Spectroscopy of Continuum States - a review

for FRIBTA symposium:
"Connecting Bound States to the Continuum"

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Which Continuum?

- The continuum appears in several ways:
  - Part of expansion of bound states;
    - eg needed for weakly bound states
  - Dominated by resonances;
    - These 'unbound states' identified eg with shell model eigenstates above threshold
  - In non-resonant continuum;
    - eg in breakup reactions, or low-energy capture.
  - Many compound-nucleus resonances to be averaged over;
    - e.g. neutron reactions with unresolved resonances
- ALL important parts of nuclear structure!!

Probing the continuum:
Structure overlaps → reaction dynamics

- Reaction models need few-body degrees of freedom in structure models.
  - Solve a few-body model directly, or
  - Extract few-particle degrees-of-freedom from microscopic model
    - Difficult for: GFMC,
    - for HFB, QRPA and RMF structure models
    - Do we transfer quasi-particles, or particles?

Measuring the Continuum?

1.	Ab-initio models	if Hamiltonian known. Sofia's talk.
2.	Average x-s for pseudo-states	like Shell Model
3.	Lifetimes / widths	measure times / widths
4.	Phase shifts	1 channel only
5.	S-matrix / eigenvalues	multi-channel
6.	Resonances as complex poles	complex energy scattering
7.	Inelastic / transfer reactions	measure Q-value spreads
8.	Peaks in response functions	DWBA
9.	Spectroscopic factors (?)	DWBA
10.	Average cross-sections for bins	CDCC / XCDCC
11.	Widths and partial widths	R-matrix theory
12.	Averaged branching ratios	Hauser-Feshbach theory

Continuum three-body wave functions

- Three-body scattering at energy E:

  hypermomentum $\kappa = \sqrt{k_x^2 + k_y^2} = \sqrt{2mE/\hbar^2}$,

  hyperangle $\alpha_\kappa = \mathrm{atan}(k_x/k_y)$

- Plane wave 3-3 scattering states:

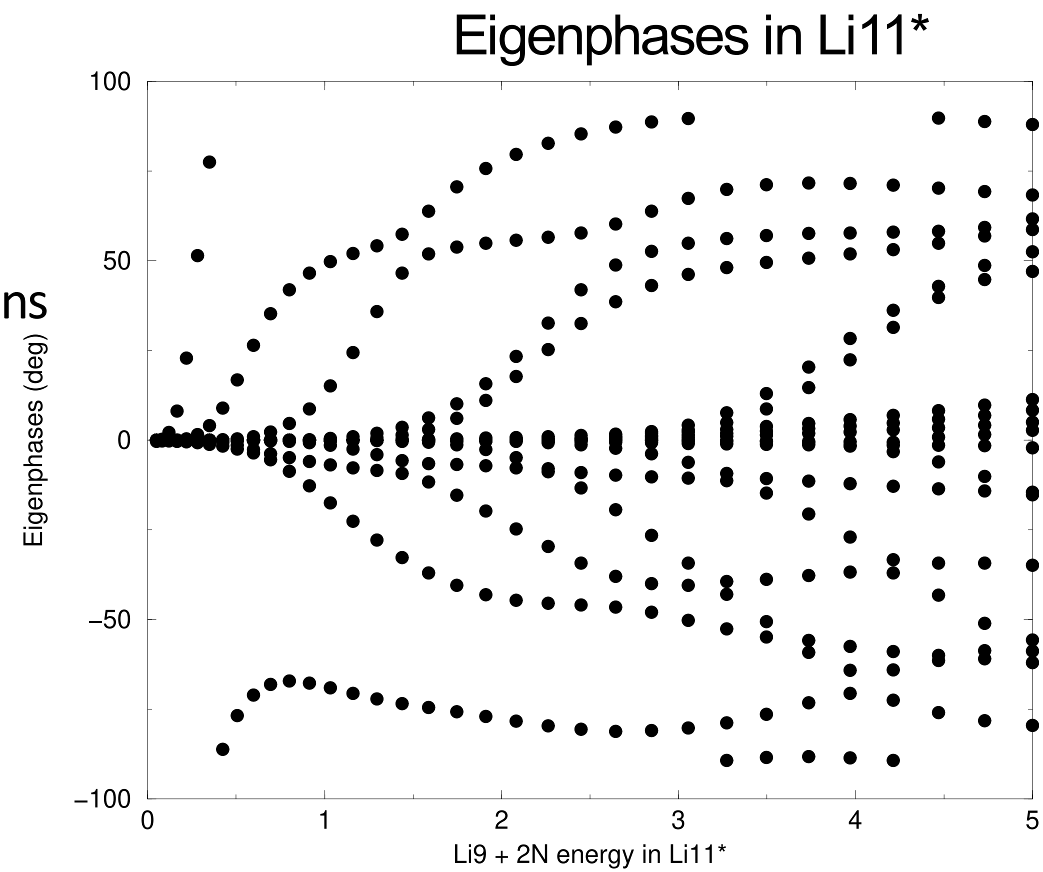
$$\begin{aligned} & (2\pi)^{-3} \exp[i(\mathbf{k_x}\cdot\mathbf{x} + \mathbf{k_y}\cdot\mathbf{y})] \\ = \; & (\kappa\rho)^{-2} \sum_{KLM_L l_x l_y} i^K J_{K+2}(\kappa\rho) \mathcal{Y}^{l_x l_y}_{KLM_L}(\Omega_5^\rho) \; \mathcal{Y}^{l_x l_y}_{KLM_L}(\Omega_5^\kappa)^* \end{aligned}$$

- Dynamical solutions for scattering states:

$$\begin{aligned} \mathbf{\Psi}^T_{\kappa JM}(\mathbf{x}, \mathbf{y}, \hat{\mathbf{k}}_x, \hat{\mathbf{k}}_y, \alpha_\kappa) = (\kappa\rho)^{-5/2} & \sum_{K\gamma, K'\gamma'} \psi^J_{K\gamma,K'\gamma'}(\kappa\rho) \Upsilon^{K\gamma}_{JM}(\Omega_5^\rho) \\ & \sum_{M'_L M'_S} \langle L' M'_L S' M'_S \mid JM \rangle \mathcal{Y}^{l'_x l'_y}_{K'L'M'_L}(\Omega_5^\kappa) \; X_T \end{aligned}$$

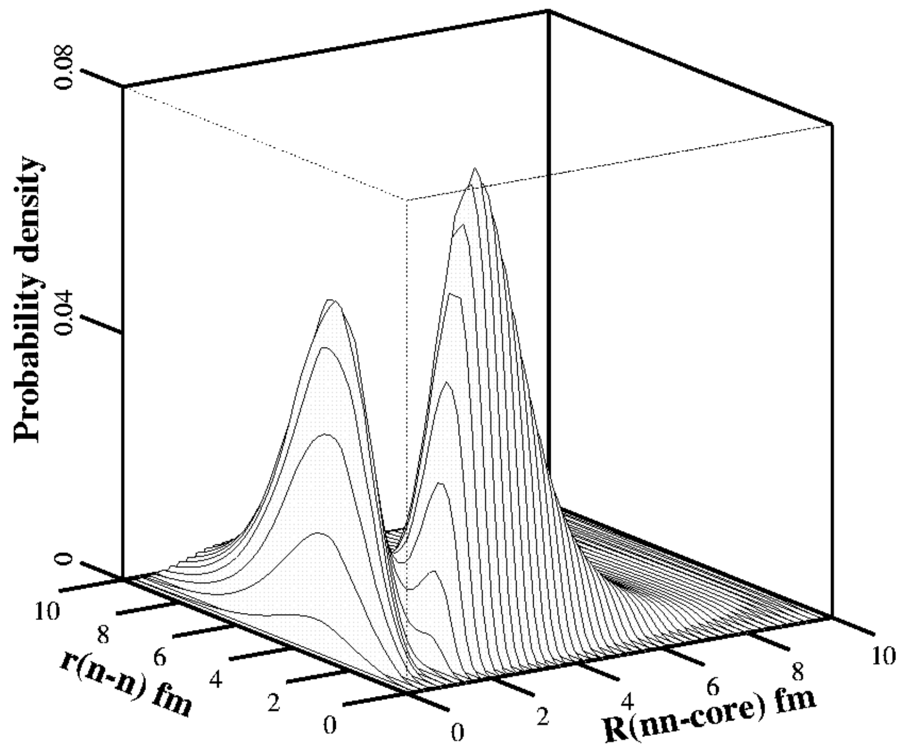
S-matrix and phases

- 1-channel: e.g. n + α
  - — phase shifts
- Many-channel: e.g. 2n + α
  - — Diagonal phase shifts ?
    - Sometimes $|S_{ii}|$ is small!
  - — Eigenphase shifts
    - Difficult to keep track of solutions

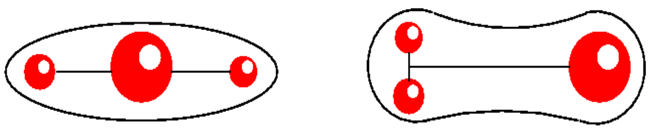


Wave functions of ^{6}He

- Ground state wave function:
- Solution of coupled equations for E ~ – 0.97 MeV.



