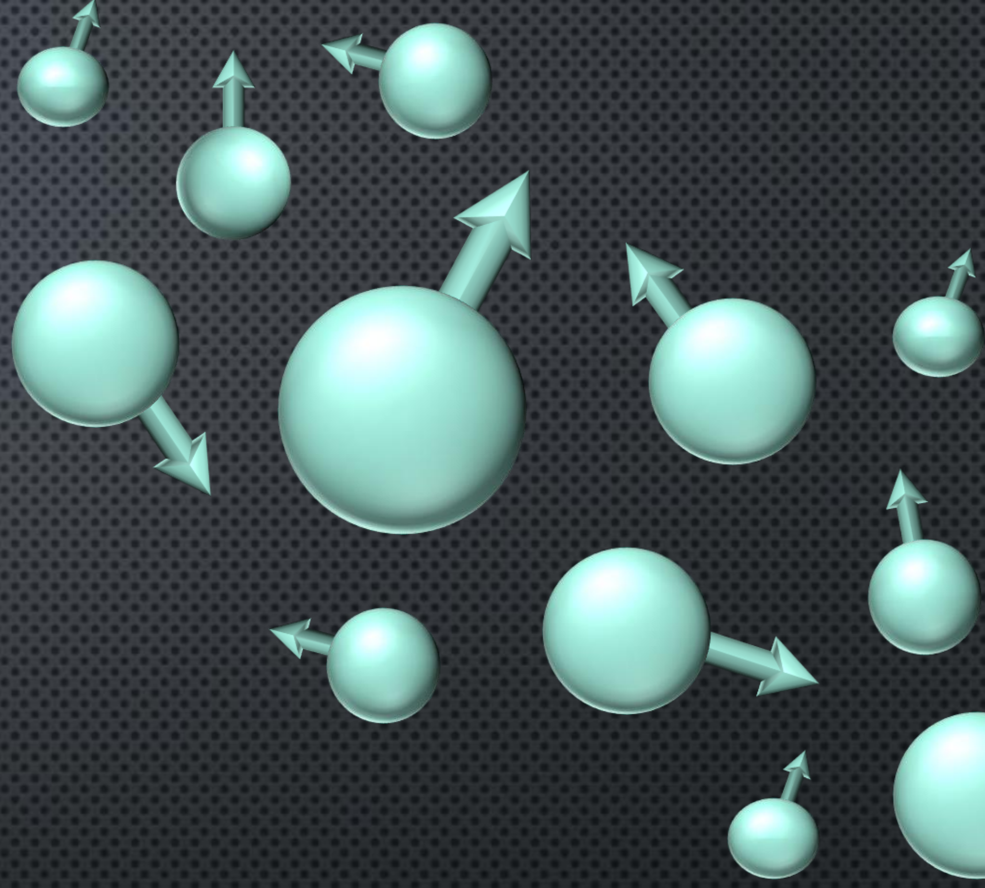
Digitization of Scalar Fields for NISQ-era Quantum Computing



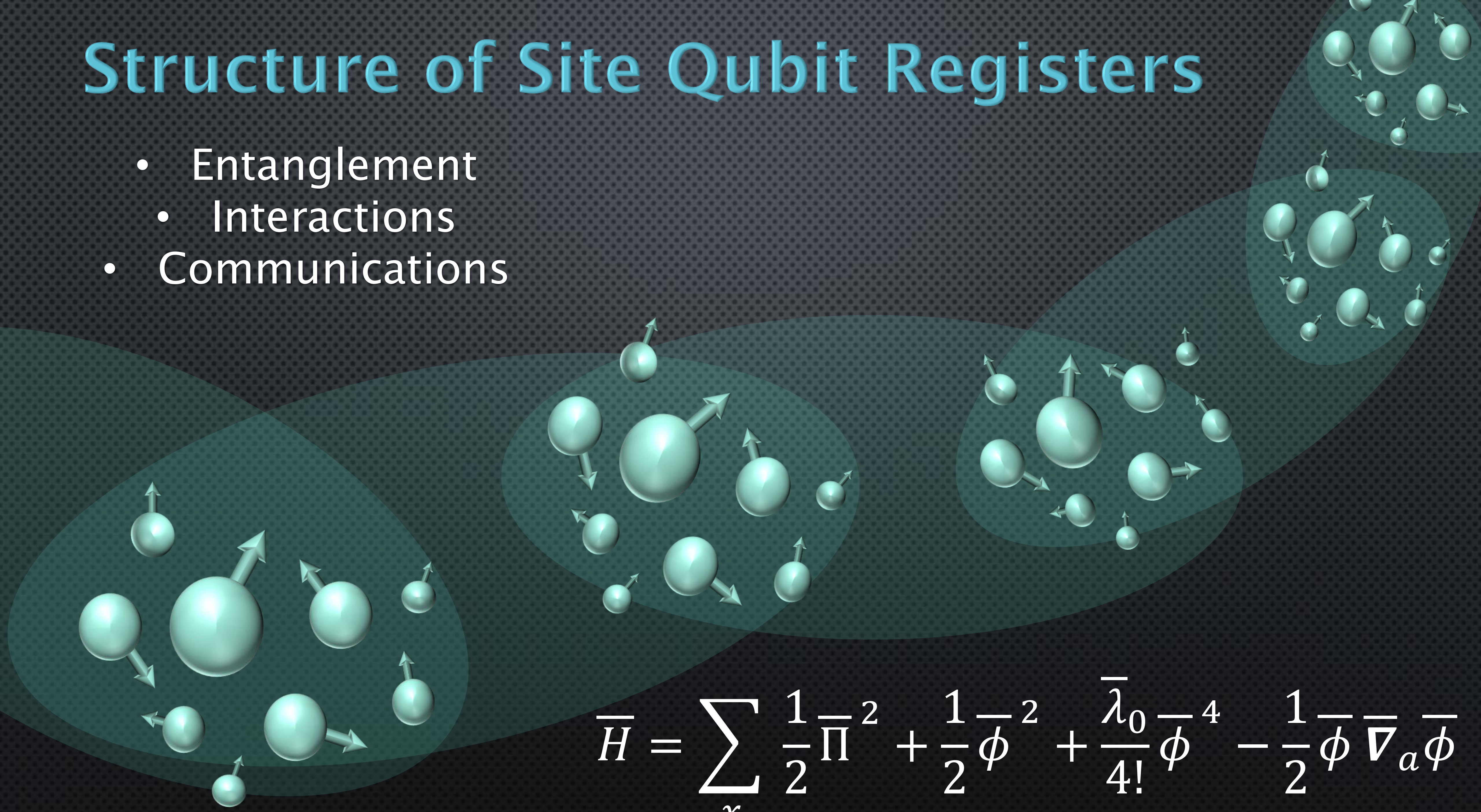
arXiv 1808.10378: NK and Martin J. Savage

Next Steps in Quantum Science for HEP
Fermilab, September 12-14, 2018

Natalie Klco

Structure of Site Qubit Registers

- Entanglement
- Interactions
- Communications



$$\overline{H} = \sum_{x} \frac{1}{2}\overline{\Pi}^{2} + \frac{1}{2}\overline{\phi}^{2} + \frac{\overline{\lambda}_{0}}{4!}\overline{\phi}^{4} - \frac{1}{2}\overline{\phi}\,\overline{\boldsymbol{\nabla}}_{a}\overline{\phi}$$

Digitization Bases

Jordan-Lee-Preskill

arXiv: 1111.3633

arXiv: 1112.4833

$$\overline{\phi} = \frac{\overline{\phi}_{max}}{2^{n_Q}-1}\sum_{j=0}^{n_Q-1} 2^j \sigma_j^Z$$

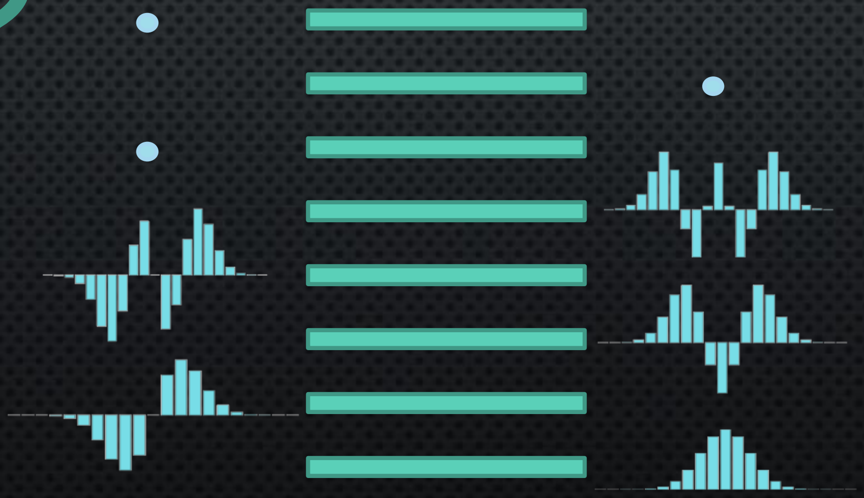
$$\delta_\phi = \frac{2\overline{\phi}_{max}}{2^{n_Q}-1}$$

