Spectroscopy of Continuum States - a review

for FRIBTA symposium: “Connecting Bound States to the Continuum”

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Which Continuum?

- The continuum appears in several ways:
  - Part of expansion of bound states;
    - eg needed for weakly bound states
  - Dominated by resonances;
    - These ‘unbound states’ identified eg with shell model eigenstates above threshold
  - In non-resonant continuum;
    - eg in breakup reactions, or low-energy capture.
  - Many compound-nucleus resonances to be averaged over;
    - e.g. neutron reactions with unresolved resonances

- ALL important parts of nuclear structure!!
Probing the continuum: Structure overlaps $\rightarrow$ reaction dynamics

- Reaction models need few-body degrees of freedom in structure models.
  - Solve a few-body model directly, or
  - Extract few-particle degrees-of-freedom from microscopic model
    - Difficult for: GFMC,
    - for HFB, QRPA and RMF structure models
    - Do we transfer quasi-particles, or particles?
# Measuring the Continuum?

1. **Ab-initio models**
   - if Hamiltonian known. Sofia’s talk.

2. **Average x-s for pseudo-states**
   - like Shell Model

3. **Lifetimes / widths**
   - measure times / widths

4. **Phase shifts**
   - 1 channel only

5. **S-matrix / eigenvalues**
   - multi-channel

6. **Resonances as complex poles**
   - complex energy scattering

7. **Inelastic / transfer reactions**
   - measure Q-value spreads

8. **Peaks in response functions**
   - DWBA

9. **Spectroscopic factors (?)**
   - DWBA

10. **Average cross-sections for bins**
    - CDCC / XCDCC

11. **Widths and partial widths**
    - R-matrix theory

12. **Averaged branching ratios**
    - Hauser-Feshbach theory
Continuum three-body wave functions

- Three-body scattering at energy $E$:
  \[
  \kappa = \sqrt{k_x^2 + k_y^2} = \sqrt{2mE/\hbar^2},
  \]
  \[
  \alpha_\kappa = \text{atan}(k_x/k_y)
  \]

- Plane wave 3-3 scattering states:
  \[
  \frac{(2\pi)^{-3}}{2} \exp[i(k_x \cdot x + k_y \cdot y)]
  \]
  \[
  = (\kappa \rho)^{-2} \sum_{KLM_L l_x l_y} i^K J_{K+2}(\kappa \rho) \mathcal{Y}_{KLM_L}^{l_x l_y} (\Omega_5^\rho) \mathcal{Y}_{KLM_L}^{l_x l_y} (\Omega_5^\kappa)^*
  \]

- Dynamical solutions for scattering states:
  \[
  \Psi_{\kappa J M}^T(x, y, \hat{k}_x, \hat{k}_y, \alpha_\kappa) = (\kappa \rho)^{-5/2} \sum_{K\gamma, K'\gamma'} \psi_{K\gamma, K'\gamma'}^J(\kappa \rho) \gamma_{J M}^{K\gamma}(\Omega_5^\rho)
  \]
  \[
  \sum_{M'L'M'_S M'_S} \langle L' M'_L S' M'_S | J M \rangle \mathcal{Y}_{K'L'M'_L}^{l_x l_y} (\Omega_5^\kappa) X_T
  \]
S-matrix and phases

- **1-channel:** e.g. \( n + \alpha \)
  - Phase shifts

- **Many-channel:** e.g. \( 2n + \alpha \)
  - Diagonal phase shifts?
    - Sometimes \( |S_{ii}| \) is small!
  - Eigenphase shifts
    - Difficult to keep track of solutions

![Eigenphases in Li11*](image)
Wave functions of $^6$He

- Ground state wave function:
- Solution of coupled equations for $E \sim -0.97$ MeV.

Nuclei such as $^6$He have highly correlated cluster structures.
Now average scattering wave functions over angles of $k_x$ and $k_y$, to see spatial correlations in continuum states in $^6$He:

from B. Danilin, I. Thompson, PRC 69, 024609 (2004)
‘True’ 3-body resonances?