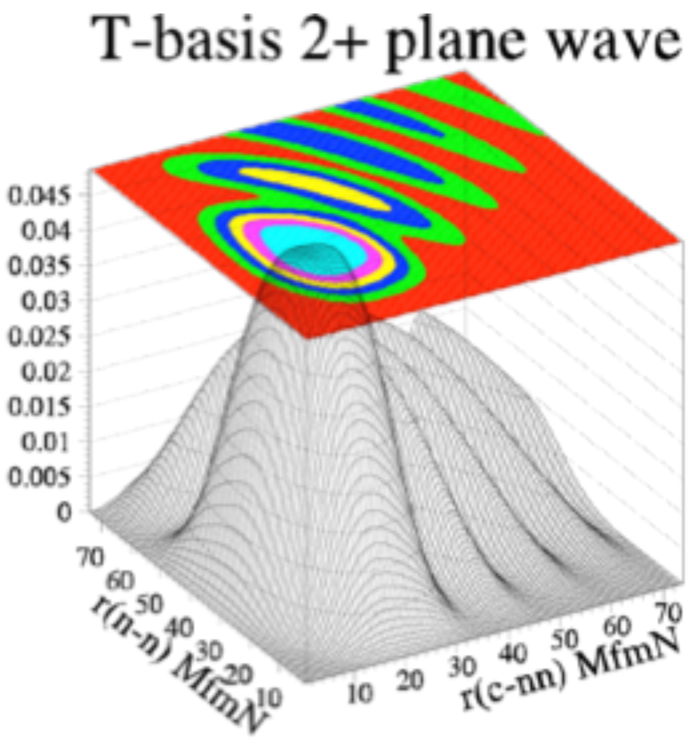
Nuclei such as ^{6}He have highly correlated cluster structures

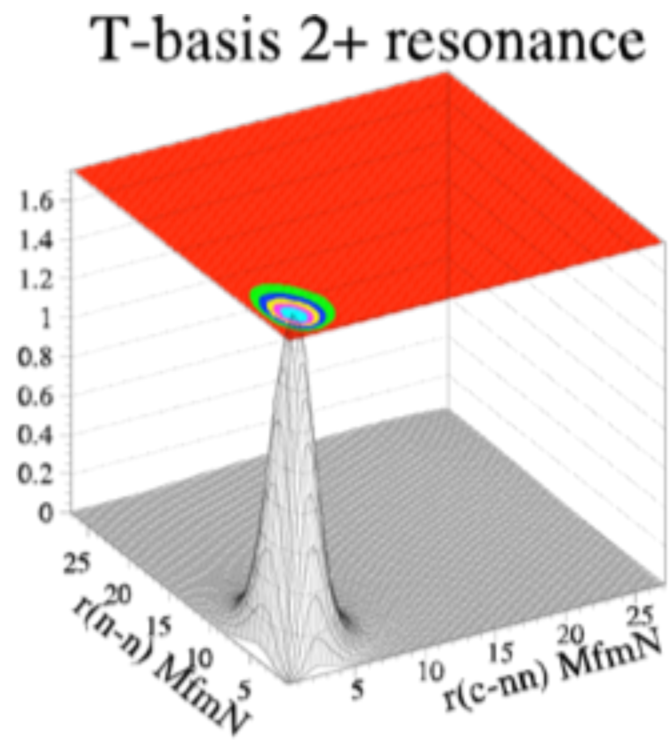


Continuum Spatial Correlations

from B. Danilin, I. Thompson, PRC 69, 024609 (2004)

- Now average scattering wave functions over angles of $\mathbf{k}_x$ and $\mathbf{k}_y$, to see spatial correlations in continuum states in ^{6}He:





‘True’ 3-body resonances?

- Expect continuum wave functions like:

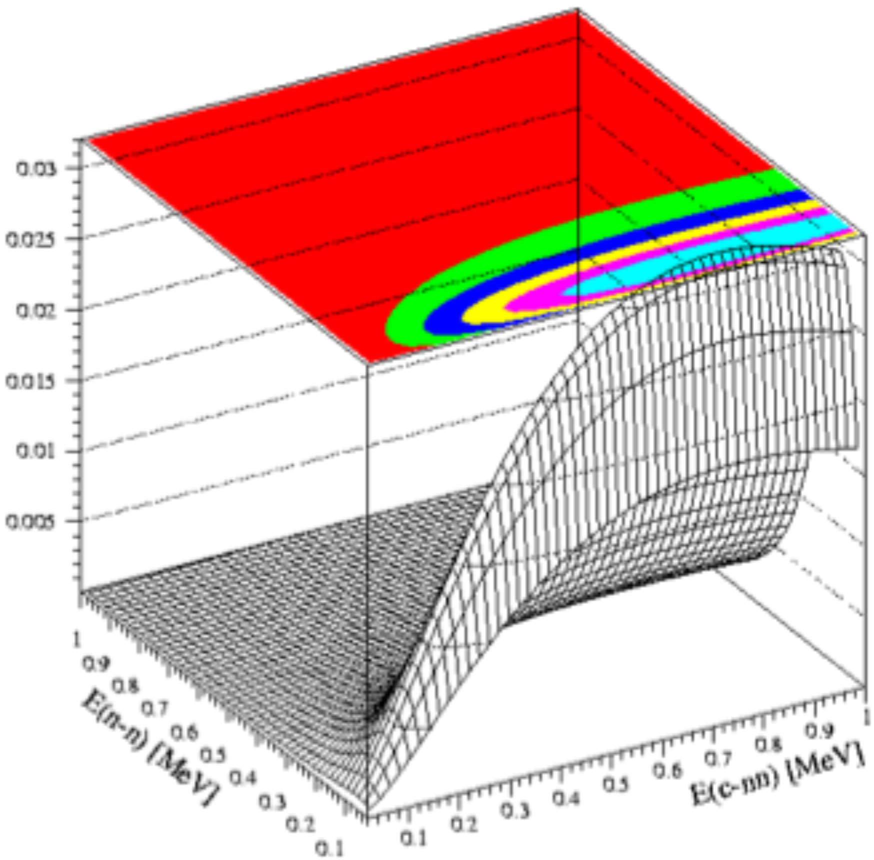
$$\psi(\rho\ \Omega_5^\rho, E\ \Omega_5^\kappa) \propto \frac{1}{(\kappa\rho)^{5/2}} \sum_{K,\gamma} C_{K\gamma}(E)\ \ \psi^R_{K\gamma}(\rho)\ Y_{K\gamma}(\Omega_5^\rho)\ Y_{K\gamma}(\Omega_5^\kappa)$$

