Phase Calculation and Systematic Effects

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Outline

- Overview of phase difference calculation
- Relevant systematics
- Calculation for simple 1 atom model
- Discussion

Assumptions

- Single atom
- Under influence of gravity and laser only
- Semi-classical picture
  - Propagation phases are calculated classically
  - Laser phase imprinting and separation phase are treated quantum mechanically

Phase Difference

Total phase, $\Delta\phi$, is decomposed into 3 parts

- Propagation phase
- Laser phase
- Separation phase

Propagation Phase

The phase built up while the state travels along the upper and lower arm of the interferometry paths i.e. the excited state = upper, ground state = lower.

$$\Delta\phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_i}^{t_f} (L_c - E_i)\mathrm{d}t \right) - \sum_{\text{lower}} \left(\int_{t_i}^{t_f} (L_c - E_i)\mathrm{d}t \right)$$

Laser Phase

The phase imprinted on the atom during interaction

$$\Delta\phi_{\text{laser}} = \left(\sum_j \pm\phi_L(t_j, \mathbf{x}_u(t_j))\right)_{\text{upper}} - \left(\sum_j \pm\phi_L(t_j, \mathbf{x}_l(t_j))\right)_{\text{lower}}$$

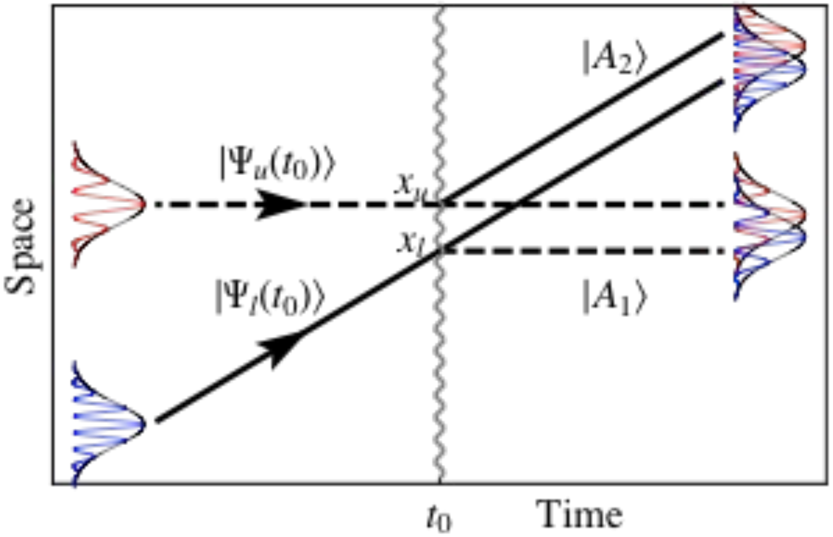
$$\phi_L(\mathbf{x}, t) = \mathbf{k}_{\text{eff}} \cdot \mathbf{x} - \omega_{\text{eff}} t + \phi_{\text{eff}}$$

Separation Phase

Phase difference caused by a displacement in the atom states at the output port.

$$\Delta\phi_{\text{separation}} = \langle \mathbf{p} \rangle \cdot \Delta\mathbf{x}$$

$$\Delta x \equiv x_l - x_u$$



Equality of Detection Outputs

The total phase difference at the outputs A_1 and A_2

$$\Delta\phi_1 \equiv \theta_u - (\theta_l - \phi_L(\mathbf{x}_l)) + \bar{\mathbf{p}}_1 \cdot \Delta\mathbf{x} + \Delta\mathbf{p} \cdot (\mathbf{x} - \bar{\mathbf{x}})$$

$$\Delta\phi_2 \equiv (\theta_u + \phi_L(\mathbf{x}_u)) - \theta_l + \bar{\mathbf{p}}_2 \cdot \Delta\mathbf{x} + \Delta\mathbf{p} \cdot (\mathbf{x} - \bar{\mathbf{x}})$$

$$\bar{\mathbf{p}}_1 = \frac{\mathbf{p}_u + (\mathbf{p}_l - \mathbf{k})}{2}$$

$$\bar{\mathbf{p}}_2 = \frac{(\mathbf{p}_u + \mathbf{k}) + \mathbf{p}_l}{2}$$

$$\Delta\phi_1 - \Delta\phi_2 = \phi_L(\mathbf{x}_l) - \phi_L(\mathbf{x}_u) + \bar{\mathbf{p}}_1 \cdot \Delta\mathbf{x} - \bar{\mathbf{p}}_2 \cdot \Delta\mathbf{x} = \mathbf{k} \cdot \Delta\mathbf{x} - \mathbf{k} \cdot \Delta\mathbf{x} = 0$$