Somma arXiv: 1503.06319

Macridin-Spentzouris-Amundson-Harnik

arXiv: 1802.07347

arXiv: 1805.09928



Harmonic Oscillator

$\vec{k}$-space mode expansion

Yeter-Aydeniz and Siopsis

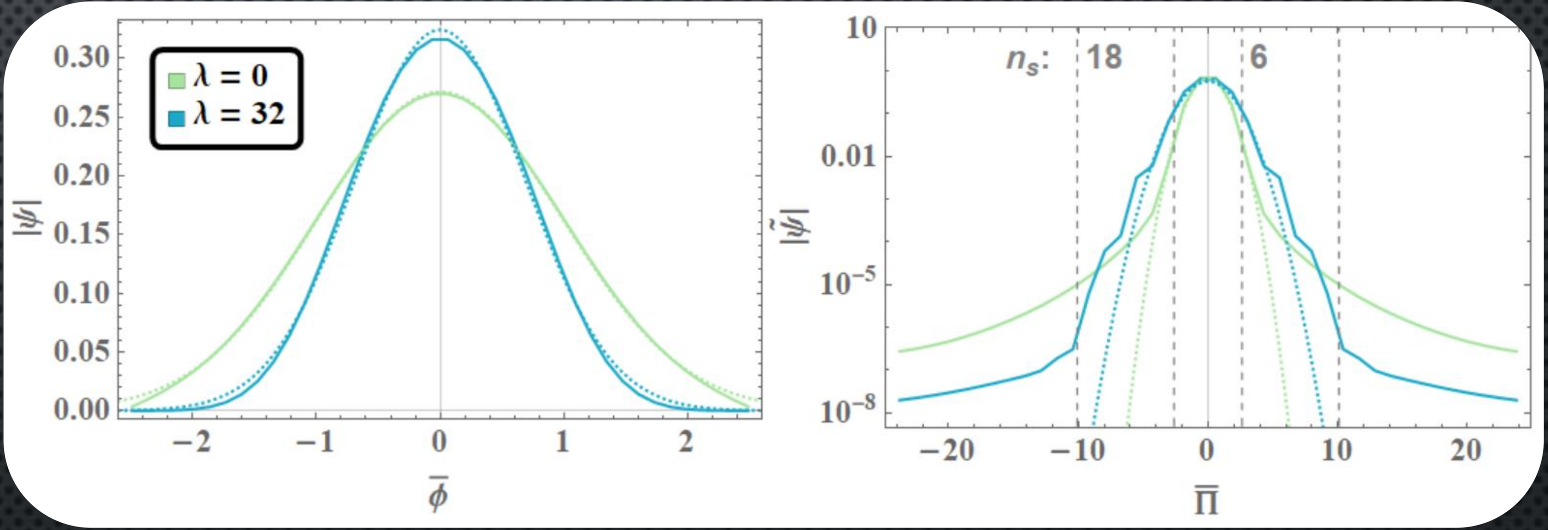
arXiv: 1709.02355

Nyquist-Shannon Sampling Theorem

(field)
continuous signal with
finite bandwidth
(conjugate-momentum)