with

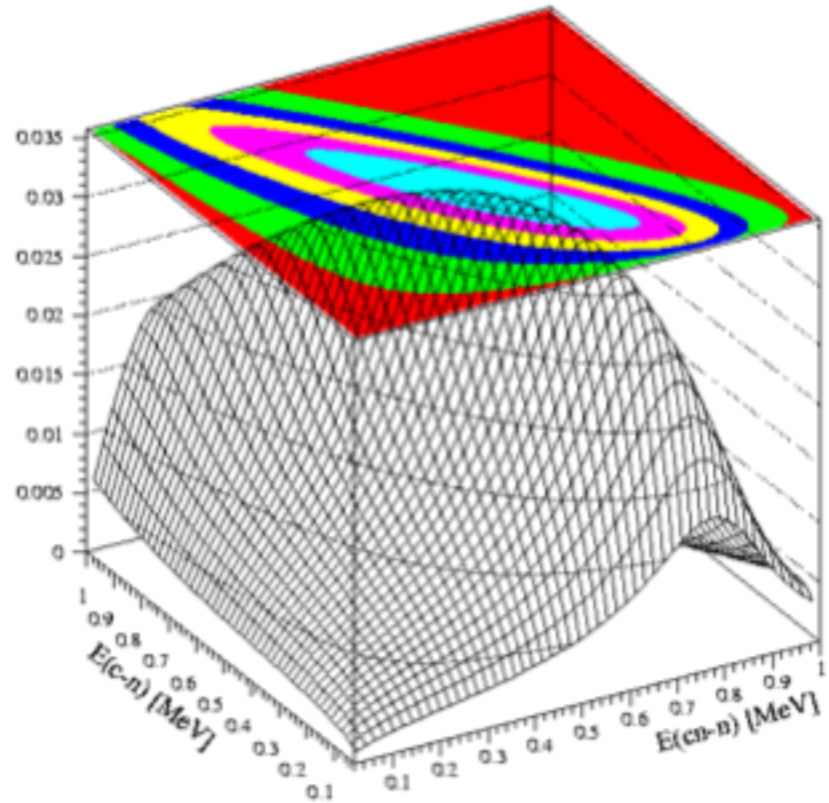
$$|C_{K\gamma}(E)|^2 = \frac{\Gamma_{K\gamma}}{(E-E_0)^2 + \Gamma^2/4}$$

Virtual states & Resonances

from B. Danilin, I. Thompson, et al



Virtual n-n pole

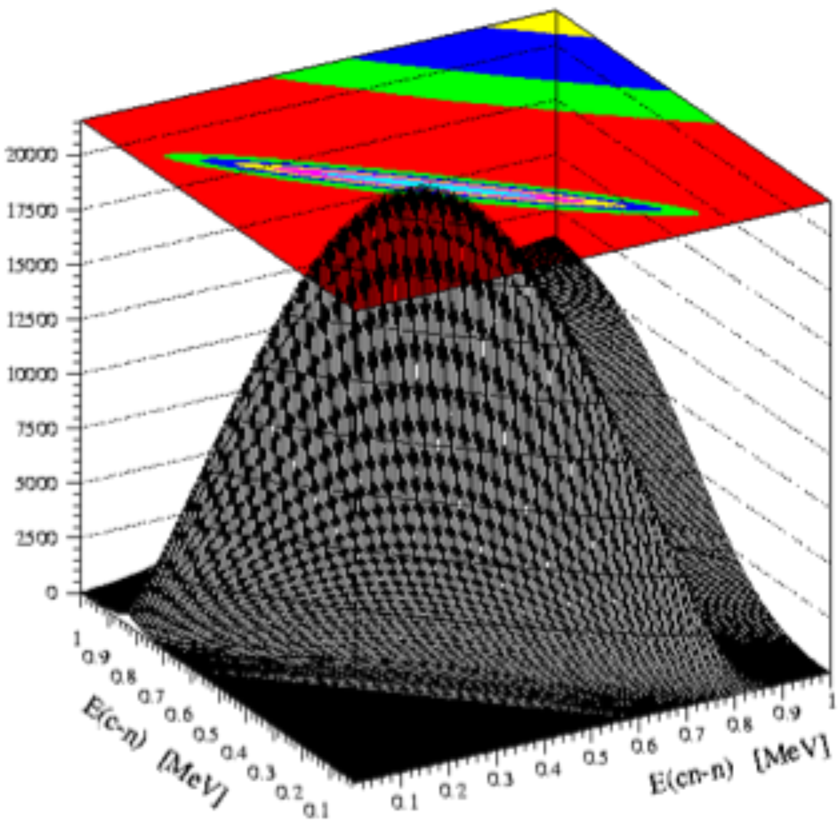


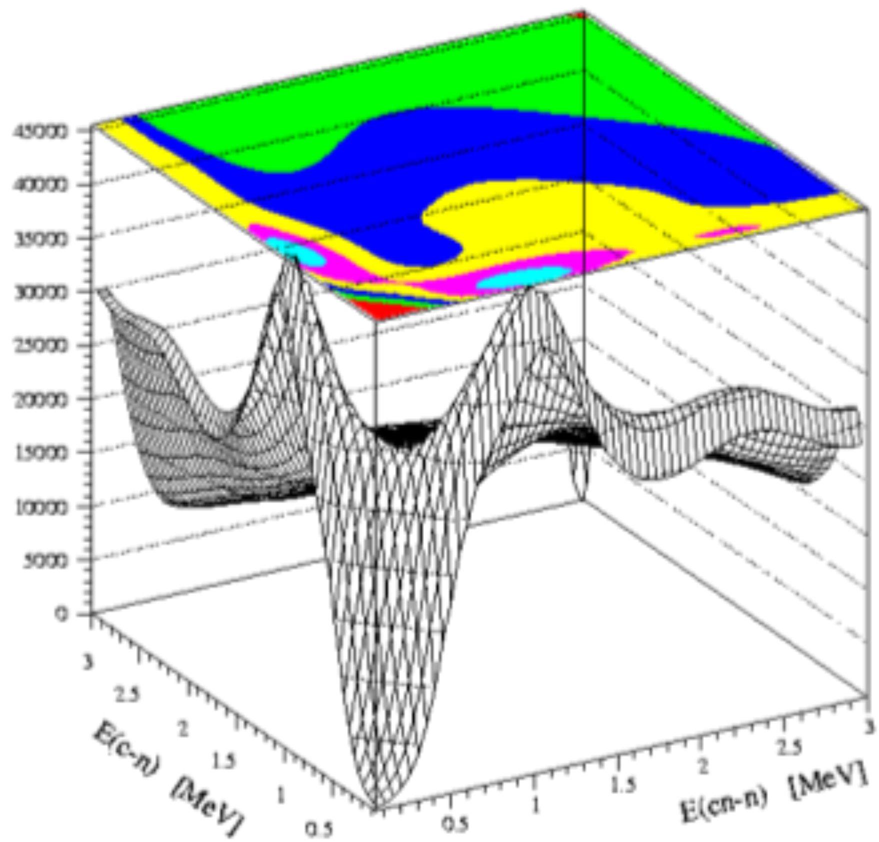
Effect of n-n 'resonance' in E(c-n), E(cn-n) coordinates

Continuum Energy Correlations

- Now average scattering wave functions over angles of $\mathbf{k}_x$ and $\mathbf{k}_y$, for fixed three-body energy E.
- Obtain similar plots for continuum energies.
- Continuum momentum & angular correlations also possible

^{6}He excitations & resonances





Pronounced 2^{+} resonance

No pronounced 1^{-} resonance

What a good reaction theory needs:

- Distinguish resonances and their backgrounds
- Recoil & finite-range of projectile vertex.
- Interference between exit partial waves
- Nuclear and Coulomb mechanisms
- Core excitation (static and/or dynamic)
- Final-state interactions:
  — between projectile fragments (needed if resonances)
  — between fragments and target (needed if close in)
- Multistep Pprocesses (higher order effects)

Elastic Breakup of 2N halo

- Elastic Breakup = Diffraction Dissociation:
  - all nuclear fragments survive along with the target in its ground state,
  - probes continuum excited states of nucleus.

- Need correlations in the three-body continuum of Borromean nuclei.

