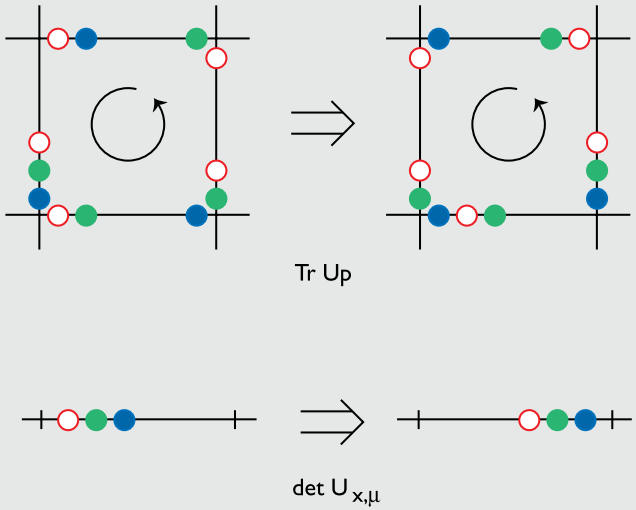
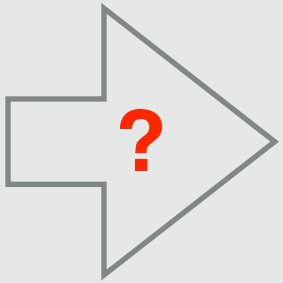
Quantum Link Lattice Field Theory

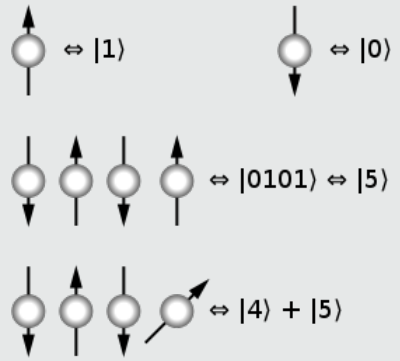
(Circa 1998: An accidental approach to Quantum Algorithms ?)

QCC Abacus





Fermionic Qbit Algorithm ?



qubits can be in a superposition of all the clasically allowed states

Rich Brower, Boston University

Next step in Quantum Science for HEP Sept 14 2018 Fermi Lab

QUANTUM LINKS (AKA D-THEORY)

Early efforts of new approach to "efficient classical computing in Lattice Gauge Theory"!

- *D. Horn, Finite Matrix Model with Continuous Local Gauge Inv. Phys. Lett. B100 (1981)*
- *P. Orland, D. Rohrlich, Lattice Gauge Magnets: Local Isospin From Spin Nucl. Phys. B338 (1990) 647*
- *S. Chandrasekharan, U-J Wiese Quantum links models: A discrete approach to gauge theories Nucl. Phys. B492 (1997)*
- *R. C. Brower, S. Chandrasekharan, U-J Wise , QCD as quantum link model, Phys. Rev D 60 (1999).*
- *R. C. Brower, The QCD Abacus: APCTP-ICPT Conference, Seoul, Korea, May (1997)*
- *R. C. Brower, S. Chandrasekharan, U-J Wiese, D-theory: Field quantization … discrete variable Nucl. Phys. B (2004).*
- *U-J Wiese, See recent talk at FNAL QC workshop Dec 6-7, 2017 & lots of published work and many others.*

For bosons I will use the "Fermionic pairs" (or what Uwe-Jens calls "rishons")

Collecting Tools & Pebbles

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

FERMIONIC QUBIT GATES

Each Fermion

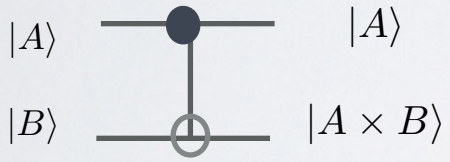
$$a^\dagger a + a a^\dagger = 1 \qquad aa = a^\dagger a^\dagger = 0$$

