What have we learned about quantum gravity from quantum error correction?

Daniel Harlow

Massachusetts Institute of Technology

September 12, 2018

Our Favorite Theory of Quantum Gravity

- The Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence, our best-understood theory of quantum gravity, is now twenty years old!

Our Favorite Theory of Quantum Gravity

- The Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence, our best-understood theory of quantum gravity, is now twenty years old!
- Until recently, most of the follow-up work has used classical gravity on the bulk side to learn about strongly-coupled QFT on the boundary side.

Our Favorite Theory of Quantum Gravity

- The Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence, our best-understood theory of quantum gravity, is now twenty years old!
- Until recently, most of the follow-up work has used classical gravity on the bulk side to learn about strongly-coupled QFT on the boundary side.
- This has been a reasonably successful approach (strongly-coupled plasmas, new understanding of transport in CMT, hydrodynamic anomalies, etc), but it is unlikely to tell us anything interesting about the deep puzzles of quantum gravity.

- In the last five years, we have seen that by re-purposing and expanding ideas from quantum information theory, we can make progress on running the correspondence in the “right” direction: learning about quantum gravity in the bulk.

- In the last five years, we have seen that by re-purposing and expanding ideas from quantum information theory, we can make progress on running the correspondence in the "right" direction: learning about quantum gravity in the bulk.
- Today I will review some of what we have learned from one aspect of this: a new interpretation of the holographic map that tells us which states and operators in the bulk AdS get mapped to which states and operators in the boundary CFT as a quantum-error-correcting code.

  Almheiri/Dong/Harlow 14, Pastawski/Yoshida/Harlow/Preskill 15, Dong/Harlow/Wall 16, Harlow 16,Harlow/Ooguri 18

Things we will learn about:

Things we will learn about:

- Exactly solvable "tensor network" models of holography.

Things we will learn about:

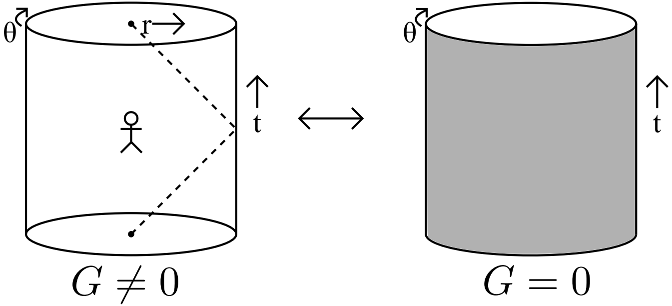
- Exactly solvable "tensor network" models of holography.
- Seeing into the black hole interior using "entanglement wedge reconstruction".

Things we will learn about:

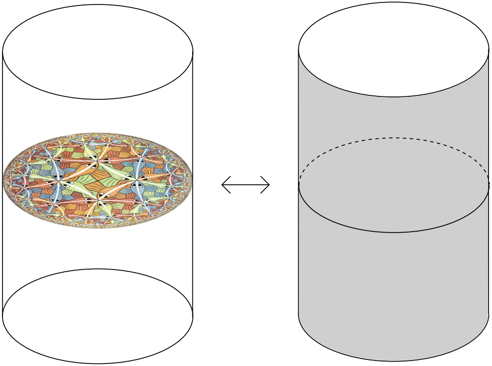
- Exactly solvable “tensor network” models of holography.
- Seeing into the black hole interior using “entanglement wedge reconstruction”.
- Showing that (at least in AdS/CFT), there are no global symmetries (eg $B - L$) in quantum gravity.

AdS/CFT Review

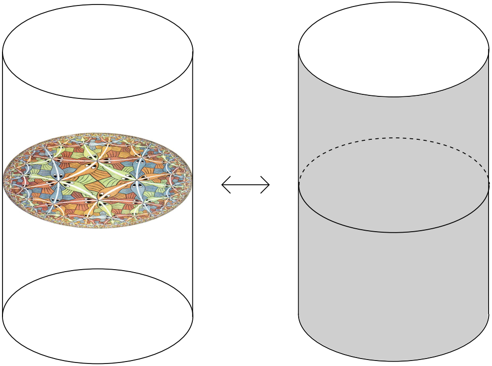
AdS/CFT says that quantum gravity in asymptotically AdS space is equivalent to conformal field theory on its boundary:



This correspondence is a quantum correspondence:

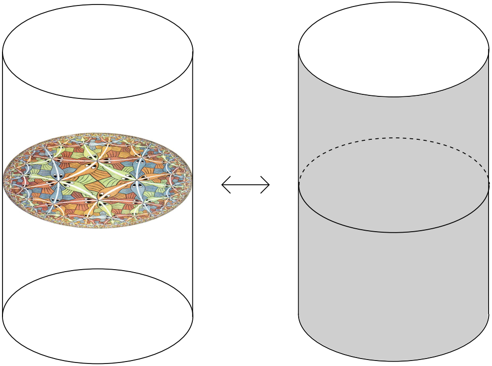


This correspondence is a quantum correspondence:



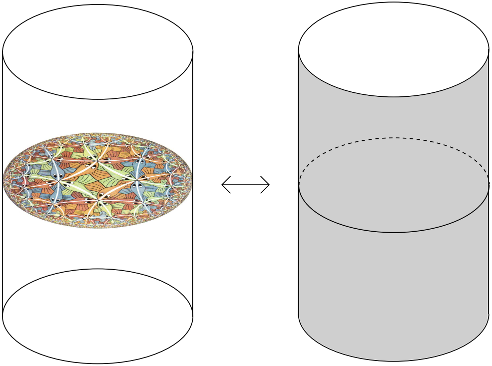
- $|\psi_{bulk}\rangle \longleftrightarrow |\psi_{boundary}\rangle$

This correspondence is a quantum correspondence:



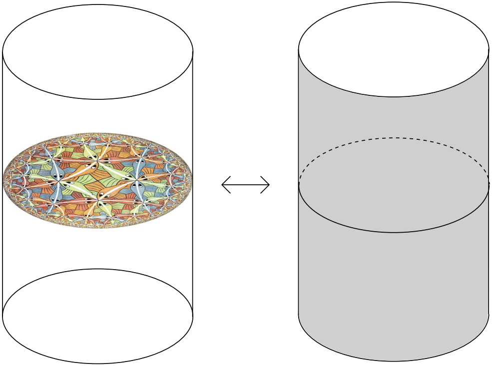
- $|\psi_{bulk}\rangle \longleftrightarrow |\psi_{boundary}\rangle$
- $H, J, \ldots \longleftrightarrow H, J, \ldots$

This correspondence is a quantum correspondence:



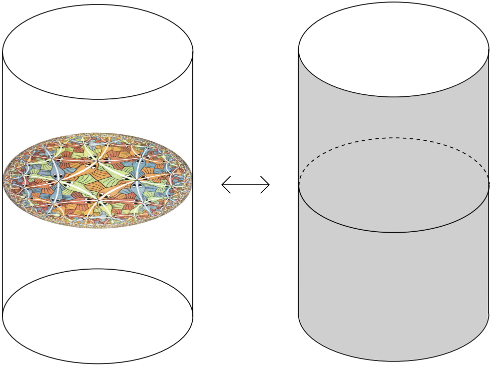
- $|\psi_{bulk}\rangle \longleftrightarrow |\psi_{boundary}\rangle$
- $H, J, \ldots \longleftrightarrow H, J, \ldots$
- $\lim_{r\to\infty} r^{\Delta}\phi(r,t,\Omega) \longleftrightarrow \mathcal{O}(t,\Omega)$.

This correspondence is a quantum correspondence:



- $|\psi_{bulk}\rangle \longleftrightarrow |\psi_{boundary}\rangle$
- $H, J, \ldots \longleftrightarrow H, J, \ldots$
- $\lim_{r\to\infty} r^{\Delta}\phi(r,t,\Omega) \longleftrightarrow \mathcal{O}(t,\Omega)$.
- Vacuum perturbations $\longleftrightarrow$ low-energy states

This correspondence is a quantum correspondence:



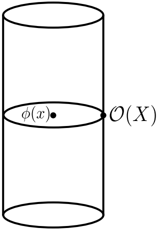
- $|\psi_{bulk}\rangle \longleftrightarrow |\psi_{boundary}\rangle$
- $H, J, \ldots \longleftrightarrow H, J, \ldots$
- $\lim_{r\to\infty} r^{\Delta}\phi(r, t, \Omega) \longleftrightarrow \mathcal{O}(t, \Omega)$.
- Vacuum perturbations $\longleftrightarrow$ low-energy states
- Black holes $\longleftrightarrow$ high-energy states

- This proposal immediately runs into the following puzzle however: if everything is made out of local degrees of freedom at the boundary, and those degrees of freedom correspond to things in the bulk which are near the boundary, how can there be anything in the interior of the bulk which is independent of what is happening near the boundary?

