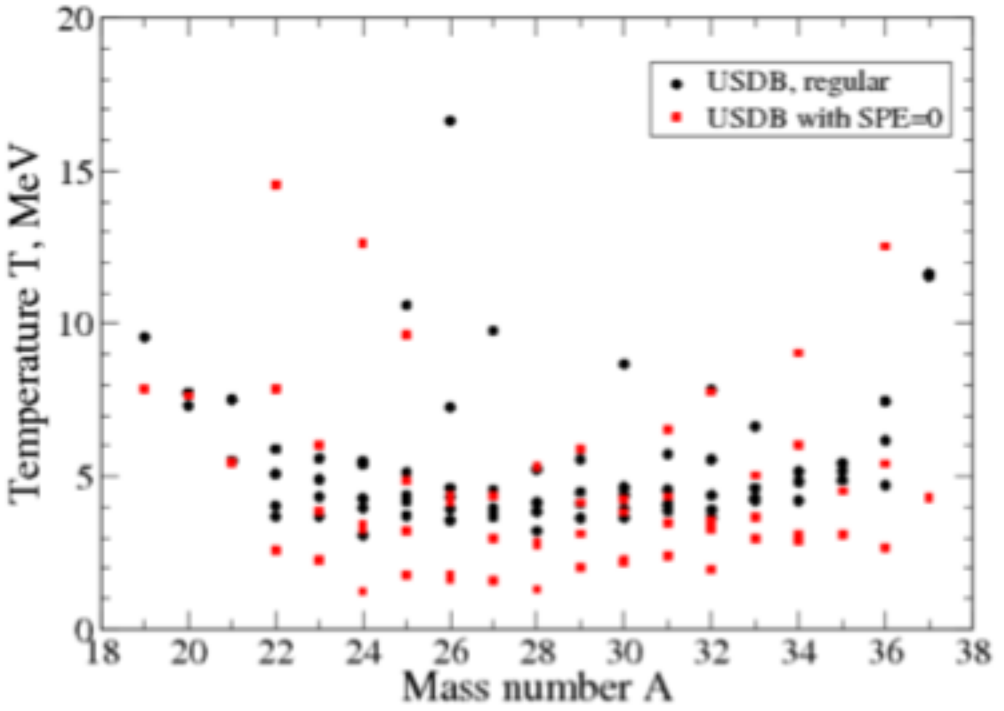
$_{14}$Si
T (MeV)
3.8
3.6
3.4
3.2
3
8
10
12
14
16
18
20
N

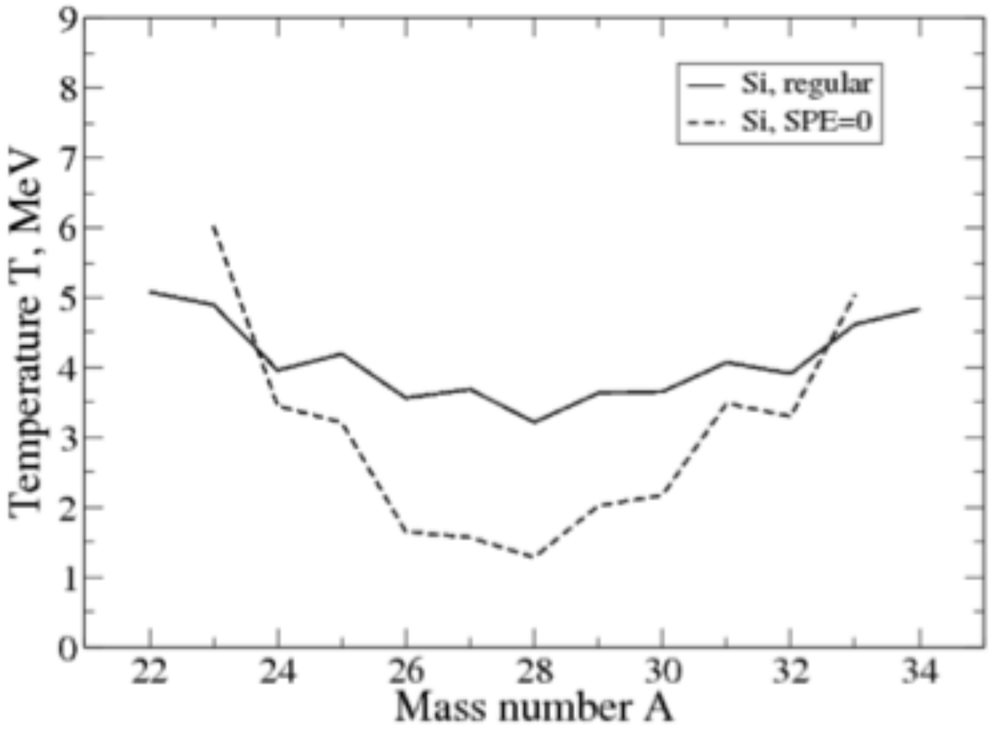


Eliminating pairing interaction

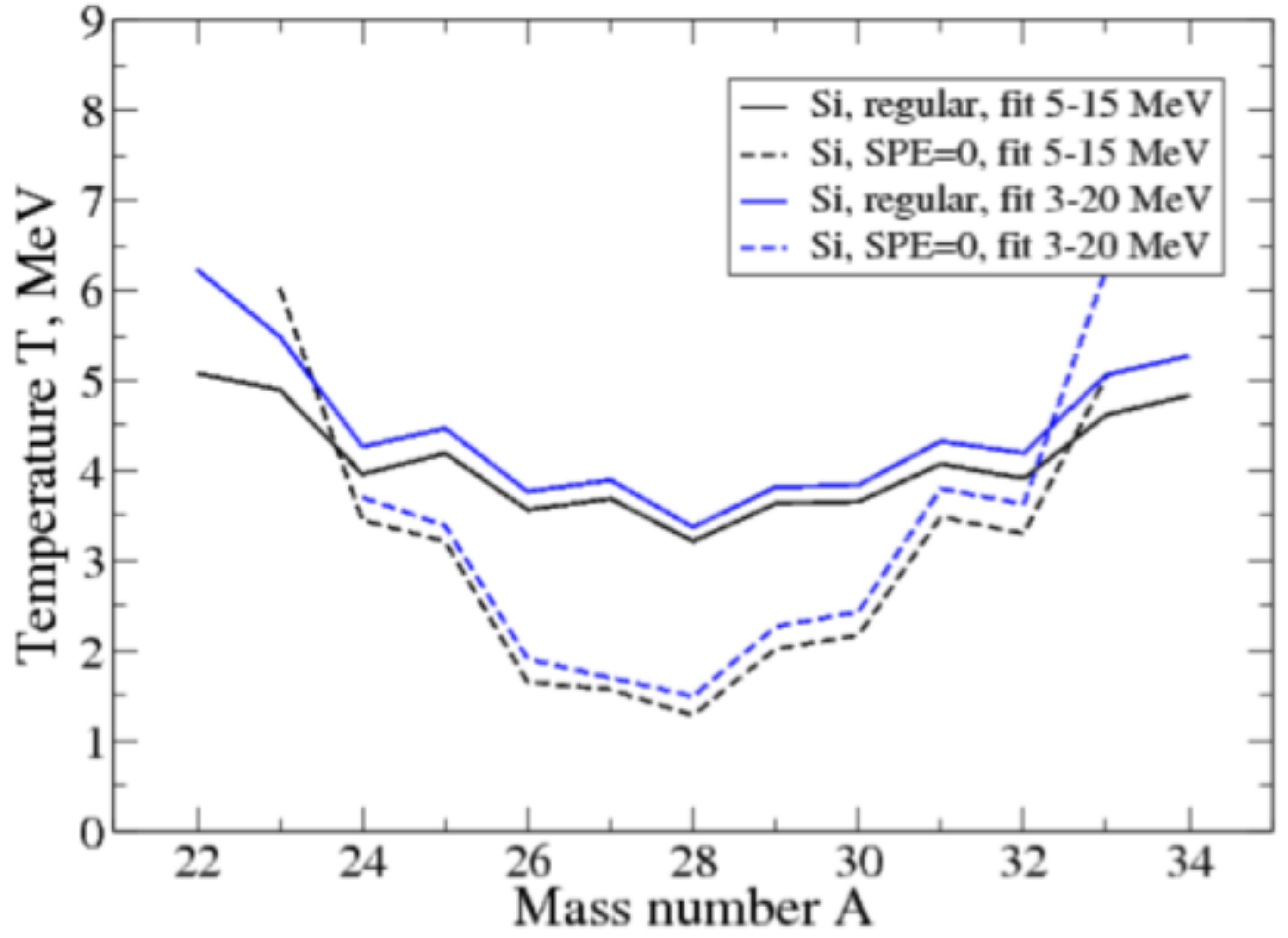


k(1) < 0 "antipairing"

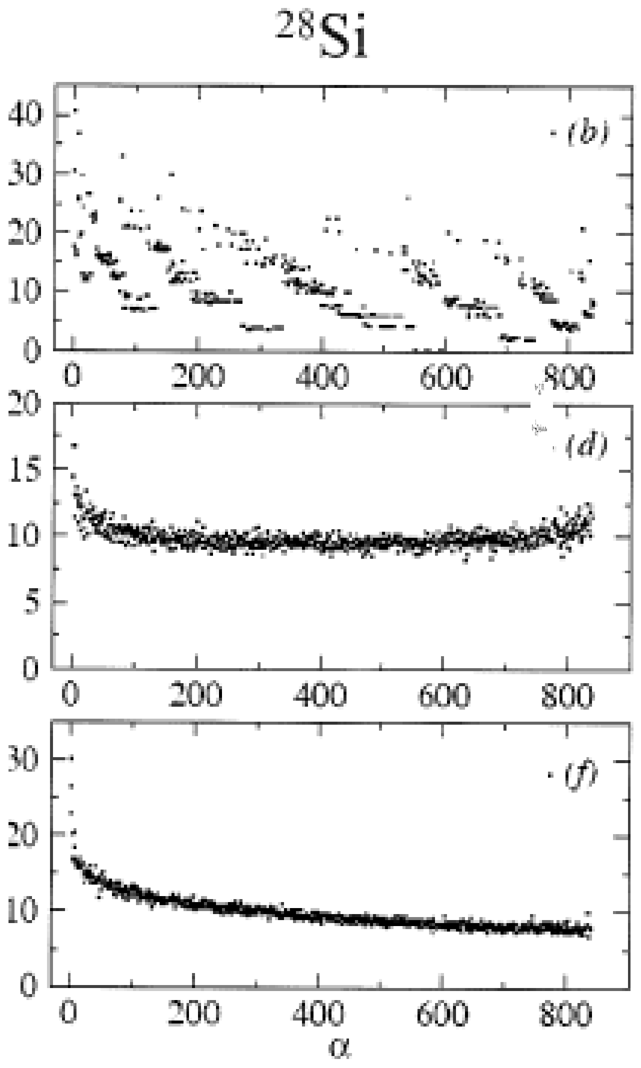




Degenerate single-particle levels – smaller T (faster chaotization)