On Each qubit

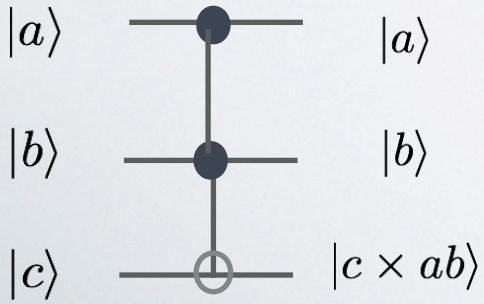
$$a^\dagger(\alpha|1\rangle + \beta|0\rangle) = \beta|1\rangle \qquad a(\alpha|1\rangle + \beta|0\rangle) = \alpha|0\rangle$$

$$a^\dagger + a \implies X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (a + a^\dagger)^2 - 1 \implies H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$(a^\dagger + a)/i \implies Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad (2a^\dagger a - 1) \implies Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



$$(1 - a^\dagger a) + a^\dagger a(b + b^\dagger) \implies \text{ContrNOT}$$



$$(1 - a^\dagger a b^\dagger b) + a^\dagger a b^\dagger b(c^\dagger + c) \implies \text{Toffoli}$$

Classical Reversible Computing Conserving Energy R. Landauer IBM J journal of Research and Development, vol. 5, pp. 183-191, 1961

UNIVERSALITY: Tools & Tricks

1. *Lattice: Discretization: (Graphs & Simplicial Structure)*

    - Subgroups of Space-Time Isometries

2. *"Field Truncation" (Group Algebra & IR expansion)*

    - Discrete Manifolds for Global/Gauge/Spinors

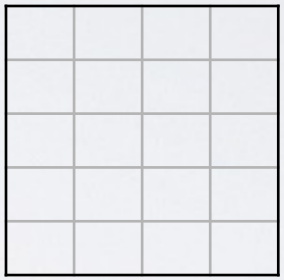
3. **"Dualities" (Hodge star & Dim Reduction)**

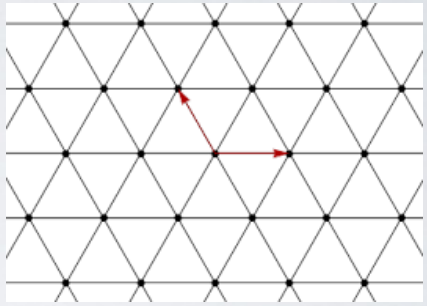
    - Gauge/Spin & Ads/CFT & Gauge/Gravity etc

Discretization/Truncation vs Universality

- There are many lattice microstructures that represent the SAME field theory in the continuum

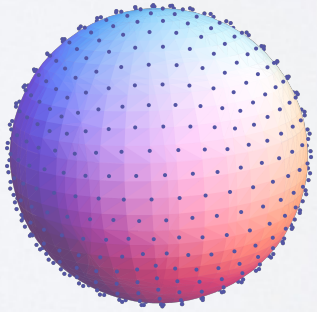
- *The Wilson Fisher c = 1/2 CFT is the same on square, triangle or spherical lattice!*

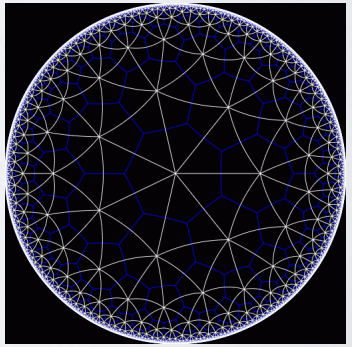




- *Using Ising Spins or phi 4th fields*

$$s \in \pm 1 \iff \phi_x \in \mathbb{R}$$





- *The bosonic Sine Gordon Theory is Fermionic Thirring Theory*

- *N = 4 SUSY dual AdS Gravity*

QUANTUM SPINS: O(3)

CLASSICAL O(3) MODEL

$$Z = \int \mathcal{D}\vec{s}\ \exp(-\frac{1}{g}S[\vec{s}]) \qquad S[\vec{s}] = -\sum_{\langle x,y\rangle} \vec{s}_x \cdot \vec{s}_y \quad , \quad \vec{s}_x \cdot s_x = 0$$

$$\vec{s}_x \to a_i^\dagger(x)\vec{\sigma}^{ij}a_j(x) \quad \text{or} \quad \hat{S}_{ij} = a_i^\dagger(x)a_j(x) \quad \text{with} \quad Tr[\hat{S}_x] = 0;$$

$$\{a_i(x), a_j^\dagger(x)\} = \delta_{ij}\delta_{xy} \quad , \quad \{a_i^\dagger(x)a_j^\dagger(x)\} = 0 \quad , \quad \{a_i(x), a_j(x)\} = 0$$

