Approaching Lattice Gauge Theories with Matrix Product States and Gaussian States

Stefan Kühn

PERIMETER INSTITUTE

based on works with

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J. Ignacio Cirac	Max Planck Institute of Quantum Optics
Eugene Demler	Harvard University
Pablo Sala	Technical University of Munich
Tao Shi	Chinese Academy of Science

Motivation

Lattice gauge theories

- Nonperturbative regime
- Analytical access hard

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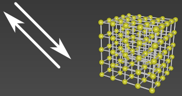

Classical simulation

- Monte Carlo methods in euclidean space-time

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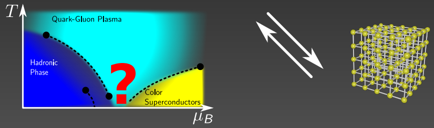

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- Monte Carlo methods in euclidean space-time
  - No real-time dynamics
  - Sign problem

K. Fukushima, T. Hatsuda, Rep. Prog. Phys. **74**, 014001 (2011)

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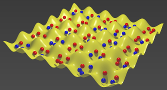

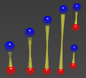

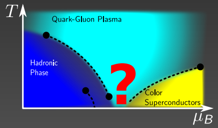

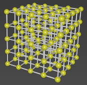

Quantum simulation

- Many promising experimental platforms
- Free from purely numerical limitations

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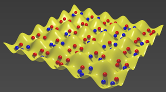

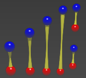

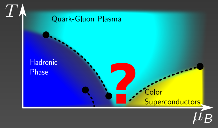

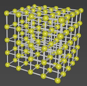

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Classical simulation

- Tensor Networks
- Variational ansatz based on Gaussian states

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Outline

1. Motivation
2. Hamiltonian lattice formulation
3. Matrix Product States for exploring a quantum simulator
4. Variational ansatz based on Gaussian States
5. Summary & Outlook

2.

Hamiltonian lattice formulation

Lattice Hamiltonian formulation

- Kogut-Susskind staggered fermions in temporal gauge $A^0 = 0$

$$H = \varepsilon \sum_{n=1}^{N-1} \left(\phi_n^\dagger U_n \phi_{n+1} + \text{H.c.} \right) + m \sum_{n=1}^{N} (-1)^n \phi_n^\dagger \phi_n + \frac{g^2}{2} \sum_{n=1}^{N-1} \mathbf{L}_n^2$$

Kinetic part + coupling to gauge field — Staggered mass term — (Color)-electric energy

J. Kogut and L. Susskind, Phys. Rev. D 11, 395

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- Gauss law for physical states $G_n^a |\psi\rangle = 0$

$$G_n^a = L_n^a - R_{n-1}^a - \mathcal{Q}_n^a, \quad \mathcal{Q}_n = \mathbf{Q}_n + \mathbf{q}_n$$

dynamical charge — external charge

J. Kogut and L. Susskind, Phys. Rev. D 11, 395

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U(1), Schwinger model

- ▷ Single component fermionic field: ϕ_n
- ▷ $Q_n = \phi_n^\dagger \phi_n - \frac{1}{2}(1 - (-1)^n)$
- ▷ $q_n \in \mathbb{R}$

SU(2)

- ▷ Two colors of fermions: $\phi_n^\dagger = (\phi_n^{r,\dagger}, \phi_n^{g,\dagger})$
- ▷ $Q_n^a = \frac{1}{2}\phi_n^\dagger \sigma^a \phi_n$
- ▷ $q_n^a = \frac{1}{2}\sigma^a$

J. Kogut and L. Susskind, Phys. Rev. D 11, 395

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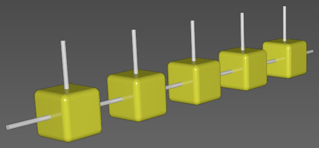
3. **Matrix Product States for exploring a quantum simulator**
4. Variational ansatz based on Gaussian States
5. Summary & Outlook

Matrix Product States



Matrix Product States (MPS)