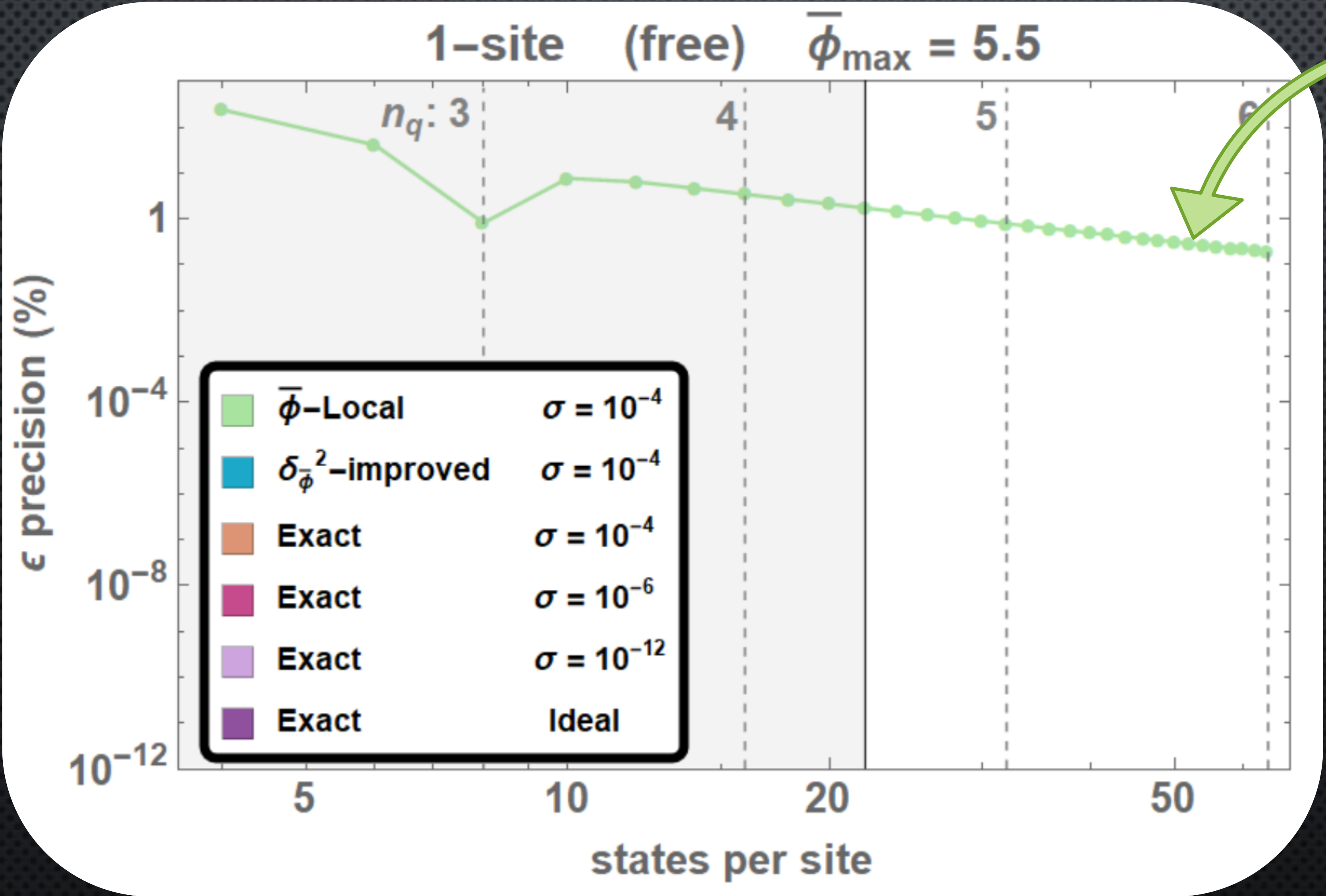
⟷

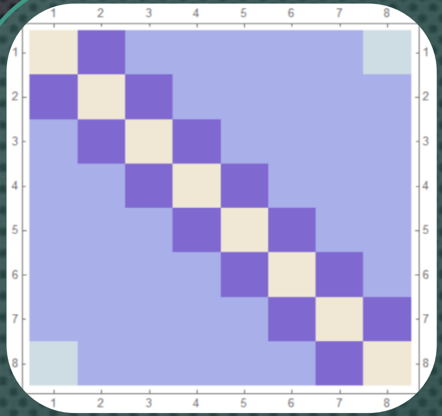
finite temporal sample rate
sufficient



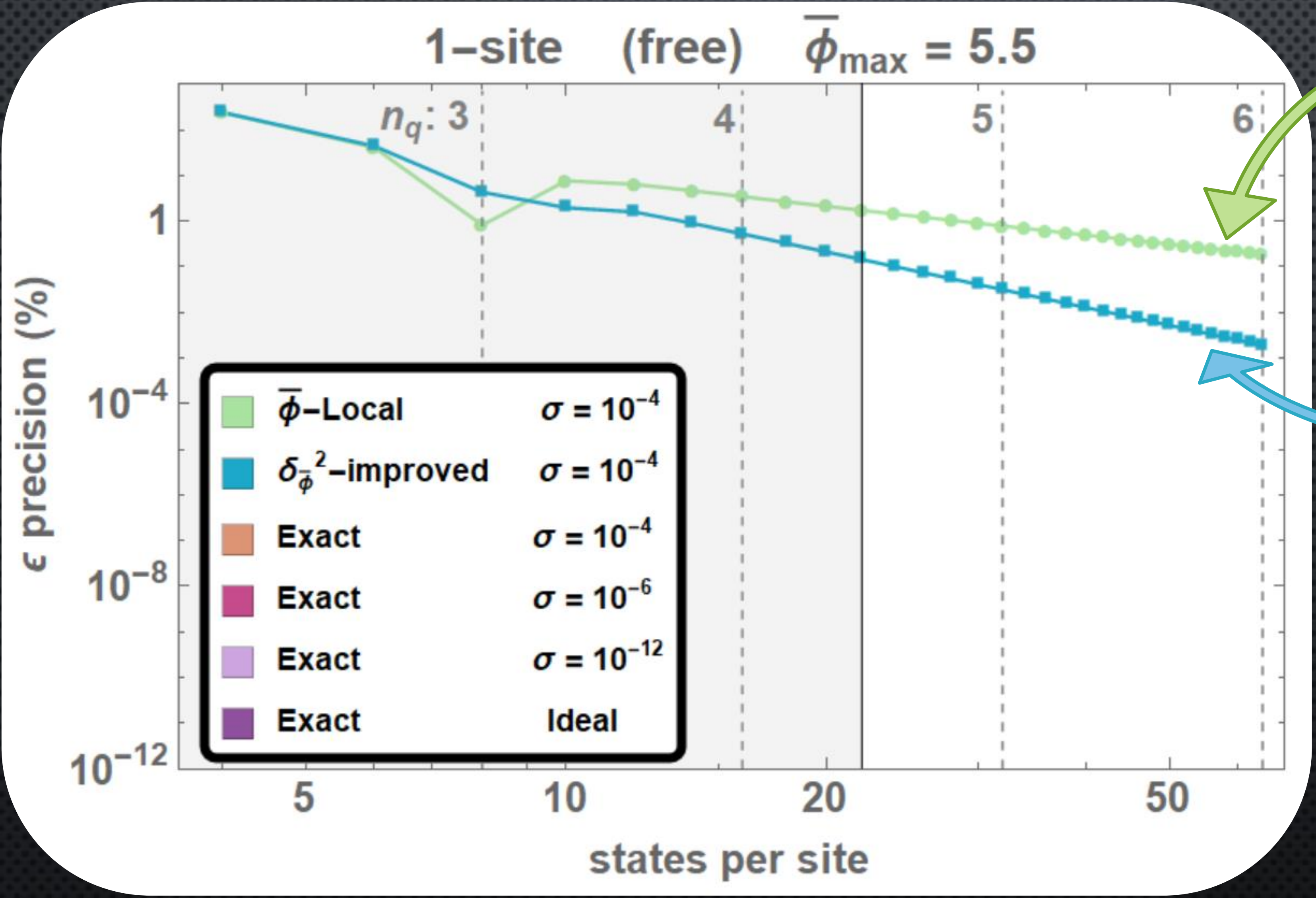
Informs optimal use of Quantum Resources