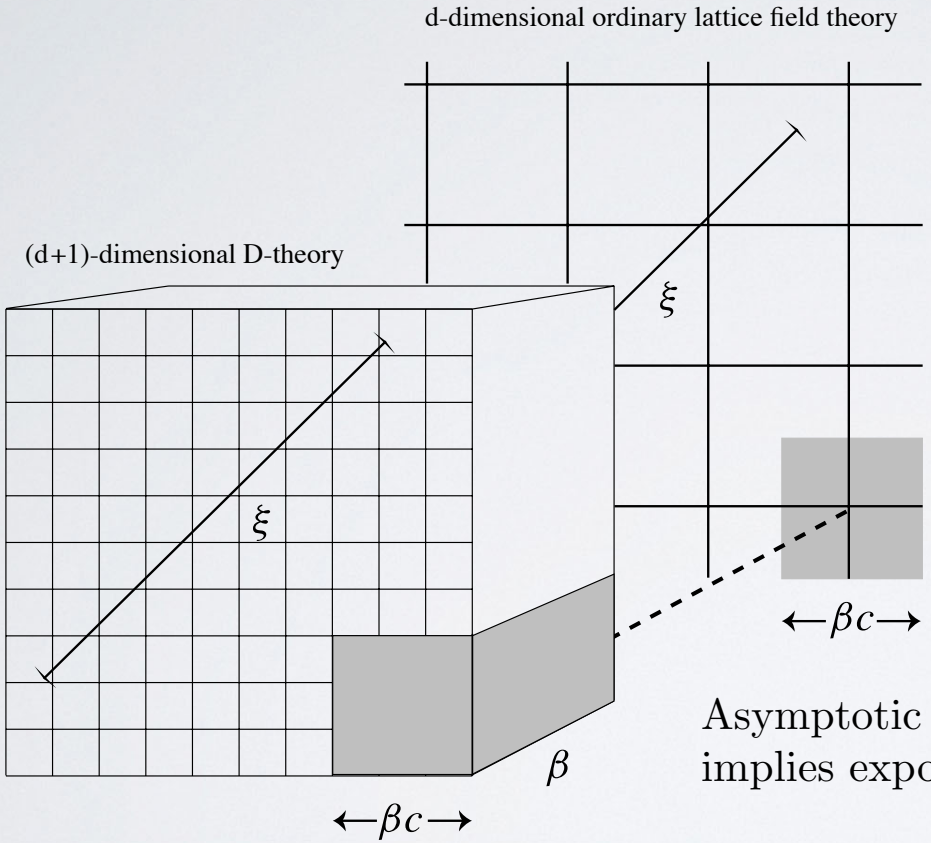
QUANTUM O(3)

$$Z = \mathrm{Tr}\exp(-\beta\hat{H}). \qquad \hat{H} = \sum_{\langle x,y\rangle} Tr[\hat{S}_x\hat{S}_y] \quad , \quad Tr[\hat{S}_x] = 0$$

Global Rotation: $\vec{J} = \sum_x Tr[\vec{\sigma}\hat{S}_x] \implies [\vec{J}, \hat{H}] = 0$

Local Fermion No: $\hat{F}_x = Tr[\hat{S}_x] \implies [\hat{F}_x, \hat{H}] = 0$

D-THEORY: FERMIONIC GAUGE, SCALAR AND DIRAC LATTICES



$$Z = \mathrm{Tr}\exp(-\beta\hat{H}).$$

Original Motivation: MAKE
Easy Bosonic into Hard Fermionic
Theories with maybe smart cluster
(aka worm) algorithms?

$$\xi = 1/am \sim \exp[c\beta/g^2]$$

Asymptotic Freedom of 2d spin and 4d gauge implies exponential dimensional reduction as $\beta \to \infty$

see Square-Lattice Heisenberg Antiferromagnet at Very Large Correlation Lengths
B. B. Beard, R. J. Birgeneau, M. Greven, and U.-J. Wiese Phys. Rev. Lett. **80**, 1742 (1998)

OPERATOR QUANTUM GAUGE LINK

On each link (x,x + μ) introduce $2N_c$complex fermion

$a, a_i^\dagger$ right(+) moving and $b_j, b_j^\dagger$ left(-) moving fluxon

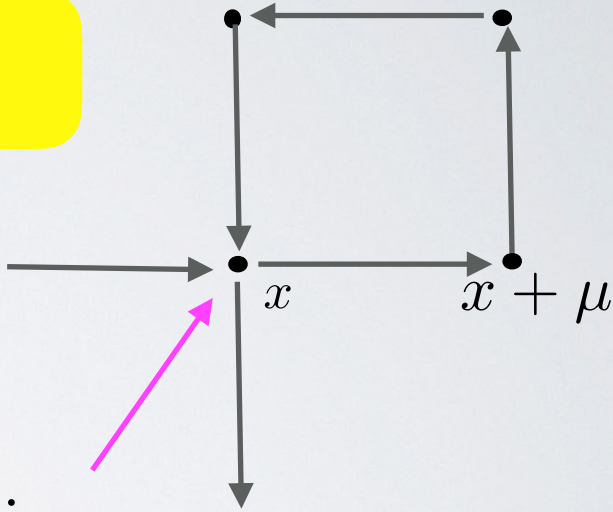
LINK:

$a_i \longrightarrow b_j^\dagger$

$$U_{ij}(x, x+\mu) \quad \rightarrow \quad \hat{U}_{ij} = a_i(x)\, b_j^\dagger(x+\mu)$$

$$\{a_i, a_j^\dagger\} = \delta_{ij} \qquad \{b_i, b_j^\dagger\} = \delta_{ij}$$

Local Gaue Operators $\Omega_{ij}(x) = a_i^\dagger(x) a_j(x) + \cdots$



For SU(3) QCD have SU(6) per Link Lie Algebra

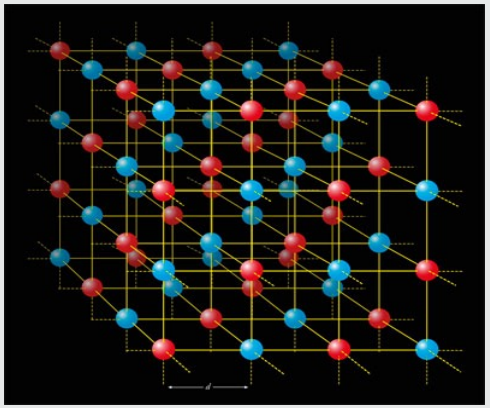
$$\begin{bmatrix} a_i a_j^\dagger & a_i b_j^\dagger \\ b_i a_j^\dagger & b_i b_j^\dagger \end{bmatrix}$$

Hilbert Space is a large qubit array of color vectors in 4d space-time

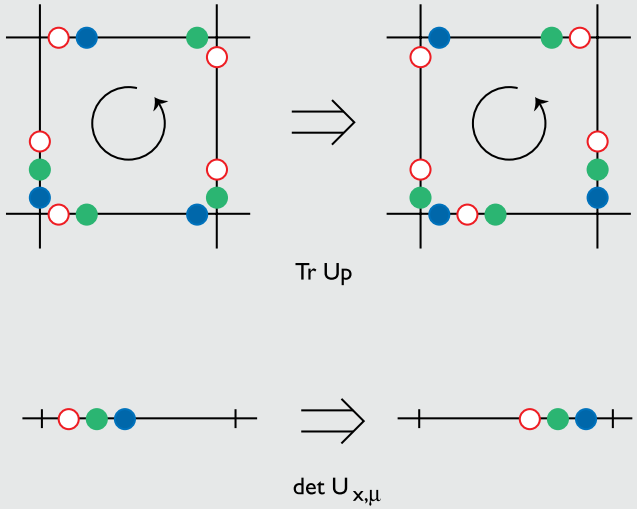
THE QCD ABACUS

$$\hat{H} = \beta \sum_{x,\mu\neq\nu} \mathrm{Tr}[\hat{U}_{x,\mu}\hat{U}_{x+\hat{\mu},\nu}\hat{U}^{\dagger}_{x+\hat{\nu},\mu}\hat{U}^{\dagger}_{x,\nu}] + \sum_{x,\mu} [\mathrm{det}\hat{U}_{x,\mu} + \mathrm{det}\hat{U}^{\dagger}_{x,\mu}]$$