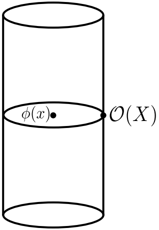
- This proposal immediately runs into the following puzzle however: if everything is made out of local degrees of freedom at the boundary, and those degrees of freedom correspond to things in the bulk which are near the boundary, how can there be anything in the interior of the bulk which is independent of what is happening near the boundary?
- In other words, how can a theory which is d dimensional possibly be equivalent to a theory which is $d+1$ dimensional?

- This proposal immediately runs into the following puzzle however: if everything is made out of local degrees of freedom at the boundary, and those degrees of freedom correspond to things in the bulk which are near the boundary, how can there be anything in the interior of the bulk which is independent of what is happening near the boundary?
- In other words, how can a theory which is d dimensional possibly be equivalent to a theory which is $d + 1$ dimensional?

We can phrase this more precisely as follows:



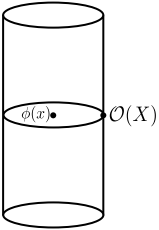
We can phrase this more precisely as follows:



x and X are spacelike-separated in the bulk, so we might expect that

$$[\phi(x), \mathcal{O}(X)] = 0.$$

We can phrase this more precisely as follows:



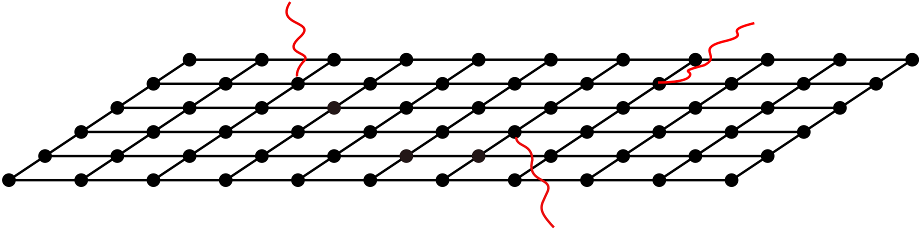
x and X are spacelike-separated in the bulk, so we might expect that

$$[\phi(x), \mathcal{O}(X)] = 0.$$

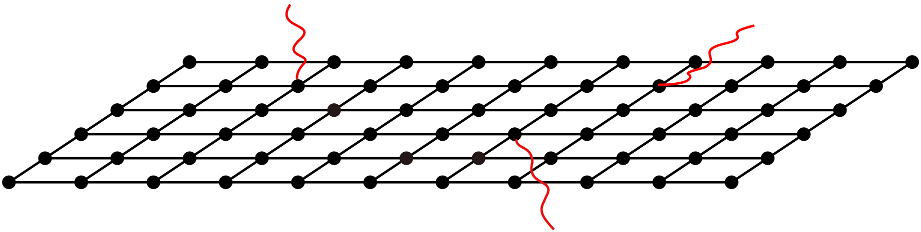
But in the boundary CFT this is impossible, since an operator which commutes with all $\mathcal{O}(X)$ must be trivial!

In fact this problem is closely related to one which is familiar to people working on quantum computers.

In fact this problem is closely related to one which is familiar to people working on quantum computers.
Any quantum memory will be built out of an array of small quantum systems, which often are arranged in a lattice like this:



In fact this problem is closely related to one which is familiar to people working on quantum computers.
Any quantum memory will be built out of an array of small quantum systems, which often are arranged in a lattice like this:



Errors tend to act locally on these systems, so we need the state we store to be independent of any particular one. But then how can it be nontrivial?

- Any quantum information theorist will tell you that this second problem is solved by *quantum error correction*, and what we have learned in the last few years is that this also solves the first problem!

- Any quantum information theorist will tell you that this second problem is solved by *quantum error correction*, and what we have learned in the last few years is that this also solves the first problem!
- In other words, we should view the information in the center of the bulk as being the "logical information" of a quantum-error-correcting code, and the boundary CFT as the "physical degrees of freedom" the memory is made out of.

Quantum Error Correction

- The basic idea of any error-correcting code, quantum or classical, is to store the information redundantly.

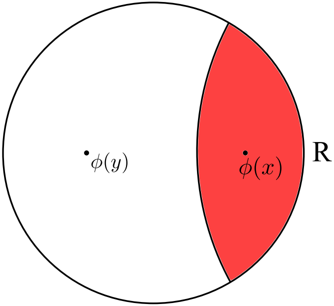
Quantum Error Correction

- The basic idea of any error-correcting code, quantum or classical, is to store the information redundantly.
- For example in the obvious "repetition code", we just send many copies of the message we want to transmit. Even if a few get lost or corrupted on the way, the receiver can still figure out with high probability what the message is.

