MICHIGAN STATE
UNIVERSITY

Storage ring studies at the Muon $g-2$ Experiment

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Budker Seminar

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1. g-2 Storage Ring Simulation

2. Betatron resonances
    - Optimal configurations to increase statistics and reduce lost muons rates
    - HV~18kV peak as a probe to determine EQS misalignments
    - HV~17kV peak to understand interplay of magnetic and electric resonances

3. Tunes and Betatron frequencies
    - Measurements of tunes for model calibration
    - Betatron frequency time-evolution vulnerable to hardware damages

4. Momentum distribution and collimation
    - Special collimation to address beam momentum distribution

Snapshot of work progress

-> “Lost Muons – Traditional Scraping Studies.”
g-2 Beam Dynamics Workshop, November 2017

-> “COSY Studies - Lost muons with measured B-field and some oomph.”
g-2 Beam Dynamics Workshop, March 2018

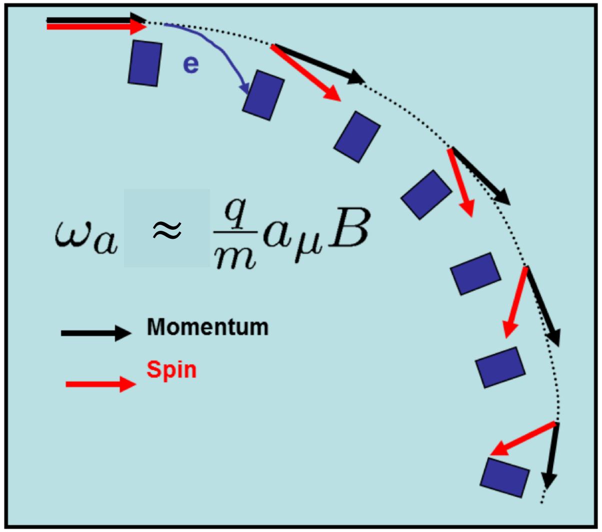
-> “Tune/Resonances”
Plenary Talk Muon g-2 Collaboration Meeting, July 2018

-> Other internal talks at g-2 Beam Dynamics meetings…

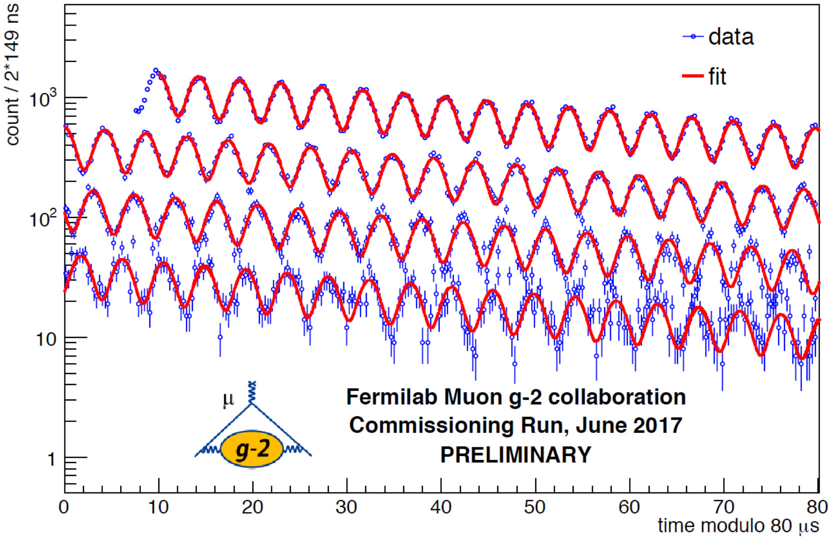
-> “Transverse beam phase-space and dispersion measurement technique at Muon Campus at Fermilab”
ICAP Conference, October 2018…

-> “Muon losses from betatron resonances at the Muon g-2 Experiment at Fermilab”
CPO Conference, October 2018…