circa 2400 b.c Abacus

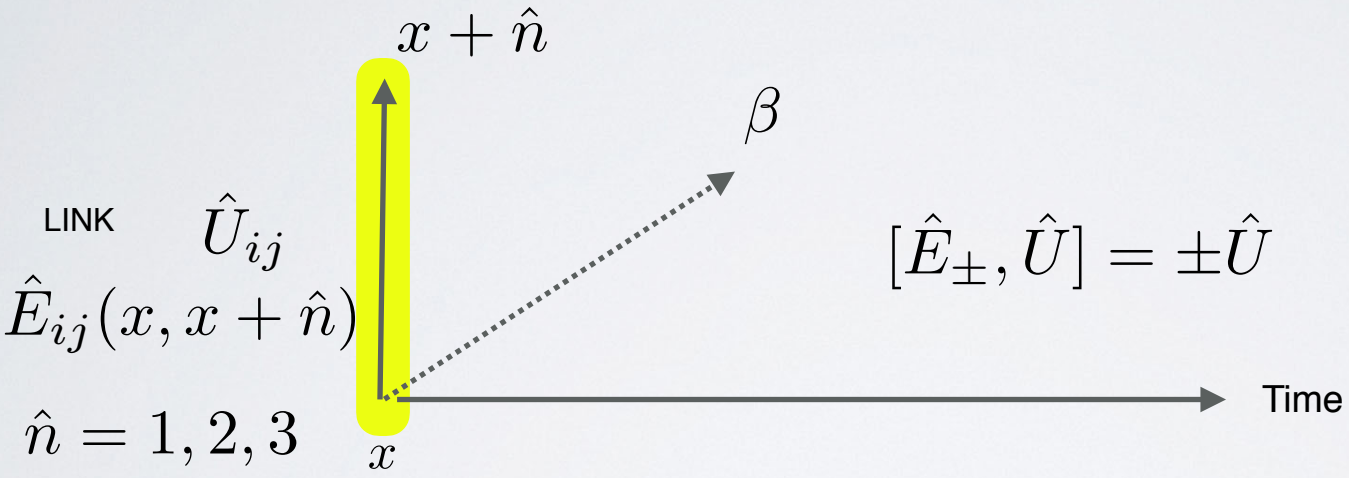


circa 20xx a.d.



QC MINKOWSKI DYAMICS

- Discrete 5th Axis Ls Layers (with Domain Wall Fermions)
- Change mu = 4 axis to Time with "Domain Wall" Hamiltonian.



$$[\hat{E}_{\pm}, \hat{U}] = \pm\hat{U}$$

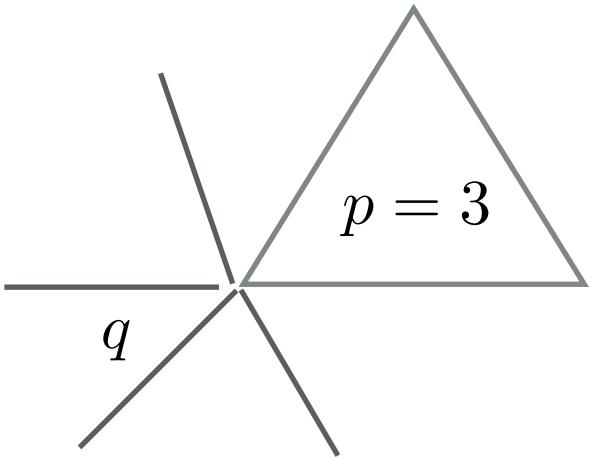
$$\hat{H}_0 = -\frac{1}{4g^2}\sum_{\square}\mathrm{Tr}[\hat{U}_{\square} + \hat{U}_{\square}^{\dagger}] + g^2\sum_{\langle x,y\rangle} Tr[\hat{E}_{+}^{2}(x,y) + \hat{E}_{-}^{2}(x,y] + \text{quarks}$$

SEE: Uwe-Jens Wiese, Ultracold Quantum Gases and Lattice Systems: Quantum Simulation of Lattice Gauge Theories Nucl. Phys. A931 (2014) 246-256