Inelastic reactions

- ^{11}Li(d,d') ^{11}Li* measurements (Kanungo et al, PRL **114**, 192502, 2015)

- See also:
  - ^{11}Li(p,p') ^{11}Li* measurements (Tanaka, et al, PLB 774, 268, 2017),
  - ^{6}He(^{12}C, ^{12}C)^{6}He(2^{+}) measurements (Kikuchi, PRC **88**, 021602R, 2013)

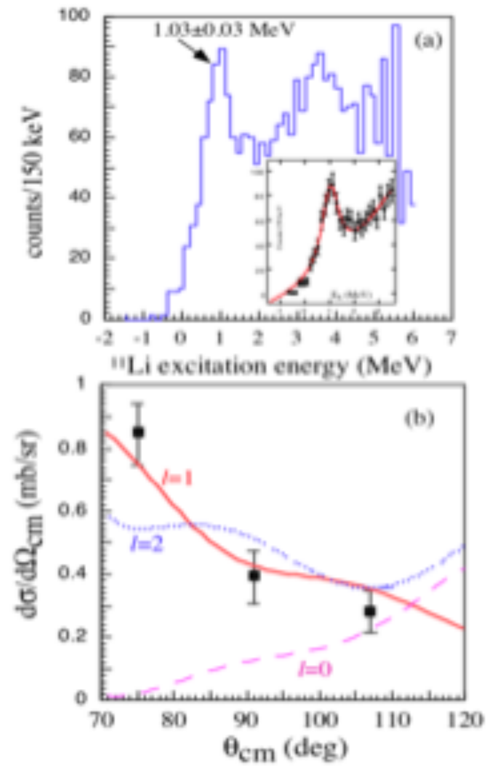


FIG. 3: (a) Inelastic scattering excitation energy spectrum for deuterons in coincidence with ^{9}Li. (b) The inelastic scattering angular distribution data for the resonance peak at E_{ex}=1.03 MeV. The curves are DWBA calculations for L=0 (pink dashed-line), L=1(red solid line), L=2(blue dotted line).

Transfer Reactions

Sanetullaev et al
PLB **755**, 481 (2016)

5.7A MeV ^{11}Li beam, DWBA calculations

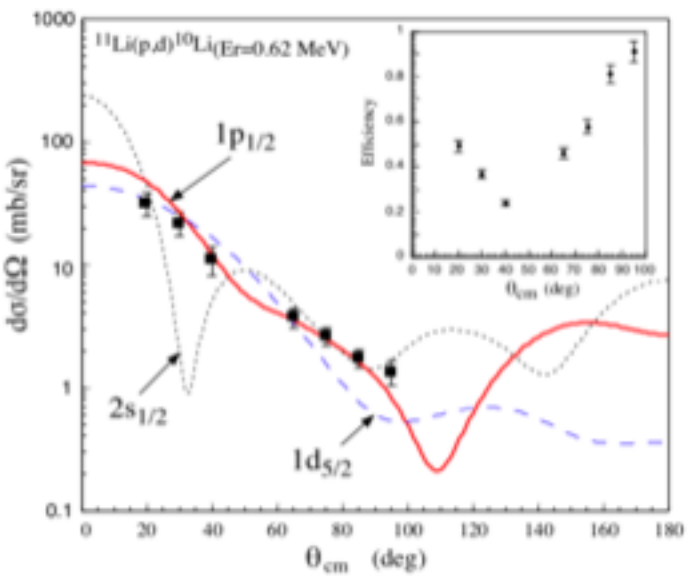
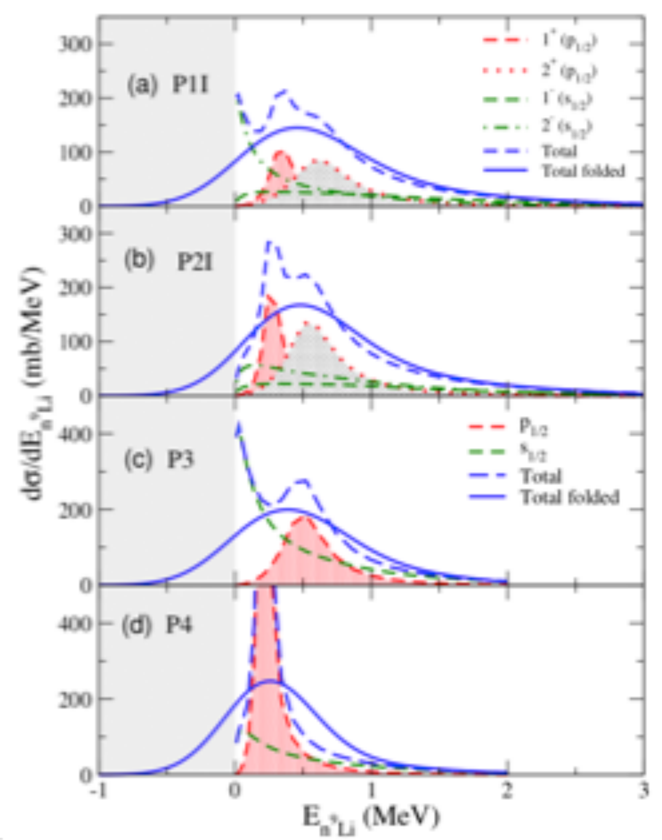


Figure 4: The angular distribution data for ^{11}Li(p,d)^{10}Li$_{E_r=0.62MeV}$. The curves show the DWBA calculations. The solid (red) curve / dashed (blue) curve / dotted (black) curve represents one neutron transfer from the $1p_{1/2}$ / $1d_{5/2}$ / $2s_{1/2}$ orbital. The inset shows the detection efficiency.

Casal et al, PLB, 767, 307 (2017)

Better: using structure overlaps from a full three-body model

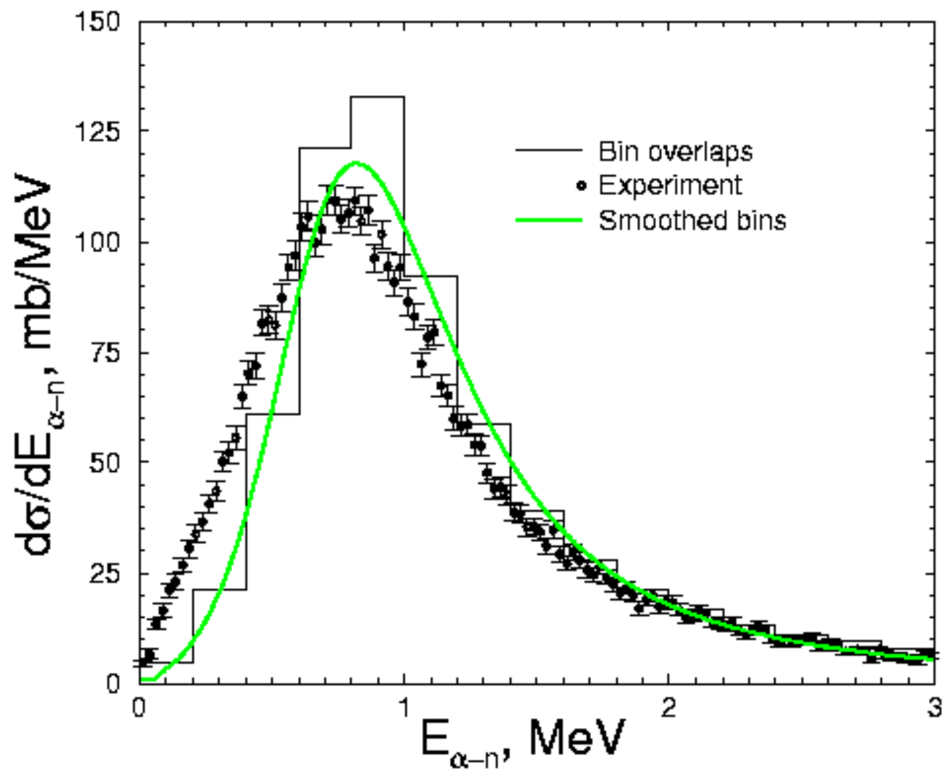


Stripping Reactions
1-N from 3-body Borromean Nuclei

- Removal of a neutron from ^{6}He, ^{11}Li, ^{14}Be,
  - populates states of ^{5}He, ^{10}Li or ^{13}Be.
  - Experiments measure decay spectrum of $^{5}He = {}^{4}He + n$, $^{13}Be = {}^{12}Be + n$, etc
- Can we predict any energy and angular correlations by Glauber model?
- Can we relate these correlations to the structure of the A+1 or the A+2 nucleus?

1N stripping from ^{6}He g.s.