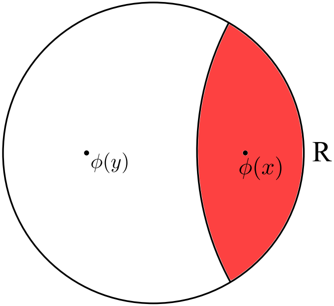
Quantum Error Correction

- The basic idea of any error-correcting code, quantum or classical, is to store the information redundantly.
- For example in the obvious "repetition code", we just send many copies of the message we want to transmit. Even if a few get lost or corrupted on the way, the receiver can still figure out with high probability what the message is.
- The repetition code cannot work for quantum messages, due to the no-cloning theorem, but there is an alternative which works beautifully: we encode the information nonlocally in the entanglement between the physical degrees of freedom!

This redundancy also has an avatar in AdS/CFT: using simple bulk methods we can show that given any boundary spatial subregion R, there is a bulk subregion W_R such that any bulk operator ϕ in W_R can be represented by an operator in the CFT with support only in R:

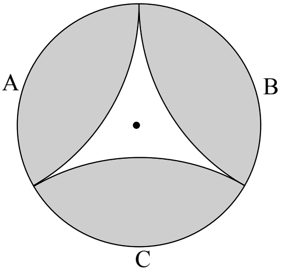


This redundancy also has an avatar in AdS/CFT: using simple bulk methods we can show that given any boundary spatial subregion R, there is a bulk subregion W_R such that any bulk operator ϕ in W_R can be represented by an operator in the CFT with support only in R:

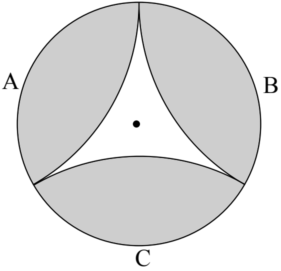


The operator $\phi(x)$ can be represented on R, but the operator $\phi(y)$ cannot.

This leads to some surprising situations:

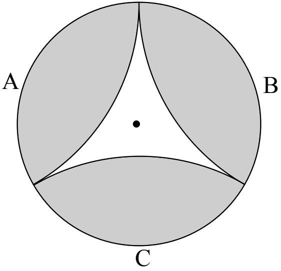


This leads to some surprising situations:



The operator in the center has no representation on A, B, or C, but it does have a representation either on AB, AC, or BC!

This leads to some surprising situations:



The operator in the center has no representation on A, B, or C, but it does have a representation either on AB, AC, or BC!
Where is the information?

Three Qutrits

The simplest quantum error correcting code is the three qutrit code, which embeds a single "logical" qutrit into three "physical" qutrits as follows:

Cleve/Gottesman/Lo

$$|\widetilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$
$$|\widetilde{1}\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$
$$|\widetilde{2}\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right).$$

Three Qutrits

The simplest quantum error correcting code is the three qutrit code, which embeds a single "logical" qutrit into three "physical" qutrits as follows:

Cleve/Gottesman/Lo

$$|\widetilde{0}\rangle = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$
$$|\widetilde{1}\rangle = \frac{1}{\sqrt{3}}\left(|012\rangle + |120\rangle + |201\rangle\right)$$
$$|\widetilde{2}\rangle = \frac{1}{\sqrt{3}}\left(|021\rangle + |102\rangle + |210\rangle\right).$$

This subspace is symmetric under cyclic permutations of the physical qutrits, and there is a lot of entanglement in all three states.

One way of understanding this code is to note that there is a unitary on the first two physical qutrits, U_{12}, such that

$$|\widetilde{i}\rangle = U_{12}^{\dagger}\left(|i\rangle_1 \otimes |\chi\rangle_{23}\right),$$

where

$$|\chi\rangle \equiv \frac{1}{\sqrt{3}}\left(|00\rangle + |11\rangle + |22\rangle\right).$$

One way of understanding this code is to note that there is a unitary on the first two physical qutrits, U_{12}, such that

$$|\widetilde{i}\rangle = U_{12}^{\dagger} \left(|i\rangle_1 \otimes |\chi\rangle_{23} \right),$$

where

$$|\chi\rangle \equiv \frac{1}{\sqrt{3}} \left(|00\rangle + |11\rangle + |22\rangle \right).$$

Explicitly

$$\begin{array}{lll} |00\rangle \to |00\rangle & |11\rangle \to |01\rangle & |22\rangle \to |02\rangle \\ |01\rangle \to |12\rangle & |12\rangle \to |10\rangle & |20\rangle \to |11\rangle \\ |02\rangle \to |21\rangle & |10\rangle \to |22\rangle & |21\rangle \to |20\rangle \end{array} .$$