Muon g-2 Experiment: Measurement Principle



$$a_\mu \approx \frac{m_\mu \omega_a}{eB}$$



Fermilab Muon g-2 collaboration
Commissioning Run, June 2017
PRELIMINARY

- $\omega_a = \omega_s - \omega_c$ obtained from fit to the wiggle plot
- Goal of E989 is to measure a_μ to 0.14ppm precision or less
- Reduction of statistical and systematic uncertainties essential.

$$N(t, E_{th}) = N_0(E_{th}) \exp^{-t/\gamma\tau_\mu} \left[1 + A(E_{th}) \cos\left(\omega_a t + \varphi_a(E_{th})\right)\right]$$

Muon g-2 Storage Ring: Simulation

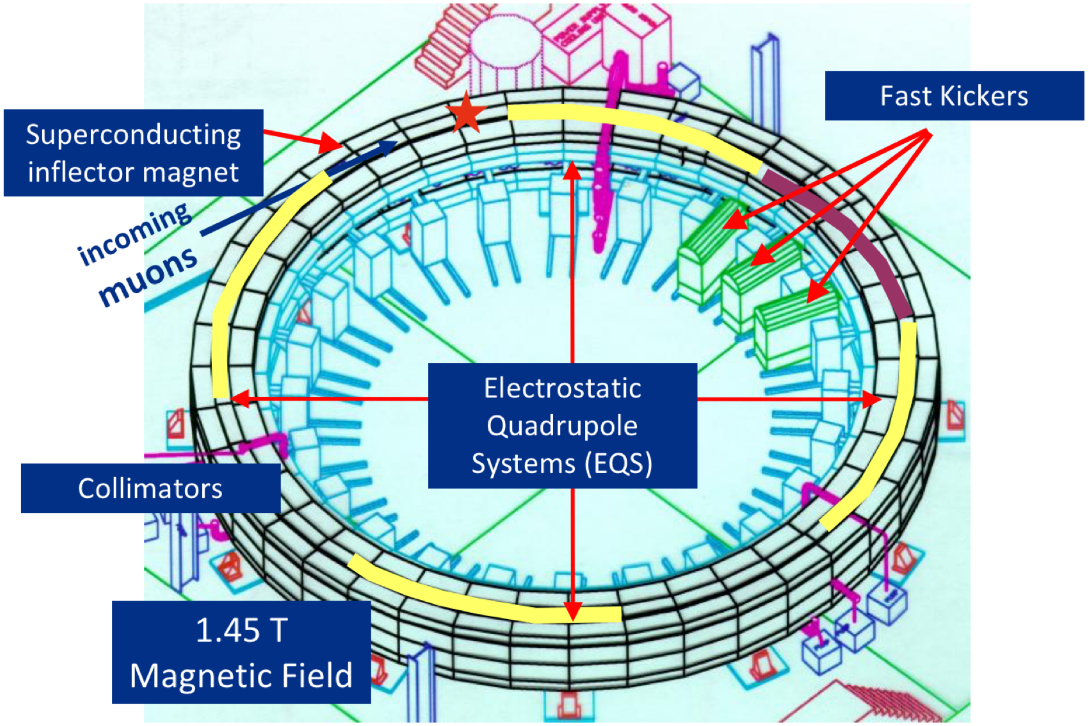
Muon g-2 Storage Ring: Simulation-> COSY INFINITY

- Preparation of high-order transport map from Runge Kutta ODE integrator (use of Differential algebra) for symplectic tracking

$$\mathcal{M}(\vec{z}_0) = \vec{z}_f$$

- $\vec{z}$: array made of (x, a, y, b, l, δ) ray vectors
- $\mathcal{M}$: Map containing $(x|x^{l_{x1}}a^{l_{x2}}y^{l_{x3}}b^{l_{x4}}l^{l_{x5}}\delta^{l_{x6}})$, $\left(a\middle|x^{l_{a1}}a^{l_{a2}}y^{l_{a3}}b^{l_{a4}}l^{l_{a5}}\delta^{l_{a6}}\right)$, …

