Tensor Networks for Fine-Graining Lattice Gauge Theory, and also Path Integral Geometry

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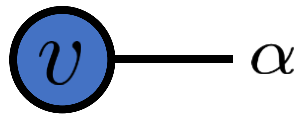
TENSOR NETWORKS INITIATIVE

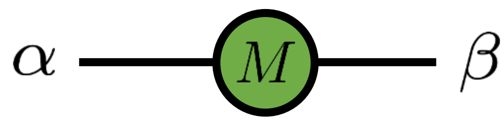
Simons Collaboration on the Many Electron Problem

tensors

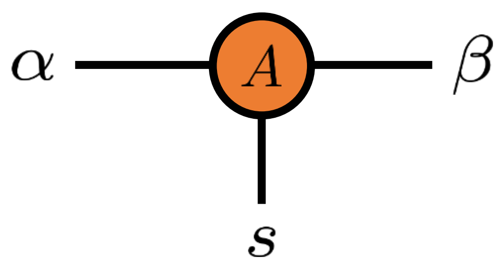
$$v_\alpha =$$



$$M_{\alpha,\beta} =$$

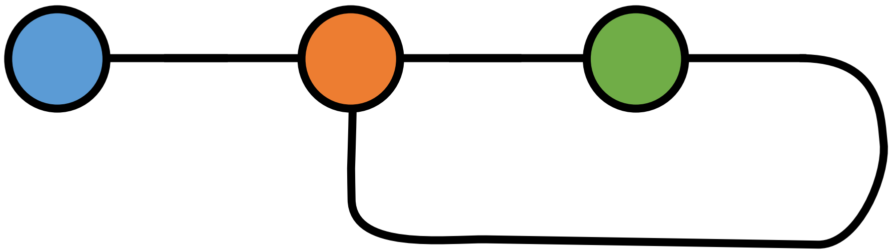


$$A^s_{\alpha\beta} =$$



tensor networks

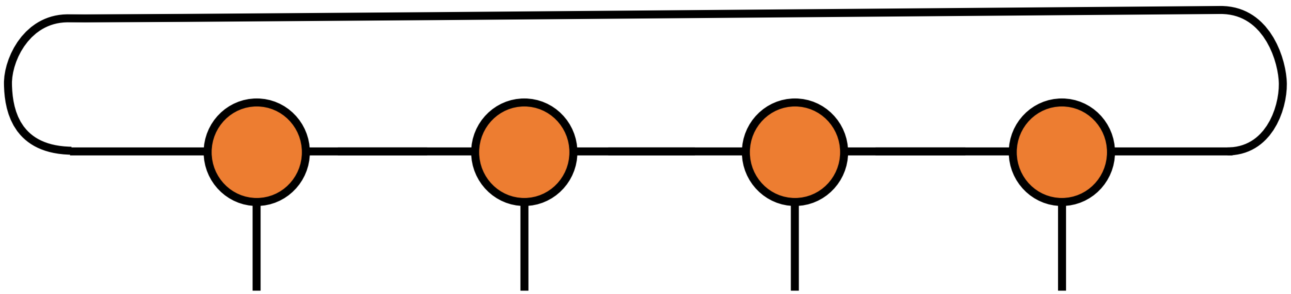
a number



$$= v_\alpha A^s_{\alpha\beta} M_{\beta s}$$

tensor networks

a state

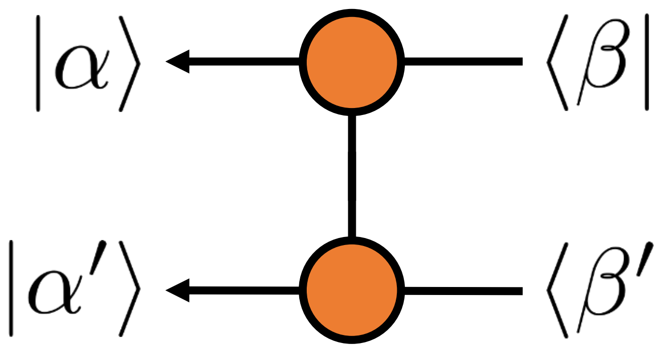


$$|s_1\rangle \otimes |s_2\rangle \otimes |s_3\rangle \otimes |s_4\rangle$$

$$= \mathrm{tr}(A^{s_1} A^{s_2} A^{s_3} A^{s_4})|s_1 s_2 s_3 s_4\rangle$$

tensor networks

a linear map, or gate

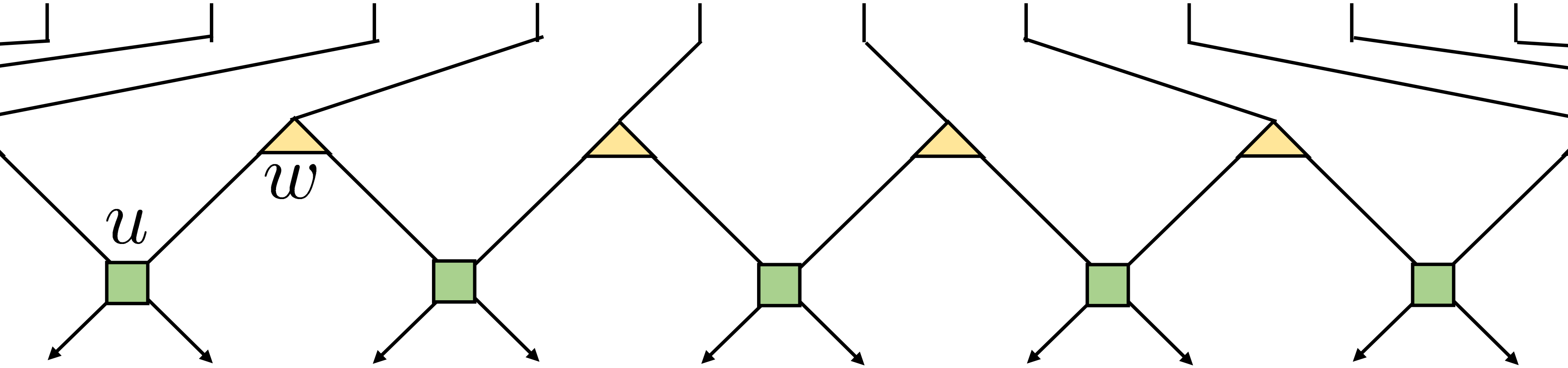


tensor networks for fine-graining lattice gauge theory

AM & Tobias J. Osborne

Phys. Rev. D 98, 014505 (2018)