One way of understanding this code is to note that there is a unitary on the first two physical qutrits, U_{12}, such that

$$|\widetilde{i}\rangle = U_{12}^{\dagger}\left(|i\rangle_1 \otimes |\chi\rangle_{23}\right),$$

where

$$|\chi\rangle \equiv \frac{1}{\sqrt{3}}\left(|00\rangle + |11\rangle + |22\rangle\right).$$

Explicitly

$$\begin{array}{lll} |00\rangle \to |00\rangle & |11\rangle \to |01\rangle & |22\rangle \to |02\rangle \\ |01\rangle \to |12\rangle & |12\rangle \to |10\rangle & |20\rangle \to |11\rangle \\ |02\rangle \to |21\rangle & |10\rangle \to |22\rangle & |21\rangle \to |20\rangle \end{array}.$$

This means that we can recover any logical state $|\widetilde{\psi}\rangle$ from just the first two qutrits:

$$U_{12}|\widetilde{\psi}\rangle = |\psi\rangle_1 \otimes |\chi\rangle_{23}.$$

One way of understanding this code is to note that there is a unitary on the first two physical qutrits, U_{12}, such that

$$|\widetilde{i}\rangle = U_{12}^{\dagger} \left(|i\rangle_1 \otimes |\chi\rangle_{23} \right),$$

where

$$|\chi\rangle \equiv \frac{1}{\sqrt{3}} \left(|00\rangle + |11\rangle + |22\rangle \right).$$

Explicitly

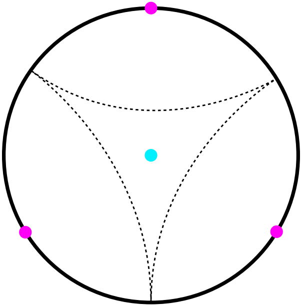
$$\begin{array}{ccc} |00\rangle \to |00\rangle & |11\rangle \to |01\rangle & |22\rangle \to |02\rangle \\ |01\rangle \to |12\rangle & |12\rangle \to |10\rangle & |20\rangle \to |11\rangle \\ |02\rangle \to |21\rangle & |10\rangle \to |22\rangle & |21\rangle \to |20\rangle \end{array} .$$

This means that we can recover any logical state $|\widetilde{\psi}\rangle$ from just the first two qutrits:

$$U_{12}|\widetilde{\psi}\rangle = |\psi\rangle_1 \otimes |\chi\rangle_{23}.$$

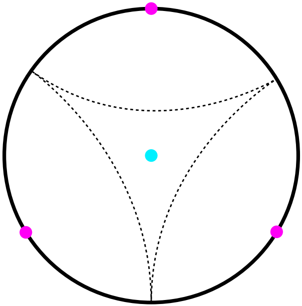
By symmetry there is also a U_{13} and U_{23}.

Let's make the analogy to holography precise:



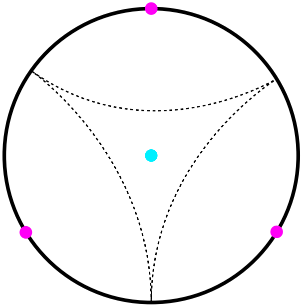
- Three "physical" qutrits are local CFT degrees of freedom on the boundary

Let's make the analogy to holography precise:



- Three "physical" qutrits are local CFT degrees of freedom on the boundary
- One "logical" qutrit is a field in the center of the bulk

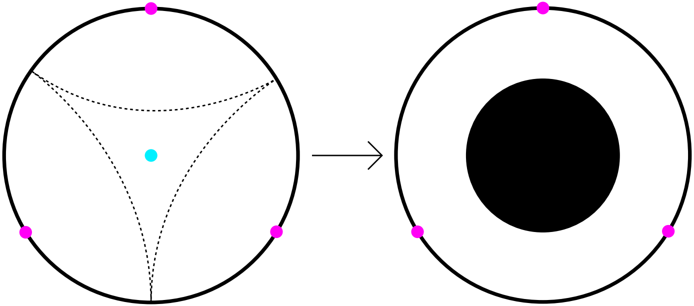
Let's make the analogy to holography precise:



- Three "physical" qutrits are local CFT degrees of freedom on the boundary
- One "logical" qutrit is a field in the center of the bulk
- The correctability we just discussed ensures that subregion duality holds provided we say that our bulk point lies in the entanglement wedge of any two boundary qutrits.

You may wonder about the rest of the states: there is a a 24-dimensional subspace orthogonal to the code subspace, what about bulk locality in those states?

You may wonder about the rest of the states: there is a a 24-dimensional subspace orthogonal to the code subspace, what about bulk locality in those states?
This is where gravity comes to the rescue: these states are the microstates of a black hole that has swallowed our bulk point!



Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.

Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.

Such a model does exist, and I'll now tell you about it

Harlow/Pastawski/Preskill/Yoshida, Hayden/Nezami/Qi/Thomas/Walter/Yang

Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.
Such a model does exist, and I'll now tell you about it

Harlow/Pastawski/Preskill/Yoshida, Hayden/Nezami/Qi/Thomas/Walter/Yang

- The idea is to replace the CFT by a chain of n qubits.

Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.
Such a model does exist, and I'll now tell you about it

Harlow/Pastawski/Preskill/Yoshida, Hayden/Nezami/Qi/Thomas/Walter/Yang

- The idea is to replace the CFT by a chain of n qubits.
- We then consider a 2^k dimensional subspace of states of the qubits, which we interpret corresponding to the set of "low energy" states in the CFT.

Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.
Such a model does exist, and I'll now tell you about it

Harlow/Pastawski/Preskill/Yoshida, Hayden/Nezami/Qi/Thomas/Walter/Yang

- The idea is to replace the CFT by a chain of n qubits.
- We then consider a 2^k dimensional subspace of states of the qubits, which we interpret corresponding to the set of "low energy" states in the CFT.
- The subspace is defined by a big tensor $T_{i_1 \ldots i_n, j_1 \ldots j_k}$, via

$$\langle i_1 \ldots i_n | \widetilde{j_1 \ldots j_k} \rangle = T_{i_1 \ldots i_n, j_1 \ldots j_k}.$$

Tensor Networks and Toy Holography

Of course three qutrits is still quite different from a full boundary CFT, and it would be nice to have a model where at least there is a volume's worth of degrees of freedom in the bulk.
Such a model does exist, and I'll now tell you about it

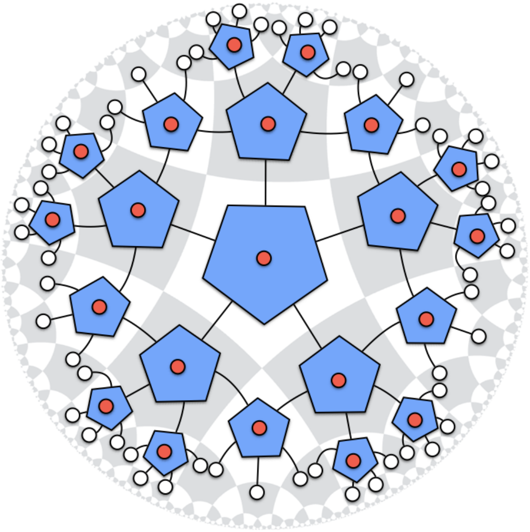
Harlow/Pastawski/Preskill/Yoshida, Hayden/Nezami/Qi/Thomas/Walter/Yang

- The idea is to replace the CFT by a chain of n qubits.
- We then consider a 2^k dimensional subspace of states of the qubits, which we interpret corresponding to the set of "low energy" states in the CFT.
- The subspace is defined by a big tensor $T_{i_1 \ldots i_n, j_1 \ldots j_k}$, via

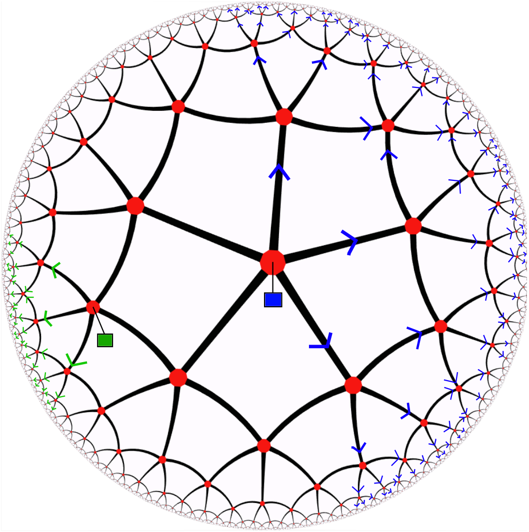
$$\langle i_1 \ldots i_n | \widetilde{j_1 \ldots j_k} \rangle = T_{i_1 \ldots i_n, j_1 \ldots j_k}.$$

- We construct this tensor using a *tensor network*, which is a way of building big tensors out of little ones.