- Expect continuum wave functions like:

\[ \psi(\rho \ \Omega_5^\rho, E \ \Omega_5^\kappa) \]

\[ \propto \frac{1}{(\kappa \rho)^{5/2}} \sum_{K,\gamma} C_{K\gamma}(E) \ \psi_{K\gamma}^R(\rho) \ Y_{K\gamma}(\Omega_5^\rho) \ Y_{K\gamma}(\Omega_5^\kappa) \]

with

\[ |C_{K\gamma}(E)|^2 = \frac{\Gamma_{K\gamma}}{(E - E_0)^2 + \Gamma^2/4} \]
Virtual states & Resonances

from B. Danilin, I. Thompson, et al

Virtual n-n pole

Effect of n-n ‘resonance’ in E(c-n), E(cn-n) coordinates
Continuum Energy Correlations

- Now average scattering wave functions over angles of $k_x$ and $k_y$, for fixed three-body energy $E$.

- Obtain similar plots for continuum energies.

- Continuum momentum & angular correlations also possible
$^6$He excitations & resonances

**Pronounced $2^+$ resonance**

**No pronounced $1^-$ resonance**
What a good reaction theory needs:

- Distinguish resonances and their backgrounds
- Recoil & finite-range of projectile vertex.
- Interference between exit partial waves
- Nuclear and Coulomb mechanisms
- Core excitation (static and/or dynamic)
- Final-state interactions:
  - between projectile fragments (needed if resonances)
  - between fragments and target (needed if close in)
- Multistep Processes (higher order effects)
Elastic Breakup of 2N halo

- Elastic Breakup = **Diffraction Dissociation:**
  — all nuclear fragments survive along with the target in its ground state,
  — probes continuum excited states of nucleus.

- Need correlations in the three-body continuum of Borromean nuclei.
Inelastic reactions

- $^{11}$Li(d,d$'$) $^{11}$Li* measurements (Kanungo et al, PRL 114, 192502, 2015)

- See also:
  - $^{11}$Li(p,p$'$) $^{11}$Li* measurements (Tanaka, et al, PLB 774, 268, 2017),
  - $^6$He($^{12}$C, $^{12}$C)$^6$He(2$^+$) measurements (Kikuchi, PRC 88, 021602R, 2013)

FIG. 3: (a) Inelastic scattering excitation energy spectrum for deuterons in coincidence with $^9$Li. (b) The inelastic scattering angular distribution data for the resonance peak at $E_{ex}=1.03$ MeV. The curves are DWBA calculations for $L=0$ (pink dashed-line), $L=1$(red solid line), $L=2$(blue dotted line).
Transfer Reactions

Sanetullaev et al
PLB 755, 481 (2016)

5.7A MeV $^{11}$Li beam, DWBA calculations

Casal et al, PLB, 767, 307 (2017)

Better: using structure overlaps from a full three-body model
Stripping Reactions
1-N from 3-body Borromean Nuclei

- Removal of a neutron from $^6\text{He}$, $^{11}\text{Li}$, $^{14}\text{Be}$,
  —populates states of $^5\text{He}$, $^{10}\text{Li}$ or $^{13}\text{Be}$.
  —Experiments measure decay spectrum of $^5\text{He} = ^4\text{He} + n$,
    $^{13}\text{Be} = ^{12}\text{Be} + n$, etc

- Can we predict any energy and angular correlations by
  Glauber model?

- Can we relate these correlations to the structure of
  the A+1 or the A+2 nucleus?
1N stripping from $^6$He g.s.

- Calculate overlaps: $<^5\text{He}(E_{\alpha-n}) | ^6\text{He}(\text{gs})>$ for a range of $^5\text{He}(E_{\alpha-n})$ bin states,
- smooth histogram of Glauber bin cross sections.
- GSI data (H. Simon)

Theory: $\sigma_{\text{str}}=137$ mb, $\sigma_{\text{diff}}=38$ mb
Expt: $\sigma_{\text{str}}=127\pm14$ mb, $\sigma_{\text{diff}}=30\pm5$ mb
from T. Tarutina thesis (Surrey)
1N stripping from $^{14}\text{Be}$ g.s.

- Calculate overlaps: $<^{13}\text{Be}(E_{\alpha-n})|^{14}\text{Be}(\text{gs})>$
- Inert-core $^{13,14}\text{Be}$ wfs.
- GSI data (H. Simon)
- See softer data, and not pronounced virtual-s and resonant-d peaks.
- New theory needed?