EQUILATERAL TRIANGULATION

Triangle case p = 3

Preserves Discrete
Subgroup of Isometries

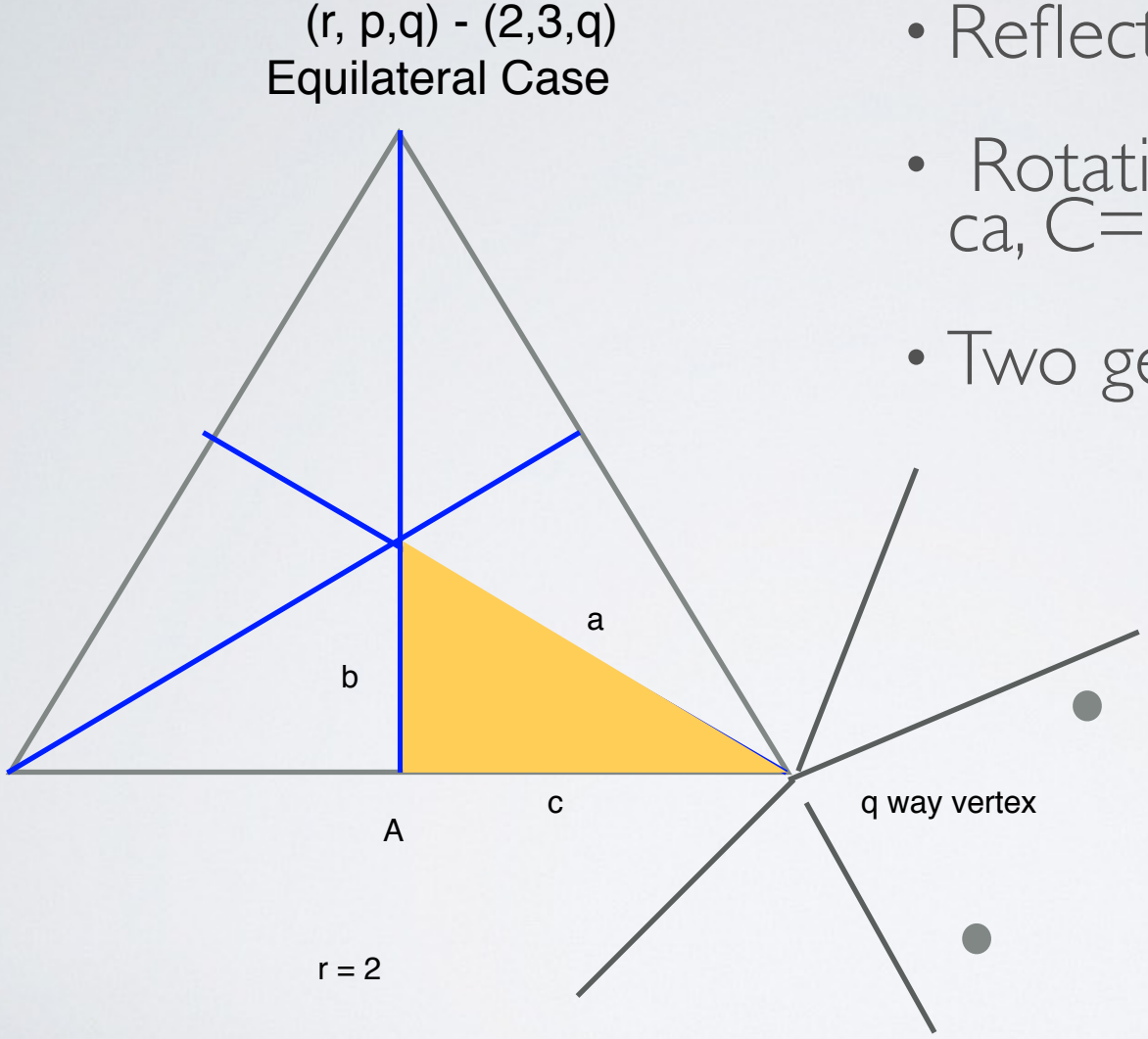


$$\frac{1}{p} + \frac{1}{q} > 1/2 \qquad \text{de Sitter} \quad \mathbb{S}^2 \qquad \text{vertex} \quad q = 3, 4, 5$$

$$\frac{1}{p} + \frac{1}{q} = 1/2 \qquad \text{flat} \quad \mathbb{T}^2 \qquad \text{vertex} \quad q = 6$$

$$\frac{1}{p} + \frac{1}{q} < 1/2 \qquad \text{Hyperbolic} \quad \mathbb{A}dS^2 \qquad \text{vertex} \quad q = 7, 8, 9, \cdots$$

Triangle Group Tiling

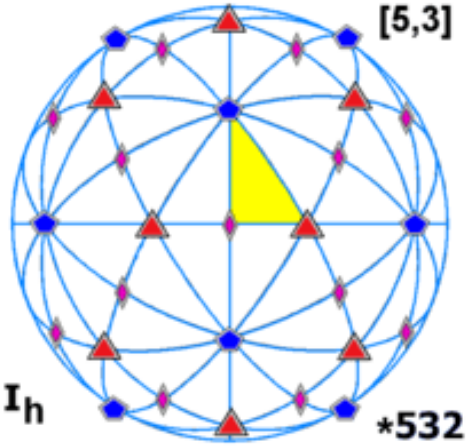


- Reflection: a, b, c
- Rotations: A = bc, B = ca, C= AB =ba
- Two generators

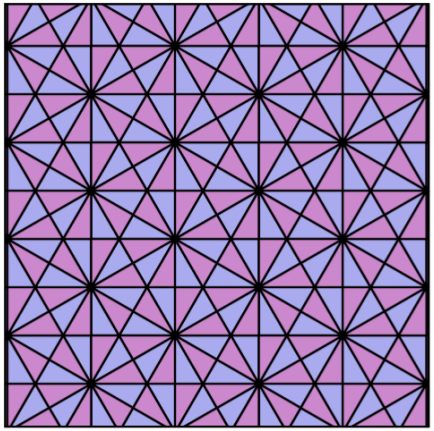
$$\Delta(p,q,r) = \{a,b,c | a^2 = b^2 = c^2 = A^p = B^q = C^r\}$$