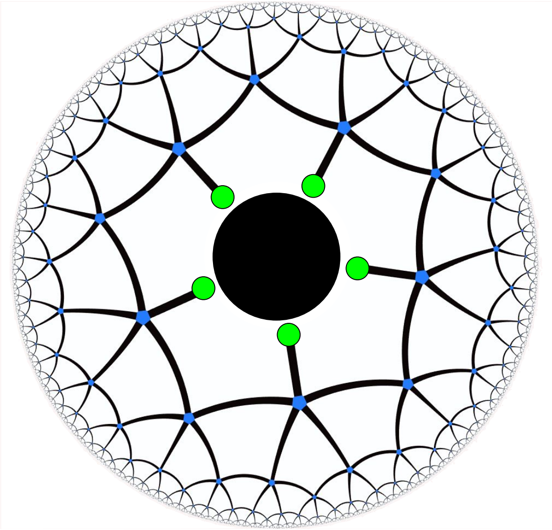
The idea is now to tile the hyperbolic plane with pentagons, each of which has one of our six-leg perfect tensors in the center:



We can then use special properties of our component tensors to do subregion duality:

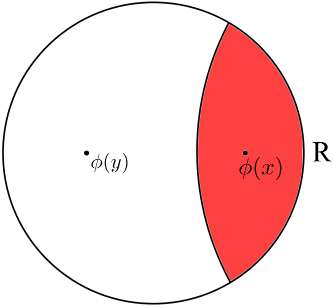


As for the qutrit code, the rest of the Hilbert space is accounted for by black holes:



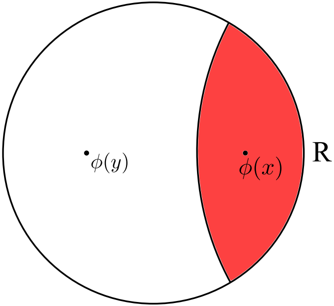
Subregion Duality

I mentioned earlier that given a boundary subregion R, there is a bulk subregion W_R such that any bulk operator in W_R can be represented as a CFT operator in R.



Subregion Duality

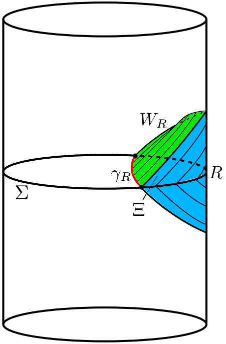
I mentioned earlier that given a boundary subregion R, there is a bulk subregion W_R such that any bulk operator in W_R can be represented as a CFT operator in R.



Until a few years ago, it was a topic of active debate how to correctly define W_R!

There were two main contenders: the *causal wedge of R* vs. the *entanglement wedge of R*.

There were two main contenders: the *causal wedge of R* vs. the *entanglement wedge of R*.
Here is a picture of the entanglement wedge of R, the causal wedge looks similar but smaller:



The details of the two definitions are not important, but there are two points to take away.

The details of the two definitions are not important, but there are two points to take away.

- The causal wedge by definition is not able to see behind a black hole horizon, while the entanglement wedge can. So this seemingly technical debate really touches on perhaps the most profound question of AdS/CFT: does the correspondence describe what is going on inside black holes, and if so how does it work?

The details of the two definitions are not important, but there are two points to take away.

- The causal wedge by definition is not able to see behind a black hole horizon, while the entanglement wedge can. So this seemingly technical debate really touches on perhaps the most profound question of AdS/CFT: does the correspondence describe what is going on inside black holes, and if so how does it work?
- Using the machinery of quantum error correction, we can actually settle this question!

The details of the two definitions are not important, but there are two points to take away.

- The causal wedge by definition is not able to see behind a black hole horizon, while the entanglement wedge can. So this seemingly technical debate really touches on perhaps the most profound question of AdS/CFT: does the correspondence describe what is going on inside black holes, and if so how does it work?
- Using the machinery of quantum error correction, we can actually settle this question!

Indeed one can show that the Ryu-Takayanagi formula for boundary von Neumann entropy as a bulk minimal area, which can be derived on independent grounds, implies that the (larger) entanglement wedge is the winner. Dong/Harlow/Wall

No global symmetries in quantum gravity

The consequences of entanglement wedge reconstruction are still being worked out, but I will close by describing one fun result which follows.

No global symmetries in quantum gravity

The consequences of entanglement wedge reconstruction are still being worked out, but I will close by describing one fun result which follows.

- There is a piece of lore going back decades that there should be no global symmetries in quantum gravity.

No global symmetries in quantum gravity

The consequences of entanglement wedge reconstruction are still being worked out, but I will close by describing one fun result which follows.

- There is a piece of lore going back decades that there should be no global symmetries in quantum gravity.
- The rough idea is that you could store charge information for such a symmetry in a black hole, and it would not come out when the black hole evaporates so the symmetry would be violated.