MPS ansatz

- MPS ansatz for system with N sites and open boundary conditions

$$|\Psi\rangle = \sum_{i_1, i_2, \ldots, i_N = 1}^{d} A_1^{i_1} A_2^{i_2} \ldots A_{N-1}^{i_{N-1}} A_N^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

$$= \sum_{i_1, i_2, \ldots, i_N = 1}^{d} \begin{pmatrix} * & * \end{pmatrix}_1^{i_1} \begin{pmatrix} * & * \\ * & * \end{pmatrix}_2^{i_2} \cdots \begin{pmatrix} * & * \\ * & * \end{pmatrix}_{N-1}^{i_{N-1}} \begin{pmatrix} * \\ * \end{pmatrix}_N^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

M. B. Hastings, J. Stat. Mech. 2007, P08024 (2007)
F. Verstraete, J.I. Cirac Phys. Rev. B 73, 094423 (2006)
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&= \sum_{i_1, i_2, \ldots, i_N = 1}^{d} \begin{pmatrix} * & * \end{pmatrix}_1^{i_1} \begin{pmatrix} * & * \\ * & * \end{pmatrix}_2^{i_2} \cdots \begin{pmatrix} * & * \\ * & * \end{pmatrix}_{N-1}^{i_{N-1}} \begin{pmatrix} * \\ * \end{pmatrix}_N^{i_N} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle
\end{aligned}
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- Size of the matrices D: Bond dimension of the MPS

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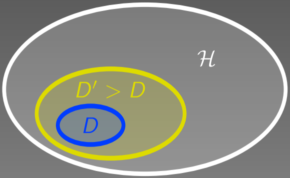
- Size of the matrices D: Bond dimension of the MPS
- Number of parameters: $\mathcal{O}(ND^2 d)$



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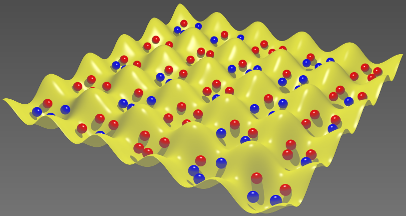
Exploring a quantum simulator for the Schwinger model

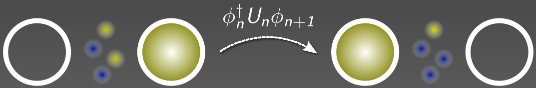


Schwinger model: Quantum simulation

cQED model

- Links are populated by two bosonic species
- Link operators (Schwinger boson representation)

$$U_n = \frac{a_n^\dagger b_n}{\sqrt{\frac{N_0}{2}\left(\frac{N_0}{2}+1\right)}}, \qquad L_n = \frac{1}{2}\left(a_n^\dagger a_n - b_n^\dagger b_n\right)$$



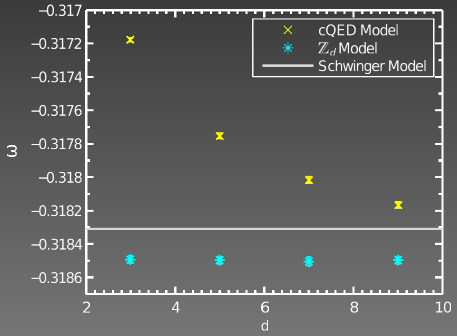
- Number of particles per link is fixed to an even number N_0
- Local Hilbert space dimension $d = N_0 + 1$
- Same $U(1)$ symmetry as the Schwinger model

E. Zohar, J.I. Cirac, B. Reznik, Phys. Rev. A 88, 023617 (2013)

Schwinger model: Quantum simulation

Spectral properties

- Ground state for various lattice bond dimensions, system sizes and lattice spacing
- Extrapolation to the thermodynamic limit and subsequently to the continuum limit



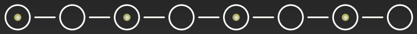
SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)

Schwinger model: Quantum simulation

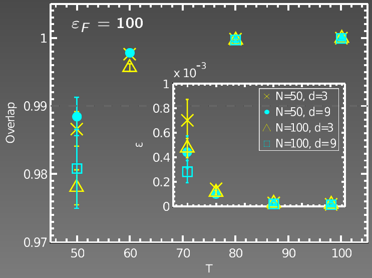
Adiabatic ground state preparation

- Strong-coupling vacuum

  $\varepsilon = 0$

  $\Rightarrow$ State fulfills Gauss Law
- Ramp ε during time T
- Monitor the overlap with the variationally computed state



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  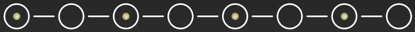