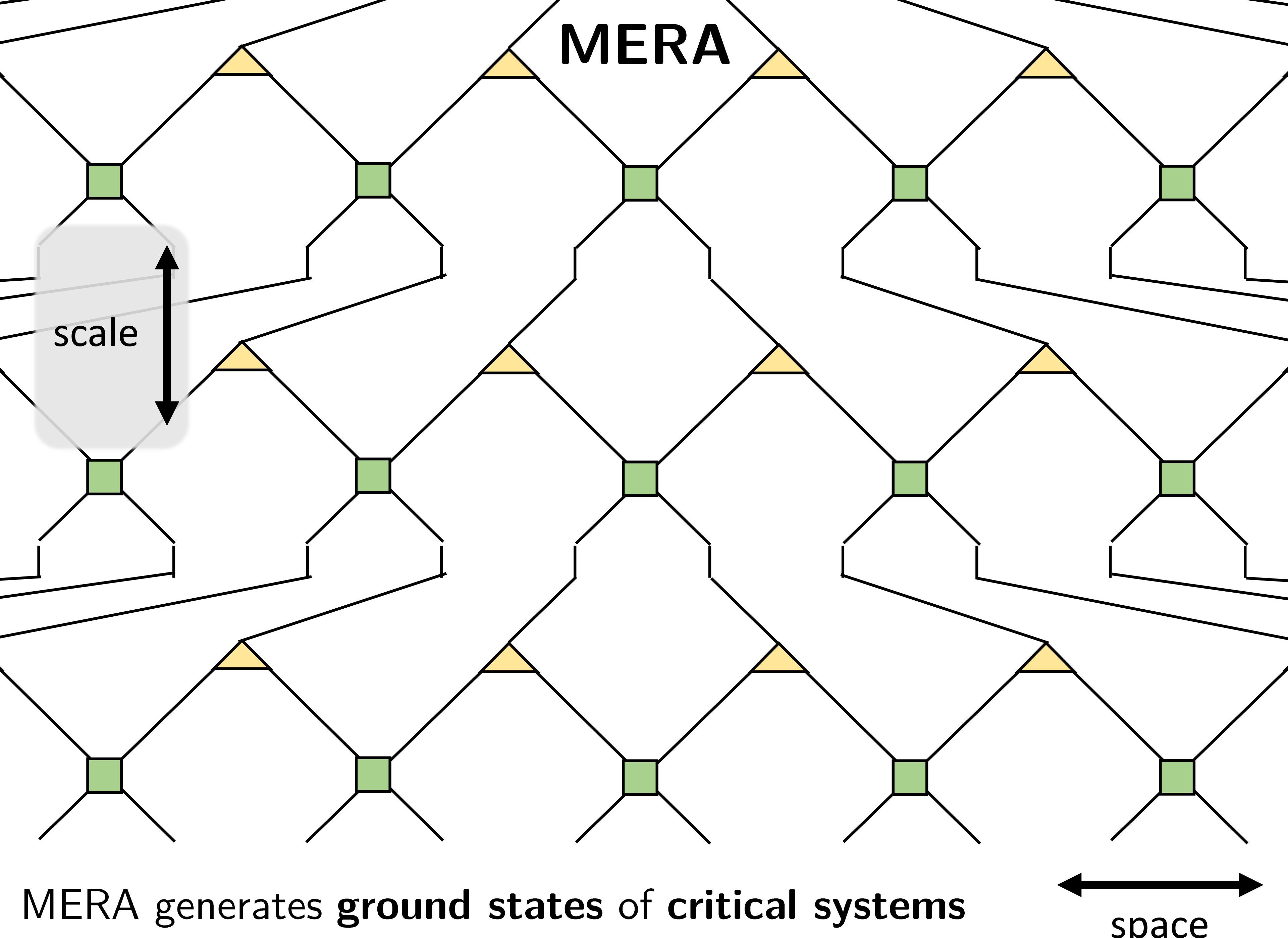
MERA



a layer of **MERA** is a **dilation**

$$\sim \exp(-\mathrm{i} s D)$$

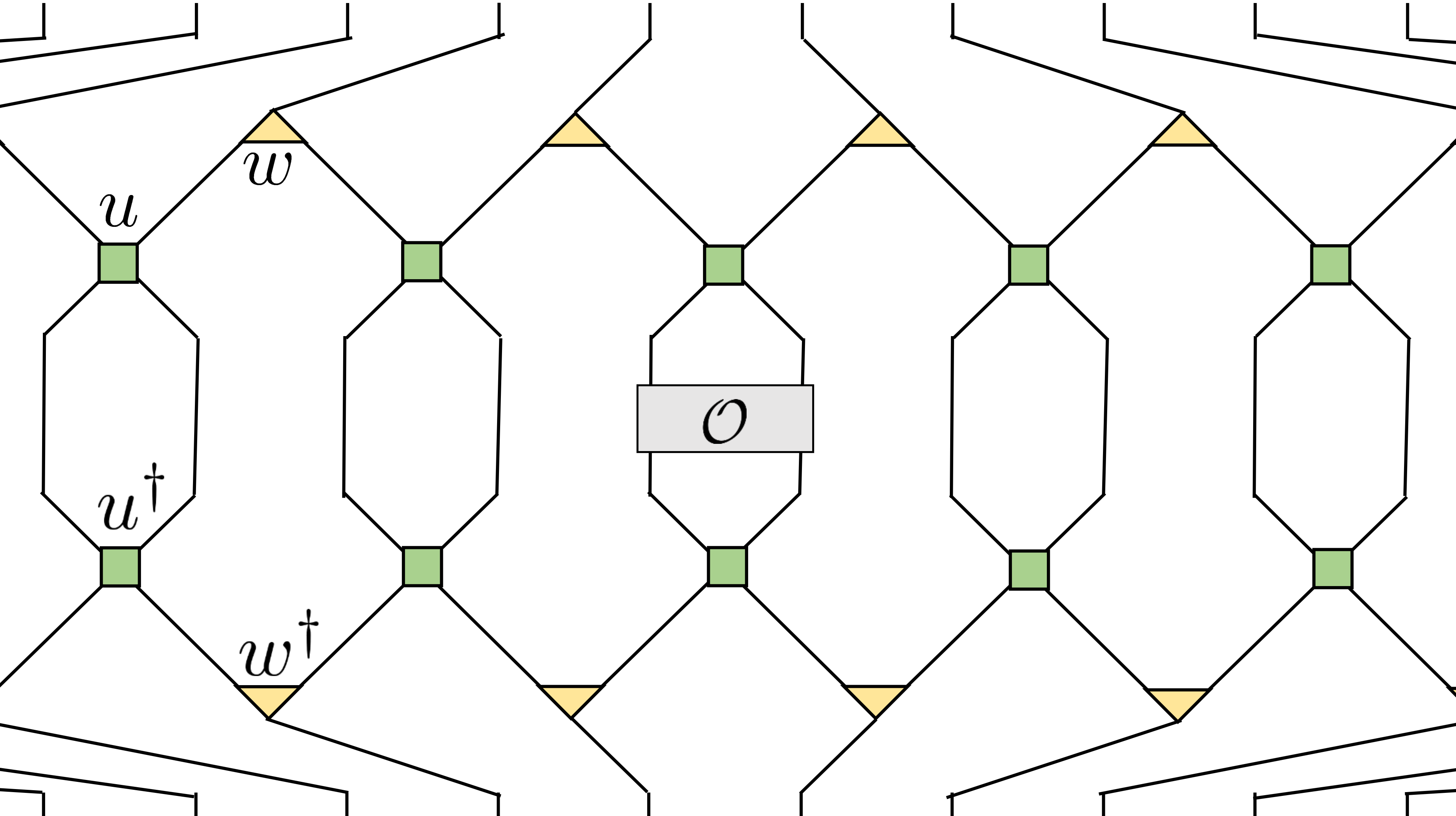
G. Vidal 2007, R. Pfeifer et al. 2009



MERA generates **ground states** of **critical systems**

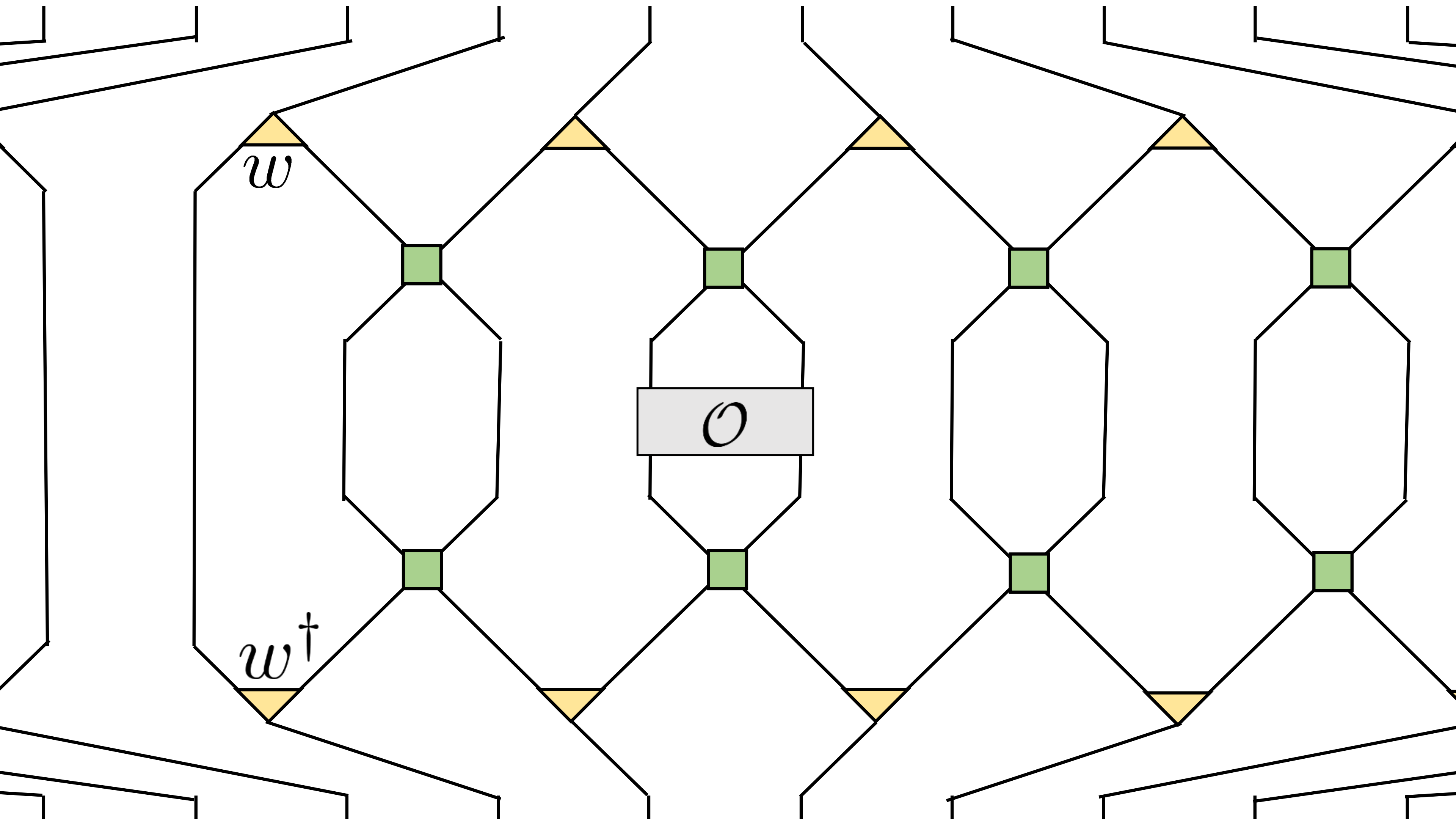
MERA

efficiently contractible



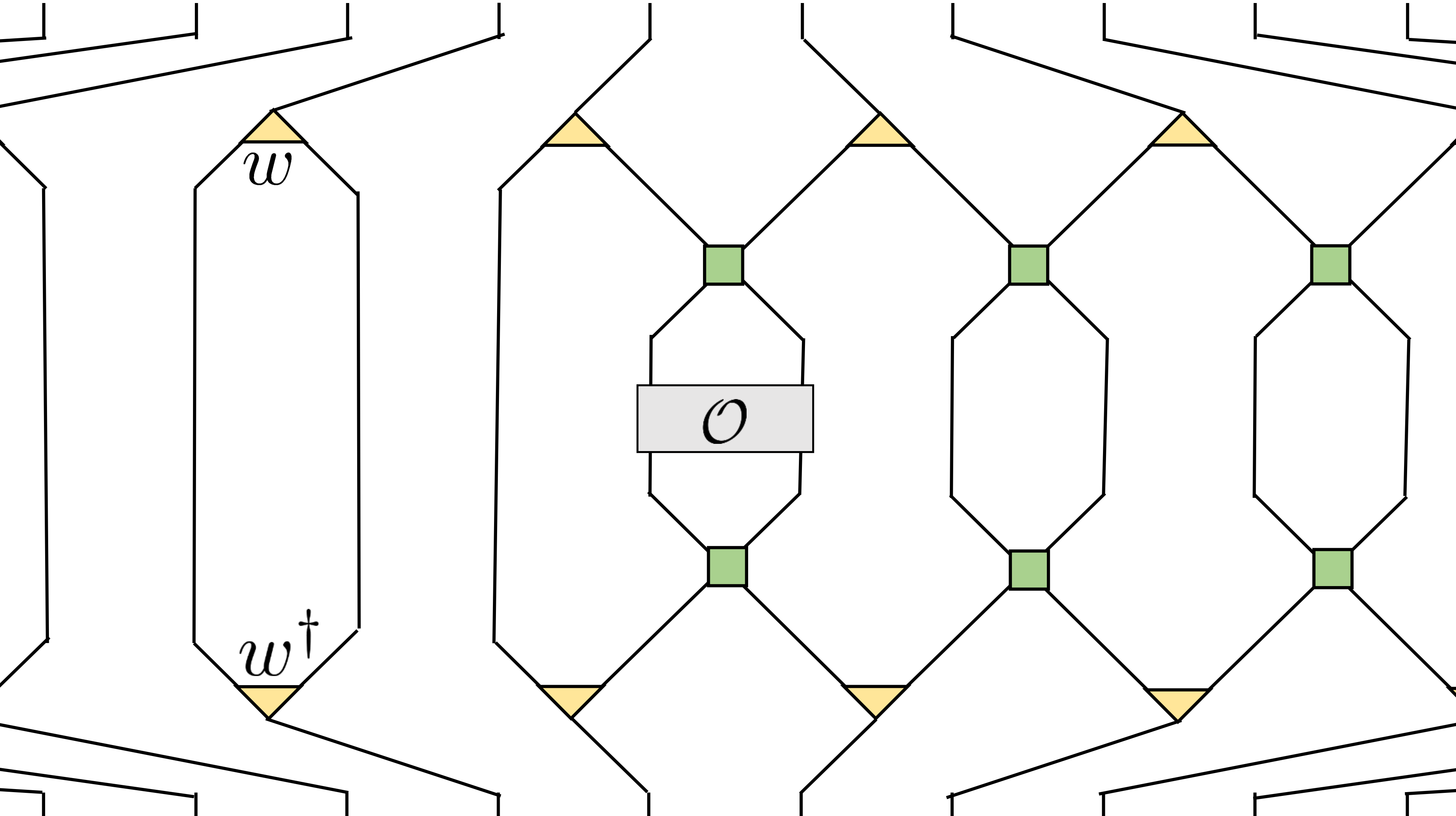
MERA

efficiently contractible



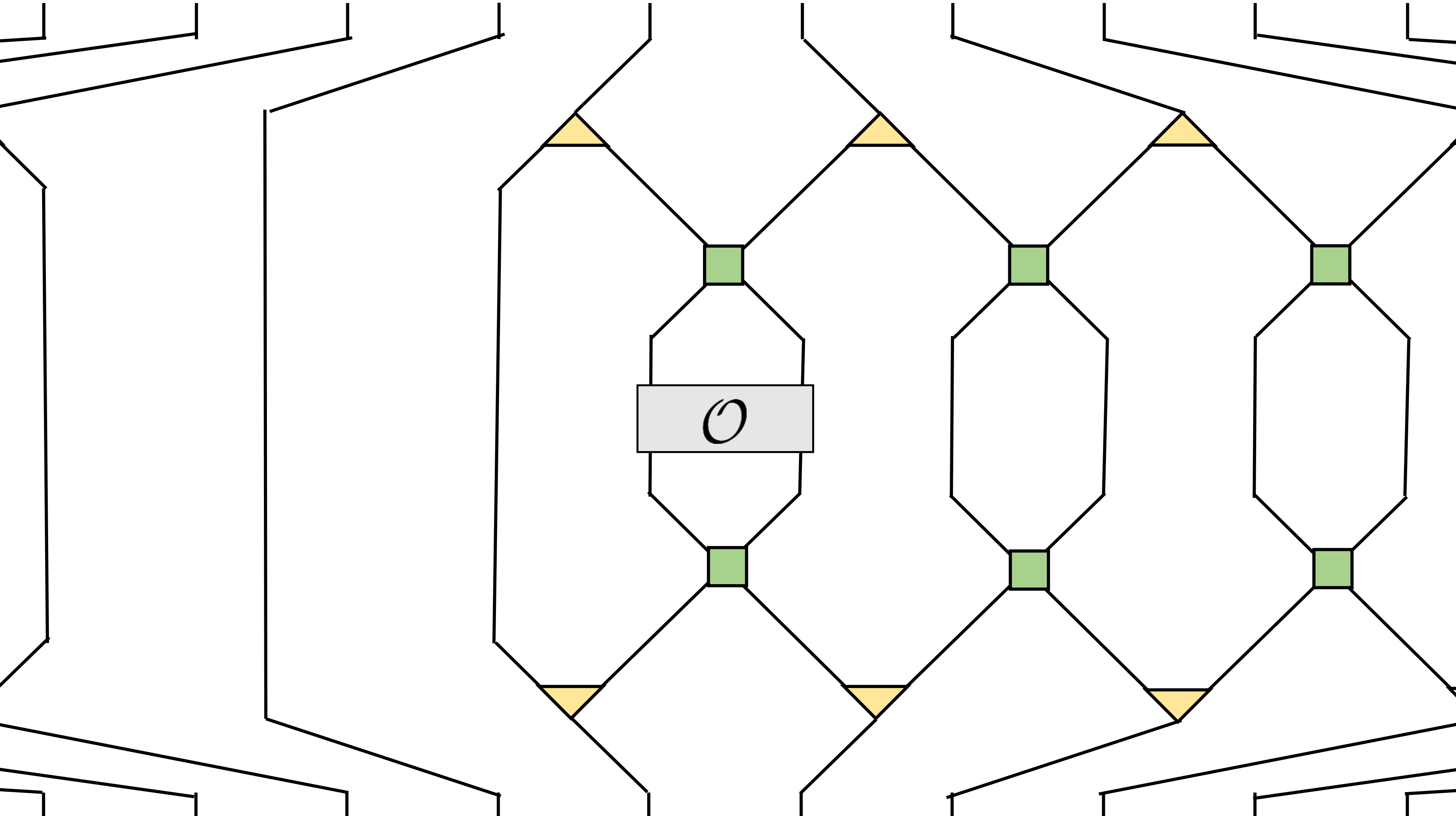
MERA

efficiently contractible