Systematic Effects

Lagrangian for our simple model

- r(t) is the position, $\boldsymbol{\Omega}$ is the frame rotation velocity
- Taylor expand ϕ (gravitational potential)

$$L = \frac{1}{2}m(\dot{\mathbf{r}} + \boldsymbol{\Omega} \times (\mathbf{r} + \mathbf{R}_e))^2 - m\phi(\mathbf{r} + \mathbf{R}_e) - \frac{1}{2}\alpha\mathbf{B}(\mathbf{r})^2$$

Systematic Effects

Numerically calculated systematics

- Earth's rotation
- Gravity perturbations
- Magnetic fields

	Phase shift	Size (rad)	Fractional size
1	$-k_{\text{eff}} g T^2$	-2.85×10^{8}	1.00
2	$k_{\text{eff}} R_e \Omega_y^2 T^2$	6.18×10^{5}	2.17×10^{-3}
3	$-k_{\text{eff}} T_{zz} v_z T^3$	1.58×10^{3}	5.54×10^{-6}
4	$\frac{7}{12} k_{\text{eff}} g T_{zz} T^4$	-9.21×10^{2}	3.23×10^{-6}
5	$-3k_{\text{eff}} v_z \Omega_y^2 T^3$	-5.14	1.80×10^{-8}
6	$2k_{\text{eff}} v_x \Omega_y T^2$	3.35	1.18×10^{-8}
7	$\frac{7}{4} k_{\text{eff}} g \Omega_y^2 T^4$	3.00	1.05×10^{-8}
8	$-\frac{7}{12} k_{\text{eff}} R_e T_{zz} \Omega_y^2 T^4$	2.00	7.01×10^{-9}
9	$-\frac{\hbar k_{\text{eff}}^2}{2m} T_{zz} T^3$	7.05×10^{-1}	2.48×10^{-9}
10	$\frac{3}{4} k_{\text{eff}} g Q_{zzz} v_z T^5$	9.84×10^{-3}	3.46×10^{-11}
11	$-\frac{7}{12} k_{\text{eff}} Q_{zzz} v_z^2 T^4$	-7.66×10^{-3}	2.69×10^{-11}
12	$-\frac{7}{4} k_{\text{eff}} R_e \Omega_y^4 T^4$	-6.50×10^{-3}	2.28×10^{-11}
13	$-\frac{7}{4} k_{\text{eff}} R_e \Omega_y^2 \Omega_z^2 T^4$	-3.81×10^{-3}	1.34×10^{-11}
14	$-\frac{31}{120} k_{\text{eff}} g^2 Q_{zzz} T^6$	-3.39×10^{-3}	1.19×10^{-11}
15	$-\frac{3\hbar k_{\text{eff}}^2}{2m} \Omega_y^2 T^3$	-2.30×10^{-3}	8.06×10^{-12}
16	$\frac{1}{4} k_{\text{eff}} T_{zz}^2 v_z T^5$	2.19×10^{-3}	7.68×10^{-12}
17	$-\frac{31}{360} k_{\text{eff}} g T_{zz}^2 T^6$	-7.53×10^{-4}	2.65×10^{-12}
18	$3k_{\text{eff}} v_y \Omega_y \Omega_z T^3$	2.98×10^{-4}	1.05×10^{-12}
19	$-k_{\text{eff}} \Omega_y \Omega_z y_0 T^2$	-7.41×10^{-5}	2.60×10^{-13}
20	$-\frac{3}{4} k_{\text{eff}} R_e Q_{zzz} v_z \Omega_y^2 T^5$	-2.14×10^{-5}	7.50×10^{-14}
21	$\frac{31}{60} k_{\text{eff}} g R_e Q_{zzz} \Omega_y^2 T^6$	1.47×10^{-5}	5.17×10^{-14}
22	$\frac{3}{2} k_{\text{eff}} T_{zz} v_z \Omega_y^2 T^5$	-1.42×10^{-5}	5.00×10^{-14}
23	$-\frac{7}{6} k_{\text{eff}} T_{zz} v_x \Omega_y T^4$	1.08×10^{-5}	3.81×10^{-14}
24	$-2k_{\text{eff}} T_{xx} \Omega_y x_0 T^3$	-6.92×10^{-6}	2.43×10^{-14}
25	$-\frac{7\hbar k_{\text{eff}}^2}{12m} Q_{zzz} v_z T^4$	-6.84×10^{-6}	2.40×10^{-14}
26	$-\frac{7}{6} k_{\text{eff}} T_{xx} v_x \Omega_y T^4$	-5.42×10^{-6}	1.90×10^{-14}
27	$-\frac{31}{60} k_{\text{eff}} g T_{zz} \Omega_y^2 T^6$	4.90×10^{-6}	1.72×10^{-14}
28	$k_{\text{eff}} T_{xx} v_z \Omega_y^2 T^5$	4.75×10^{-6}	1.67×10^{-14}
29	$\frac{3\hbar k_{\text{eff}}^2}{8m} g Q_{zzz} T^5$	4.40×10^{-6}	1.55×10^{-14}
30	$\frac{31}{360} k_{\text{eff}} R_e T_{zz}^2 \Omega_y^2 T^6$	1.63×10^{-6}	5.74×10^{-15}