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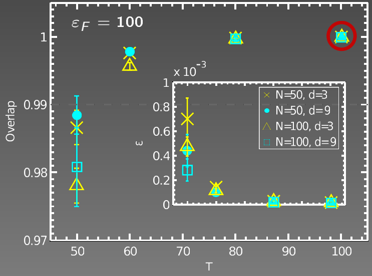

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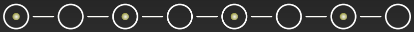

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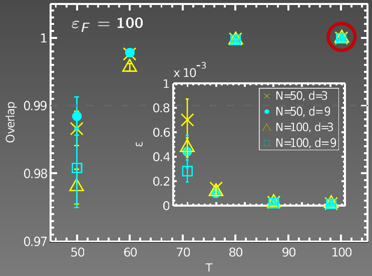
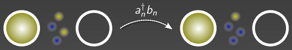


- Mimic gauge invariance breaking noise
- Assume noise strength proportional to ε:
  $\lambda\varepsilon(t)\sum_n(a_n^\dagger b_n + b_n^\dagger a_n)$



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  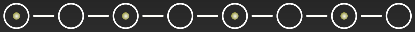
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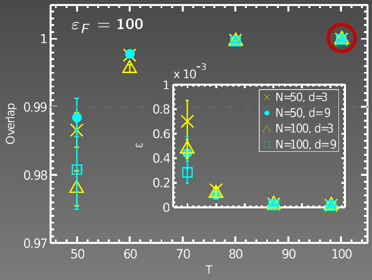

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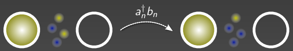

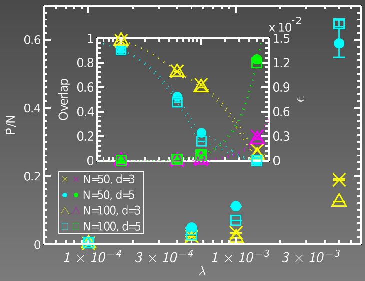

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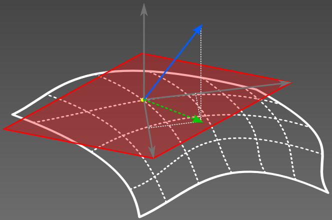
4.

Disentangling the gauge field and Gaussian States



Hamiltonian lattice formulation

Disentangling the gauge field

- Transformation disentangling the gauge degrees of freedom

$$\Theta = \overrightarrow{\prod_{k=1}} \exp\left(i\theta_k^a \sum_{m>k} \mathcal{Q}_m^a\right)$$

- Hamiltonian in the rotated frame $H_\Theta = \Theta H \Theta^\dagger$

$$H_\Theta = \varepsilon \sum_n \left(\phi_n^\dagger \phi_{n+1} + \text{H.c.}\right) + m \sum_n (-1)^n \phi_n^\dagger \phi_n + \sum_a \sum_{n,m} \mathcal{Q}_n^a V_{n,m} \mathcal{Q}_m^a$$

- In the sector of vanishing total charge $\sum_n \boldsymbol{\mathcal{Q}}_n = 0$

$$V_{n,m} = -\frac{1}{2}|n-m|$$

- H_Θ depends only on the fermionic content and is nonlocal

F. Lenz, H.W.L. Naus, M. Thies, Ann. Phys. 233, 317 (1994)
B. Bringoltz, Phys. Rev. D 79, 105021 (2009)

Variational ansatz

Variational ansatz

- Pure Gaussian state

$$|\mathrm{GS}\rangle = C \times \exp\left(-\frac{1}{2}\Phi^\dagger \xi \Phi\right)|\Omega\rangle$$

with $\Phi^\dagger = (\phi_1^\dagger, \ldots, \phi_M^\dagger, \phi_1, \ldots, \phi_M)$
$\phi_j |\Omega\rangle = 0$, ξ hermitian $2M \times 2M$ matrix