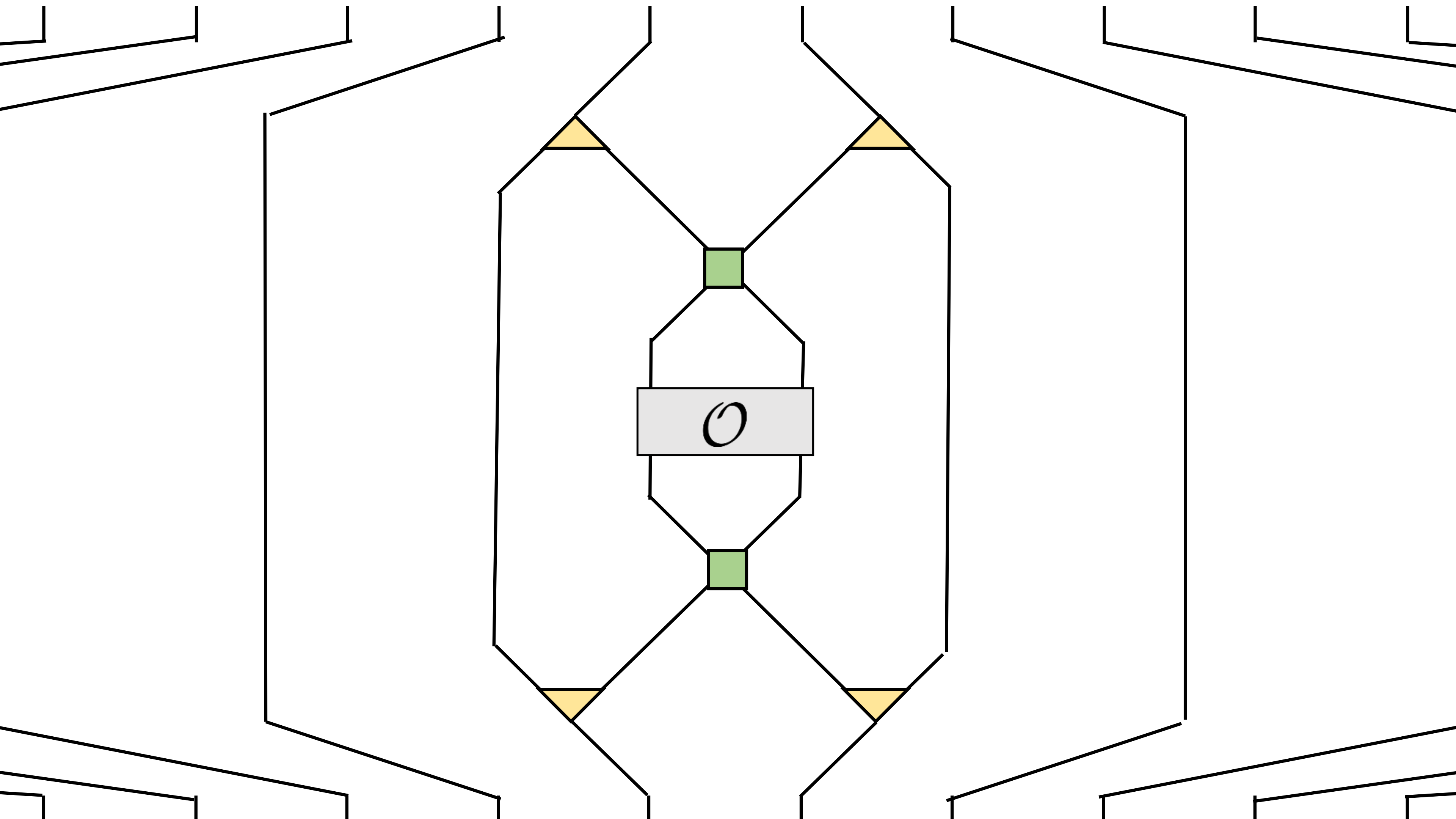
MERA

efficiently contractible



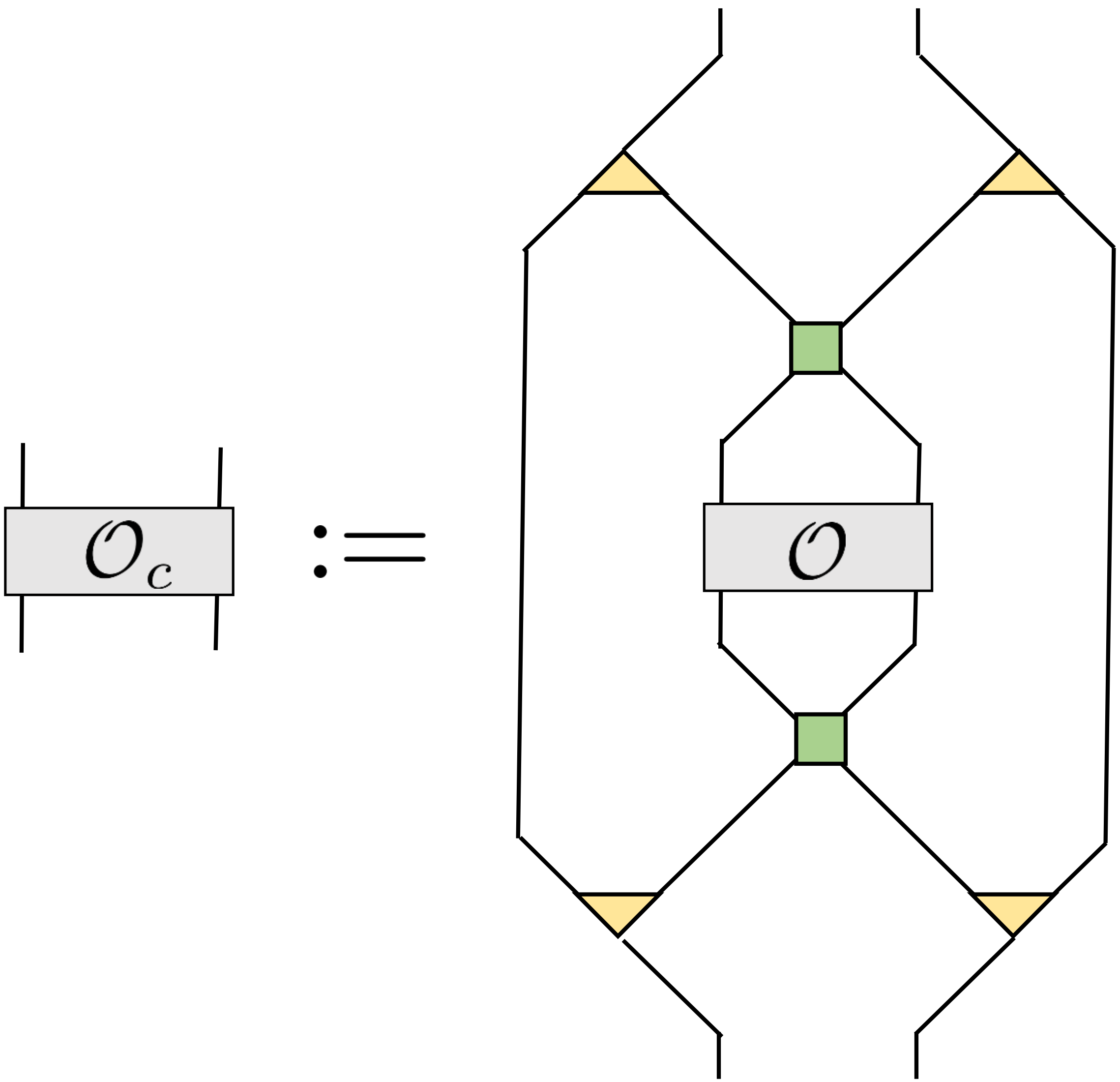
MERA

efficiently contractible



MERA

efficiently contractible



coarse-grained operator

- compute correlators
- extract conformal data
- etc.

MERA

tensors

typically found **variationally**

MERA

tensors

typically found **variationally**

we **propose** a family of MERA tensors
designed to **fine-grain** lattice gauge theory

a MERA for lattice gauge theory