Storage ring configuration



Superconducting inflector magnet

Fast Kickers

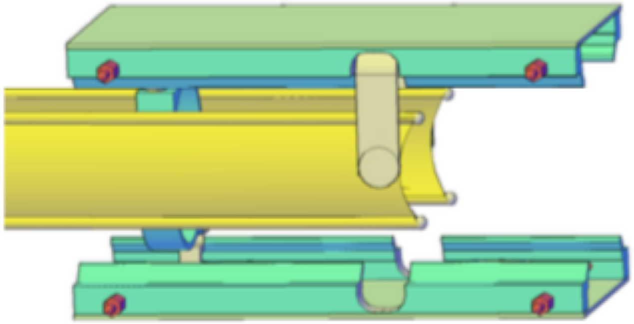
Electrostatic Quadrupole Systems (EQS)

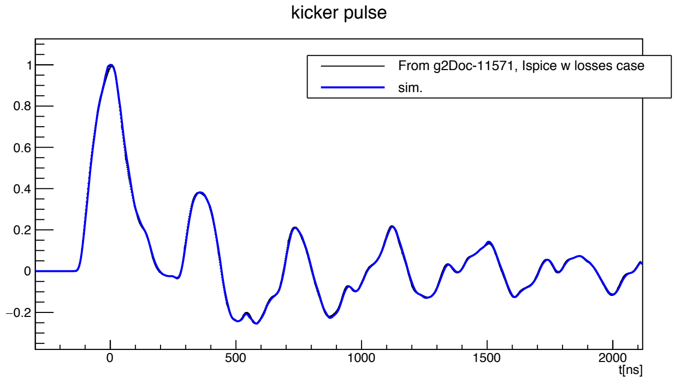
Collimators

1.45 T Magnetic Field

Muon g-2 Storage Ring: Simulation-> Injection Kicker

- Three kicker modules (1.27m long each) to ideally kick 10.8 ± 0.4mrad at $\sim 90°$ from injection point
- Current simulation recreates instantaneous kick at central kicker
- Measured time-dependent kicker pulse considered
- 75% of nominal kick and ringing signal after main kick in simulations to imitate behavior during first run



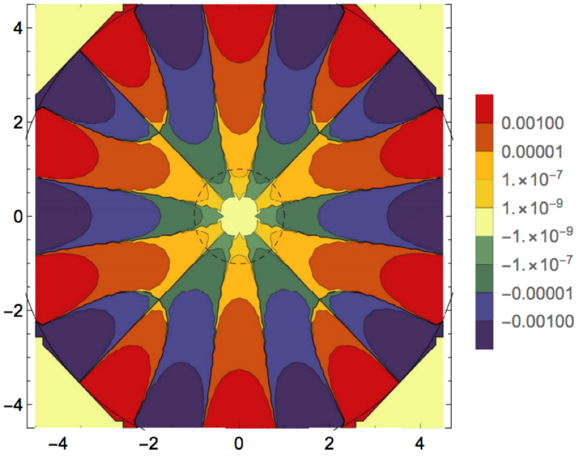


Muon g-2 Storage Ring: Simulation-> EQS

- Electrostatic multipole terms from EQS flat plates up to 20th order

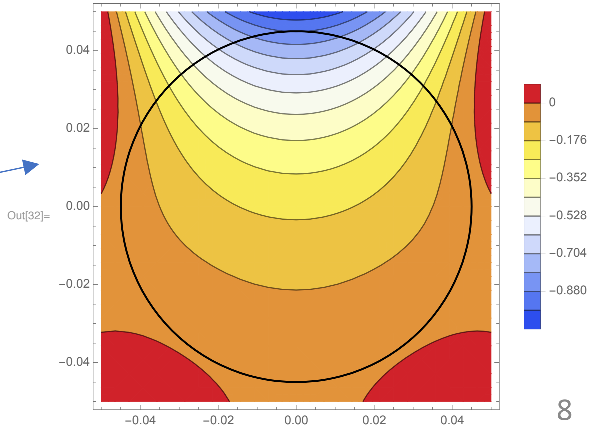
$$\phi(r,\theta) = \sum_{j=0}^{\infty} r^j \left[a_j \cos(j\theta) + b_j \sin(j\theta)\right]$$