Sensitivity to the fit interval



PAIR CORRELATOR

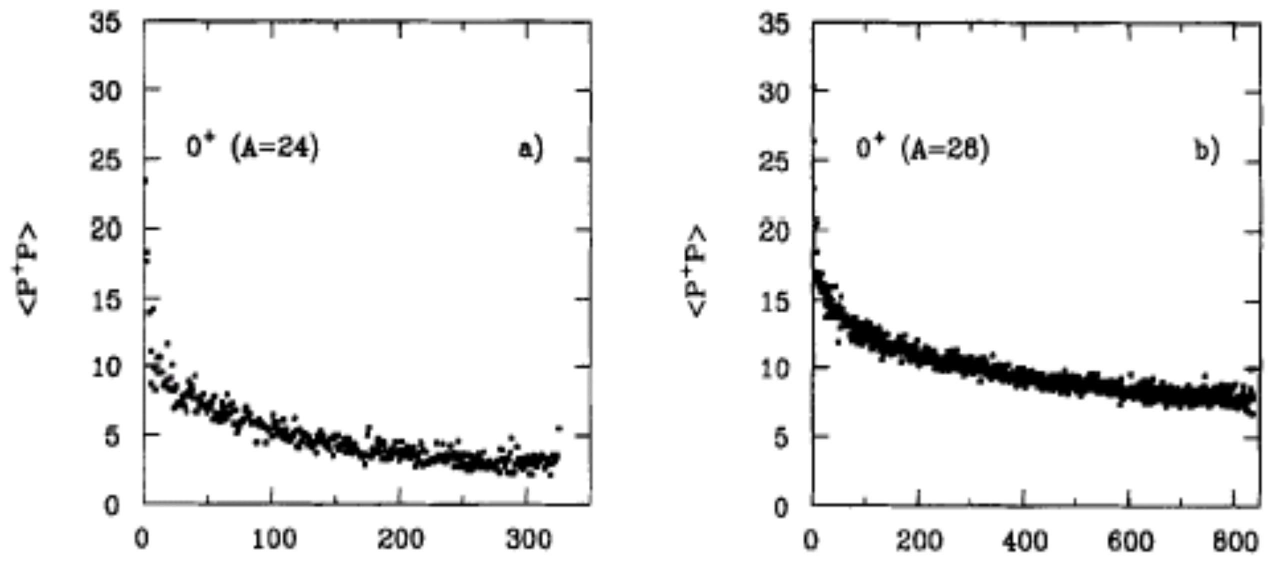
(b) Only pairing

(d) Non-pairing interactions

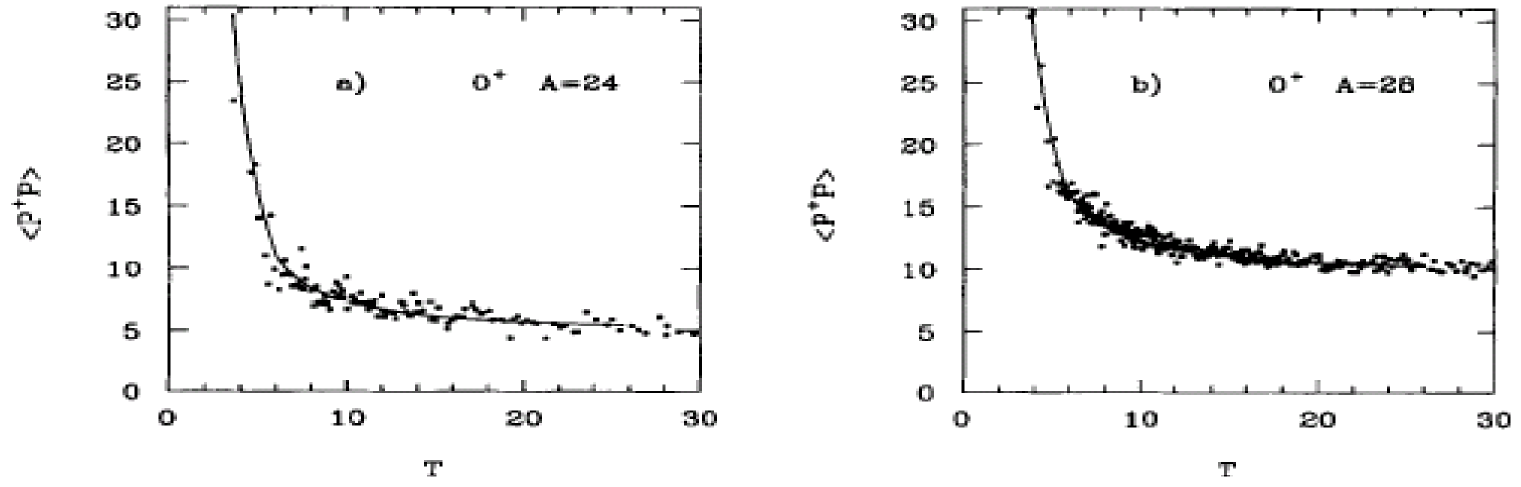
(f) All interactions

$$\mathcal{H}_P = \sum_{t=0,\pm 1} P_t^\dagger P_t$$

$$P_t = \frac{1}{\sqrt{2}} \sum_j [a_j a_j]_{J=0,T=1,T_3=t}$$

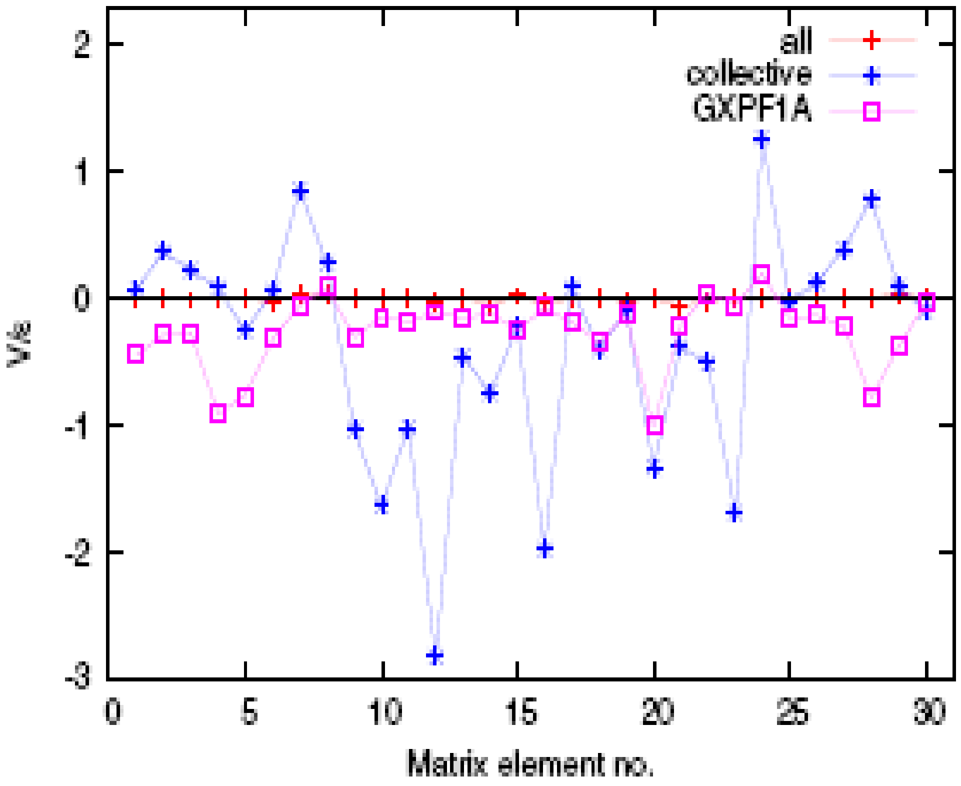


PAIRING PHASE TRANSITION



PAIR CORRELATOR as a THERMODYNAMIC FUNCTION

Strong interaction 4.0

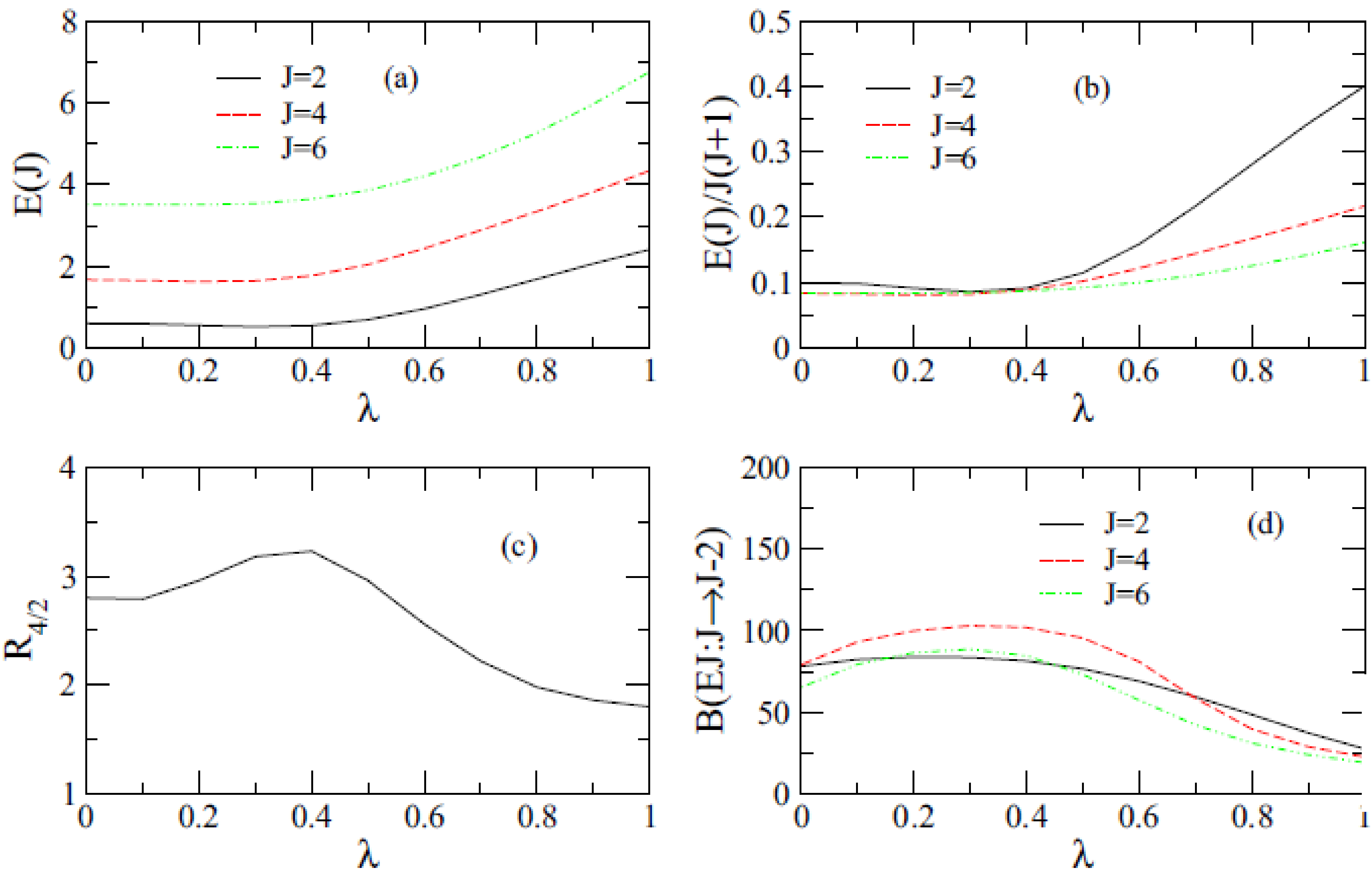


	$\langle j_1 j_2 \vert V \vert j_3 j_4 \rangle (JT)$	Full average	Prolate average
1	$\langle ff \vert V \vert ff \rangle (10)$	0.021	0.078
2	$\langle ff \vert V \vert ff \rangle (30)$	0.012	0.374
3	$\langle ff \vert V \vert ff \rangle (50)$	-0.007	0.227
4	$\langle ff \vert V \vert ff \rangle (70)$	0.007	0.089
5	$\langle ff \vert V \vert ff \rangle (01)$	0.008	-0.252
6	$\langle ff \vert V \vert ff \rangle (21)$	-0.020	0.062
7	$\langle ff \vert V \vert ff \rangle (41)$	0.026	0.869
8	$\langle ff \vert V \vert ff \rangle (61)$	0.034	0.282
9	$\langle ff \vert V \vert pf \rangle (30)$	0.004	-1.033
10	$\langle ff \vert V \vert pf \rangle (50)$	0.022	-1.630
11	$\langle ff \vert V \vert pf \rangle (21)$	0.006	-1.010
12	$\langle ff \vert V \vert pf \rangle (41)$	-0.010	-2.826
13	$\langle ff \vert V \vert pp \rangle (10)$	0.014	-0.451
14	$\langle ff \vert V \vert pp \rangle (30)$	-0.043	-0.739
15	$\langle ff \vert V \vert pp \rangle (01)$	0.025	-0.223
16	$\langle ff \vert V \vert pp \rangle (21)$	-0.036	-1.977
17	$\langle pf \vert V \vert pf \rangle (20)$	0.007	0.088
18	$\langle pf \vert V \vert pf \rangle (30)$	0.010	-0.393
19	$\langle pf \vert V \vert pf \rangle (40)$	-0.018	-0.092
20	$\langle pf \vert V \vert pf \rangle (50)$	0.004	-1.328
21	$\langle pf \vert V \vert pf \rangle (21)$	-0.052	-0.376
22	$\langle pf \vert V \vert pf \rangle (31)$	-0.019	-0.507
23	$\langle pf \vert V \vert pf \rangle (41)$	0.011	-1.685
24	$\langle pf \vert V \vert pf \rangle (51)$	-0.003	1.276
25	$\langle pf \vert V \vert pp \rangle (30)$	0.007	-0.023
26	$\langle pf \vert V \vert pp \rangle (21)$	0.014	0.133
27	$\langle pp \vert V \vert pp \rangle (10)$	0.003	0.400
28	$\langle pp \vert V \vert pp \rangle (30)$	0.003	0.779
29	$\langle pp \vert V \vert pp \rangle (01)$	0.054	0.102
30	$\langle pp \vert V \vert pp \rangle (21)$	0.005	-0.092