the programme

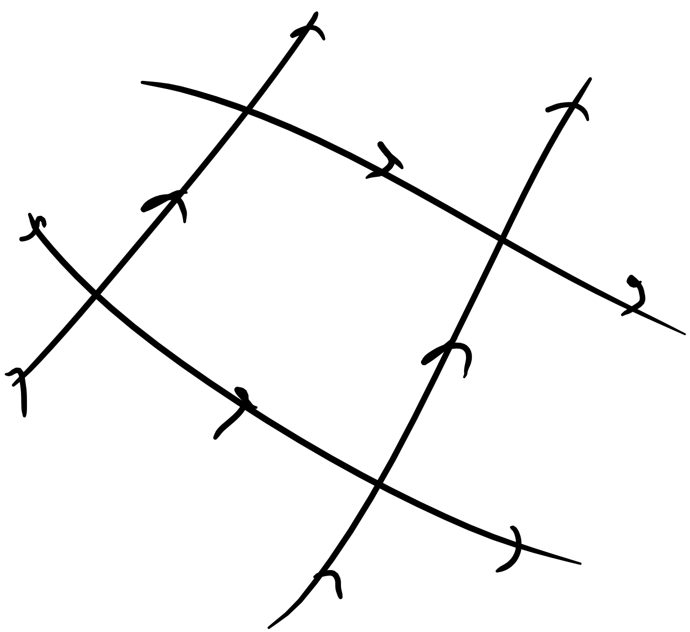
efficiently contractible representation
of the **Yang-Mills ground state**
and low-lying excitations

potential for an **analytical** continuum limit

extension to **QCD**

Lattice Yang-Mills: Kogut-Susskind Hamiltonian

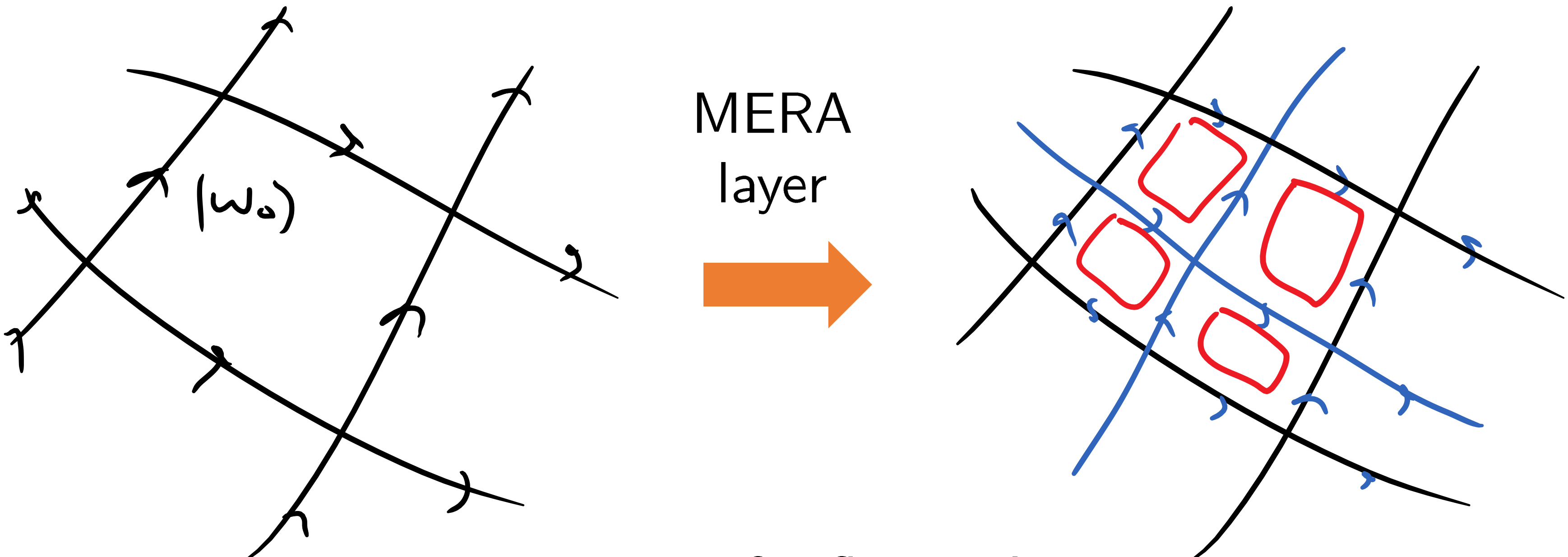
$$H(g) = \frac{g^2}{2a}\sum_{e\in E}\triangle_e + \frac{1}{g^2 a}\left(2 - \sum_{\square}\mathrm{Re}(\mathrm{tr}(\widehat{u}_{\square}))\right)$$