Theory: $\sigma_{\text{str}}=109$ mb, $\sigma_{\text{diff}}=109$ mb
Expt: $\sigma_{\text{str}}=125\pm19$ mb, $\sigma_{\text{diff}}=55\pm19$ mb
Breakup reactions

CDCC: Coupled Discretised Continuum Channels

Try CDCC:
- Proposed by Rawitscher, developed by Kamimura group.
- Treat Coulomb and Nuclear mechanisms
  - Need to check convergence of long-range Coulomb process!
- All higher-order effects with a \((r,R,L)\) reaction volume
- Can calculate fragment coincident angular distributions:
  - Predict e.g. \(d^3\sigma/dE_1d\Omega_1d\phi_{12}\) and fold with detector apertures & efficiencies

- Extended by Neil Summers to XCDCC
  - Include excited state of ‘core’ in the projectile subject to breakup

- Extended by de Diego et al, to CDCC with target excitations
  - (de Diego etal. PRC 89, 064609, 2014)
The Hamiltonian for the reaction of a projectile on a target (neglect the internal structure of the target):

\[ H = h_{\text{proj}} + h_{\text{targ}} + T_\alpha + V_\alpha \]

\[ \Rightarrow h_{\text{proj}} = h_{\text{core}} + h_{\text{frag}} + T_{\text{cf}} + V_{\text{cf}} \]

\[ \Rightarrow V_\alpha = V_{\text{core-targ}} + V_{\text{frag-targ}} \]

\[ \psi_{JM}^{\text{CDCC}}(r,R) = \sum_{l=0}^{l_{\text{max}}} \sum_{l'=1}^{N} [\phi_{i,l}(r) \otimes Y_L(\hat{R})]_{JM} \chi_{i,l,J,L}(R) \]
CDCC Formalism

The CDCC basis consists of scattering wavefunctions averaged over an energy interval

\[ h_{\text{proj}} \phi_k = \varepsilon_k \phi_k \]

\[ \phi_{i,l} = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} w_i(k) \phi_{lm}(k, r) \, dk \]

\[ N_i = \int_{k_{i-1}}^{k_i} |w_i(k)|^2 \, dk \]

\[ N_{\text{bins}} = \frac{k_{\text{max}}}{\Delta k} \]

**Coupling potentials**

\[ V_{\text{CDCC}}^{il,l'}(R) = \langle \phi_{il}(r) \mid V_\alpha(r, R) \mid \phi_{i'l'}(r) \rangle \]
Testing CDCC Convergence

- Compare, in Adiabatic Few-Body Model, with Bremstrahlung integral
- Compare, in first-order PWBA model, with semiclassical theory

Note the ‘post-acceleration’
Adiabatic CDCC: compare with Exact 3-body model

Absolute errors in CDCC for $d+^{208}\text{Pb}$ at 50 MeV, nuclear only

d$+^{208}\text{Pb}$ at 50 MeV, nuclear only
**15C + 9Be breakup at 54 MeV/u**


FIG. 4. Diagrammatic representation of the CDCC model space calculation for 15C. The left side shows the physical bound states and continuum and the right hand side the included continuum bins (10) in each n + 14C partial wave. The dashed arrows are representative of the one-way couplings included in the DWBA. The solid arrows show representative couplings for the full CDCC calculations which connect all bins, including diagonal bin couplings, with two-way couplings to all orders. Relative h waves were found to make negligible contributions.

FIG. 11. The nucleon-removal parallel momentum distributions dσ/dp_∥, for the 15C + 9Be reaction at 54 MeV/nucleon to the 14C ground state, shown on a more familiar linear scale. The solid curves assume the stripping contributions have the same form as that calculated using the CDCC. The dashed curves assume the stripping contributions have a parallel momentum distribution at all angles of the residue given by the eikonal calculation shown in Fig. 2.
$^{11}\text{Be} + ^{12}\text{C}$ breakup at 67 MeV/u

Angular distributions of $^{11}\text{Be}^*$
left: low-energy continuum
right: region of $d_{5/2}$ resonance

Energy excitation spectrum
dashed line: multiplied by 0.8

CDCC calculations of
Howell, Tostevin, Al-Khalili,
Breakup reactions

CDCC $^8\text{B} + ^{58}\text{Ni} \rightarrow ^7\text{Be} + p + ^{58}\text{Ni}$ (E$_b$=26 MeV)

Energy distributions: Excellent agreement with the data!
$^8\text{B} + ^{208}\text{Pb} \rightarrow ^7\text{Be}$ parallel momentum distributions

44 MeV/u

Dot-dashed: semiclassical Coul.
Solid: Coulomb+nuclear DWBA
Dashed: CDCC coupled channels - reduced asymmetry

CDCC calculations with scaled E2 amplitudes - need to increase asymmetry again!