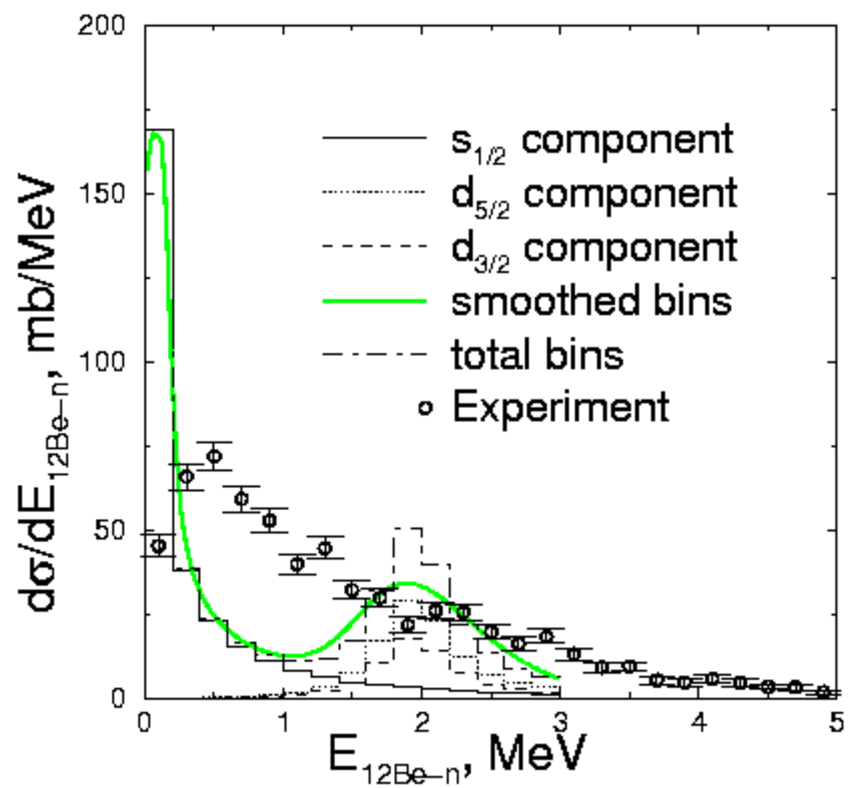
- Calculate overlaps: $<^5\text{He}(E_{\alpha\text{-}n})\ |^6\text{He(gs)}>$ for a range of $^5\text{He}(E_{\alpha\text{-}n})$ bin states,
- smooth histogram of Glauber bin cross sections.
- GSI data (H.Simon)



Theory: σ_{str}=137 mb, σ_{diff}=38 mb
Expt: σ_{str}=127±14 mb, σ_{diff}=30±5 mb
from T. Tarutina thesis (Surrey)

1N stripping from ^{14}Be g.s.

- Calculate overlaps: <^{13}Be($E_{\alpha\text{-}n}$) |^{14}Be(gs)>
- Inert-core ^{13,14}Be wfs.
- GSI data (H.Simon)
- See softer data, and not pronounced virtual-s and resonant-d peaks.
- New theory needed?



Theory: σ_{str}=109 mb, σ_{diff}=109 mb
Expt: σ_{str}=125±19 mb, σ_{diff}=55±19 mb

Breakup reactions
CDCC: Coupled Discretised Continuum Channels

Try CDCC:

- — Proposed by Rawitscher, developed by Kamimura group.
- — Treat Coulomb and Nuclear mechanisms
  - Need to check convergence of long-range Coulomb process!
- — All higher-order effects with a (r,R,L) reaction volume
- — Can calculate fragment coincident angular distributions:
  - Predict e.g. $d^3\sigma/dE_1 d\Omega_1 d\phi_{12}$ and fold with detector apertures & efficiencies

- — Extended by Neil Summers to XCDCC
  - Include excited state of 'core' in the projectile subject to breakup
  - Summers et al, PLB **650**, 124 (2007), Phys. Rev. C **74**, 014606 (2006), 069901E (2014)
- — Extended by de Diego et al, to CDCC with target excitations
  - (de Diego etal. PRC **89**, 064609, 2014)

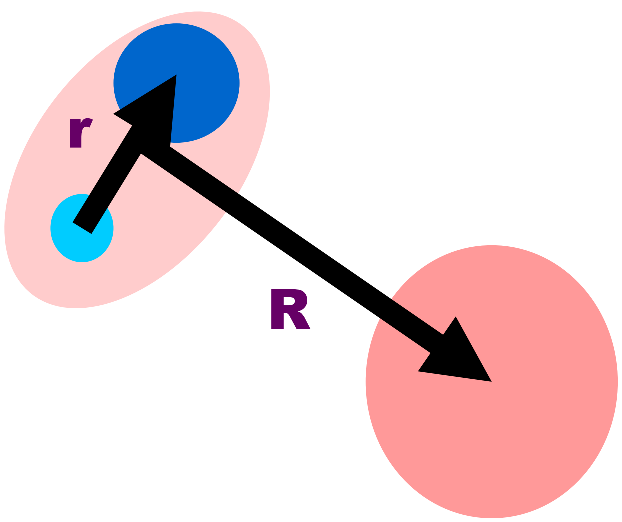
CDCC Hamiltonian & model space

The Hamiltonian for the reaction of a projectile on a target

$$H = h_{proj} + h_{targ} + T_\alpha + V_\alpha$$

$$\Rightarrow h_{proj} = h_{core} + h_{frag} + T_{cf} + V_{cf}$$

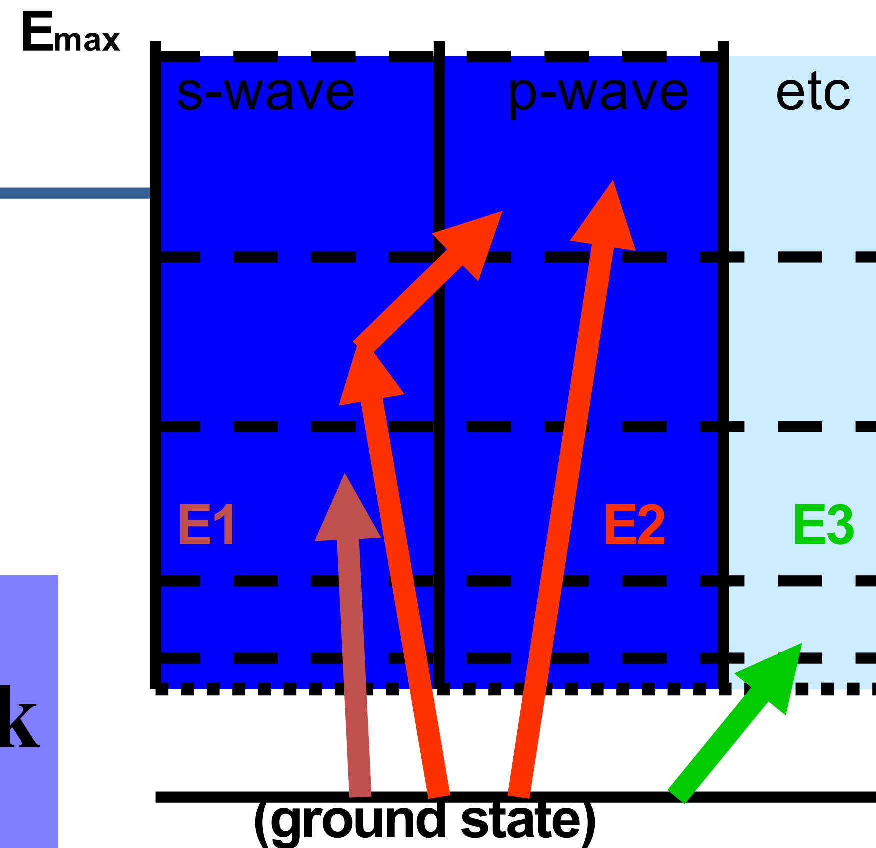
$$\Rightarrow V_\alpha = V_{core-targ} + V_{frag-targ}$$



$$\psi^{CDCC}_{JM}(r,R) = [\phi_0(r) \otimes Y_L(\hat{R})]_{JM}\, \chi^J_{0,L}(R) + \sum_{l=0}^{lmax} \sum_{L} \sum_{i=1}^{N} [\phi_{i,l}(r) \otimes Y_L(\hat{R})]_{JM}\, \chi^J_{i,l,L}(R)$$

(neglect the internal structure of the target)