Matrix elements

9-12: pf mixing,
16 : quadrupole pair transfer,
20-24: quadrupole-quadrupole forces
in particle-hole channel = formation of the mean field

Large fluctuations of non-extensive nature (the same for 10 000 and 100 000 realizations)



24 Mg

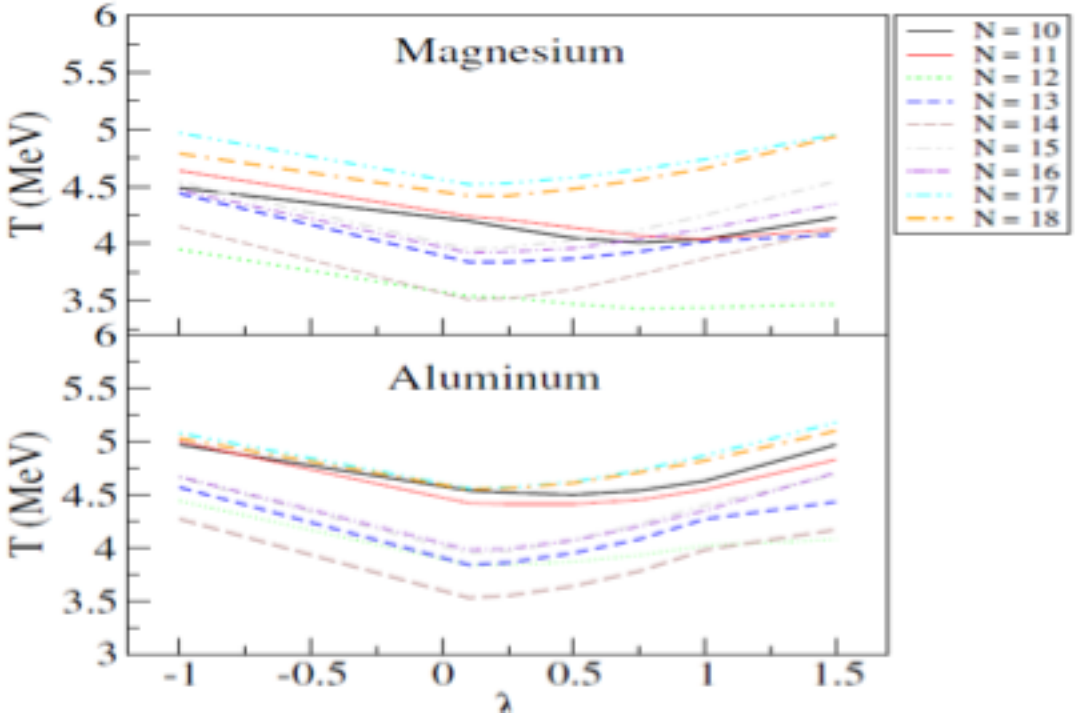
Low-lying levels
in absolute (a)
and rotational (b)
units;

Ratio E(4)/E(2) (c)

Transition rates (d)

$$H = h + (1-\lambda)V_1 + \lambda V_2$$

V(1) = matrix elements of the two-body interaction
with change of orbital momentum of one particle
by 2 units (the same parity) – way to deformation



V(1) = matrix elements of the two-body interaction
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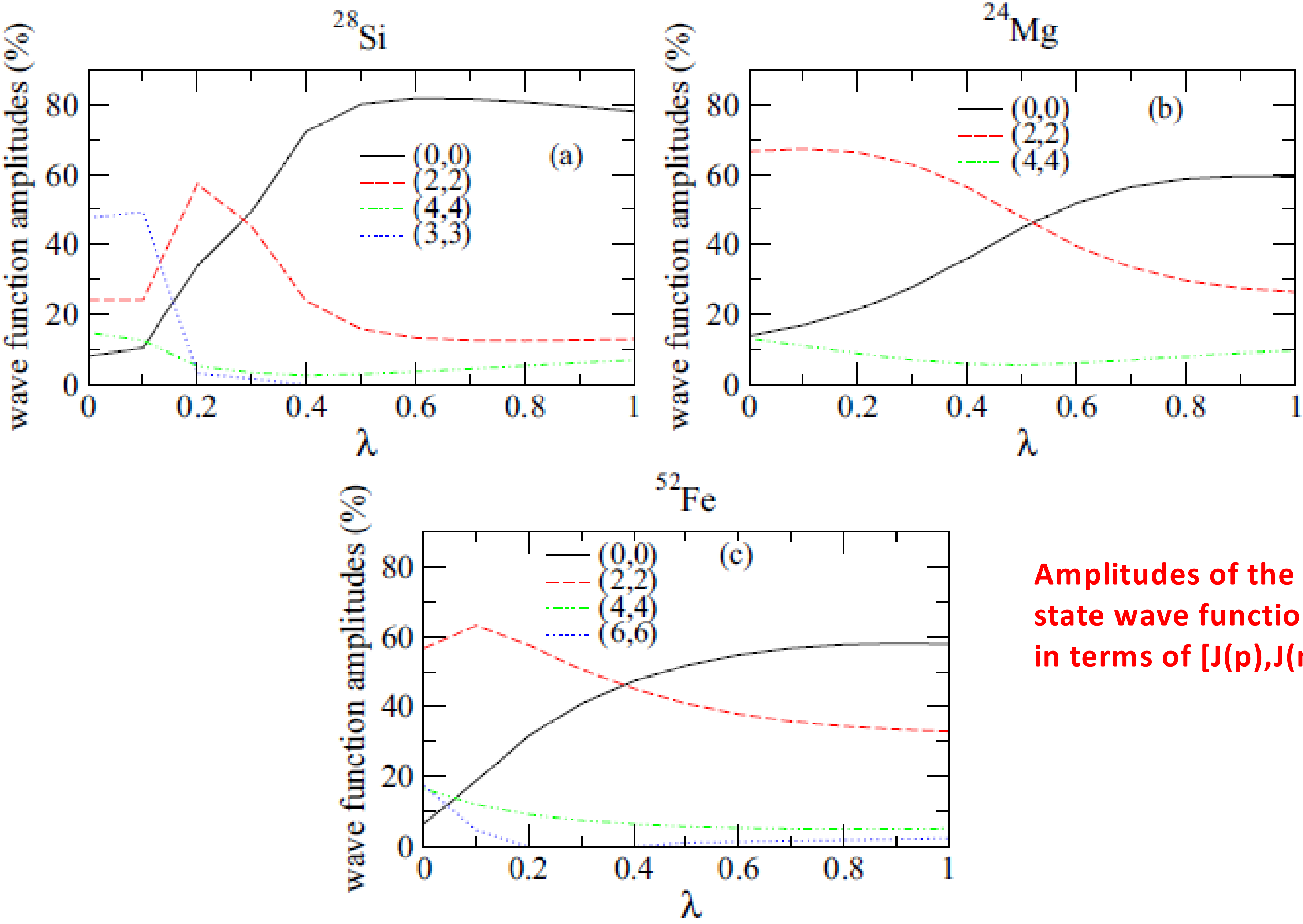
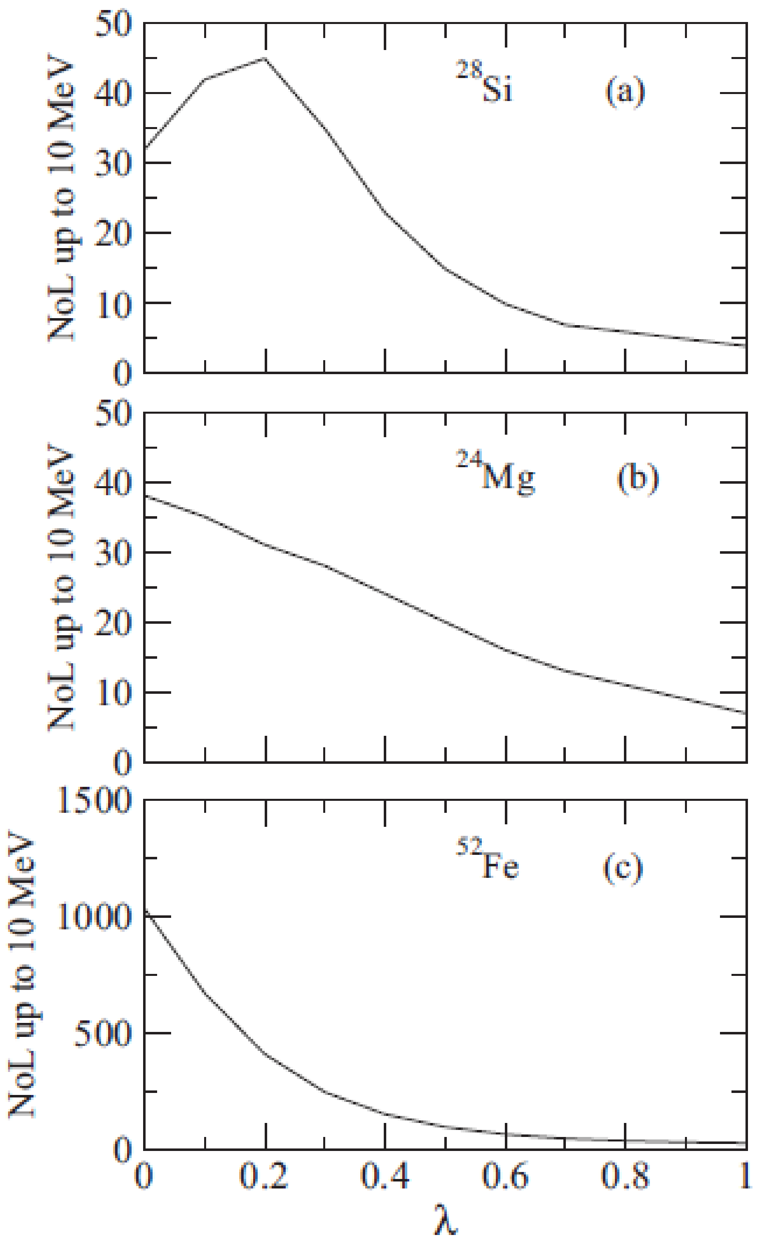
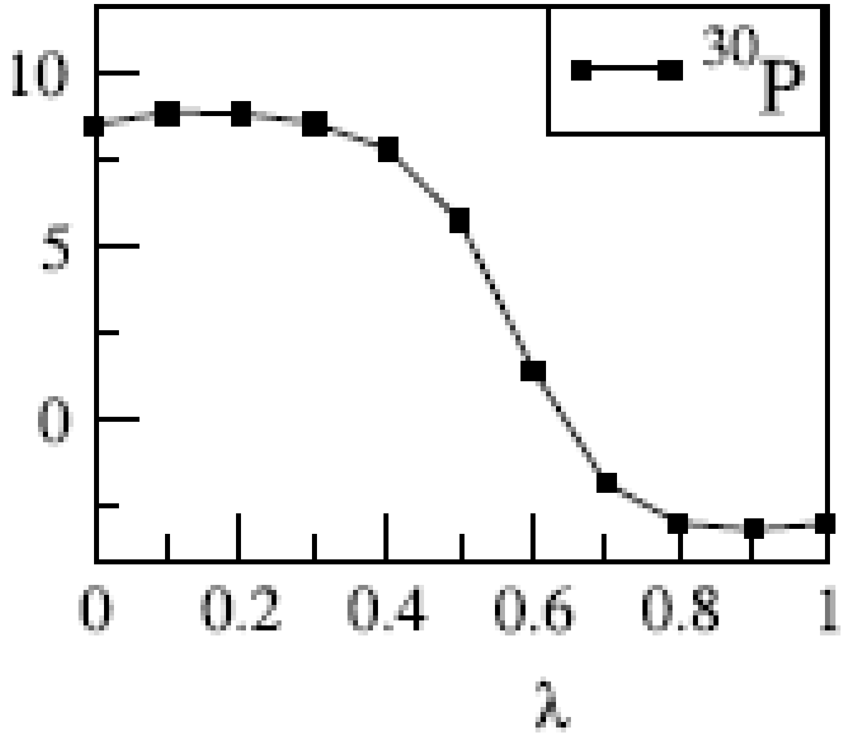


Amplitudes of the ground state wave functions in terms of [J(p),J(n)]