No global symmetries in quantum gravity

The consequences of entanglement wedge reconstruction are still being worked out, but I will close by describing one fun result which follows.

- There is a piece of lore going back decades that there should be no global symmetries in quantum gravity.
- The rough idea is that you could store charge information for such a symmetry in a black hole, and it would not come out when the black hole evaporates so the symmetry would be violated.
- This argument has various loopholes however, one prominent one being that the sharpest version of it does not apply to discrete global symmetries.

No global symmetries in quantum gravity

The consequences of entanglement wedge reconstruction are still being worked out, but I will close by describing one fun result which follows.

- There is a piece of lore going back decades that there should be no global symmetries in quantum gravity.
- The rough idea is that you could store charge information for such a symmetry in a black hole, and it would not come out when the black hole evaporates so the symmetry would be violated.
- This argument has various loopholes however, one prominent one being that the sharpest version of it does not apply to discrete global symmetries.

Recently Ooguri and I have shown that in AdS/CFT a much more robust argument can be given using entanglement wedge reconstruction.

The basic idea is that in quantum field theory the unitary operators $U(g)$ which implement any symmetry can be broken up into pieces $U(g, R_i)$, each of which implements the symmetry only in a spatial subregion R_i.

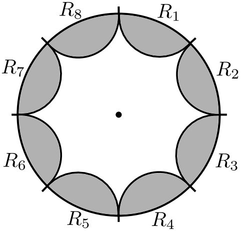
The basic idea is that in quantum field theory the unitary operators $U(g)$ which implement any symmetry can be broken up into pieces $U(g, R_i)$, each of which implements the symmetry only in a spatial subregion R_i. For example if the symmetry is continuous and has a Noether current, then we have

$$U(e^{i\theta^a T_a}, R_i) = e^{i\theta^a \int_{R_i} \star J_a}.$$

The basic idea is that in quantum field theory the unitary operators $U(g)$ which implement any symmetry can be broken up into pieces $U(g, R_i)$, each of which implements the symmetry only in a spatial subregion R_i. For example if the symmetry is continuous and has a Noether current, then we have

$$U(e^{i\theta^a T_a}, R_i) = e^{i\theta^a \int_{R_i} \star J_a}.$$

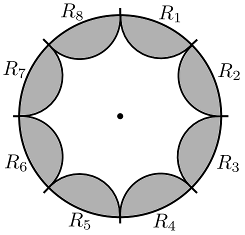
We then have a simple contradiction:



The basic idea is that in quantum field theory the unitary operators $U(g)$ which implement any symmetry can be broken up into pieces $U(g, R_i)$, each of which implements the symmetry only in a spatial subregion R_i. For example if the symmetry is continuous and has a Noether current, then we have

$$U(e^{i\theta^a T_a}, R_i) = e^{i\theta^a \int_{R_i} \star J_a}.$$

We then have a simple contradiction:



No operator in the middle of the bulk could be charged, since the entanglement wedges of the R_i cannot reach there for small enough R_i!

- The details of this argument involve many interesting subtleties, most of which have to do with defining what we really mean by global (and gauge) symmetries in quantum field theory and quantum gravity.

- The details of this argument involve many interesting subtleties, most of which have to do with defining what we really mean by global (and gauge) symmetries in quantum field theory and quantum gravity.
- This leads to a number of interesting observations about these subjects, for example since this is Fermilab I will mention that we are able to give an improved explanation of the decay $\pi_0 \to \gamma\gamma$ in the standard model, fixing the misleading explanation which is usually given in textbooks.

- The details of this argument involve many interesting subtleties, most of which have to do with defining what we really mean by global (and gauge) symmetries in quantum field theory and quantum gravity.
- This leads to a number of interesting observations about these subjects, for example since this is Fermilab I will mention that we are able to give an improved explanation of the decay $\pi_0 \to \gamma\gamma$ in the standard model, fixing the misleading explanation which is usually given in textbooks.
- My time is up, so I will finish by saying that tools from quantum information theory have proven surprisingly useful in thinking about quantum field theory and quantum gravity, and I think we are not done yet.

- The details of this argument involve many interesting subtleties, most of which have to do with defining what we really mean by global (and gauge) symmetries in quantum field theory and quantum gravity.
- This leads to a number of interesting observations about these subjects, for example since this is Fermilab I will mention that we are able to give an improved explanation of the decay $\pi_0 \to \gamma\gamma$ in the standard model, fixing the misleading explanation which is usually given in textbooks.
- My time is up, so I will finish by saying that tools from quantum information theory have proven surprisingly useful in thinking about quantum field theory and quantum gravity, and I think we are not done yet.
- Thanks!