CDCC Formalism



The CDCC basis consists of scattering wavefunctions averaged over an energy interval

$$h_{proj}\phi_k = \varepsilon_k \phi_k$$

$$\phi_{i,l} = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} w_i(k)\phi_{lm}(k,r)dk$$

$$N_i = \int_{k_{i-1}}^{k_i} |w_i(k)|^2 \, dk$$

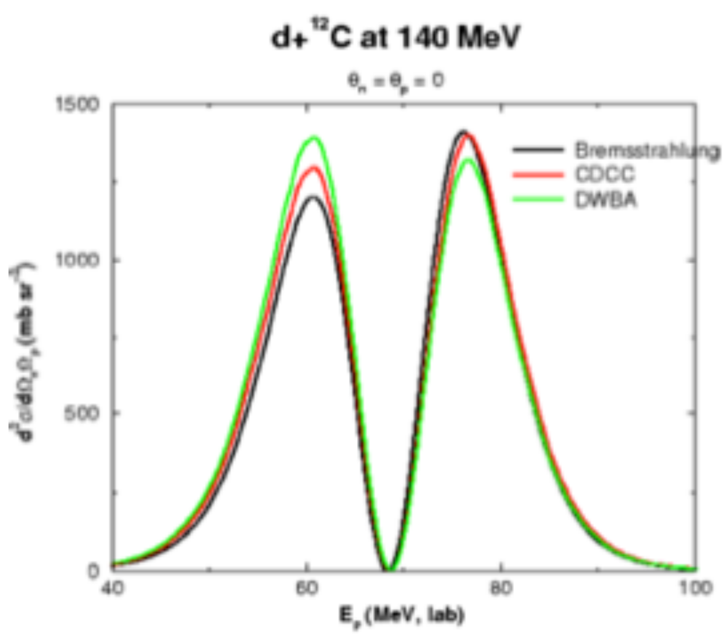
$$N_{bins} = \frac{k_{max}}{\Delta k}$$

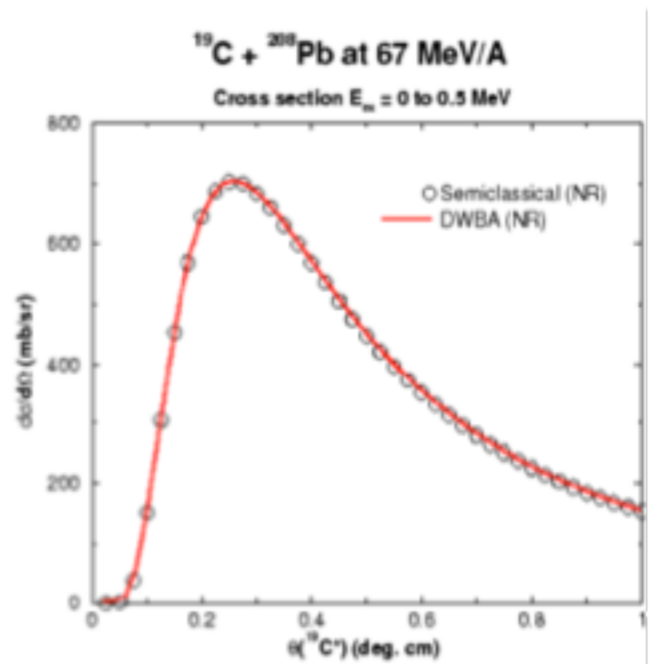
Coupling potentials

$$V_{il,i'l'}^{CDCC}(R) = <\phi_{il}(r)\,|\,V_\alpha(r,R)\,|\,\phi_{i'l'}(r)>$$

Testing CDCC Convergence

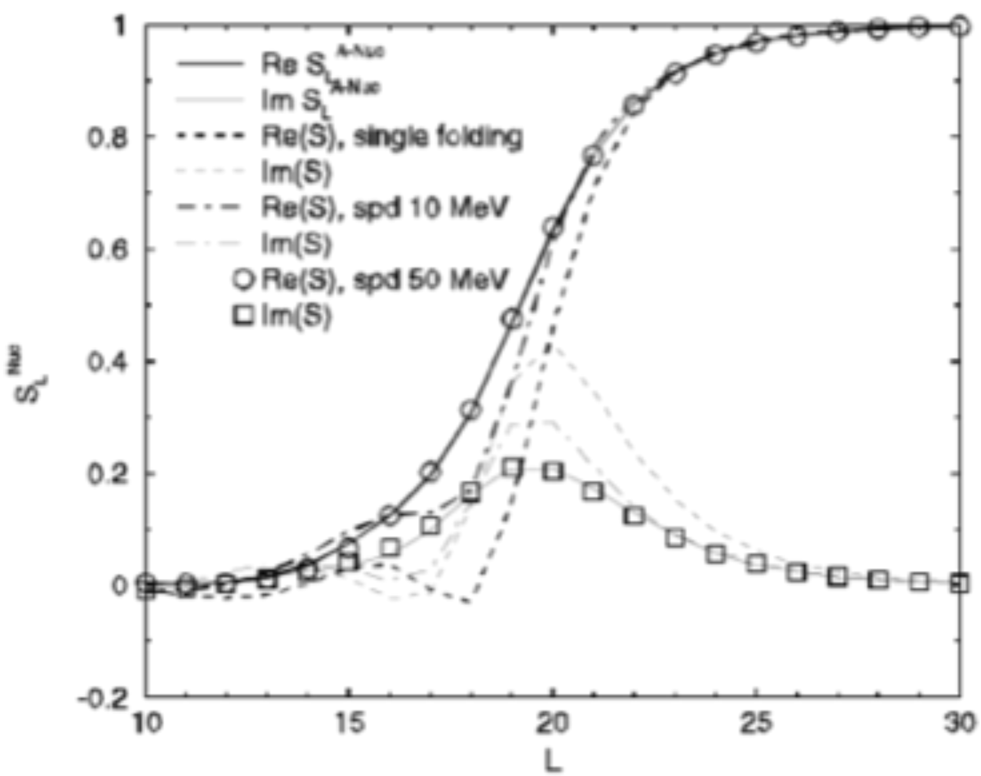
- Compare, in Adiabatic Few-Body Model, with Bremstrahlung integral
- Compare, in first-order PWBA model, with semiclassical theory



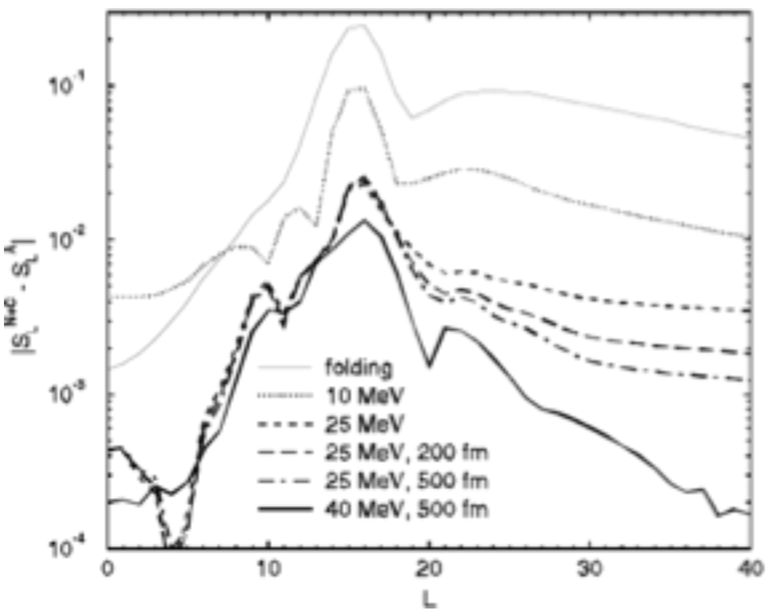


Note the ‘post-acceleration’