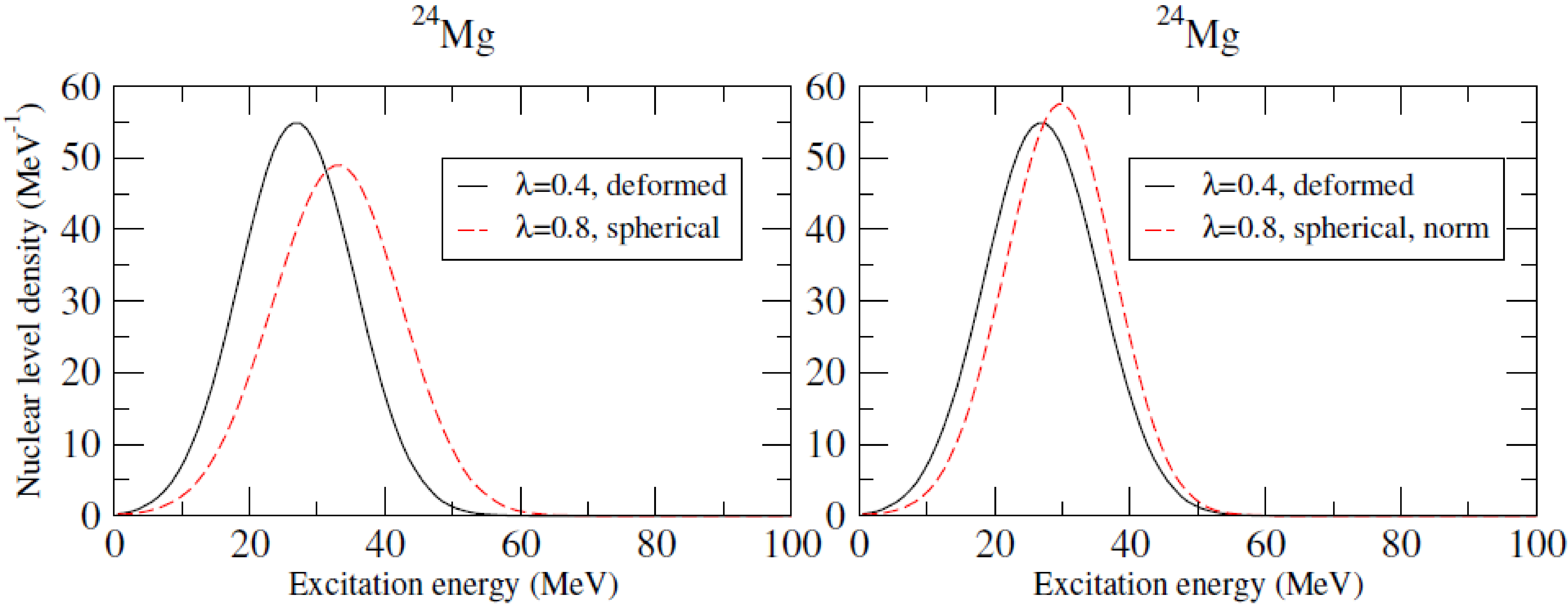
Number of 0+ levels up to energy 10 MeV



Quadrupole moment of 2+ state in 30P

as a function of the strength of

the mixing interaction strength



Level density (0+) on two sides of deformation shape transition

/"collective enhancement"/

What next?

- * Tables for **pf-shell** – and further?
- * Comparison of phenomenological Fermi-liquid description with "Constant temperature" model
- * New methods - Lanczos algorithm
  - hybrid methods
  - **random interactions**
- * **Mesoscopic applications (disordered solids)**
- * **Can we analytically derive CTM?**
- * Computational progress
- * **Continuum effects, width distribution, overlapping resonances**
- * Application to reactions

GLOBAL PROBLEMS

1. New approach to many-body theory for
   mesoscopic systems –
   instead of blunt diagonalization -
   mean field out of chaos,
   coherent modes plus
   thermalized chaotic background

2. Chaos-free scalable quantum computing
   (internal and external chaos)

V. Z., B.A. Brown, N. Frazier and M. Horoi.
The nuclear shell model as a testing ground for many-body quantum chaos.
Phys. Reports 276 (1996) 315.

V. Z.. Quantum chaos and complexity in nuclei.
Annu. Rev. Nucl. Part. Sci. 46 (1996) 237.

A.Volya and V. Z.
Invariant correlational entropy as a signature of quantum phase transitions in nuclei.
Phys. Lett. B 574 (2003) 27.

V. Z. and A. Volya.
Nuclear structure, random interactions and mesoscopic physics.
Phys. Rep. 391 (2004) 311.

F. Borgonovi, F.M. Izrailev, L.F. Santos, and V.Z.
Quantum chaos and thermalization in isolated systems of interacting particles.
Physics Reports 626 (2016) 1.

V.Z. and A. Volya.
Chaotic features of nuclear structure and dynamics: Selected topics.
Physica Scripta 91 (2016) 033006.

M. Horoi, J. Kaiser, and V. Z. Spin- and parity-dependent nuclear level densities and the exponential convergence method. Phys. Rev. C 67 (2003) 054309.

M. Horoi, M. Ghita, and V. Z. Fixed spin and parity nuclear level density for restricted shell model configurations. Phys. Rev. C 69 (2004) 041307(R).

M. Horoi and V. Z. Exact removal of the center-of-mass spurious states from level densities. Phys. Rev. Lett. 98 (2007) 262503.

R.A. Sen'kov, M. Horoi, and V.Z. High-performance algorithm for calculating non-spurious spin- and parity-dependent nuclear level densities. Phys. Lett. B 702 (2011) 413.

R.A. Sen'kov, M. Horoi, and V.Z. A high-performance Fortran code to calculate spin- and parity-dependent nuclear level densities. Computer Physics Communications 184 (2013) 215.

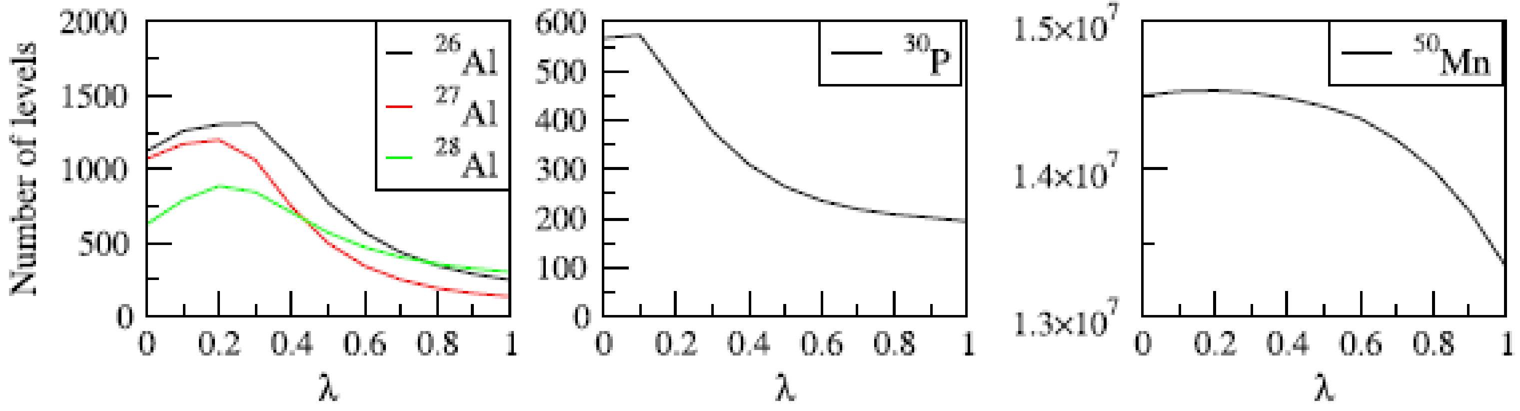
R.A. Sen'kov and V. Z. Nuclear level density: Shell-model approach. Phys. Rev. C 93 (2016) 064304.

S. Karampagia and V. Z. Nuclear shape transitions, level density, and underlying interactions. Phys. Rev. C 94 (2016) 014321.

S. Karampagia, A. Renzaglia, and V.Z. Quantum phase transitions and collective enhancement of level density in odd-A and odd-odd nuclei. Nucl. Phys. A962 (2017) 46.

S. Karampagia, R.A. Sen’kov, and V.Z. Level density in the *sd*-nuclei - statistical shell model predictions. ADNDT, 120, 1-120 (2018).

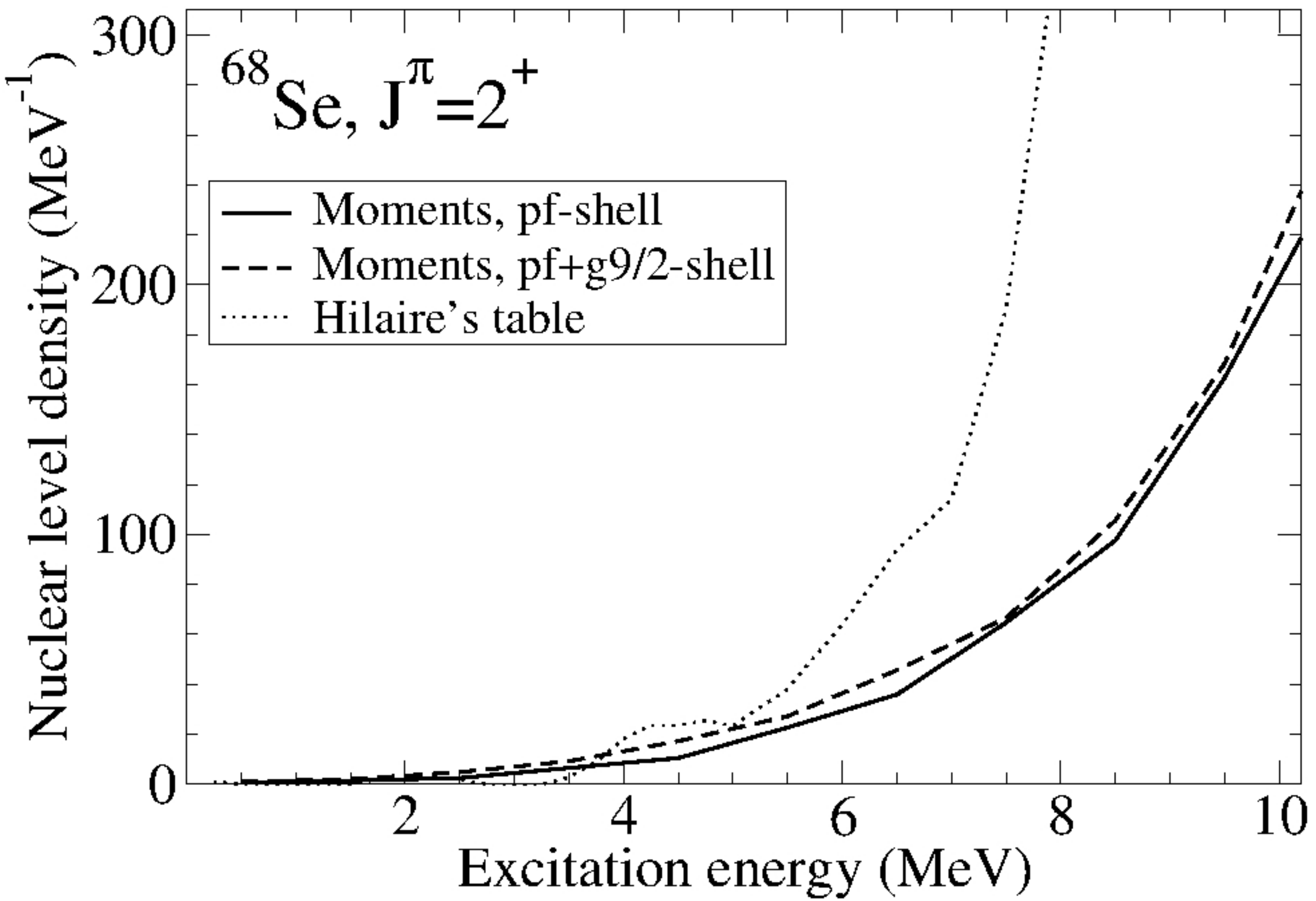
V. Z. S. Karampagia, and A. Berlaga. Constant temperature model for nuclear level density. Phys. Lett. B, in press (2018).