- Multipole terms from conformal maps method by Eremey V. and Martin B.

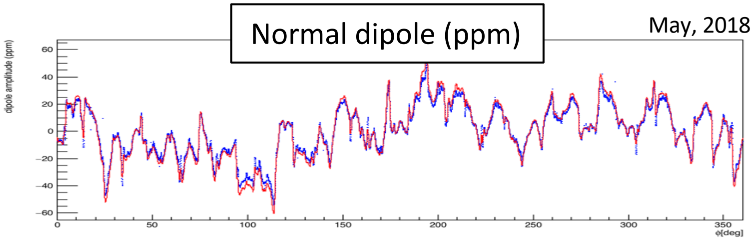


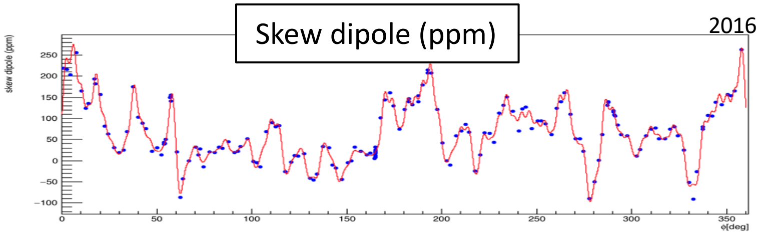
- Fringe fields falloff described with Enge functions coefficients from fitting the multipole term $M_{2,2}(s,r)$. $M_{2,2}$ obtained from 3D electrostatic potential calculation using an FFT transform along the circle of radius $R_{ref} = 3.6$cm

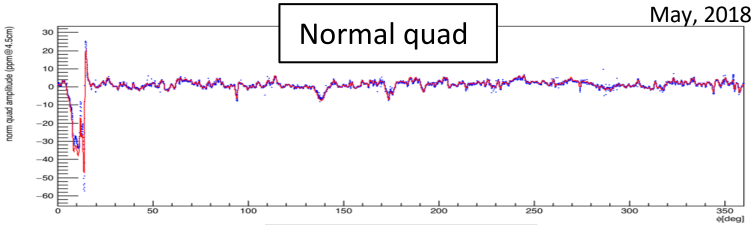
- Mispowered EQS for scraping stage by superposition and rotation of mispowered plates with nominal EQS.

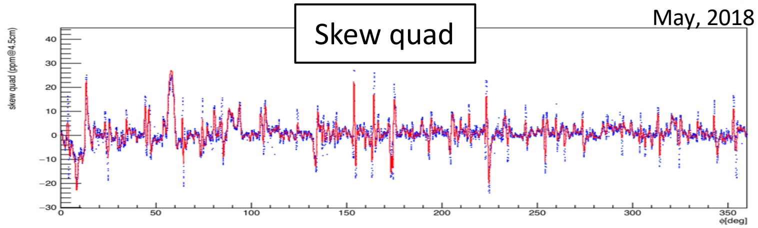


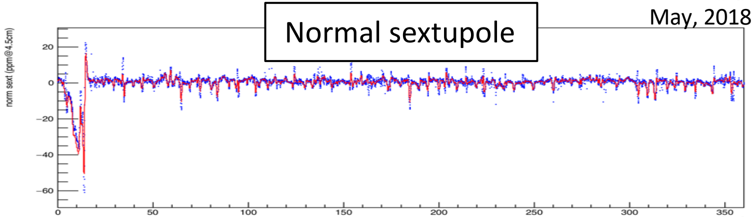
Muon g-2 Storage Ring: Simulation-> Magnetic Field