DISCRETE ISOMETRIES & THE TRIANGLE GROUP

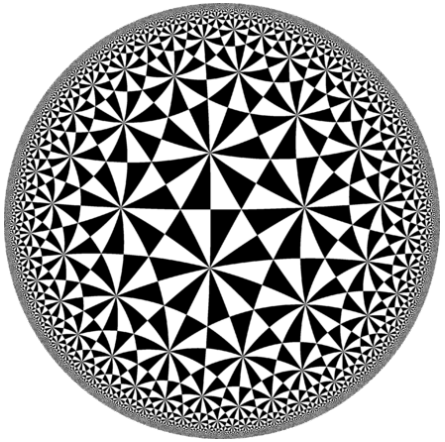
$$\frac{\pi}{p}+\frac{\pi}{q}+\frac{\pi}{r} \quad \begin{cases} > \pi & \text{Postive curvature} \\ = \pi & \text{Zero curvature} \\ < \pi & \text{Negative Curvature} \end{cases}$$



$(2,3,5)$
120 element
Icosahedral in O(3)



$(2,3,6)$
Triangle Lattice
on Euclidean $\mathbb{R}^2$

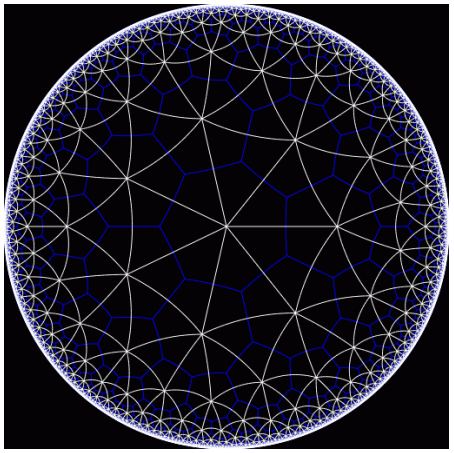


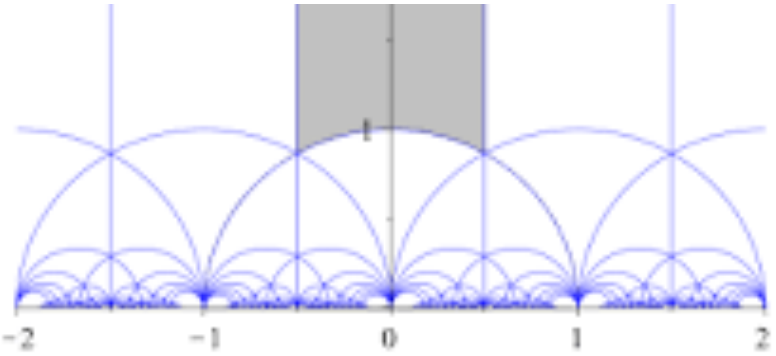
$(2,3,7)$
Subgroup of Modular
Group on $\mathbb{H}^2$