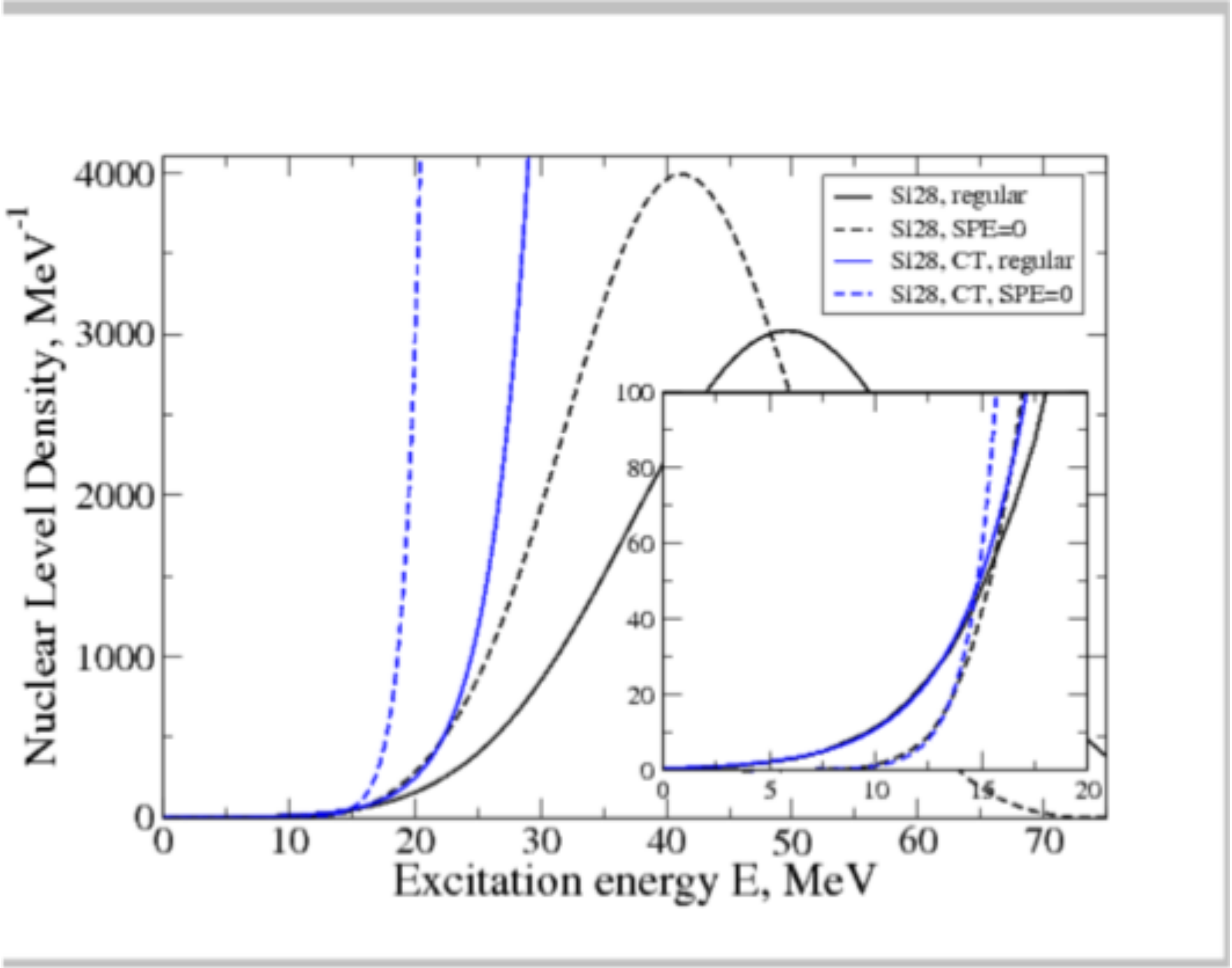
J=0 – 10 for 26 Al, 28 Al, 30 P (up to 10 MeV)

J=1/2 – 21/2 for 27 Al (up to 10 MeV)

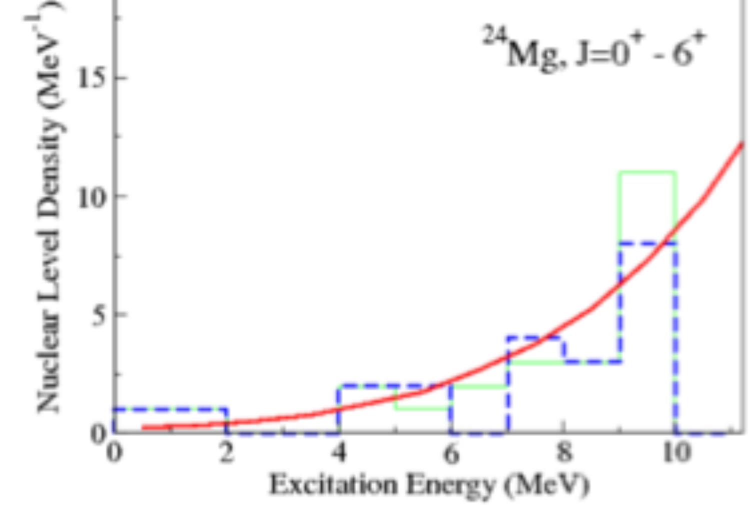
J=0 – 10 for 50 Mn (up to 60 MeV)



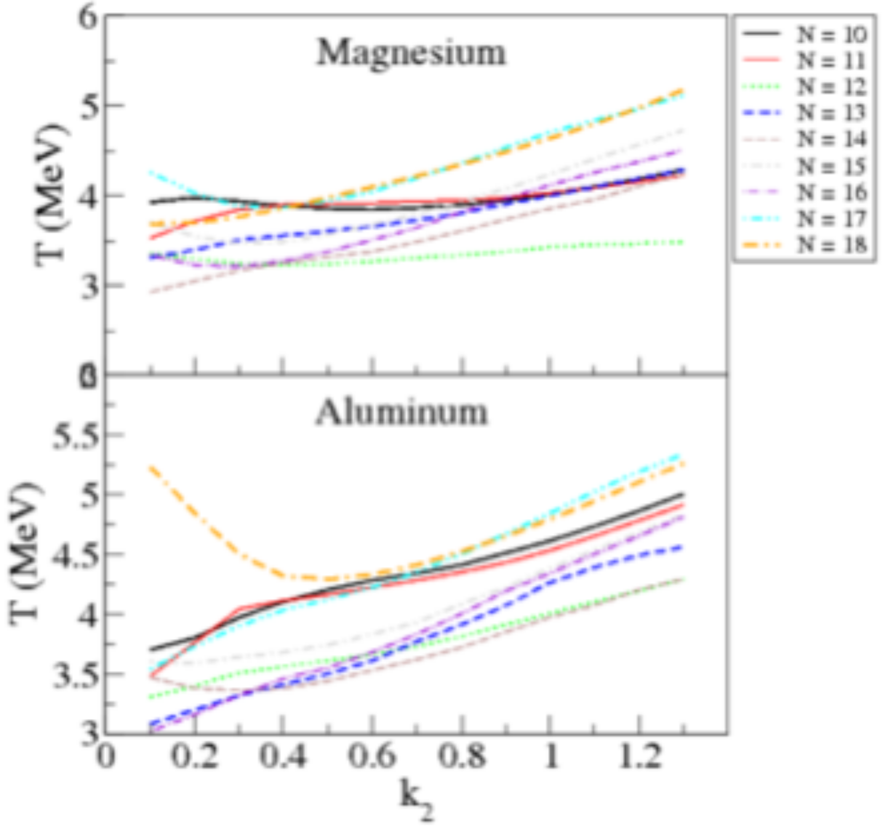
^{68}Se, $J^{\pi}=2^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
100
200
300
2
4
6
8
10



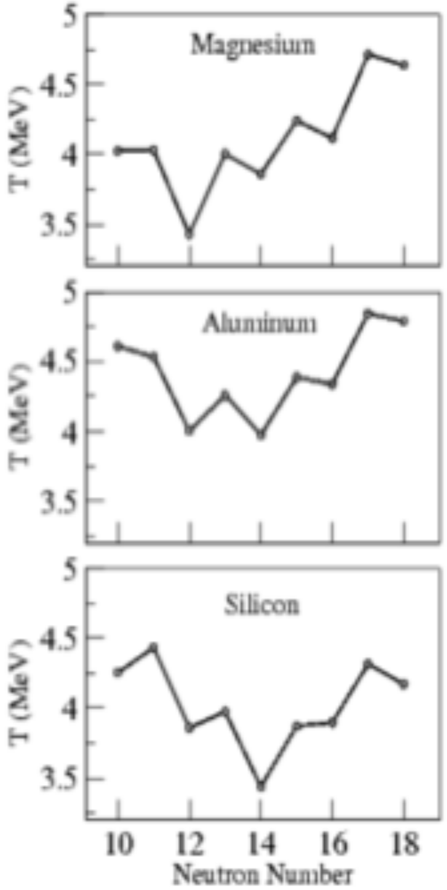
Global comparison



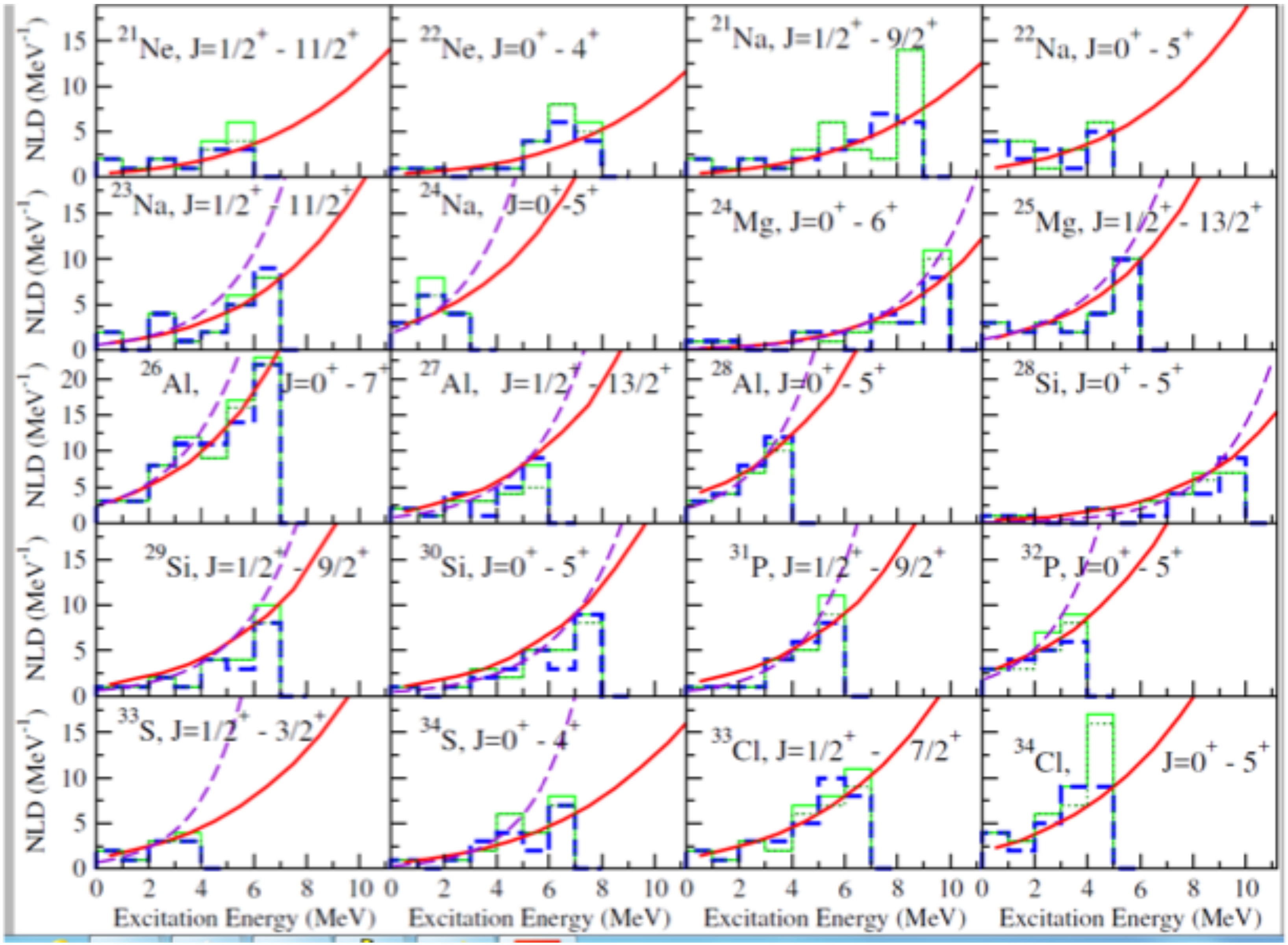
^{24}Mg, J=0$^+$ - 6$^+$
Nuclear Level Density (MeV^{-1})
Excitation Energy (MeV)
0
2
4
6
8
10
5
15