J. Kogut, L. Susskind, Phys Rev D **11**, 395 (1975)

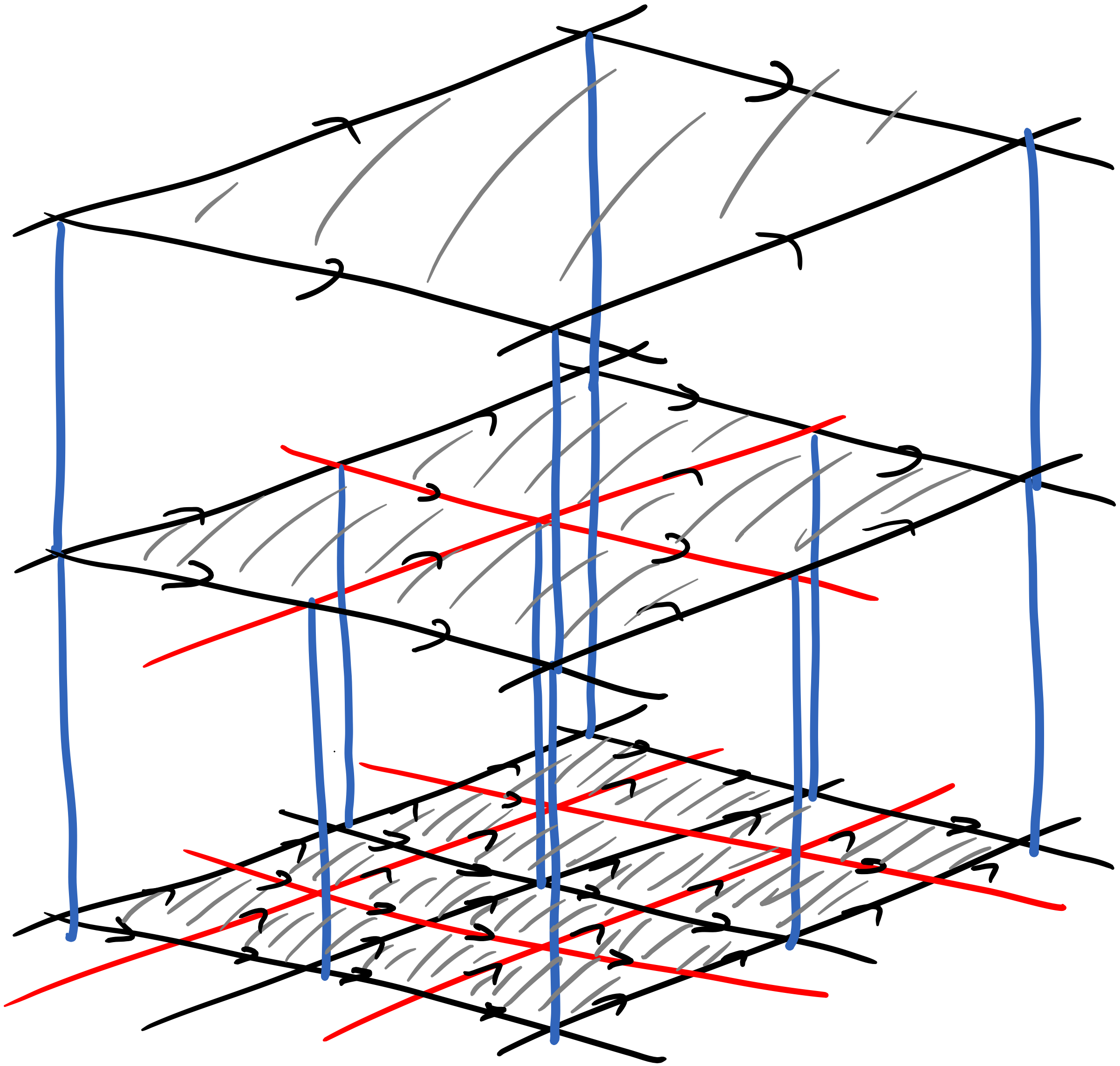
Lattice Yang-Mills: Kogut-Susskind Hamiltonian

$$H(g) = \frac{g^2}{2a} \sum_{e \in E} \triangle_e + \frac{1}{g^2 a} \left(2 - \sum_{\square} \mathrm{Re}(\mathrm{tr}(\widehat{u}_{\square})) \right)$$



optimize for flatness!

MERA ansatz for fine-graining lattice Yang-Mills



$$|\Psi_0\rangle = |\Omega(g=\infty)\rangle$$

↓ refine!

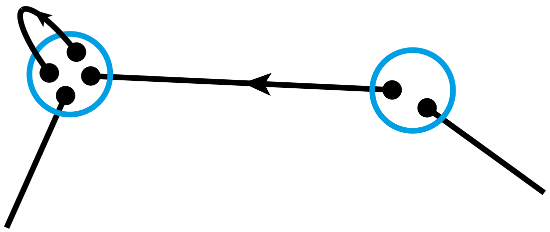
$$|\Psi'\rangle \approx |\Omega(g'_H < \infty)\rangle$$

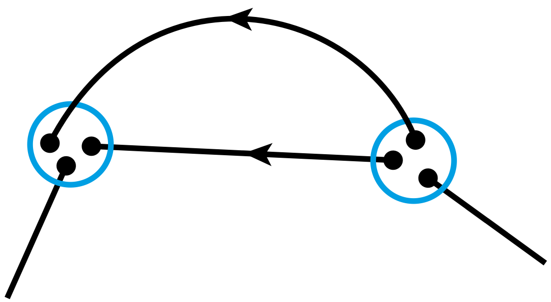
↓ refine!

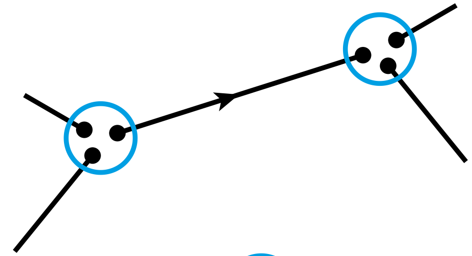
$$|\Psi''\rangle \approx |\Omega(g''_H < g'_H)\rangle$$

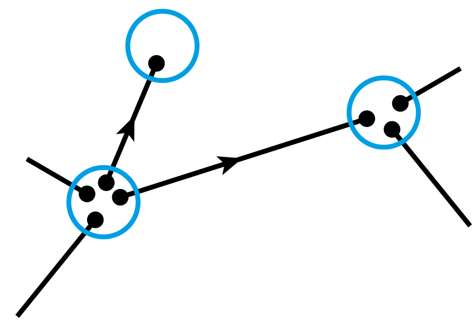
graph manipulation gates

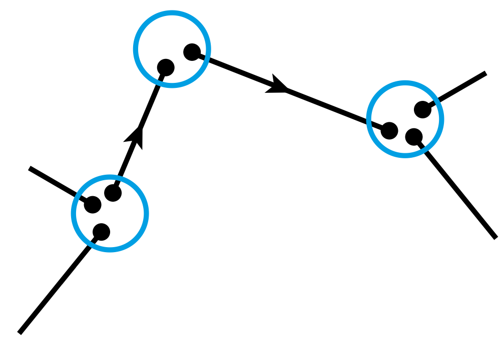
(quantum parallel transport)