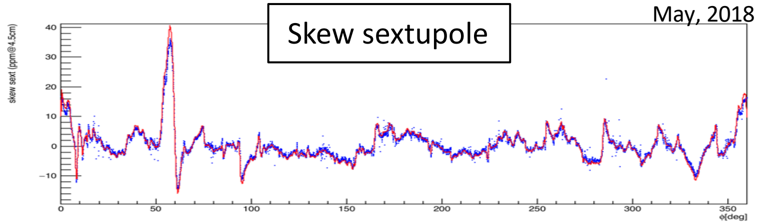

Normal dipole (ppm)
May, 2018

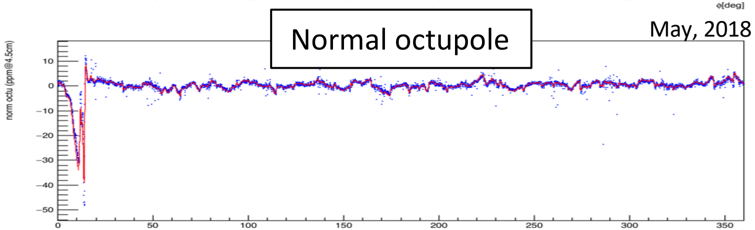

Skew dipole (ppm)
2016

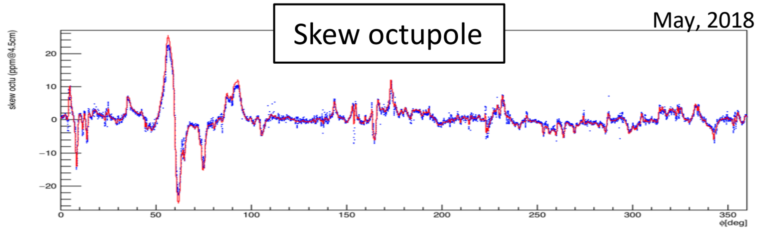

Normal quad
May, 2018

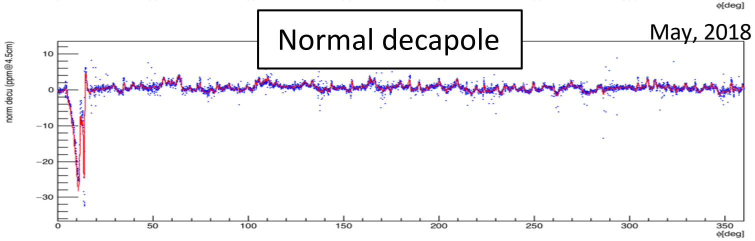

Skew quad
May, 2018

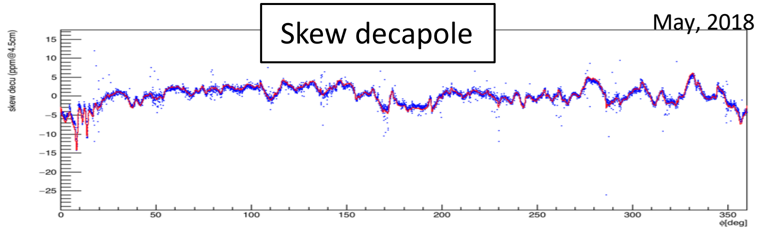

Normal sextupole
May, 2018


Skew sextupole
May, 2018


Normal octupole
May, 2018


Skew octupole
May, 2018


Normal decapole
May, 2018


Skew decapole
May, 2018

- Present radial field differs from the 2016 measurements, after the radial field was adjusted using the surface coils in 2017.