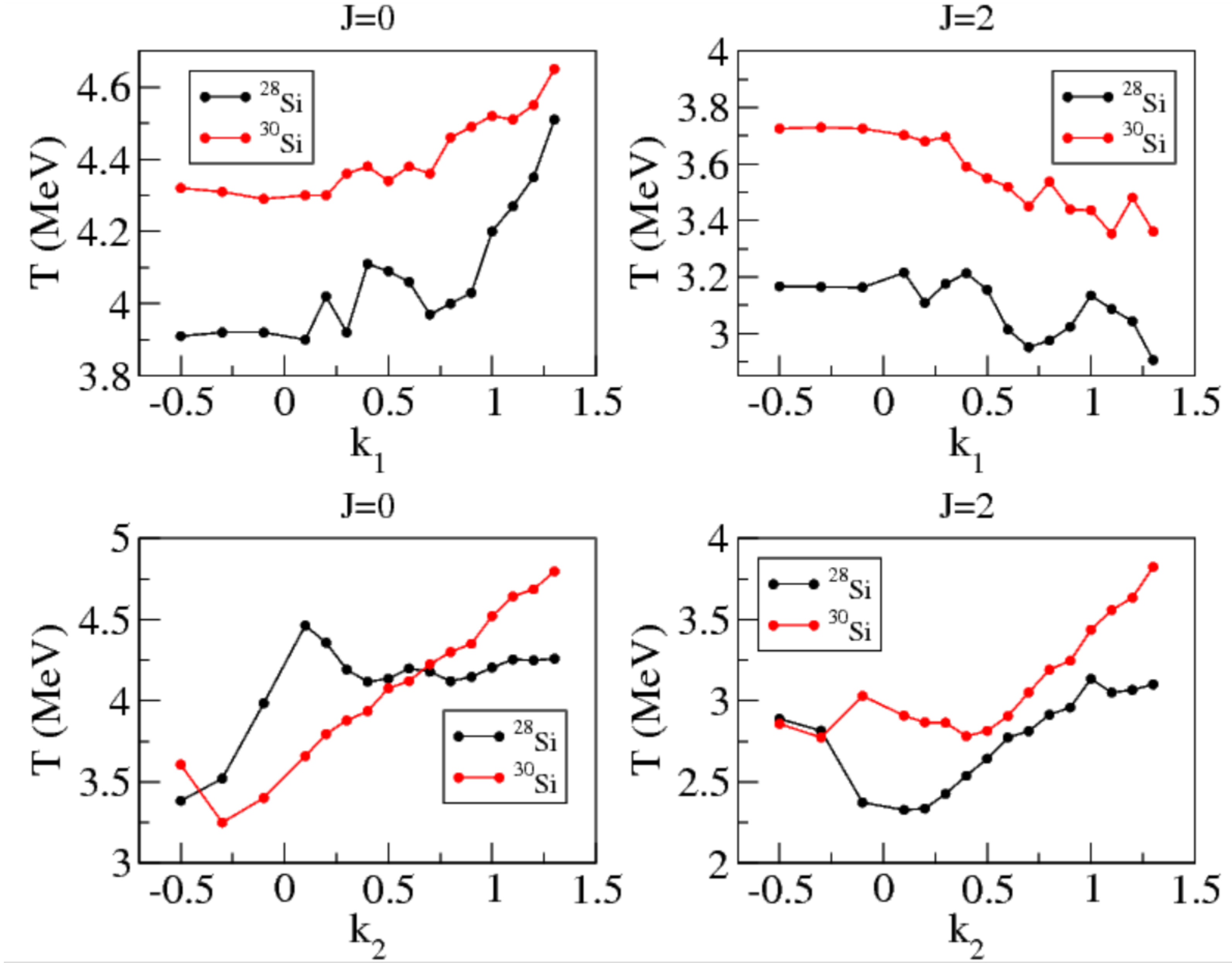
Magnesium
Aluminum
T (MeV)
k$_2$
N = 10
N = 11
N = 12
N = 13
N = 14
N = 15
N = 16
N = 17
N = 18
0
0.2
0.4
0.6
0.8
1
1.2
3
3.5
4
4.5
5
5.5
6



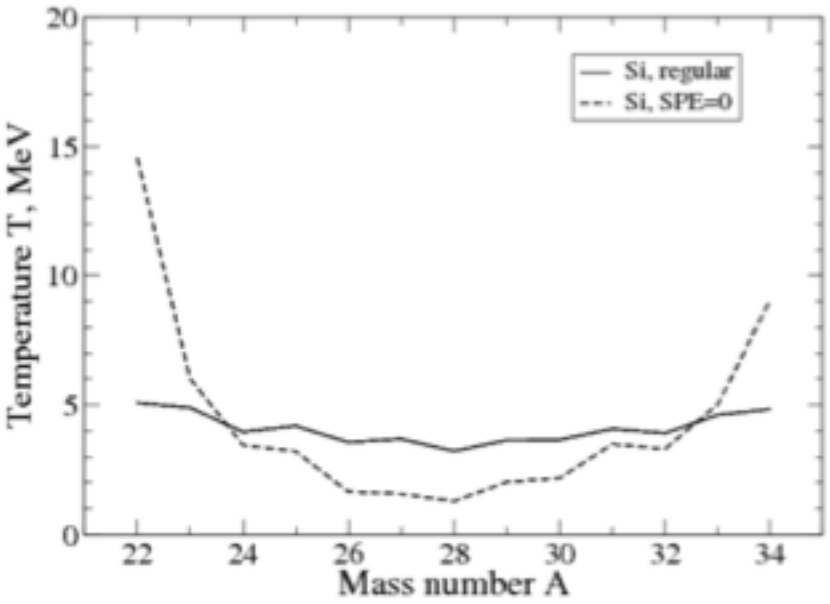
Magnesium
Aluminum
Silicon
T (MeV)
Neutron Number
10
12
14
16
18
3.5
4
4.5
5



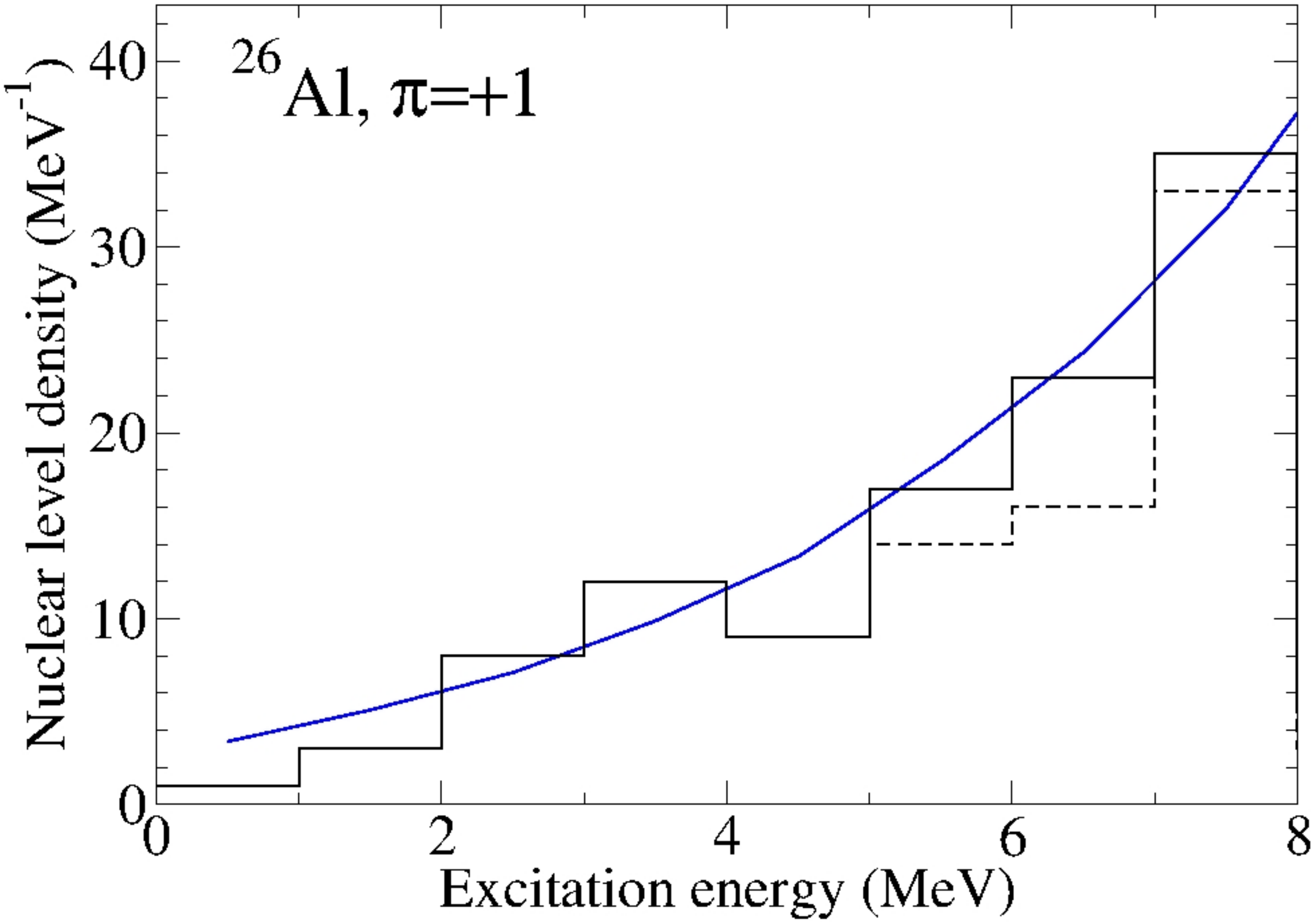
21Ne, J=1/2+ - 11/2+
22Ne, J=0+ - 4+
21Na, J=1/2+ - 9/2+
22Na, J=0+ - 5+
23Na, J=1/2+ - 11/2+
24Na, J=0+-5+
24Mg, J=0+ - 6+
25Mg, J=1/2+ - 13/2+
26Al, J=0+ - 7+
27Al, J=1/2+ - 13/2+
28Al, J=0+ - 5+
28Si, J=0+ - 5+
29Si, J=1/2+ - 9/2+
30Si, J=0+ - 5+
31P, J=1/2+ - 9/2+
32P, J=0+ - 5+
33S, J=1/2+ - 3/2+
34S, J=0+ - 4+
33Cl, J=1/2+ - 7/2+
34Cl, J=0+ - 5+
NLD (MeV-1)
Excitation Energy (MeV)



J=0
J=2
J=0
J=2
T (MeV)
k1
k2
28Si
30Si
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
5
4.5
3.5
2.5
2
-0.5
0
0.5
1
1.5



Temperature T, MeV
Mass number A
Si, regular
Si, SPE=0
0
5
10
15
20
22
24
26
28
30
32
34