- Characterized by its covariance matrix

$$\Gamma = \left\langle \Phi\Phi^\dagger \right\rangle$$

- Variational ansatz in the original frame

$$|\psi\rangle = \Theta^\dagger |\mathrm{GS}\rangle |0\rangle_{\mathrm{gauge}}$$

$\Rightarrow$ Non-Gaussian ansatz

S. Bravyi, arXiv:quant-ph/0507282 (2005)
S. Bravyi, Quantum Inf. and Comp. 5, 216 (2005)
C. Weedbrook et al., Rev. Mod. Phys. 84, 621 (2012)
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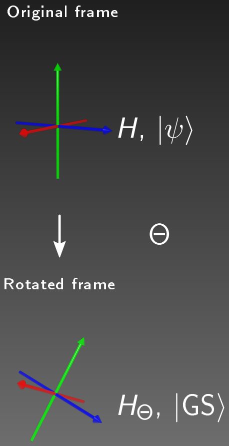

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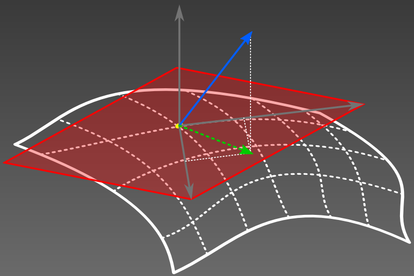
Variational ansatz

Time-dependent variational principle

- Compute the evolution of a given initial Gaussian state under H_Θ in the manifold of Gaussian States



T. Shi, E. Demler, J. I. Cirac, Ann. Phys. **390**, 245 (2018)

Variational ansatz

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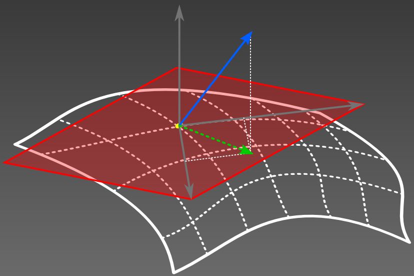
- Time-dependent variational principle applied to Gaussian States yields



- ▷ Imaginary-time evolution:

$$\frac{d}{d\tau}\Gamma(\tau) = \{\Gamma, \mathcal{H}(\Gamma)\} - 2\Gamma\,\mathcal{H}(\Gamma)\Gamma$$

- ▷ Real-time evolution:

$$i\frac{d}{dt}\Gamma(t) = [\mathcal{H}(\Gamma), \Gamma]$$

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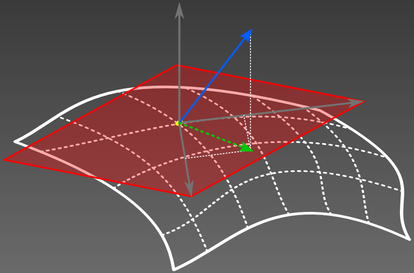

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▷ Real-time evolution:

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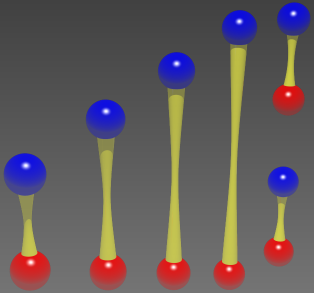
⇒ Allows us to compute ground states/real-time dynamics

- From Γ we can efficiently obtain averages of (local) observables

T. Shi, E. Demler, J. I. Cirac, Ann. Phys. 390, 245 (2018)

String breaking in the case of 1+1d U(1) and SU(2) LGT



Results: Schwinger model

Static potential

- Start from the strong-coupling vacuum

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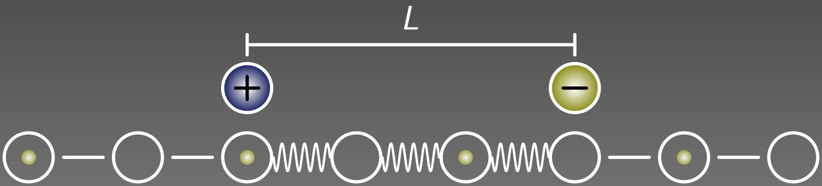
Static potential

- Start from the strong-coupling vacuum
- Impose a flux string between static external charges separated by a length L on top
- Evolve in imaginary time and determine the ground-state energy $E_Q(L)$