Muon g-2 Storage Ring: Simulation-> Magnetic Field

- Measured B-field multipoles extracted from azimuth-independent fit of NMR trolley probes measurements

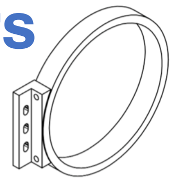
- Superimposed uniform vertical B-field, ESQ E-field and measured B-field multipoles represented as fine mesh of symplectic kicks applied to the map

- Multi-gaussian functions to include continuous multipoles amplitudes in simulation

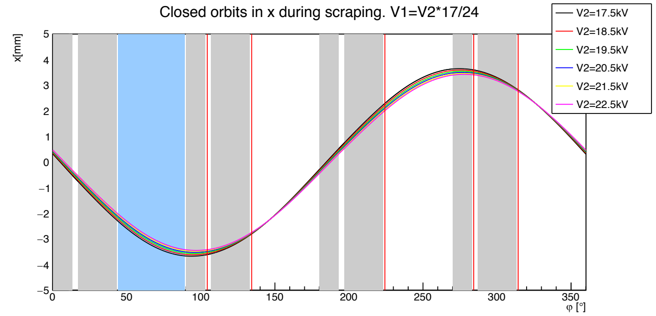
- High-order multipoles stability over time of $\sim\pm 1$ppm and azimuthally averaged dipole stability of less than 2ppm

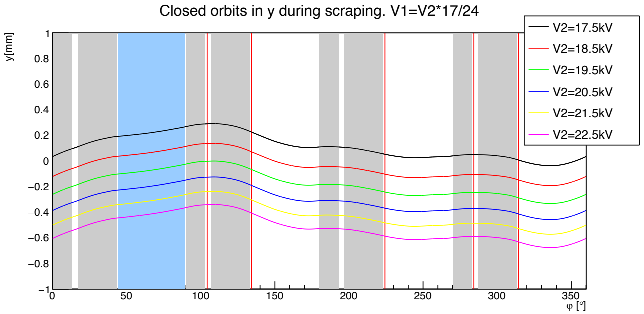
- Previously tried to recreate measured B-field from azimuth-dependent data fit in midplane. Caveat: Midplane symmetry assumed.

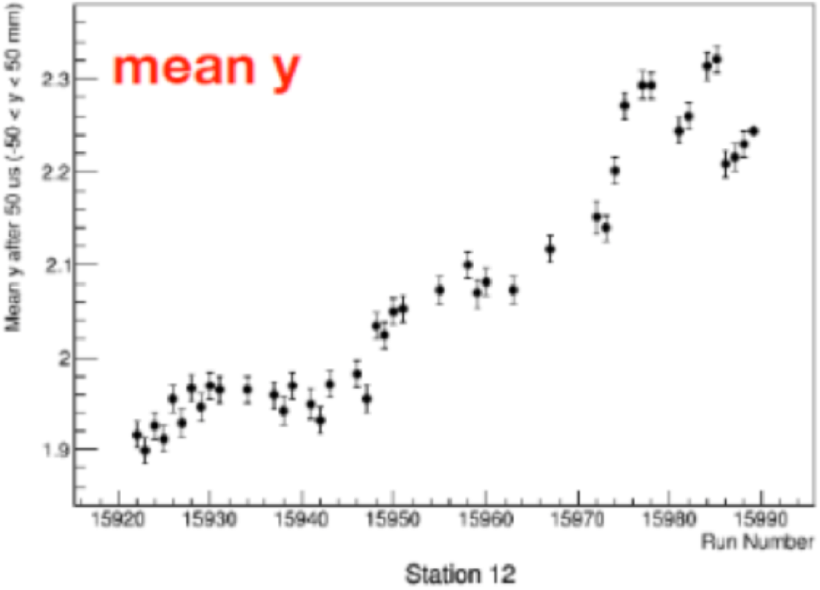
Muon g-2 Storage Ring: Simulation-> Beam collimators



- Beam collimators to provide effective scraping of the injected beam to remove muons outside the storage region