^{26}Al, π=+1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8

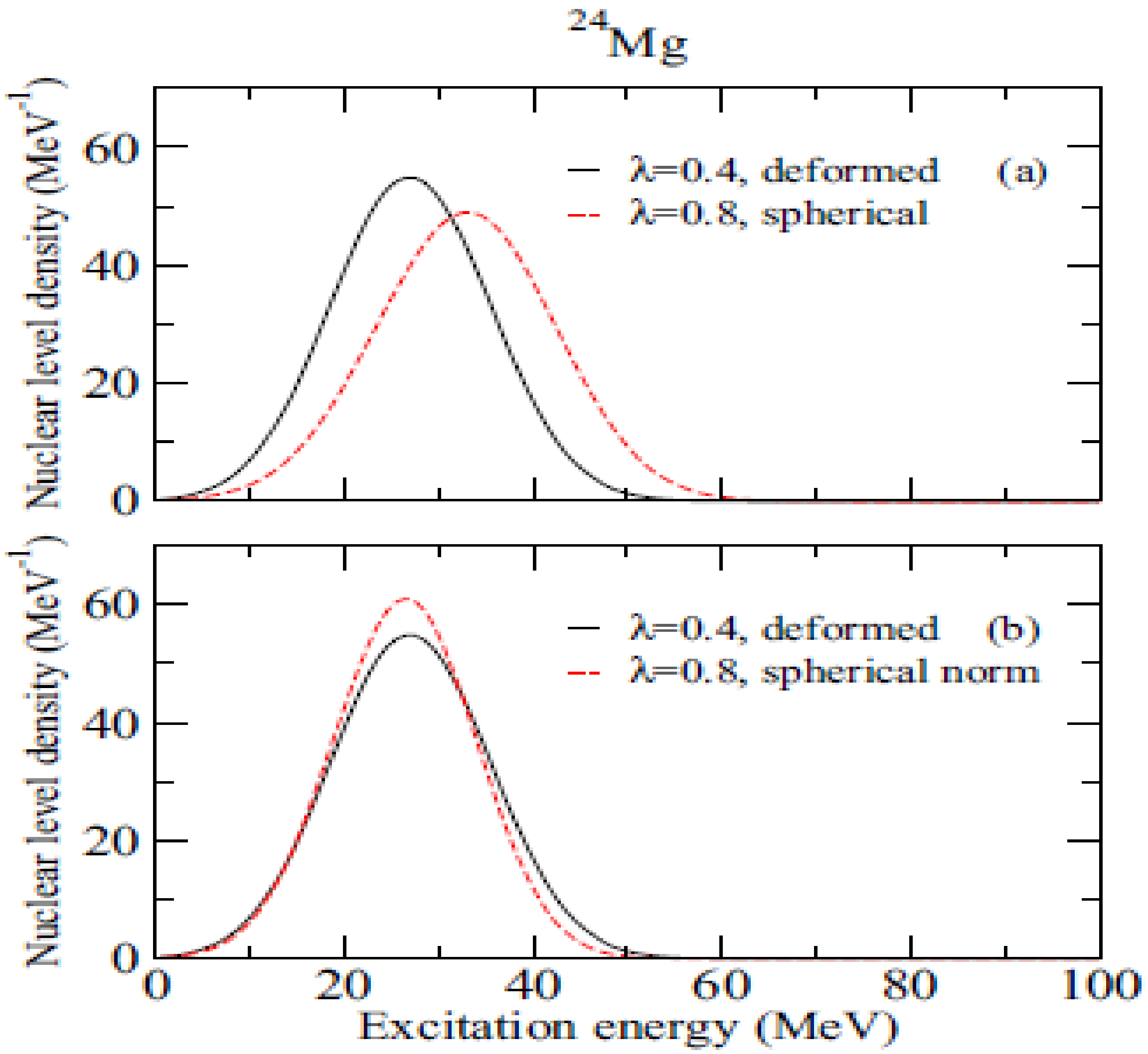
TABLE III: Cumulative Number of Levels (NoL) of $J = 0$ up to energy 10 MeV for different (k_1, k_2) combinations for ^{28}Si, ^{24}Mg and ^{52}Fe found with the moments method. The column NoL corresponds to the calculation of the moments method, while the column Renorm corresponds to the renormalized level density (NoL up to 0.4).

shape	case	nucleus	$R_{4/2}$	NoL	Renorm
deformed	$k_1 = 1.0, k_2 = 0.4$	^{28}Si	3.31	22	60
deformed	$k_1 = 1.0, k_2 = 0.5$	^{28}Si	3.33	17	54
deformed	$k_1 = 1.0, k_2 = 0.6$	^{28}Si	3.21	13	49
spherical	$k_2 = 1.0, k_1 = 0.9$	^{28}Si	2.12	5	34
deformed	$k_1 = 1.0, k_2 = 0.5$	^{24}Mg	3.20	10	24
deformed	$k_1 = 1.0, k_2 = 0.6$	^{24}Mg	3.21	8	21
spherical	$k_2 = 1.0, k_1 = 0.3$	^{24}Mg	2.03	6	18
deformed	$k_1 = 1.0, k_2 = 0.4$	^{52}Fe	3.07	236	6516
spherical	$k_2 = 1.0, k_1 = 0.0$	^{52}Fe	2.25	30	2617

H = k(1)V(1) + k(2)V(2)

V(1) – matrix elements of

single-particle transfer



Level density (0+) on two sides of deformation shape transition

/"collective enhancement"/

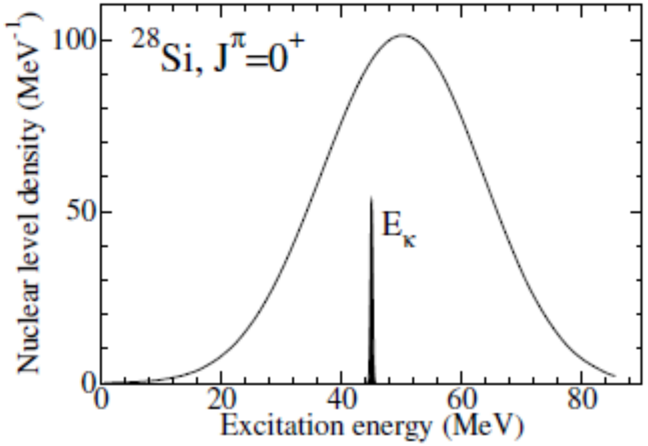
Statistical approach to Nuclear Level Density (cont.)

$$\rho(E,\beta) = \sum_{\kappa} D_{\beta\kappa} \cdot G(E - E_{\beta\kappa}, \sigma_{\beta\kappa})$$

$G(x,\sigma)$ - Gaussian distribution

$\beta = \{n, J, T_z, \pi\}$ - quantum numbers

κ - configurations



κ	$d_{\frac{5}{2}}$	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$
1	6	0	0
2	5	1	0
3	5	0	1
4	4	2	0
...	...	...	...
15	0	2	4

$D_{\beta\kappa}$ - number of many-body states with given β that can be built for a given configuration κ

Moments of H for each configuration κ:

$$E_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}$$

$$\sigma^2_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H^2]/D_{\beta\kappa} - \left(\mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}\right)^2$$

M. Horoi, M. Ghita, and V. Zelevinsky, PRC **69** (2004) 041307(R)

No diagonalization required

****

Neutron resonances

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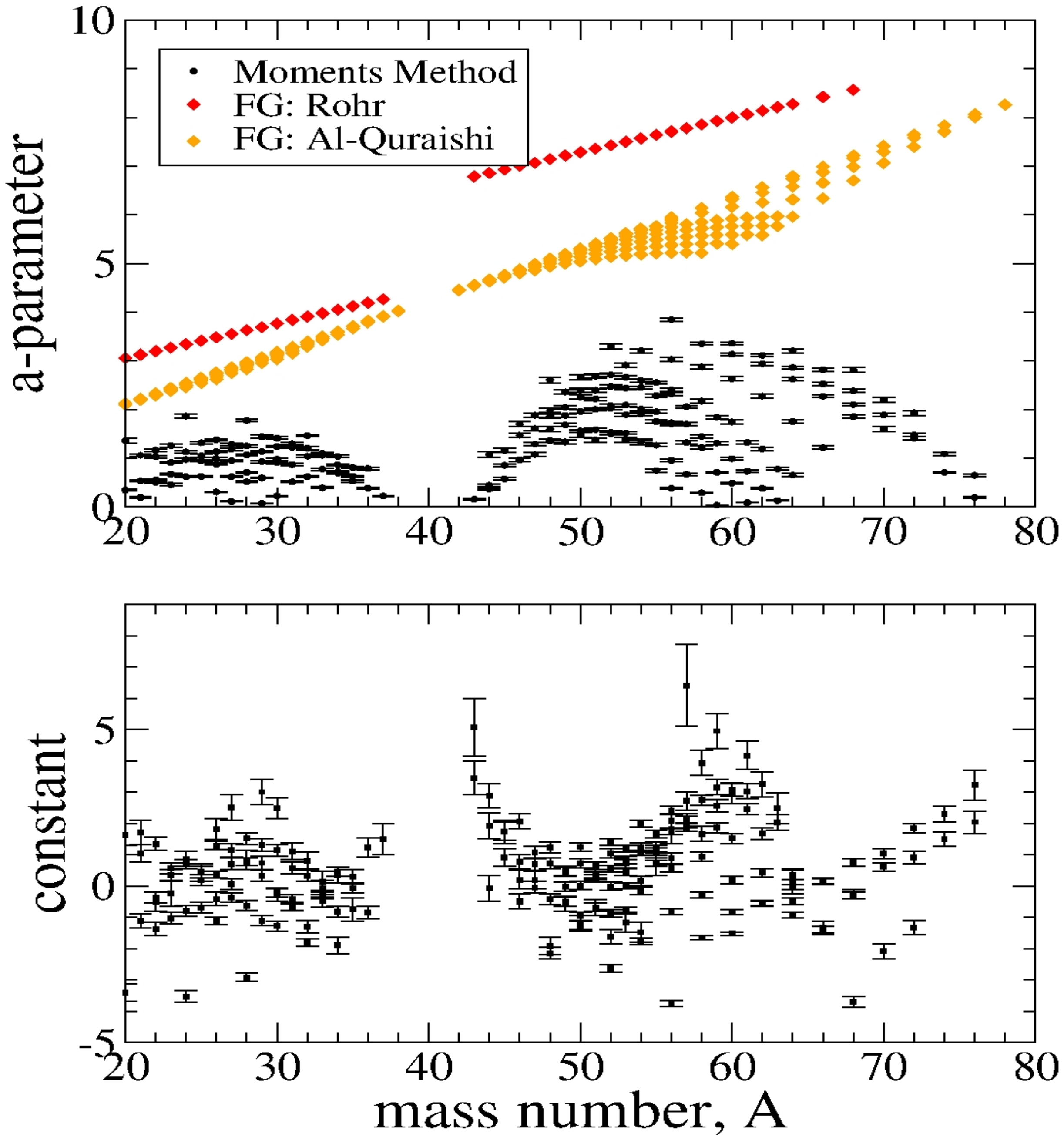
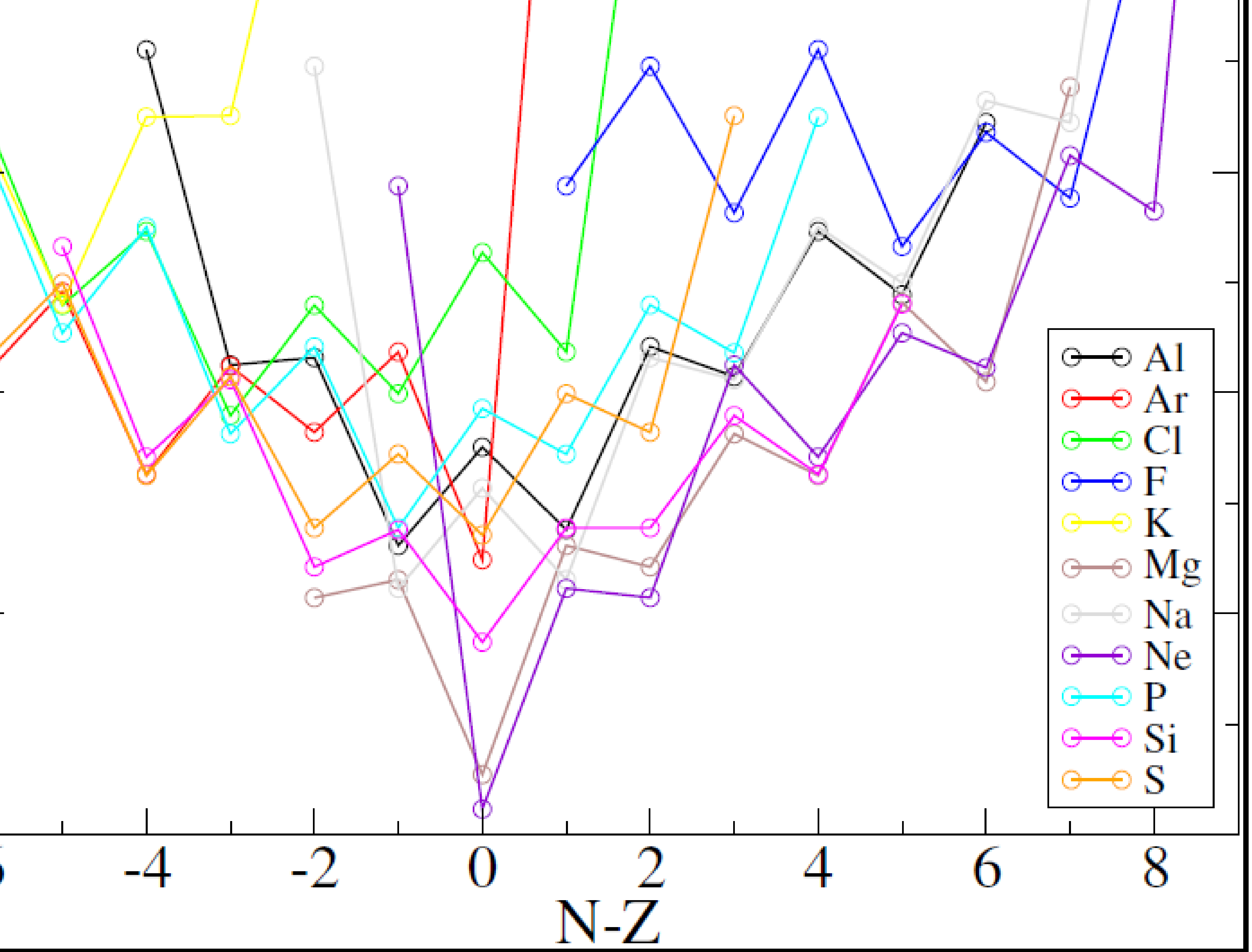
Low-lying levels