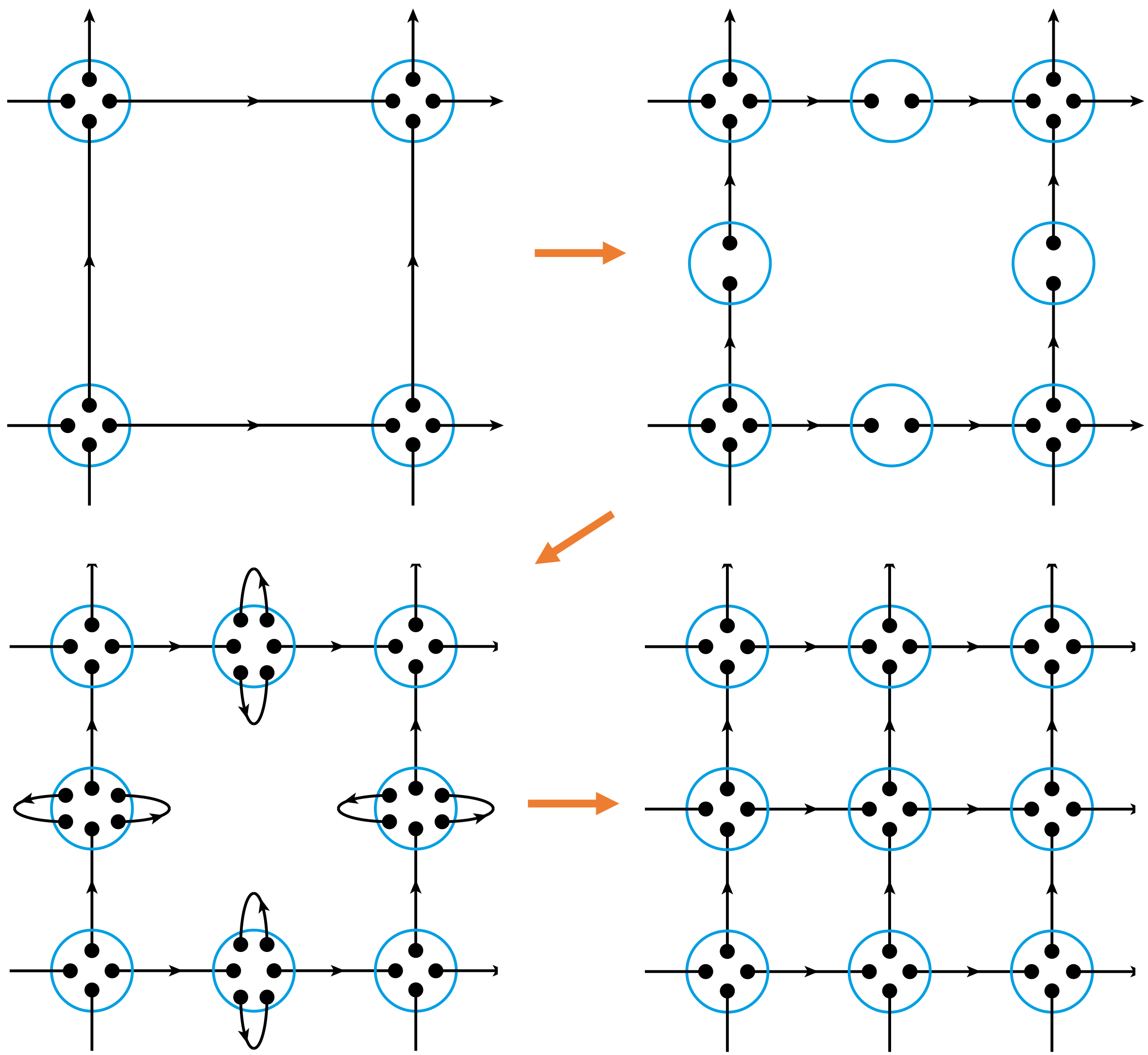




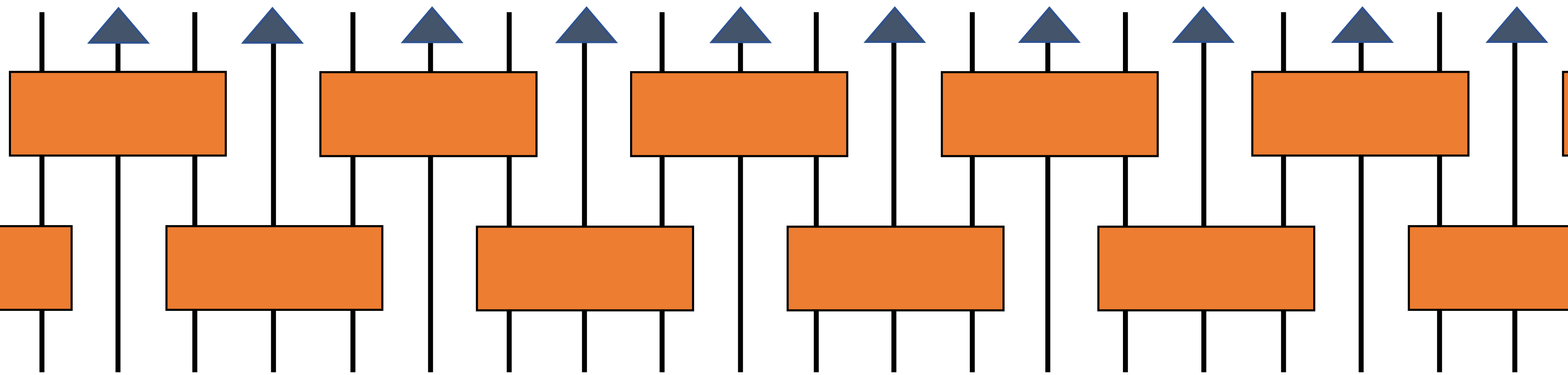


J. Baez, Adv. Math. **117**, 253-272 (1996)
E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J. Math. Phys. **43**, 4452 (2002)
M. Aguado and G. Vidal, PRL **100**, 070404 (2008)
O. Buerschaper, M. Aguado, and G. Vidal, Phys Rev B **79**, 085119 (2009)
R. König, B. W. Reichardt, and G. Vidal, Phys Rev B **79**, 195123 (2009)

curvature interpolation isometry (MERA layer)



curvature interpolation isometry (MERA layer)



(1+1D version)

properties of the ansatz

manifestly **gauge invariant** ✓

a **contractible** tensor network (MERA) ✓

Lorentz-invariant **continuum limit** ?

only **irrelevant** (UV) errors ?

quantum simulation

fine-graining to take **continuum limits** without starting fresh at each coupling strength (**state preparation**)

basis truncation needed (as usual)

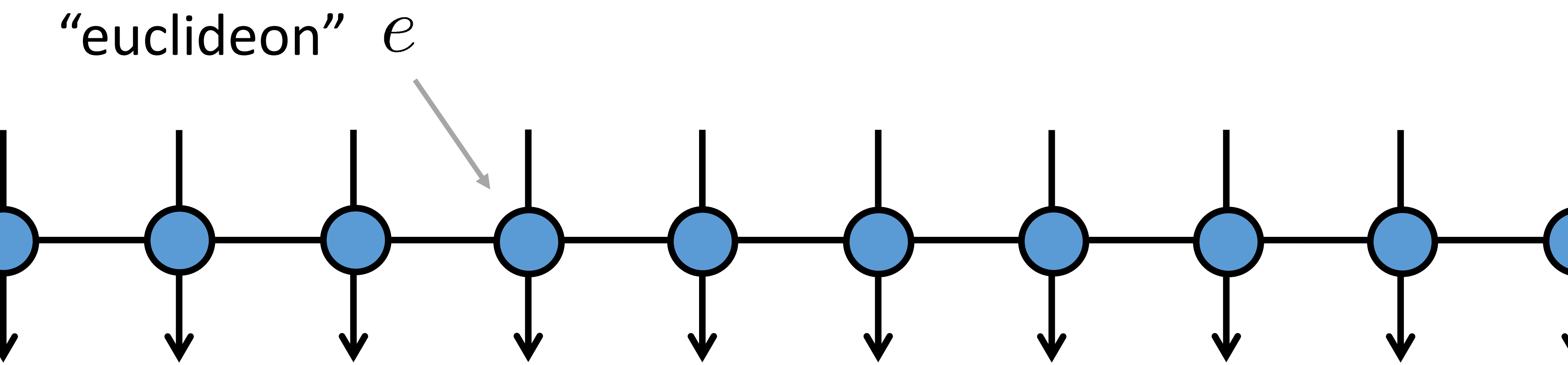
need to decompose the fine-graining isometry into a circuit of simpler gates

tensor networks as path integral geometry

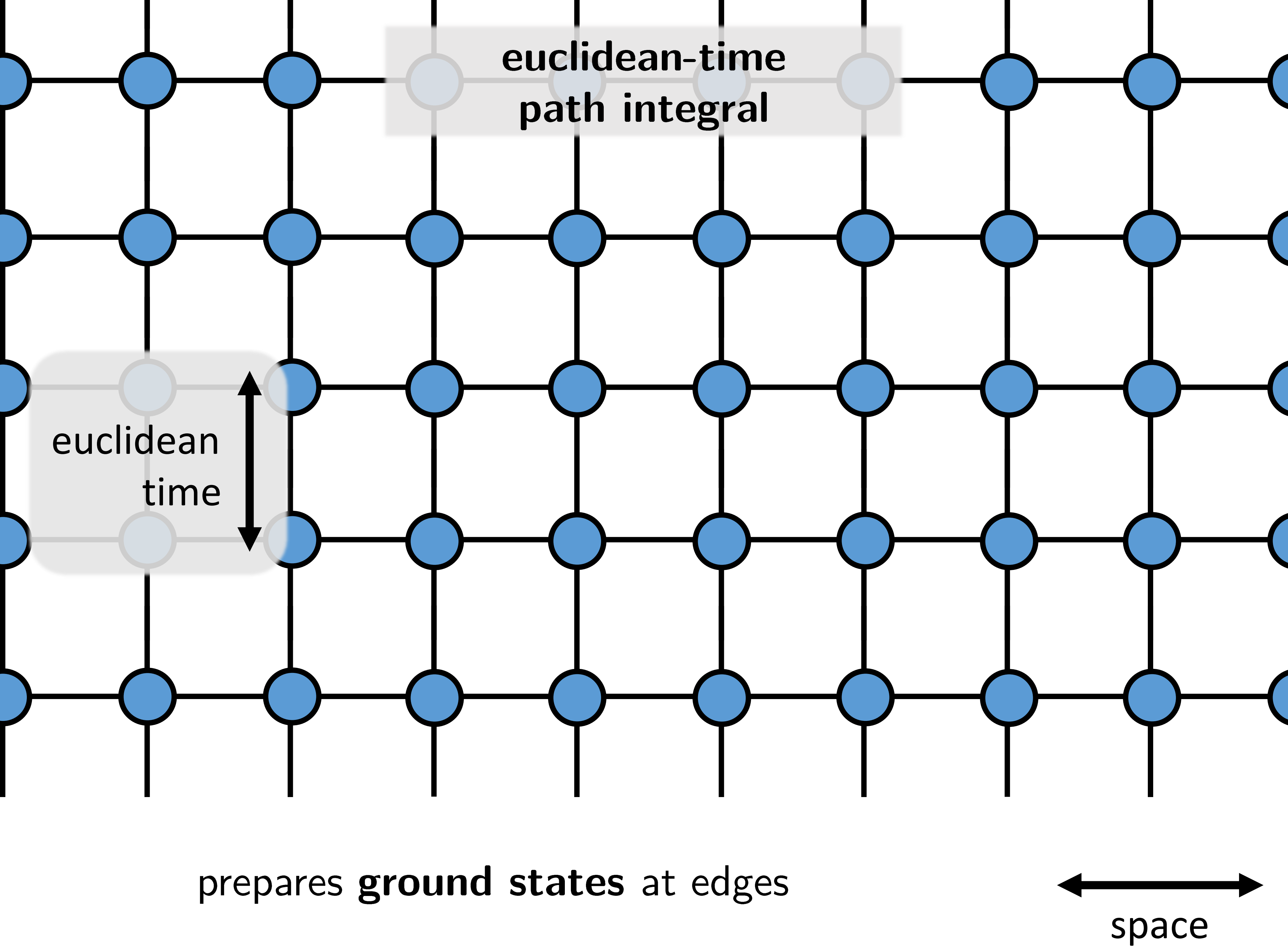
AM & Guifre Vidal

arXiv:1805.12524 (2018)
arXiv:1807.02501 (2018)