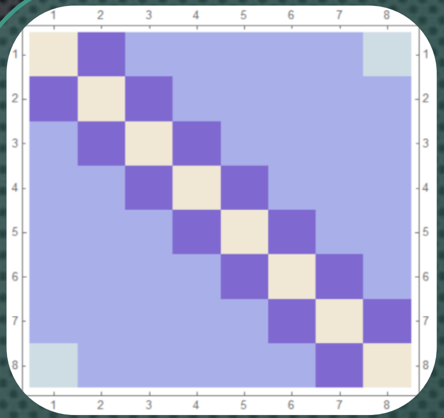
QFT and digitization Improvement

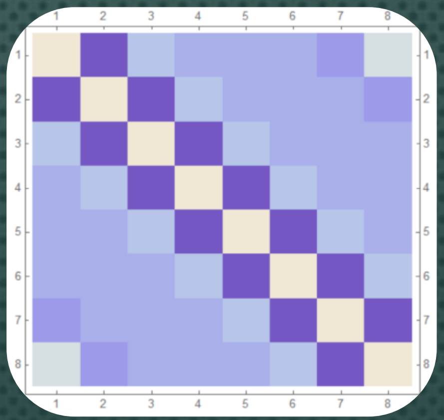




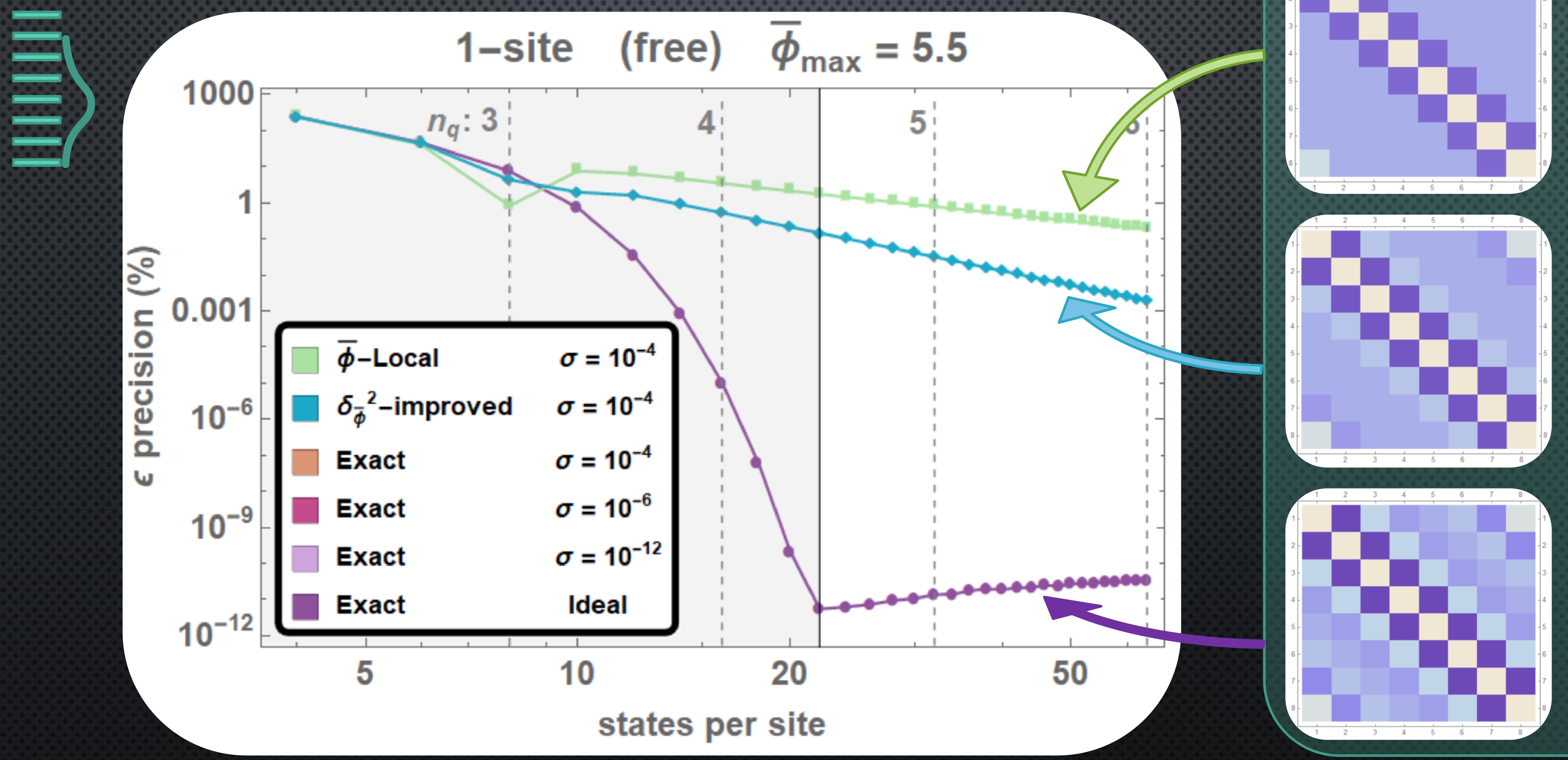
QFT and digitization Improvement



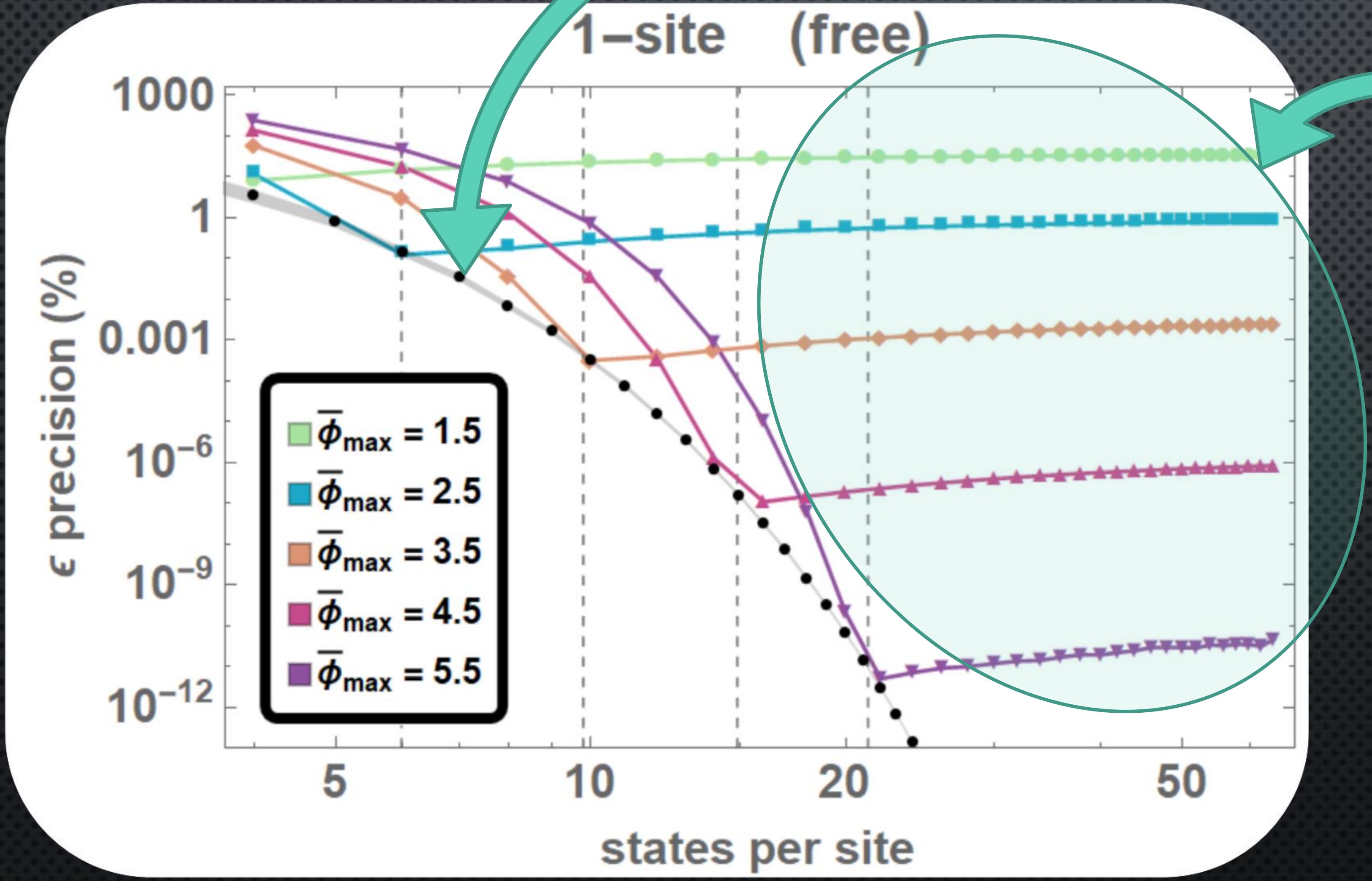




QFT and digitization Improvement