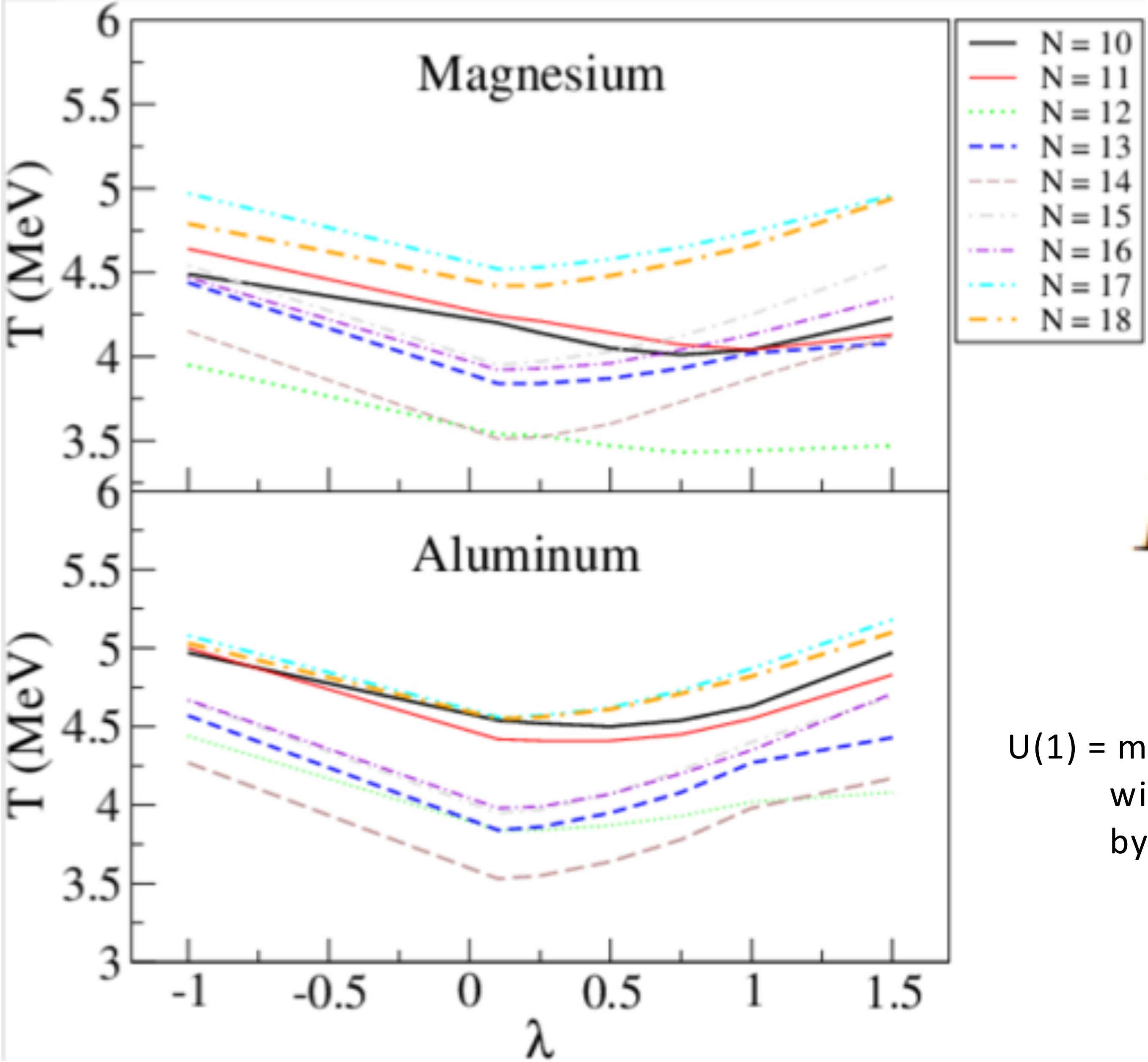




Effective temperature for the level density at low energy (up to 6 – 8 Mev)

Even-odd staggering

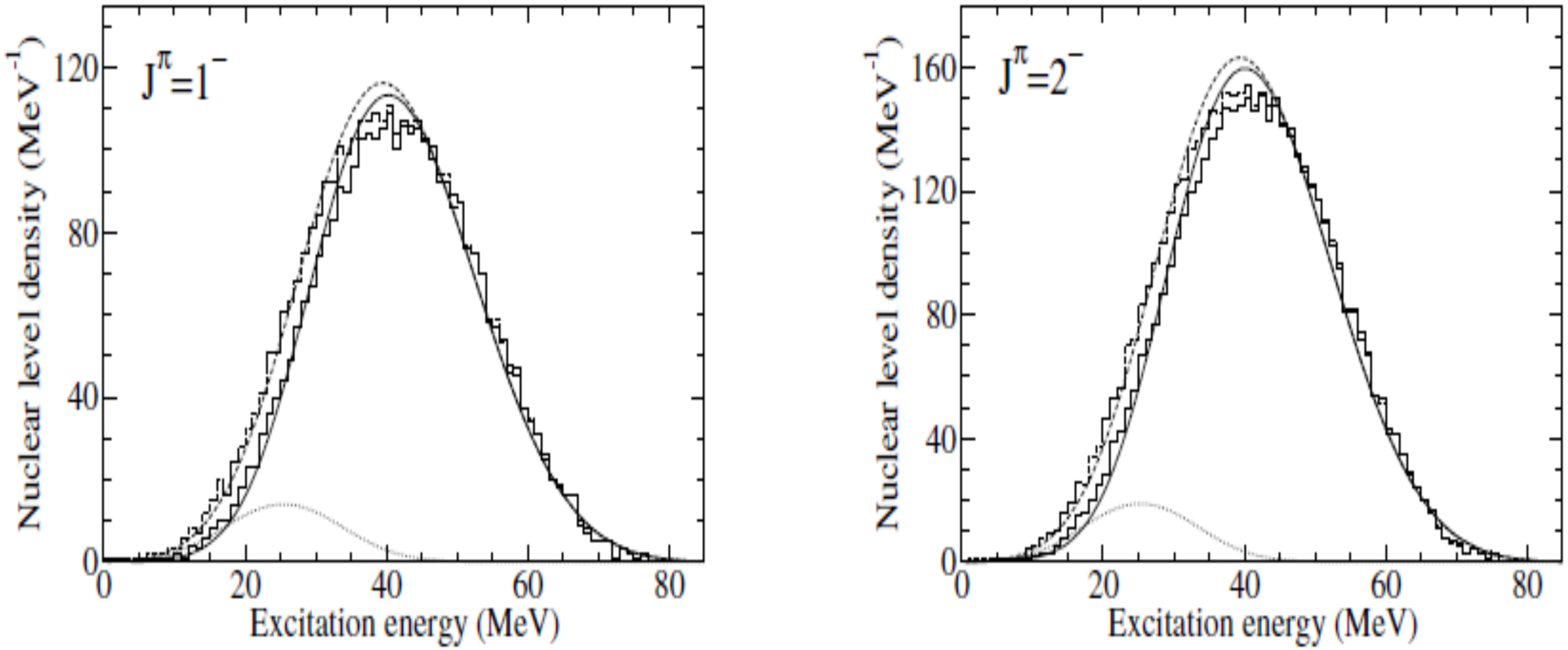
Clear minima in the vicinity of N=Z



$$H = h + \lambda U_1 + U_2.$$

U(1) = matrix elements of the two-body interaction with change of orbital momentum of one particle by 2 units (the same parity) – way to deformation

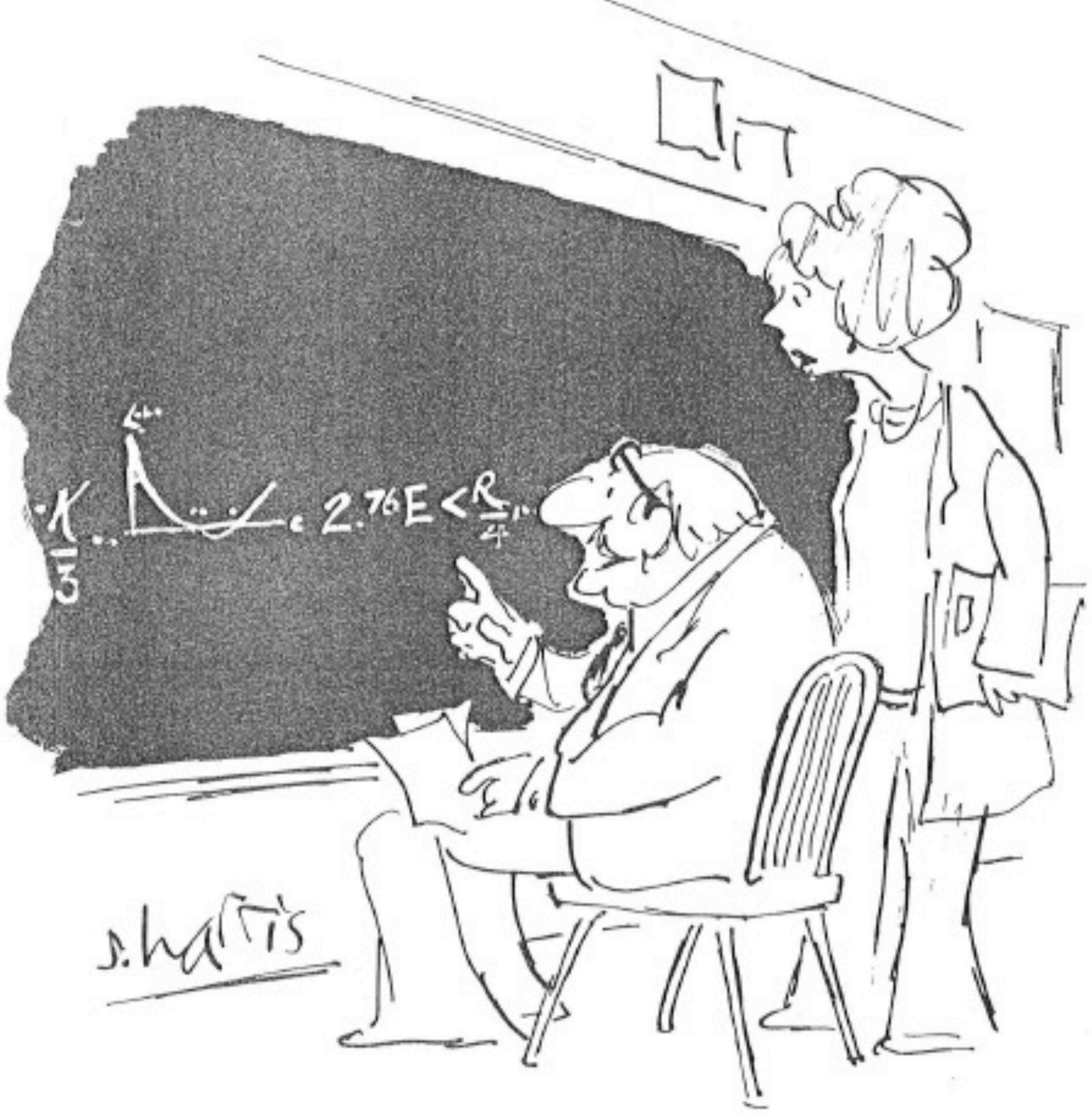
20 Ne



s + p + sd + pf shell space
WBT interaction,
negative parity

$1\hbar\omega$ subspace $H \rightarrow H' = H + \beta\left[\left(H_{CM} - \frac{3}{2}\hbar\omega\right)\frac{A}{\hbar\omega}\right]$

Exact shell model: stair-dashed (with CM) and stair-solid (no CM)
Method of moments: straight-dashed (with CM) and straight-solid (no CM)
Dotted line: spurious states



"THE BEAUTY OF THIS IS THAT IT IS ONLY OF THEORETICAL IMPORTANCE, AND THERE IS NO WAY IT CAN BE OF ANY PRACTICAL USE WHATSOEVER."