[1]

[1] Hogan, Thesis. Table 4.1

Table 4.1 Key

- Many of these are common mode and cancel out in a differential measurement scheme
- To note some important quantities
  - 1) Acceleration measurement (also intrinsic sensitivity of apparatus)
  - 2) Centrifugal force acting on the atom
  - 3,4) Gravity inhomogeneities
  - 6) Coriolis force

Systematic Effects

- After common mode cancellations
- Similar results for dual interferometers

	Phase shift	Size (rad)	Fractional size
1	$-\frac{1}{2}\left(\frac{1}{\mathrm{m}85}-\frac{1}{m_{87}}\right)\hbar k_{\mathrm{eff}}^2 T_{zz} T^3$	1.66×10^{-2}	5.83×10^{-11}
2	$2k_{\mathrm{eff}}\delta v_x \Omega_y T^2$	3.35×10^{-3}	1.18×10^{-11}
3	$-k_{\mathrm{eff}} T_{zz}\delta v_z T^3$	1.44×10^{-4}	5.05×10^{-12}
4	$-\frac{3}{2}\left(\frac{1}{m_{85}}-\frac{1}{m_{87}}\right)\hbar k_{\mathrm{eff}}^2 \Omega_y^2 T^3$	-5.40×10^{-5}	1.90×10^{-13}
5	$-3k_{\mathrm{eff}}\Omega_y^2 \delta v_z T^3$	-4.68×10^{-6}	1.64×10^{-14}
6	$-k_{\mathrm{eff}} T_{zz}\delta z T^2$	8.93×10^{-7}	3.14×10^{-15}
7	$-k_{\mathrm{eff}}\delta y \Omega_y \Omega_z T^2$	-7.41×10^{-7}	2.60×10^{-15}
8	$3k_{\mathrm{eff}}\delta v_y \Omega_y \Omega_z T^3$	2.98×10^{-7}	1.05×10^{-15}
9	$-\frac{7}{12}\left(\frac{1}{m_{85}}-\frac{1}{m_{87}}\right)\hbar k_{\mathrm{eff}}^2 Q_{zzz} v_z T^4$	-1.61×10^{-7}	5.65×10^{-16}
10	$\frac{3}{8}\left(\frac{1}{m_{85}}-\frac{1}{m_{87}}\right)\hbar k_{\mathrm{eff}}^2 g Q_{zzz} T^5$	1.03×10^{-7}	3.63×10^{-16}
11	$-\left(\frac{\alpha_{85}}{m_{85}}-\frac{\alpha_{87}}{m_{87}}\right)\hbar k_{\mathrm{eff}} B_0(\partial_z B) T^2$	-9.94×10^{-8}	3.49×10^{-16}
12	$-2k_{\mathrm{eff}} T_{xx}\delta x \Omega_y T^3$	-6.92×10^{-8}	2.43×10^{-16}

[2]

[2] Hogan, Thesis. Table 4.2

Table 4.2 Key

- Although a differential measurement of different isotopes shows effect of differential measurements on the systematics
- Important terms here
  - 2) Coriolis force (largest contribution in differential measurements)
  - 3) Gravity gradients

Phase Calculation

- Followed Jason’s method in his Thesis
- Used power series solution for trajectories of the atom
  - Solved Euler-Lagrange equations
- Computed 3 phase differences
  - Propagation phase
  - Laser phase
  - Separation phase
- Add them together for total phase difference

$$r_i(t) = \sum_n a_{i,n}(t - t_0)^n$$

Phase Calculation

- Done in Mathematica
- Can be converted to python easily for 1 atom case
- Discuss with Jason for further code necessary

Leading Order

- Coriolis term dominates others
- Gravity gradients become relevant as well

Other Noise

- Gravity gradient noise is also expected
- Seismometer measurements will give better estimate

$$h_{\mathrm{GGN}} = \begin{cases} \dfrac{G\rho_0 k_l}{2\pi f^2} \langle \xi_z \rangle \\ \dfrac{G\rho_0}{f c_R} \langle \xi_z \rangle \end{cases} \quad ; \quad f \lesseqgtr \frac{c_R}{2\pi L}$$

Conclusion

- The largest systematics have schemes for mitigation at 8-10 m scales
- 100m mitigation schemes are of great interest and will require time and effort during R&D