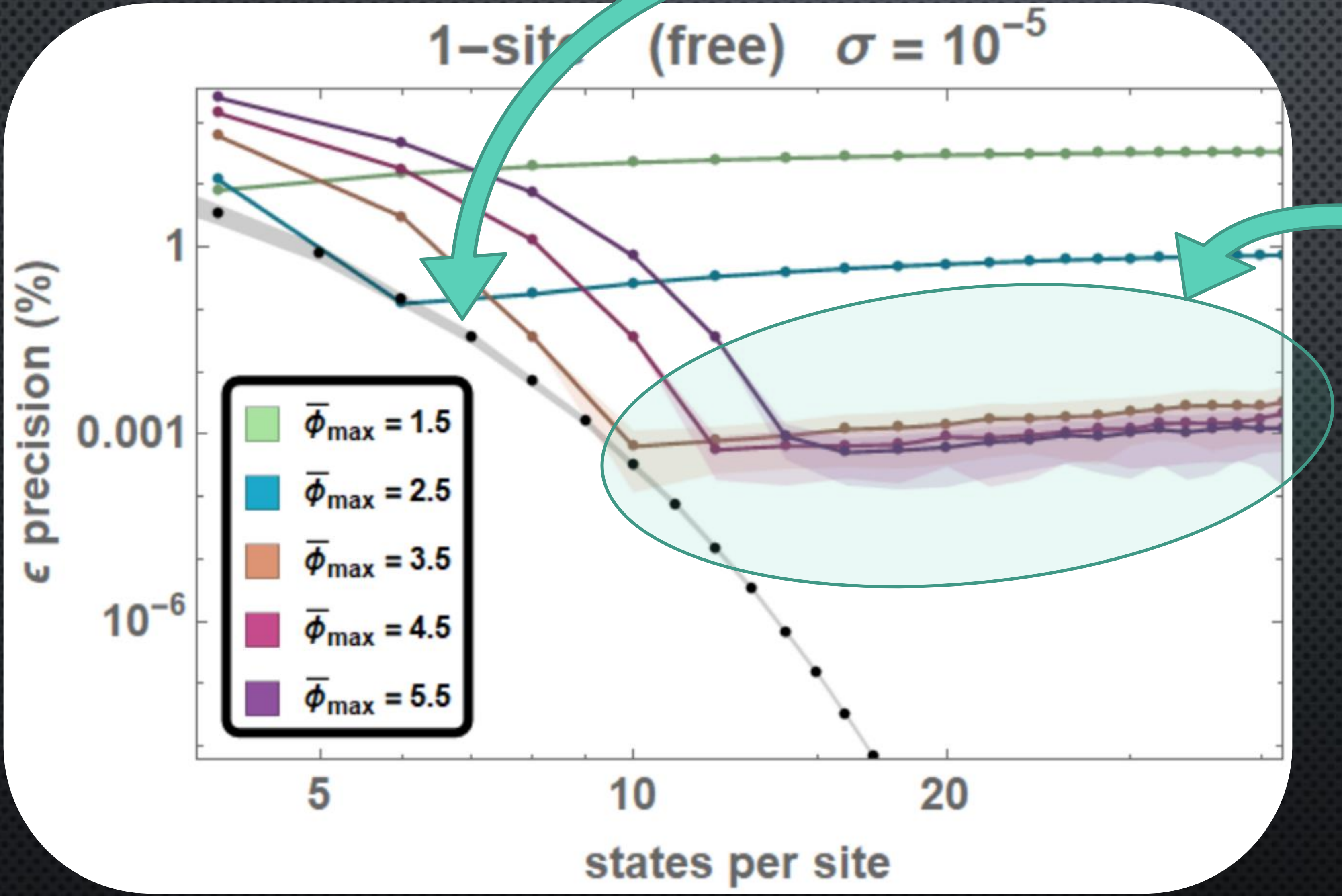


$$\varepsilon \sim (1.8(2) \times 10^3)\, 2^{-2.234(4)2^{n_Q}}$$



Improvements after NS saturation are exponentially small

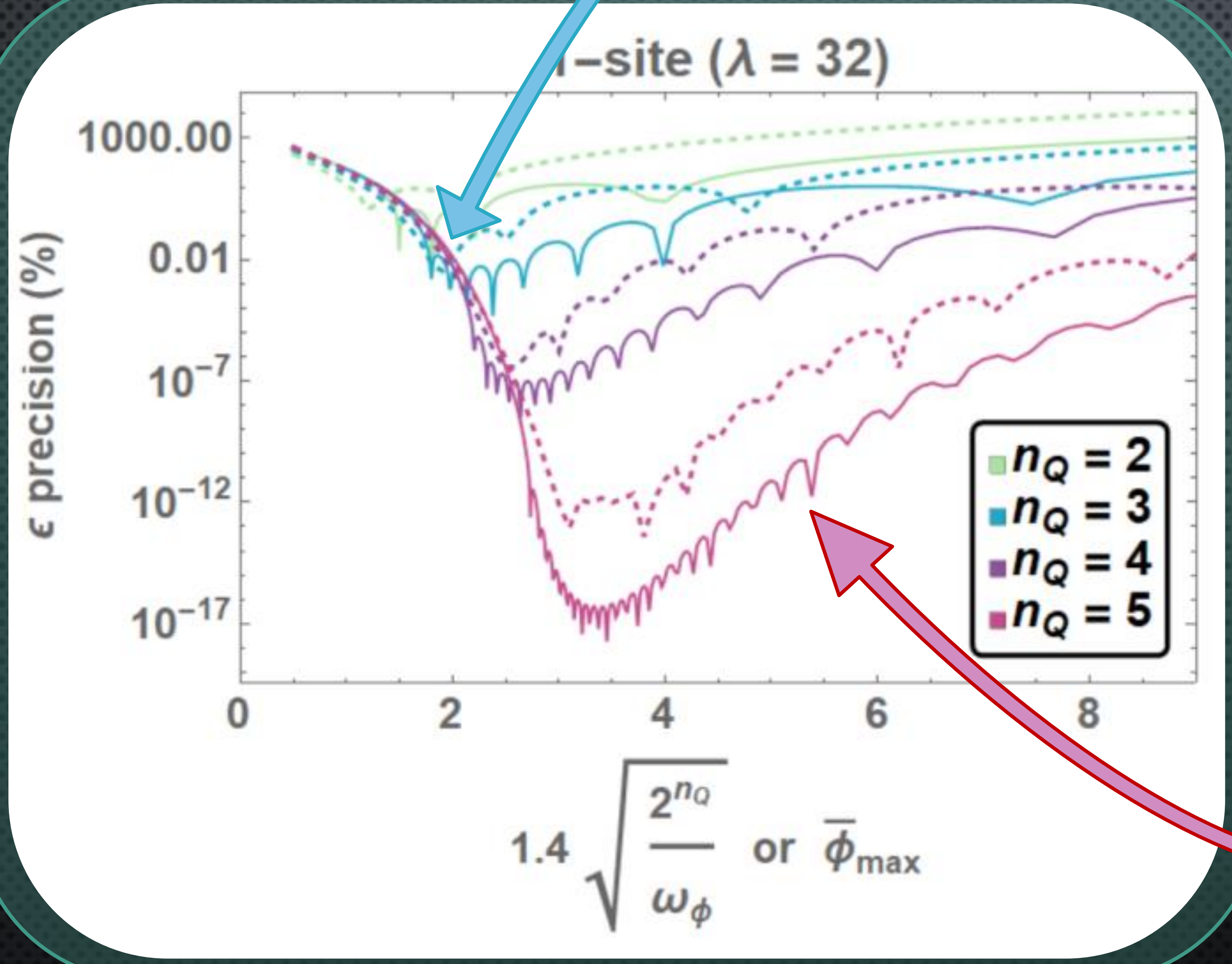
$$\varepsilon \sim (1.8(2) \times 10^3)\, 2^{-2.234(4) 2^{n_Q}}$$