https://en.wikipedia.org/wiki/(2,3,7)_triangle_group

Hyperbolic (e.g. Poincare Disk) and Global AdS

q = 7



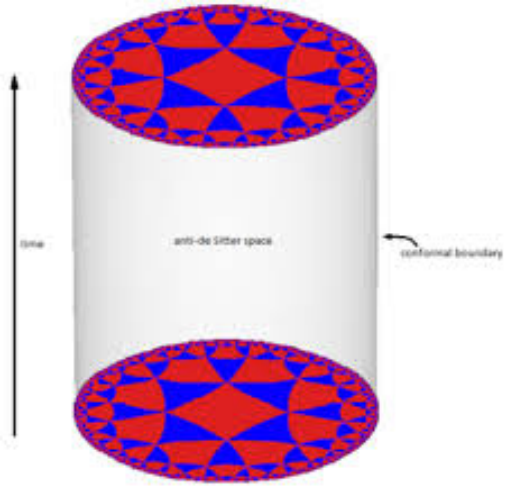


Triangle Group Tessellation: Preserve Finite subgroup of the Modular Group

1/2 + 1/3 + 1/q < 1

$$z \to \frac{az+b}{cz+d} \qquad ad-bc=1$$

$$a,b,c,d \in \mathbb{Z}\ mod\ q$$

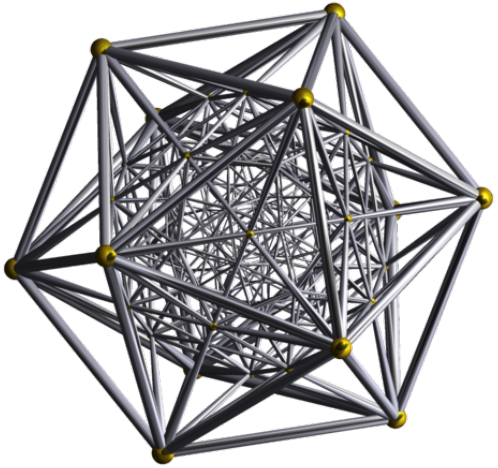


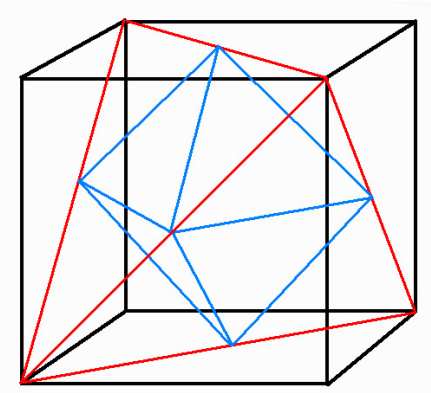
Are these Tessellation "Tensor Networks" ?
YES: See Daniel Harlow's Slide from Wednesday

Can we do QC lattice Field Theories in AdS?
Classical YES /QC Maybe

3 Spheres and 4D Radial Simplicial Lattices

$$\mathbb{S}^3 \quad \Longrightarrow \quad \mathbb{R} \times \mathbb{S}^3$$





Aristotle' s 2% Error!

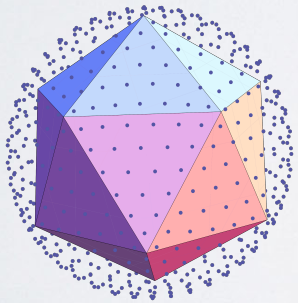
$$(2\pi - 5ArcCos[1/3])/(2\pi) = 0.0204336$$

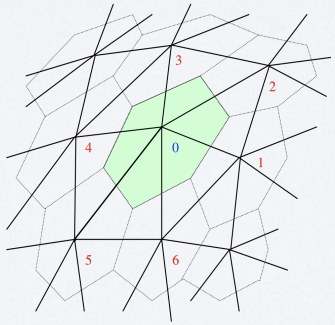
Fast Code Domains of
Regular 3D Grids on Refinement

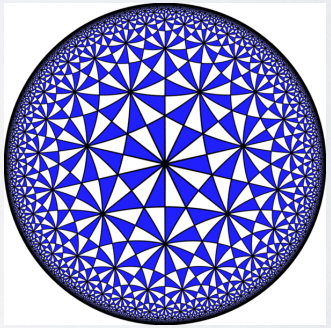
The full symmetry group of the 600-cell is the Weyl group of H_4. This is a group of order 14400. It consists of 7200 rotations and 7200 rotation-reflections. The rotations form an invariant subgroup of the full symmetry group.

QUANTUM "RISHONS" FOR GAUGE & SPIN & DIRAC ?

- HOW GENERAL IS TOTAL FERMIONIC APPROACH?
- WHAT CLASSICAL COMPLEXITY VS QC COMPLEXITY?
- START WITH 2D O(3) as H for 1+1 and 2 + 1
- More General Simplicial Lattice for CFT & Gauge/AdS ?







See https://arxiv.org/abs/1803.08512 https://arxiv.org/abs/1610.08587

Take Aways & Questions

- Universality gives a huge tool box of lattices approaches a given field theory: These maybe change of variable on existing lattice, truncation OR entirely different field theory representations.

- The best "classical" algorithms have a range of multi-grid and RG methods that are close cousins of tensor networks.

- I expect the "classical" and "quantum" to be an equally adept at "stealing" ideas for each other.

- Are good example of herbed classical/quantum algorithms?

- Are really any (even slow) real time quantum algorithms?