Adiabatic CDCC:
compare with Exact 3-body model



d+^{208}Pb at 50 MeV, nuclear only



Absolute errors in CDCC for d+^{208}Pb at 50 MeV, Nuclear+Coulomb

$^{15}C + {}^{9}Be$ breakup at 54 MeV/u

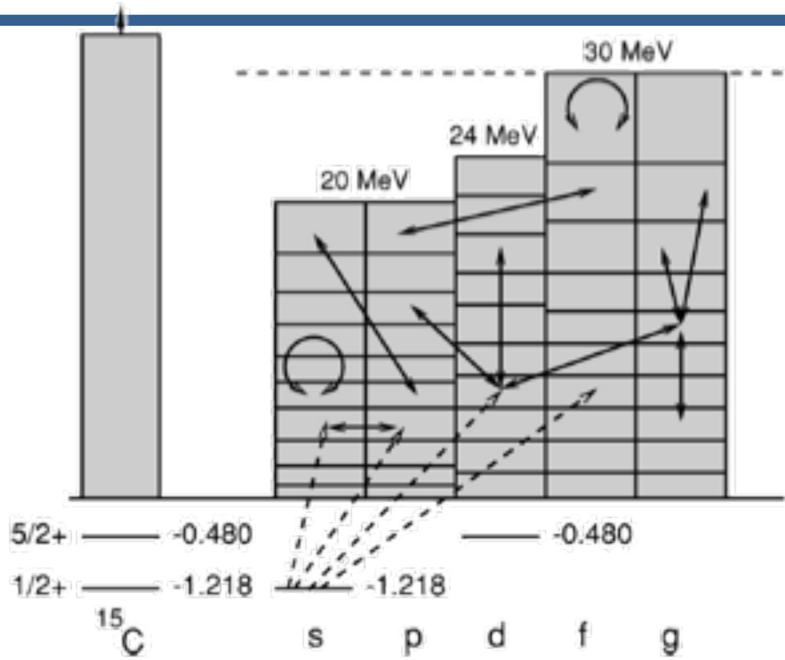


FIG. 4. Diagrammatic representation of the CDCC model space calculation for ^{15}C. The left side shows the physical bound states and continuum and the right hand side the included continuum bins (10) in each $n+{}^{14}$C partial wave. The dashed arrows are representative of the one-way couplings included in the DWBA. The solid arrows show representative couplings for the full CDCC calculations which connect all bins, including diagonal bin couplings, with two-way couplings to all orders. Relative h waves were found to make negligible contributions.

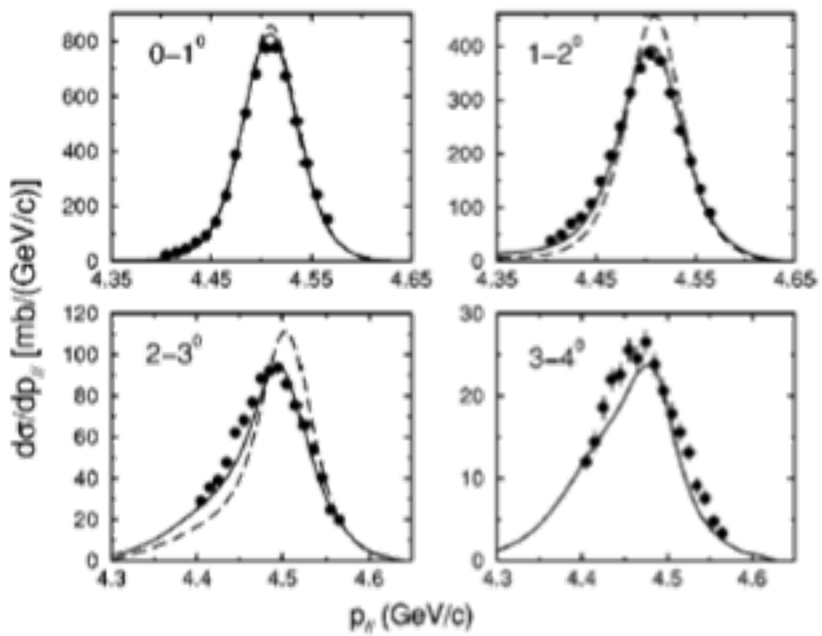
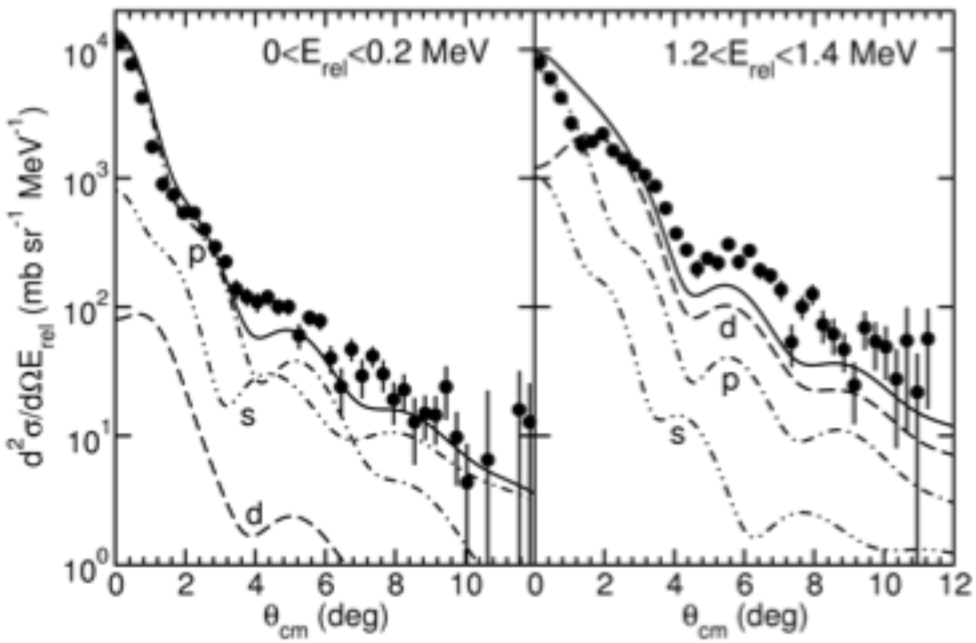


FIG. 11. The nucleon-removal parallel momentum distributions $d\sigma/dp_{\parallel}$, for the ^{15}C+^{9}Be reaction at 54 MeV/nucleon to the ^{14}C ground state, shown on a more familiar linear scale. The solid curves assume the stripping contributions have the same form as that calculated using the CDCC. The dashed curves assume the stripping contributions have a parallel momentum distribution at all angles of the residue given by the eikonal calculation shown in Fig. 2.

Tostevin et al, PRC **66**, 024607 (2002)

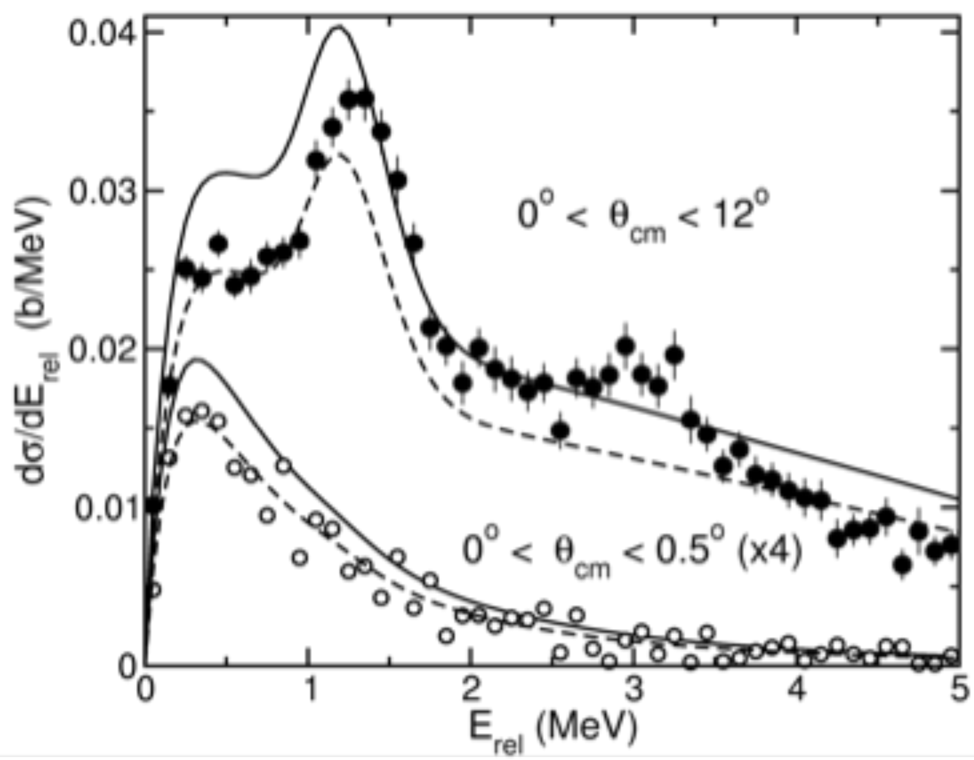
^{11}Be + ^{12}C breakup at 67 MeV/u



Angular distributions of ^{11}Be*
left: low-energy continuum
right: region of $d_{5/2}$ resonance