Quantum noise limits achievable precision

Tuning the basis

Exponentially converged in n_Q



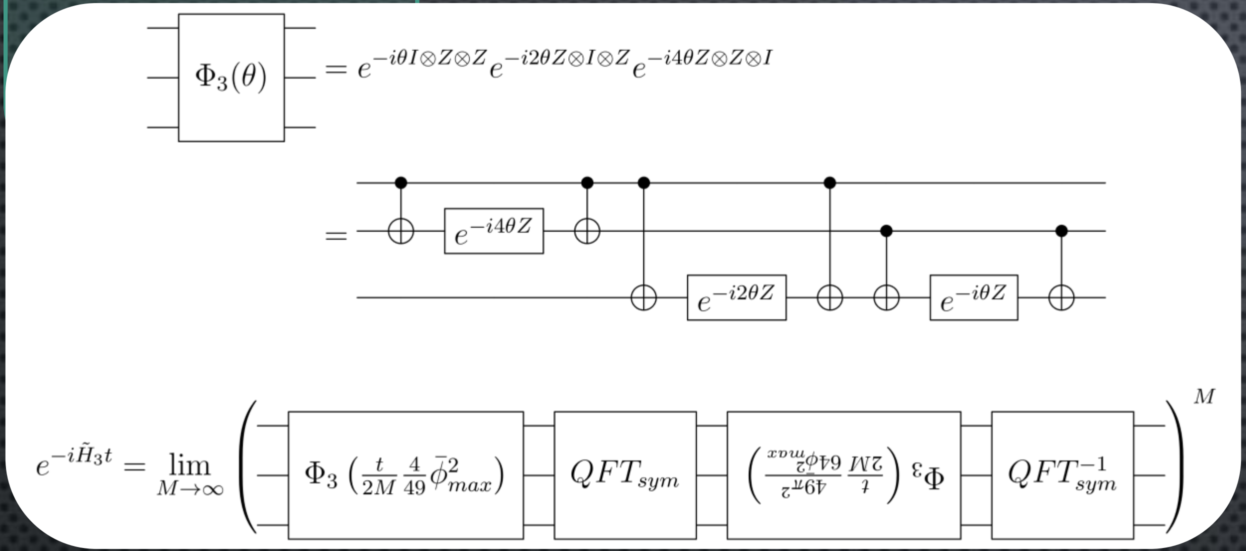
Under-sampling conjugate-momentum space

Dashed

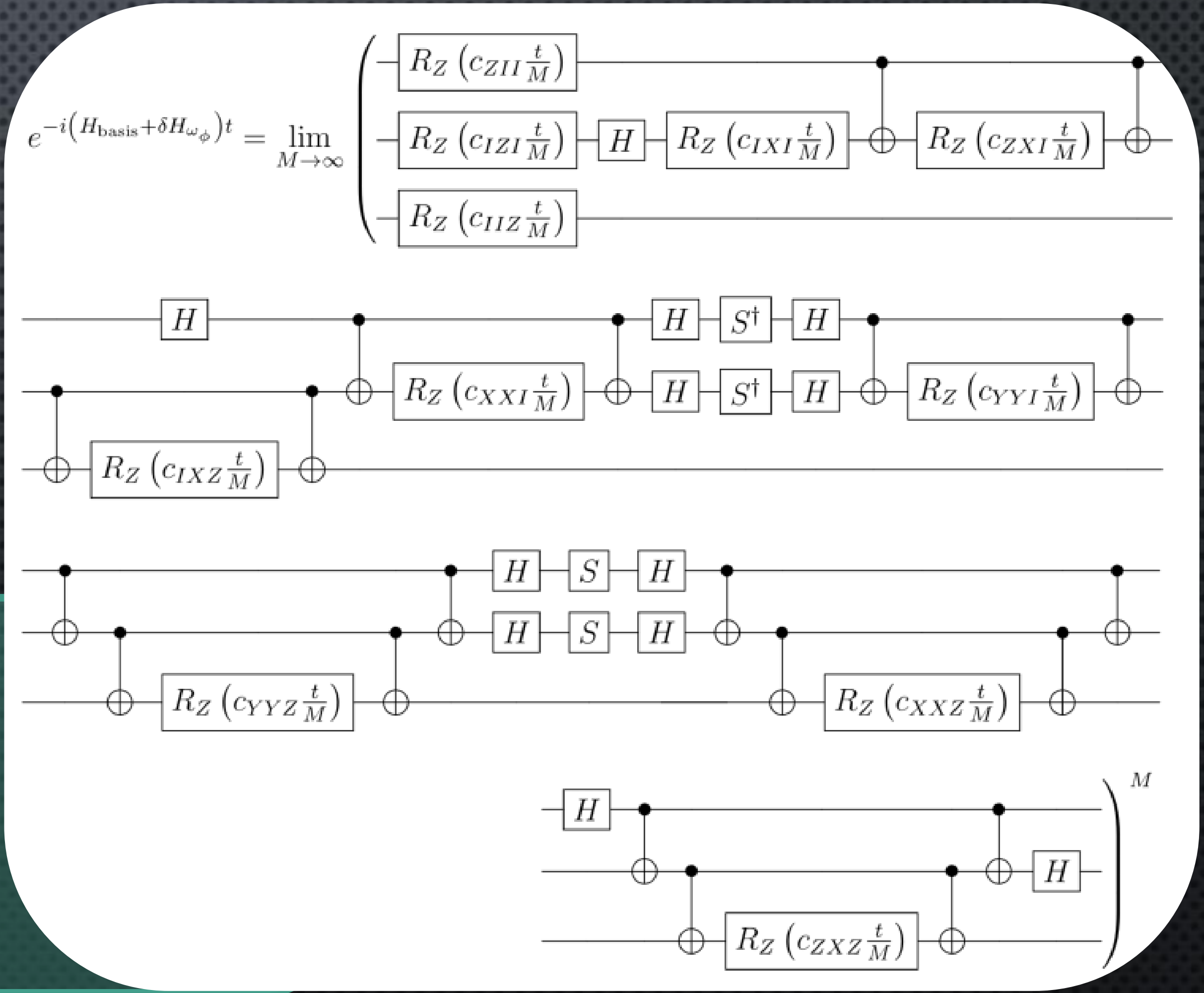
Solid

$$e^{-iHt} = = \lim_{N\to\infty}\left(\prod_j e^{-iH_j\frac{t}{N}}\right)^N$$

$\epsilon^{\lambda}_{\text{tuned}} \sim 10^{-3}$

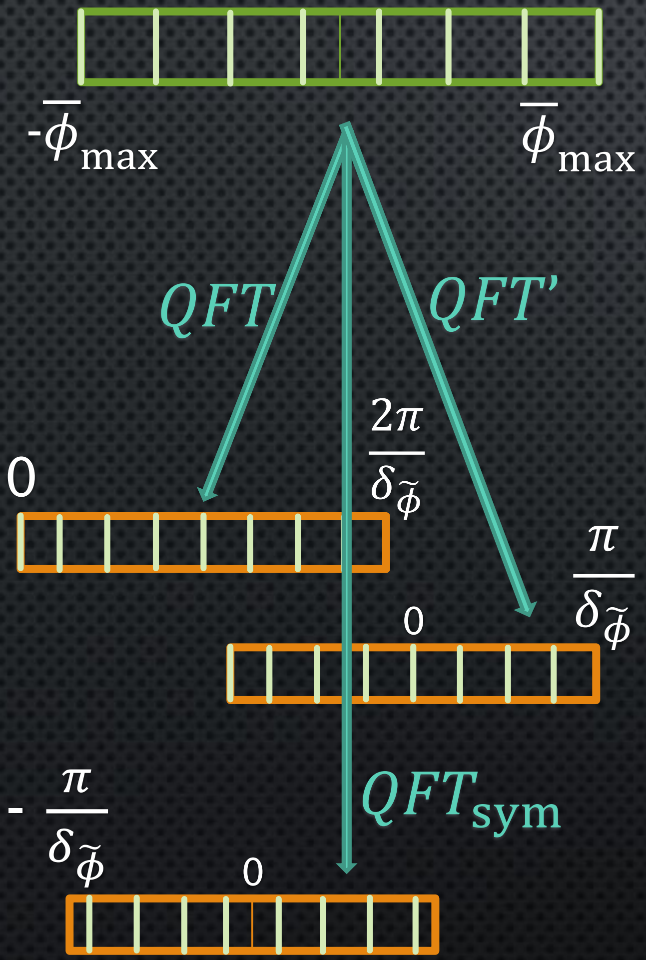


$\epsilon^{\lambda}_{\text{tuned}} \sim 10^{-4}$

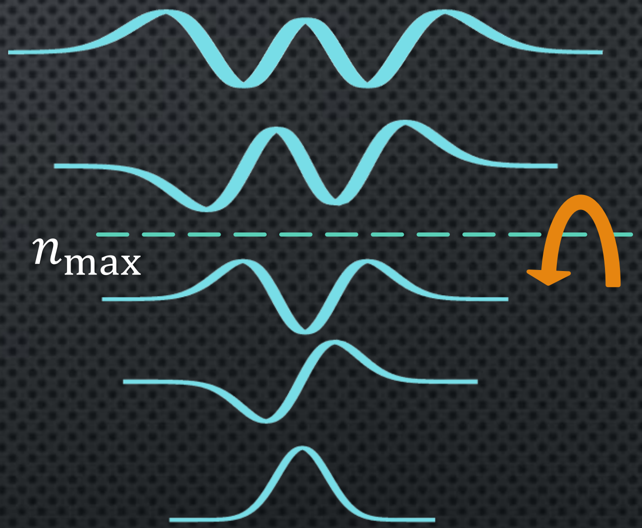


3-qubit real-time evolution

Circuits, Operators and Gates

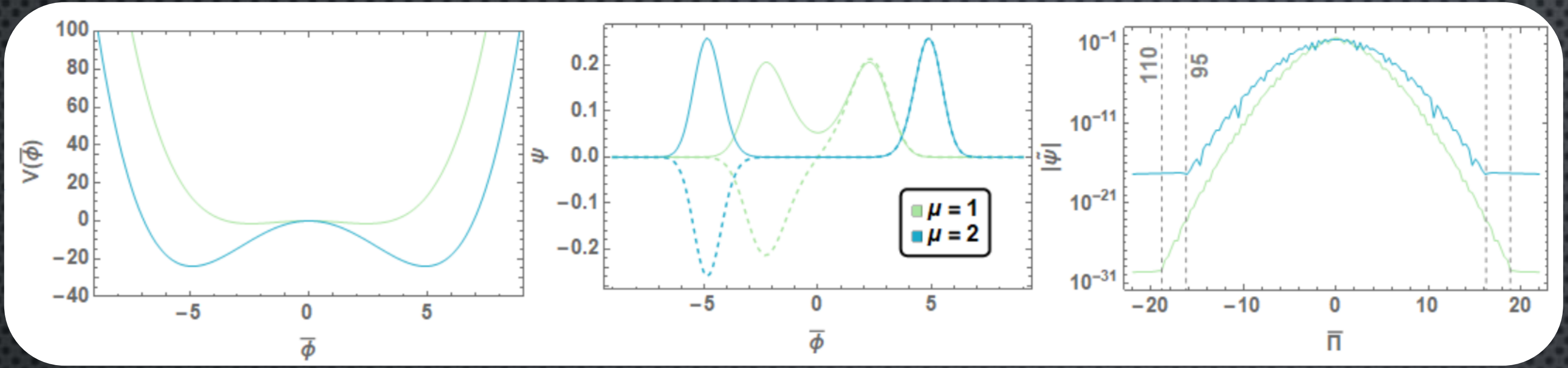


Basis	n_Q	0-body	1-body	2-body	3-body	4-body	5-body	6-body	QFT	CNOTs
	2	1	8	2					✓	8
	3	1	14	6					✓	24
JLP	4	1	20	12		1			✓	54
	5	1	26	20		5			✓	110
	6	1	32	30		15			✓	210
JLP	n_Q	1	$4n_Q - 6$	$2 * \binom{n_Q}{2}$		$\binom{n_Q}{4}$			✓	$8\binom{n_Q}{2} + 6\binom{n_Q}{4}$
	2	1	3	2						4
	3	1	5	9	4					34
HO	4	1	6	16	18	10				164
	5	1	7	22	32	44	22			612
	6	1	8	29	44	84	98	46		1982