Results: Schwinger model

Static potential

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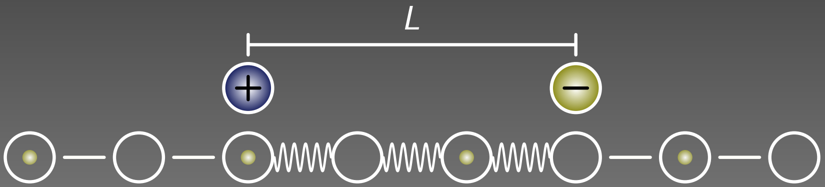
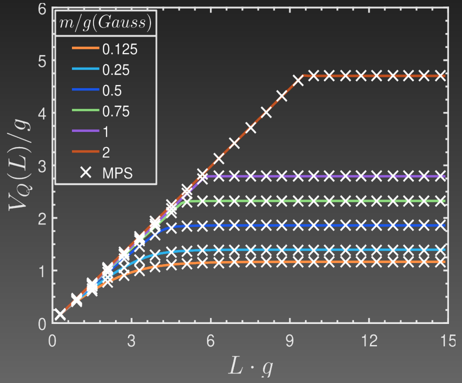
- Evolve in imaginary time and determine the ground-state energy $E_Q(L)$
- Measure the static potential $V_Q(L) = E_Q(L) - E_{\text{vac}}$



Results: Schwinger model

Static potential for $Q/g = 1$



$\Rightarrow$ Excellent agreement with results from Tensor Network calculations

B. Buyens, J. Haegeman, H. Verschelde. F. Verstraete, K. Van Acoleyen. Phys. Rev. X 6, 041040 (2016)
P. Sala, T. Shi, SK, M.C. Bañuls, E. Demler, J.I. Cirac Phys. Rev. D 98, 034505 (2018)

Results: SU(2) lattice gauge theory

Static potential

- External charges are described by $q^a = \frac{1}{2}\sigma^a$
  $\Rightarrow$ Gauge invariant color-neutral state is not a Gaussian state
- Parity symmetries of H_Θ allow for finding unitary transformations V_1 and V_2 which decouple the external charges

$$\overline{H}_\Theta(s_1, s_2) = V_2 V_1 H_\Theta V_1^\dagger V_2^\dagger$$

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$$\overline{H}_\Theta(s_1, s_2) = V_2 V_1 H_\Theta V_1^\dagger V_2^\dagger$$

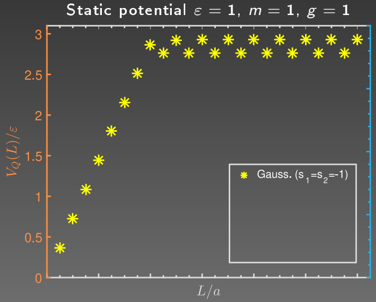
- $s_1, s_2 \in \{-1, 1\}$: Eigenvalues of the parity operators in the rotated frame
- Gauge invariant correlation function to characterize the entanglement between the static external charges

$$C_2(n_1, n_2) = \sum_{a,b} \left\langle q^a_{n_1} \left(U^{\mathrm{Adj.},\dagger}_{n_1} \cdots U^{\mathrm{Adj.},\dagger}_{n_2-1} \right)_{a,b} q^b_{n_2} \right\rangle$$

Results: SU(2) lattice gauge theory

Static potential

- Analogous to the U(1) case start with the strong-coupling vacuum
- Impose a color-flux string of length L on top
- Compute the ground state via imaginary time evolution



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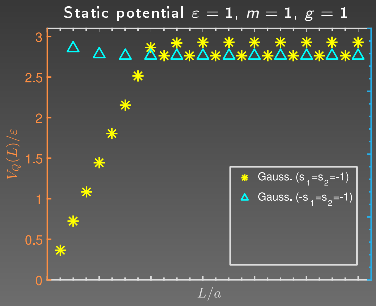

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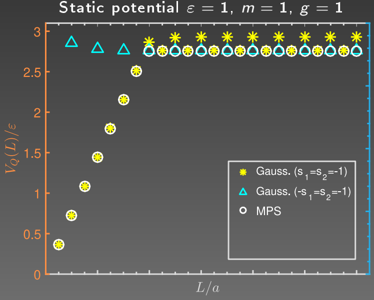

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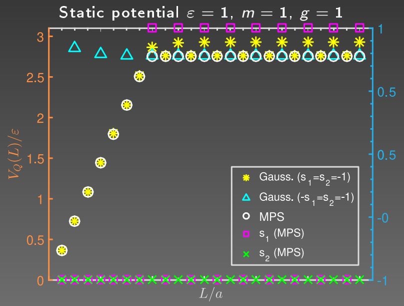