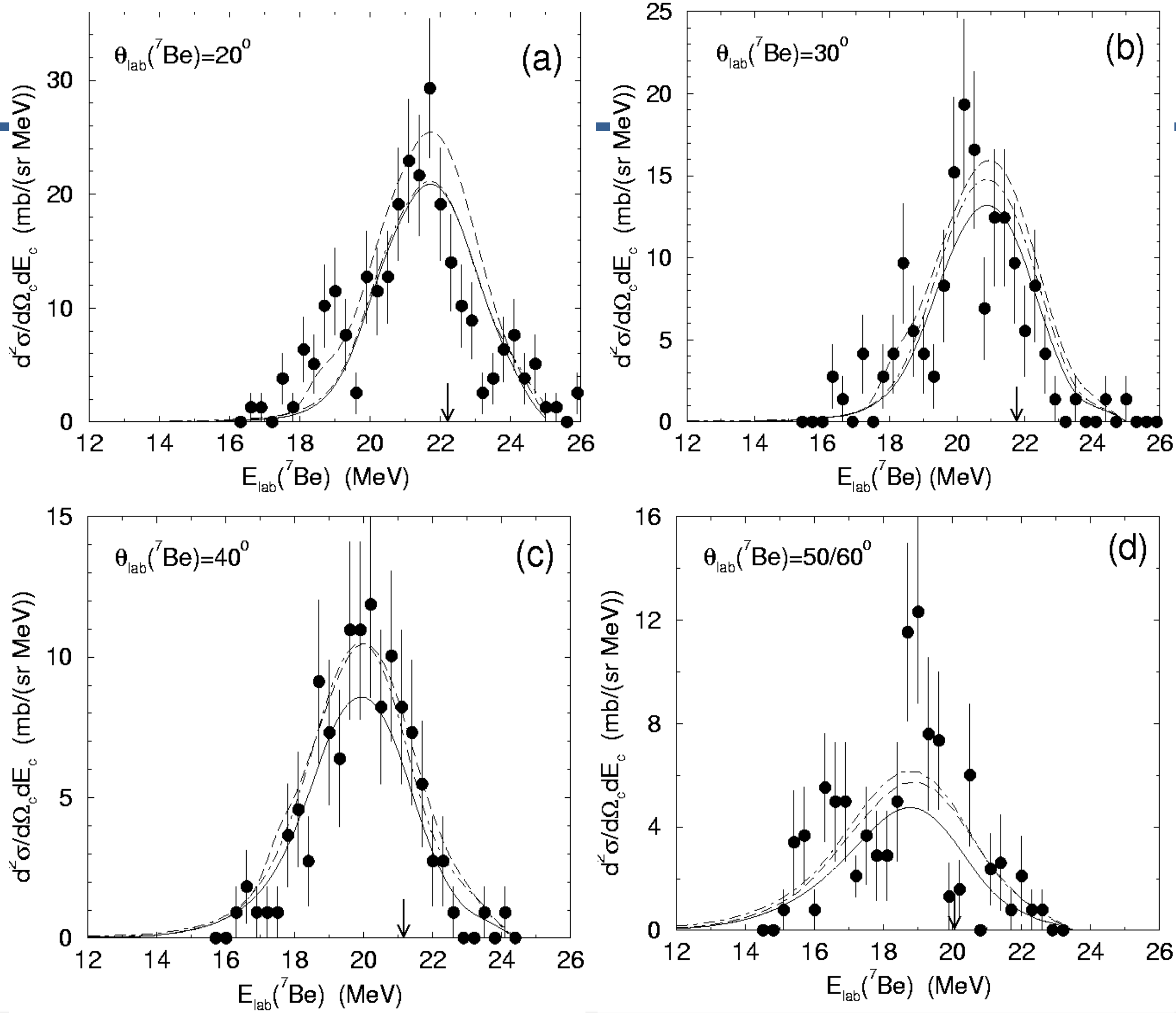
CDCC calculations of
Howell, Tostevin, Al-Khalili,
J. Phys. G **31** (2005) S1881

Energy excitation spectrum
dashed line: multiplied by 0.8



Breakup reactions

CDCC $^{8}B + ^{58}Ni \rightarrow {^{7}Be}+p + ^{58}Ni$ (E_b=26 MeV)

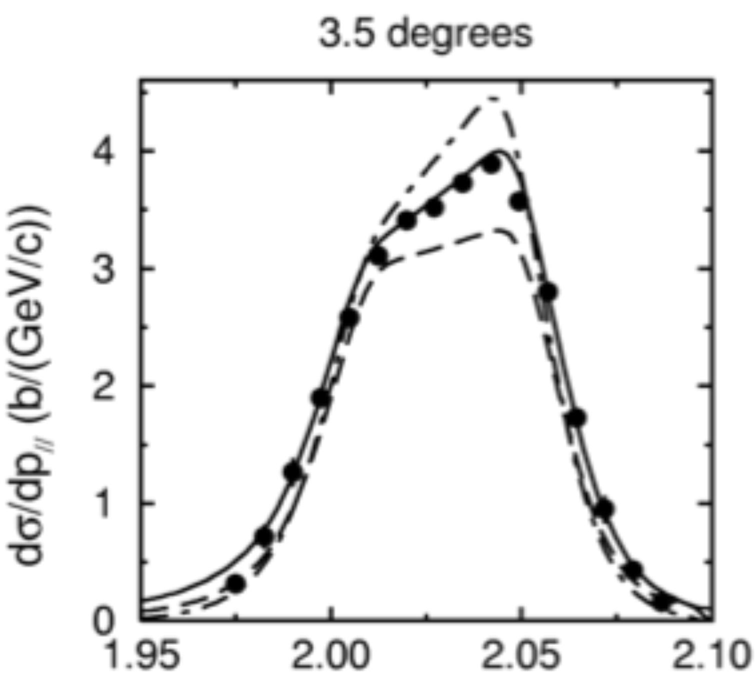


Energy distributions: Excellent agreement with the data!

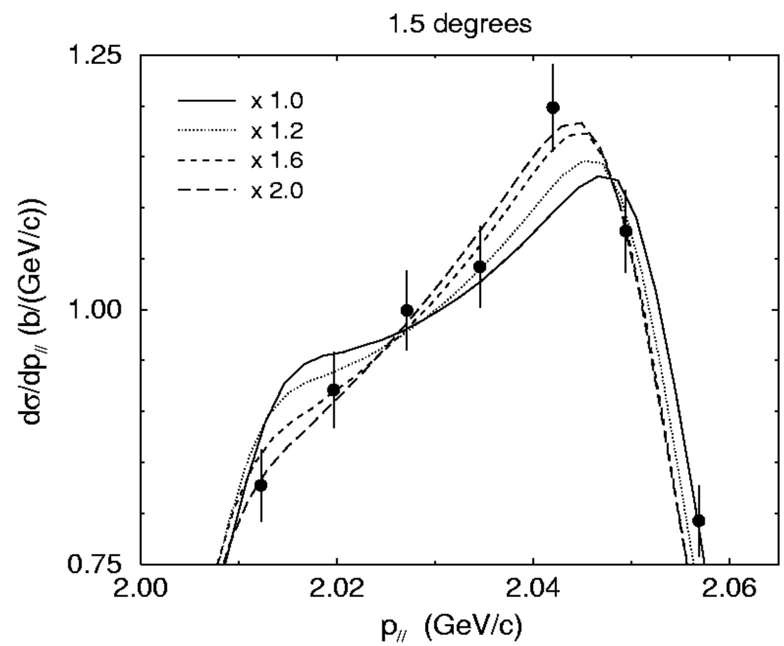
ESB+BG
KIM+BG
ESB+VG
ND data

$^{8}B + ^{208}Pb \rightarrow ^{7}Be$ parallel momentum distributions

44 MeV/u



Dot-dashed: semiclassical Coul.
Solid: Coulomb+nuclear DWBA
Dashed: CDCC coupled channels
- reduced asymmetry



CDCC calculations with scaled E2 amplitudes
- need to increase asymmetry again!

from Mortimer et al., Phys Rev C 65 (2002) 64619

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