Quantum resources sensitive to truncation details

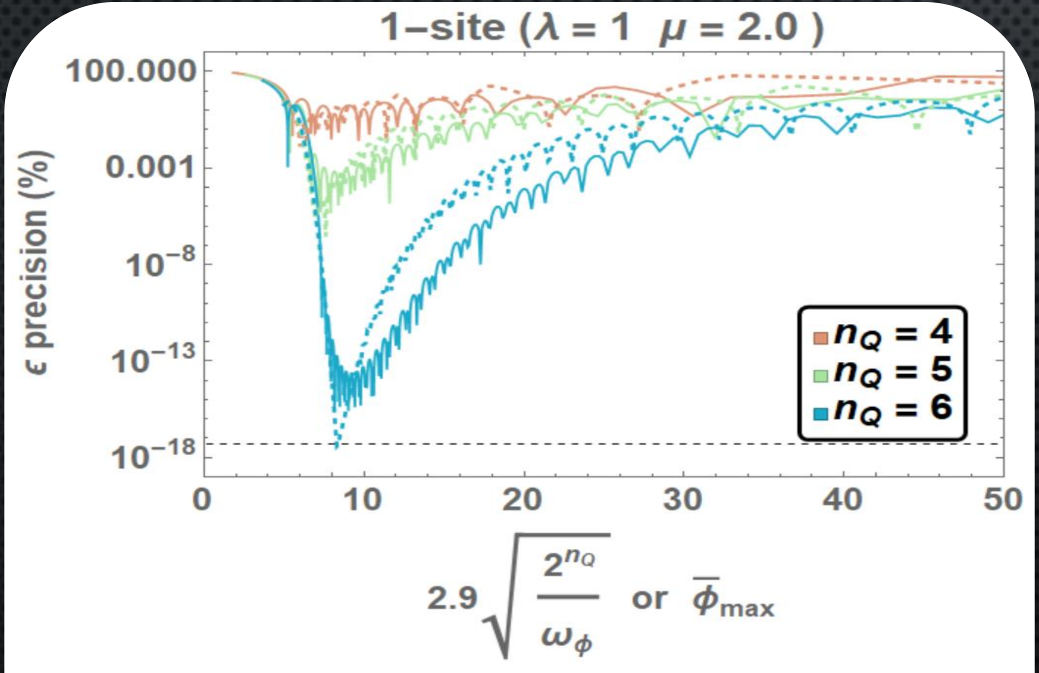
Delocalized Wavefunctions

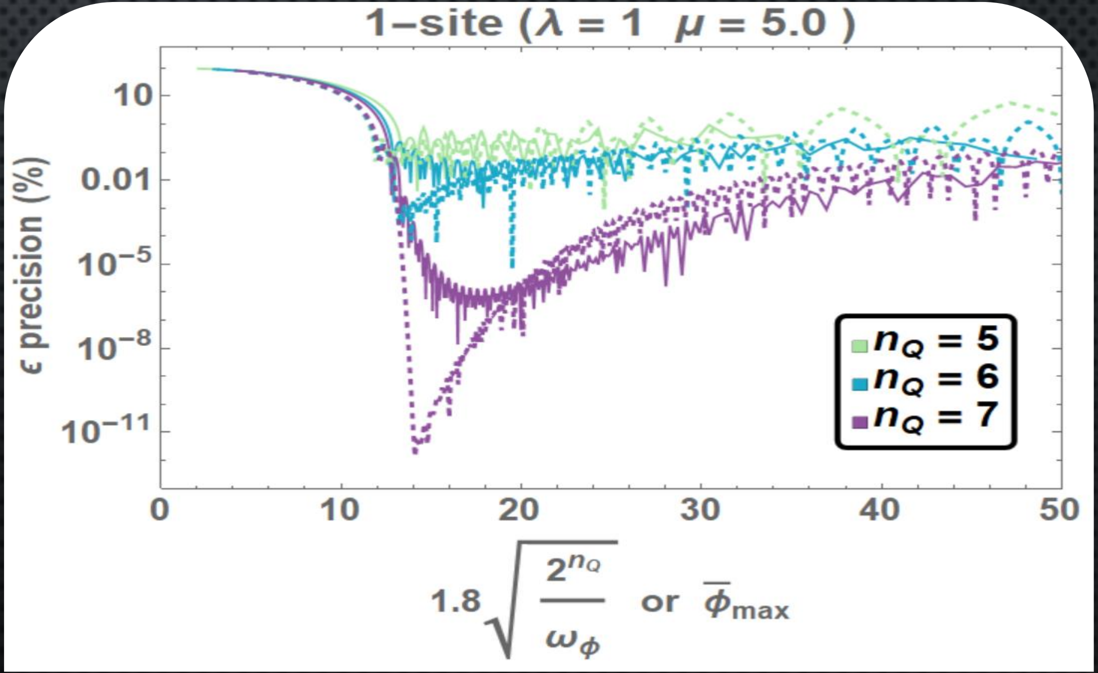


$$-\mu^2 = m^2 < 0$$

Minima: $\varphi = \pm \frac{\sqrt{3!}\mu}{\sqrt{\lambda}}$

$$\lim_{\mu \to \infty} E_1 = E_0$$

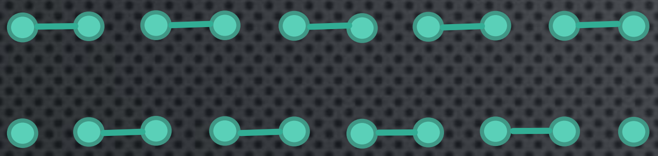




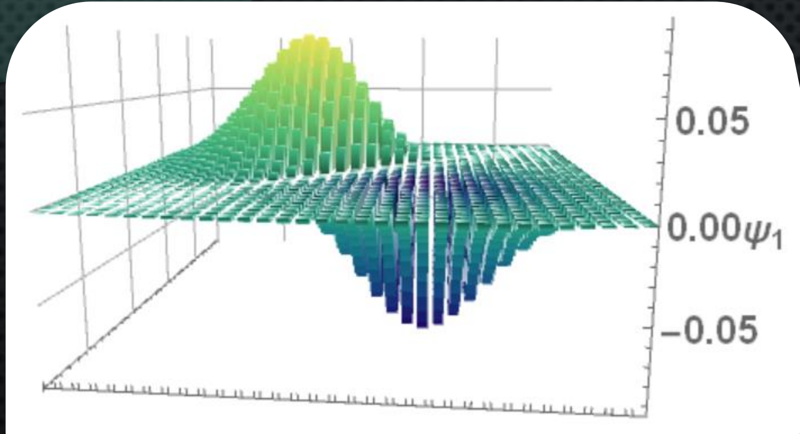
1+1 dimensions

Position-space-local interactions can be parallelized

1.)
2.)



Calculation time for real-time evolution is <u>**constant**</u> with increasing volume



Summary

- Parallel exploration of quantum implementations is vital --- optimal basis may change dramatically with hardware properties.

- Hardware-specific, informed decisions in formulating calculations for quantum device requires analysis of the multi-dimensional, correlated space of necessary qubits, operators, and gates.

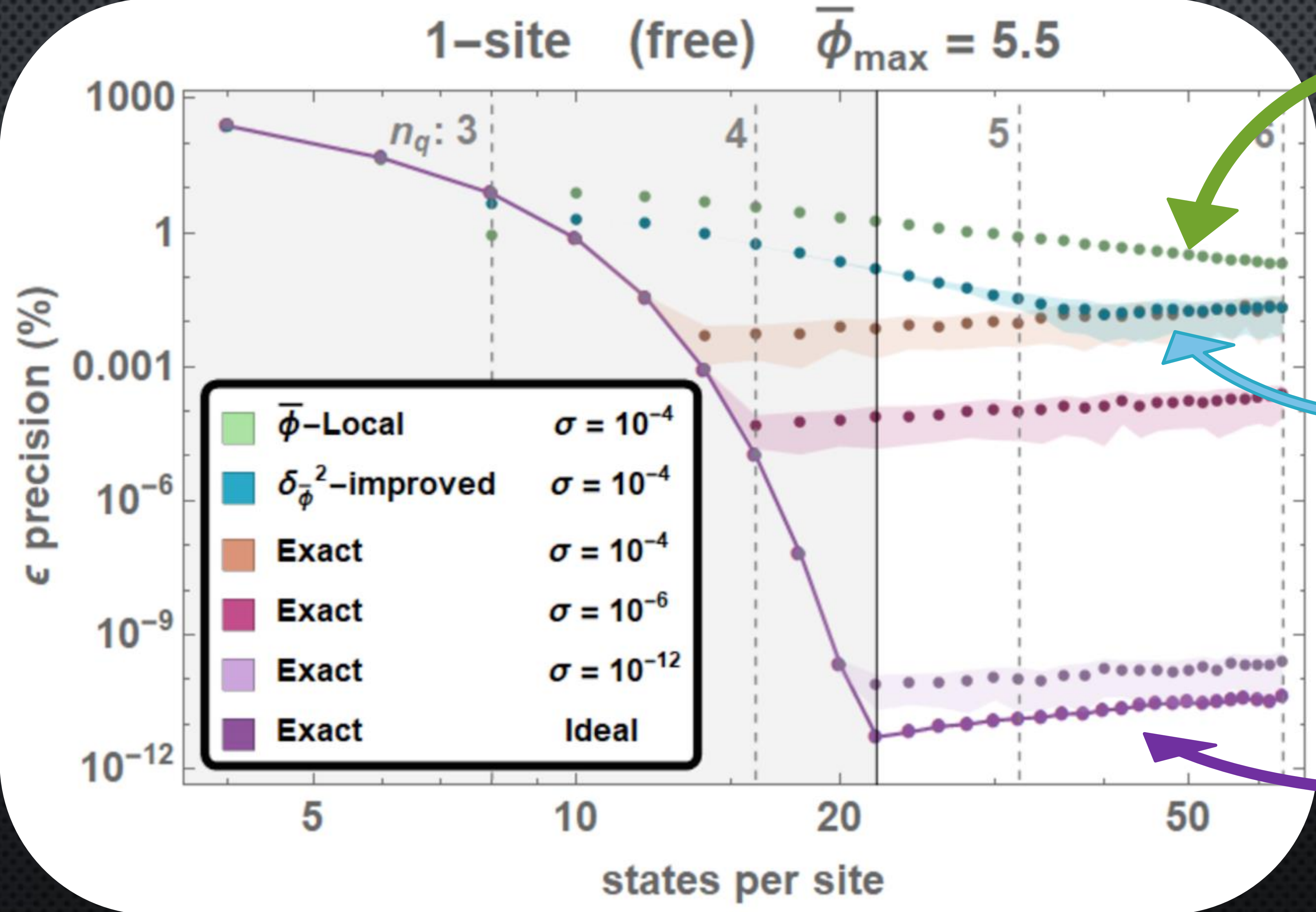
- Small scalar lattices have communication and digitization requirements amenable to precise calculation with near-term quantum devices.