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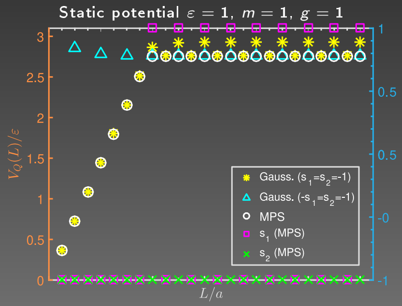

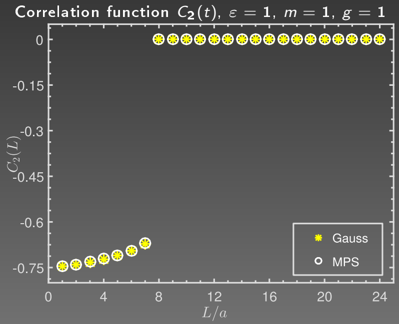

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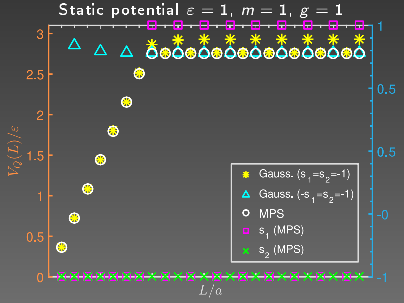
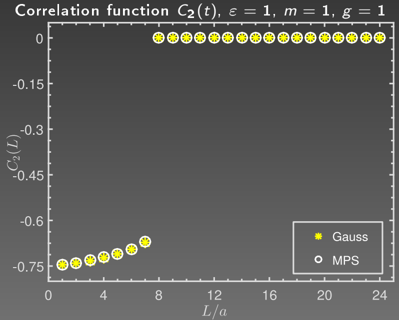




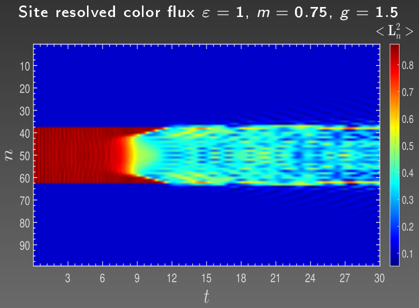
⇒ Reliable simulations of static properties also for the non-Abelian cases

P. Sala, T. Shi, SK, M.C. Bañuls, E. Demler, J.I. Cirac Phys. Rev. D 98, 034505 (2018)

Results: SU(2) lattice gauge theory

Out-of-equilibrium dynamics

- Again we compute the interacting vacuum
- Impose a string between static external charges on top



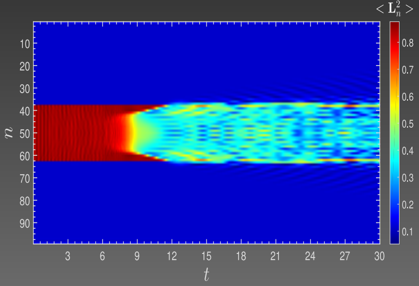
P. Sala, T. Shi, SK, M.C. Bañuls, E. Demler, J.I. Cirac Phys. Rev. D 98, 034505 (2018)

Results: SU(2) lattice gauge theory

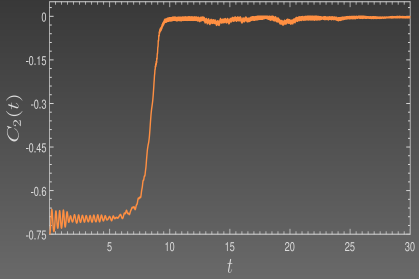
Out-of-equilibrium dynamics

- Again we compute the interacting vacuum
- Impose a string between static external charges on top

Site resolved color flux $\varepsilon = 1$, $m = 0.75$, $g = 1.5$



Correlation function $C_2(t)$ $\varepsilon = 1$, $m = 0.75$, $g = 1.5$



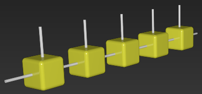
$\Rightarrow$ Reliable simulations of dynamics also for the non-Abelian cases

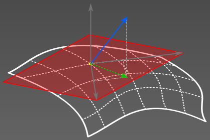
P. Sala, T. Shi, SK, M.C. Bañuls, E. Demler, J.I. Cirac Phys. Rev. D 98, 034505 (2018)