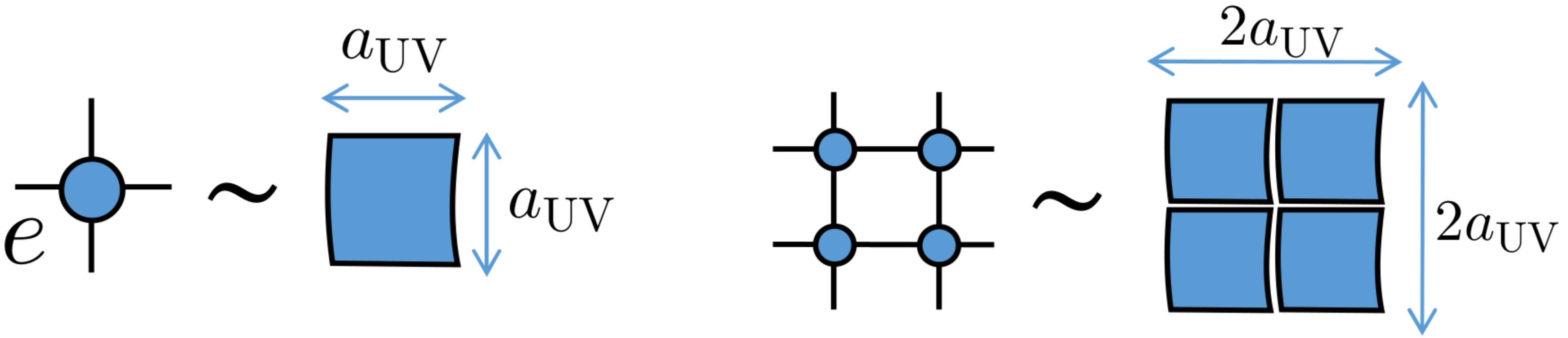
euclidean-time
transfer matrix



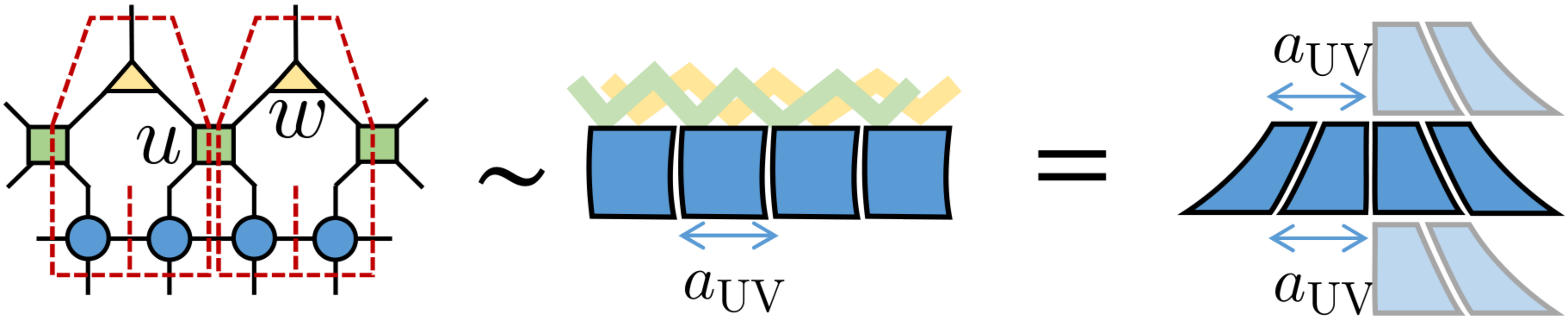
$$\exp(-\Delta\tau H)$$


euclidean-time
path integral
euclidean
time
prepares ground states at edges
space

euclideons as flat patches



MERA as rescaling



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P. Caputa et al. Phys. Rev. Lett. 119, 071602 (2017)
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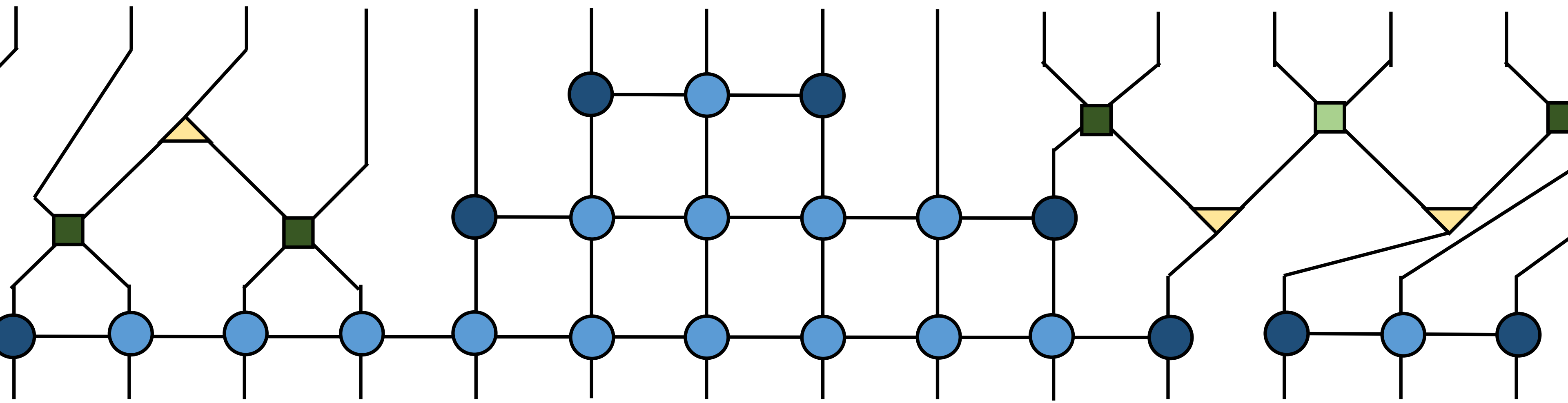
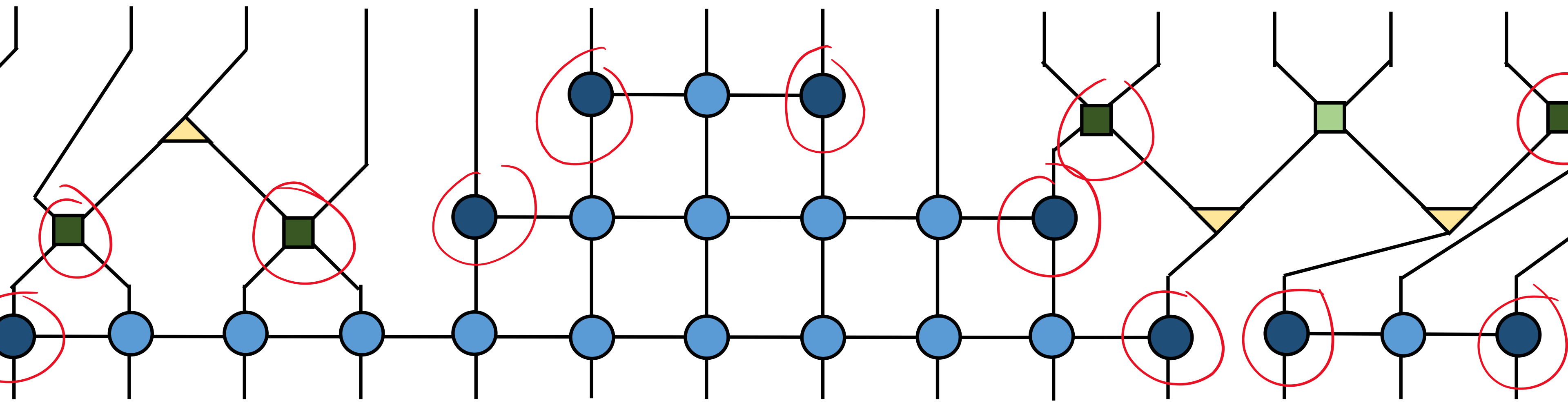
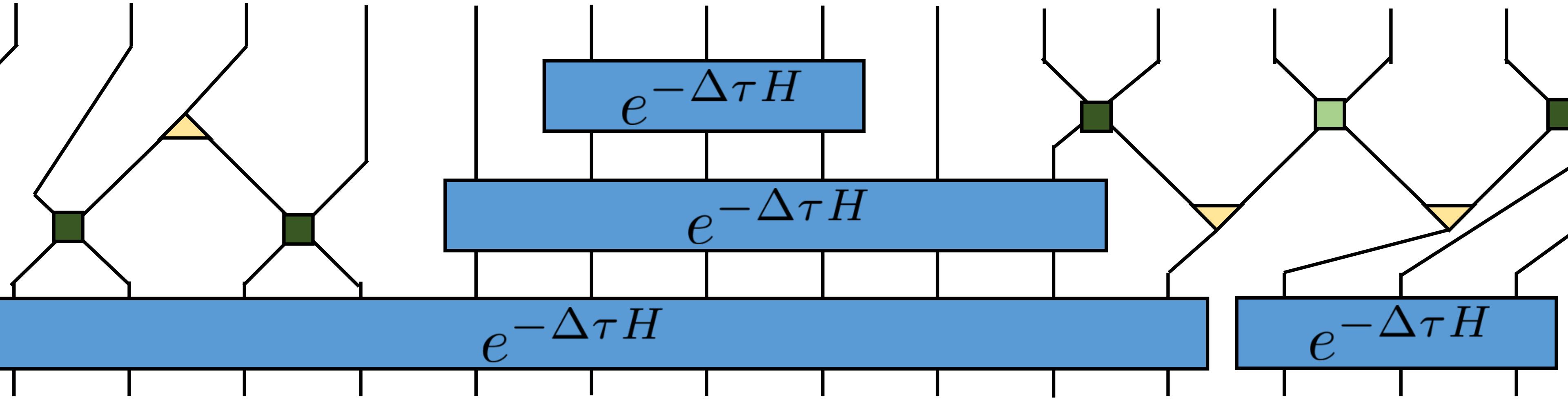
computational framework for simulations on curved spacetimes



computational framework for simulations on curved spacetimes



computational framework for simulations on curved spacetimes



our contribution

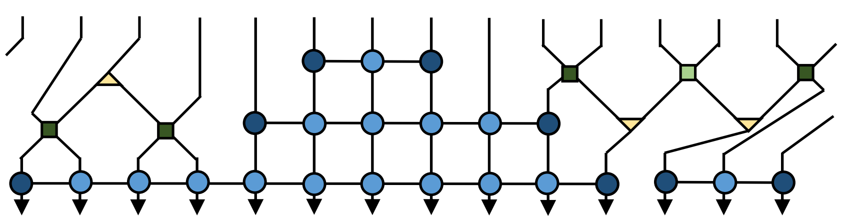
tensor network geometry ⟷ path integral geometry