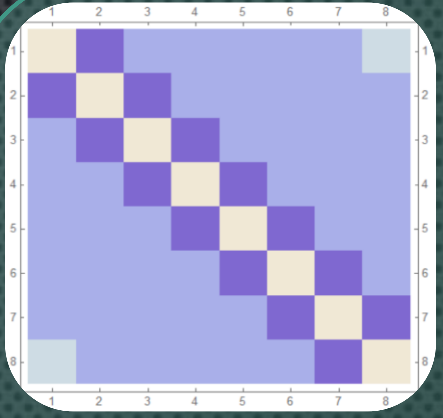
- Lessons learned in scalar field implementation will be carried forward for gauge field theories.

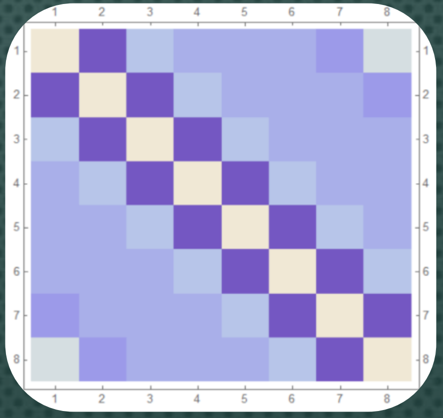
UW collaborators:

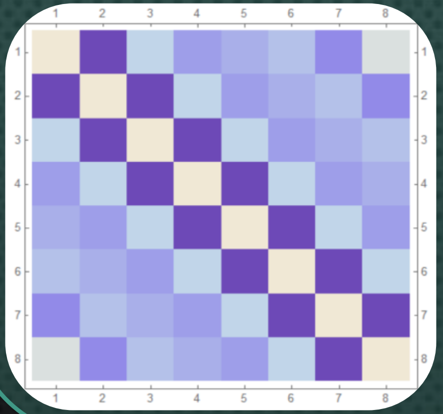
Silas Beane
David Kaplan
Kenneth Roche
Martin Savage
Jesse Stryker

Introduction of Quantum Noise





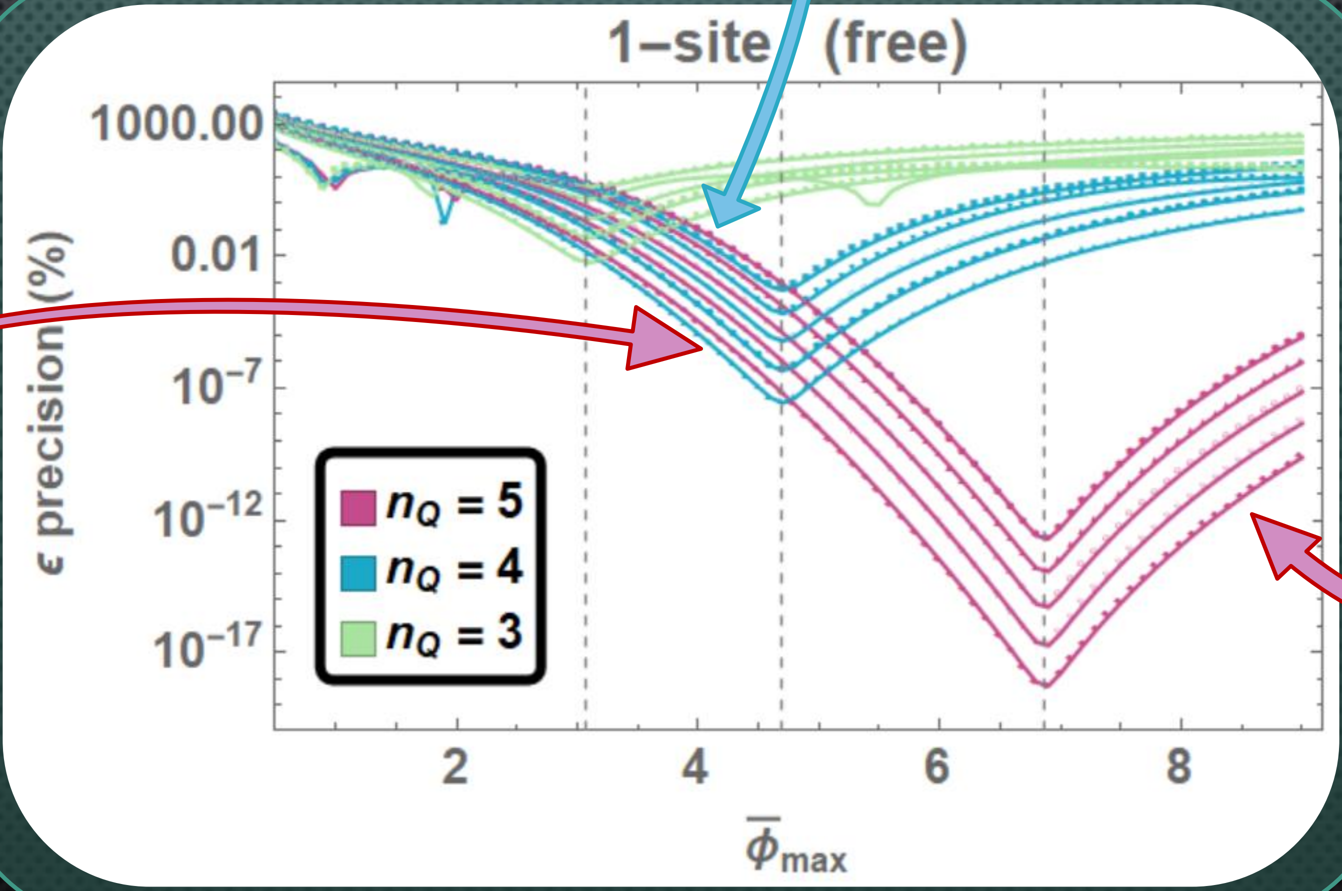




Tuning Basis for Fixed Qubits

Exponentially converged in n_Q

$\overline{\phi}_{\text{max}}$ truncation of field-space wavefunction



Under-sampling conjugate-momentum space

Gradient Operator

Basis	n_Q	2-body	3-body	4-body	5-body	6-body	7-body	8-body	9-body	10-body	11-body	12-body	CNOT
	2	4											8
	3	9											18
JLP	4	16											32
	5	25											50
	6	36											72
	n_Q	n_Q^2											$2n_Q^2$
	2	1	6	9									80
	3	1	8	30	56	49							1,152
HO	4	1	10	47	140	271	330	225					11,264
	5	1	12	68	244	630	1204	1668	1612	961			89,600
	6	1	14	93	392	1186	2772	5154	7560	8541	7182	3969	626,688

Free

Basis	n_Q	0-body	1-body	2-body	3-body	4-body	5-body	6-body	QFT	CNOTs
JLP	2	1	8	2					✓	8
	3	1	14	6					✓	24
	4	1	20	12					✓	48
	5	1	26	20					✓	80
	6	1	32	30					✓	120
JLP	n_Q	1	$6n_Q - 4$	$2 * \binom{n_Q}{2}$					✓	$8\binom{n_Q}{2}$
$\mathrm{HO}_{\omega\equiv 1}$	2	1	2							0
	3	1	3							0
	4	1	4							0
	5	1	5							0
	6	1	6							0
$\mathrm{HO}_{\omega\equiv 1}$	n_Q	1	n_Q							0
$\mathrm{HO}_{\omega\neq 1}$	2	1	3	1						2
	3	1	4	4	3					20
	4	1	5	5	11	7				96
	5	1	6	6	16	26	15			352
	6	1	7	7	22	42	57	31		1120