Summary

MPS for exploring a quantum simulator

- MPS allow for examining relevant questions
  - Effects of approximations
  - Scaling of resources
  - Necessary noise control





Variational ansatz based on Gaussian States

- Small number of variational parameters
- Ansatz captures the relevant features
  - Static properties
  - Out-of-equilibrium dynamics
- Good agreement with Tensor Network results

Summary & Outlook

Outlook

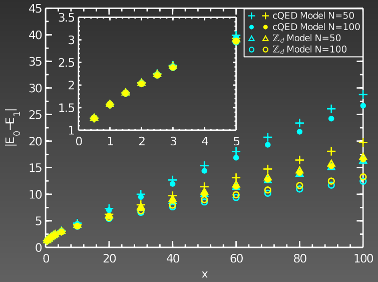
- Generalization of the variational ansatz based on Gaussian states to higher dimensions
- Observables and scenarios explored might be realizable in a quantum simulator
- Formulation $H_{\ominus}$ might be useful for other applications
  - Tensor Networks
  - Quantum simulation

Thank you for your attention!

A. Schwinger model: Quantum simulation

Adiabatic ground state preparation

- Maximum ramping speed of the interaction depends on the gap
- Numerical result for the gap



- Choice of the ramp

$$x(t) = x_F \left(\frac{t}{T}\right)^3$$

SK, J.I. Cirac, M.C. Bañuls, Phys. Rev. A 90, 042305 (2014)

B. Variational ansatz for the case of SU(2)

Parity symmetries of the Hamiltonian

- Parity symmetry of H_Θ: $[P_1, H_\Theta] = 0$
  with $P_1 = \sigma_1^z \sigma_2^z P_z$ and $P_z = \exp\left[i\frac{\pi}{2} \sum_n \phi_n^\dagger (\sigma^z + \mathbb{1}) \phi_n \right]$
- Unitary transformation rotating P_1 to σ_1^x

$$V_1 = \frac{1}{\sqrt{2}}(1 - i\sigma_1^y \sigma_2^z P_z),$$

$\Rightarrow$ In the rotated frame σ_1^x is conserved, classical variable s_1

- Parity symmetry of $P_1^\dagger H_\Theta P_1$: $[P_2, P_1^\dagger H_\Theta P_1] = 0$
  with $P_2 = \sigma_2^x P_x$ and $P_x = \exp\left[i\frac{\pi}{2} \sum_n \phi_n^\dagger (\sigma^x + \mathbb{1}) \phi_n \right]$
- Unitary transformation rotating P_2 to $-\sigma_2^z$

$$V_2 = \frac{1}{\sqrt{2}}(1 - i\sigma_2^y P_x)$$

$\Rightarrow$ In the rotated frame σ_2^z is conserved, classical variable s_2

B. Variational ansatz for the case of SU(2)

Variational ansatz

- Ansatz in the rotated frame

$$|\overline{\psi}\rangle = |\mathrm{GS}\rangle\, |s_1\rangle\, |s_2\rangle$$

- Ansatz in the original frame

$$\begin{aligned}|\psi\rangle = &\frac{1}{4\sqrt{2}}\Theta^\dagger \Big[(|\uparrow\rangle_z + s_1 |\downarrow\rangle_z)\big[(1+s_2)|\uparrow\rangle_z + (1-s_2)|\downarrow\rangle_z\big] \\ &+ s_2(s_1|\uparrow\rangle_z - |\downarrow\rangle_z)\big[(1+s_2)|\uparrow\rangle_z + (1-s_2)|\downarrow\rangle_z\big] P_z \\ &- s_2(|\uparrow\rangle_z + s_1|\downarrow\rangle_z)\big[(1-s_2)|\uparrow\rangle_z + (1+s_2)|\downarrow\rangle_z\big] P_x \\ &+ (s_1|\uparrow\rangle_z - |\downarrow\rangle_z)\big[(1-s_2)|\uparrow\rangle_z + (1+s_2)|\downarrow\rangle_z\big] i^{\mathcal{N}} P_y \Big] \\ &\times |\mathrm{GS}\rangle\, |0\rangle_{\mathrm{gauge}}\end{aligned}$$

- If $|\mathrm{GS}\rangle$ is chosen to be the Dirac sea
  - Singlet between external charges for $s_1 = s_2 = -1$
  - Triplet for any other choice of s_1 and s_2