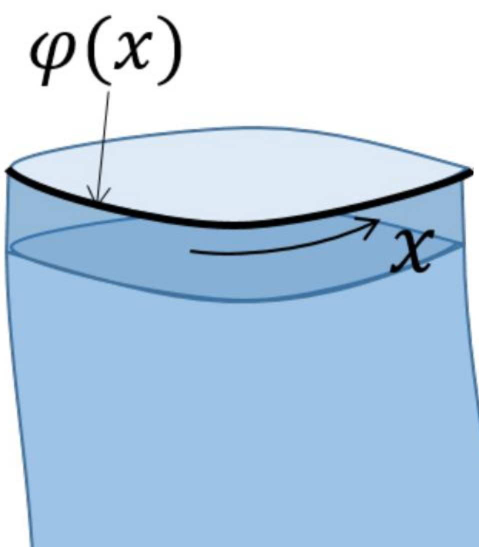
our contribution

tensor network geometry ⟷ path integral geometry

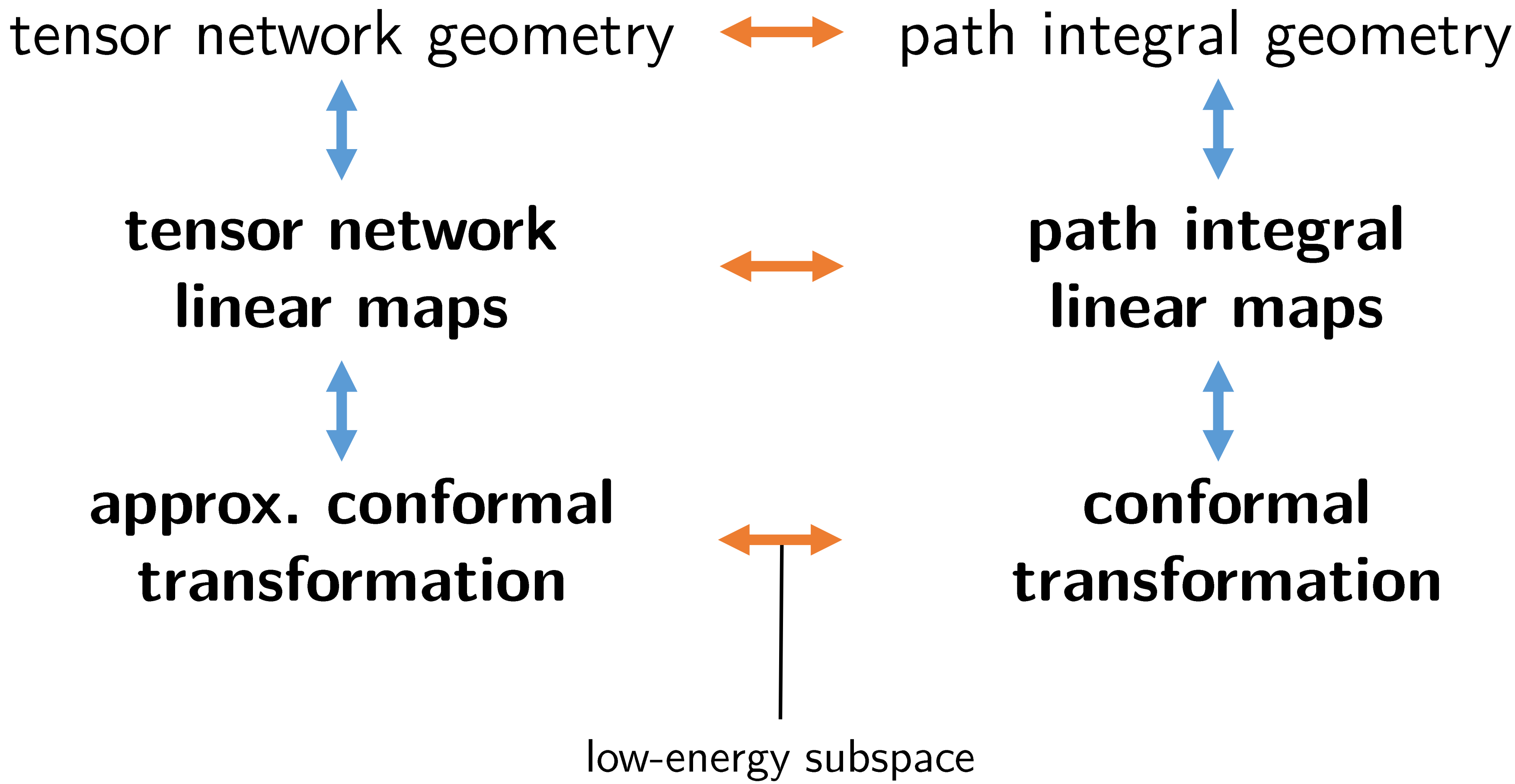
↕ ↕

tensor network linear maps ⟷ **path integral linear maps**

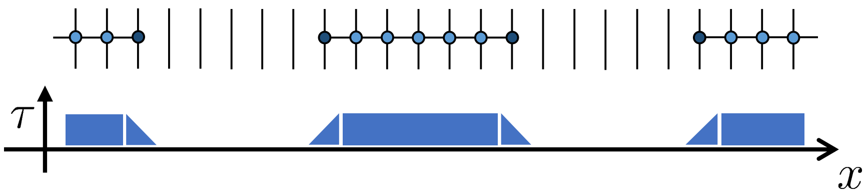


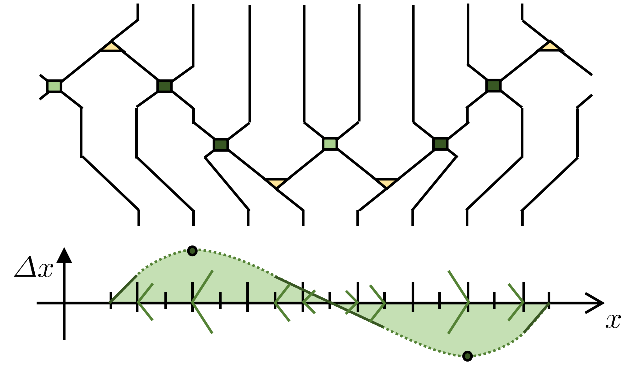


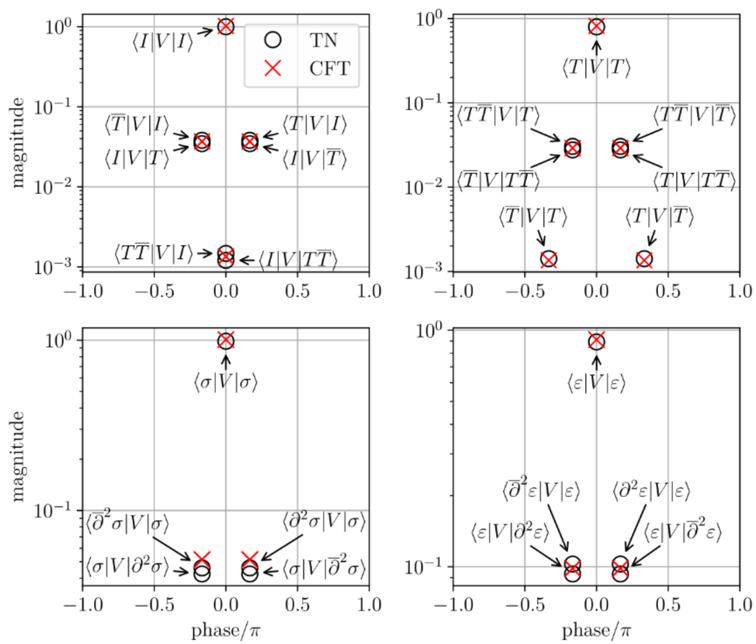
our contribution

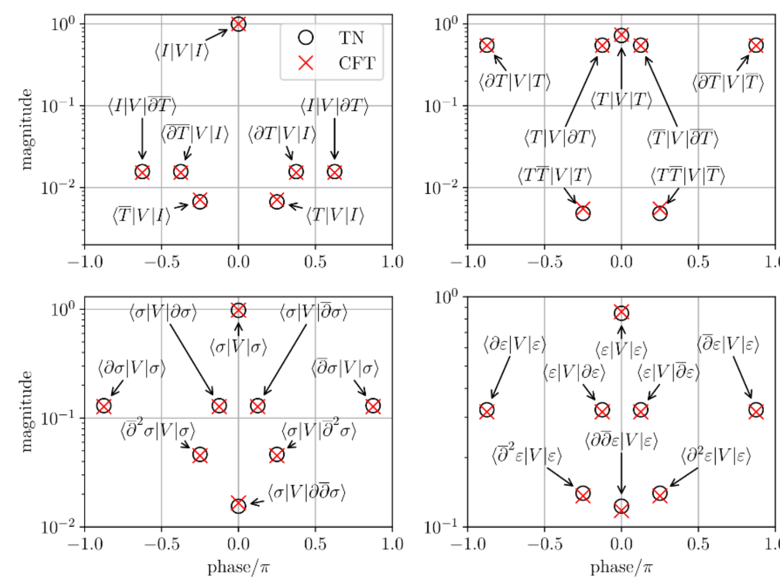


can check numerically









thank you!