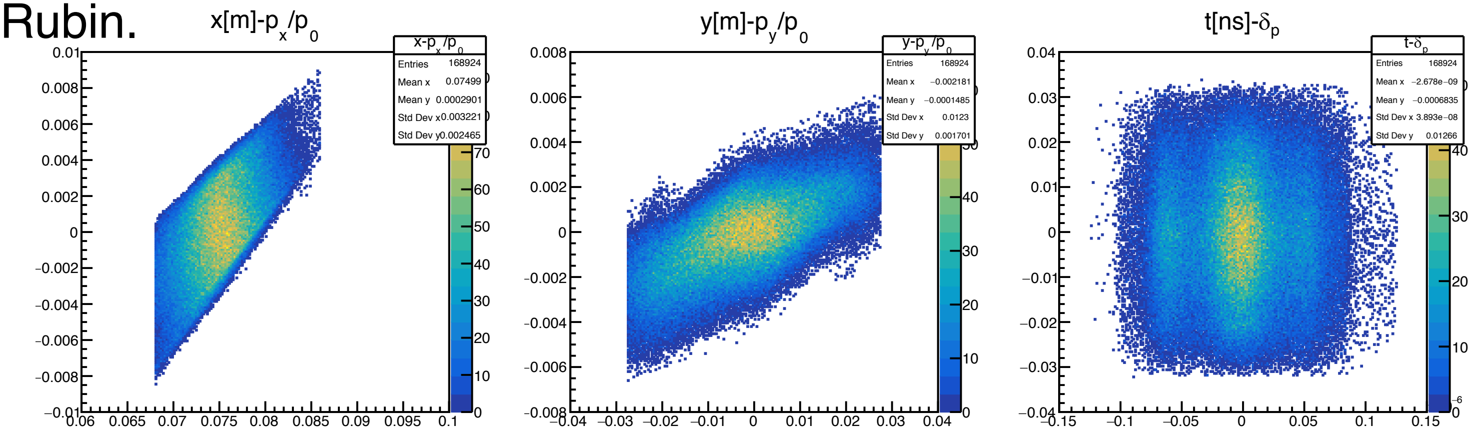




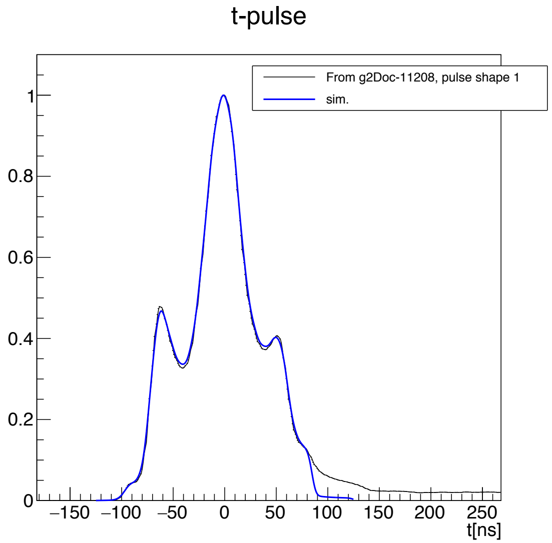
- Simulation: If muon location is beyond collimators apertures at their corresponding azimuth positions, it is lost.

Muon g-2 Storage Ring: Simulation-> Initial beam dist.

- $x, p_x/p_0$, $y, p_y/p_0$ and δ_p distributions at $t=0$ (at infl. exit) taken from BMAD sims by D. Rubin.



- t distribution based on measurements.



- Initial distribution at t=0 (> 6M muons) generated after "inflating" low-statistics distributions, where for each original muon, 40 are randomly created within the vicinity of the original muon for the uniform range [$-\boldsymbol{\sigma}/10, \boldsymbol{\sigma}/10$]. This method preserves averages and correlations of the original distribution.

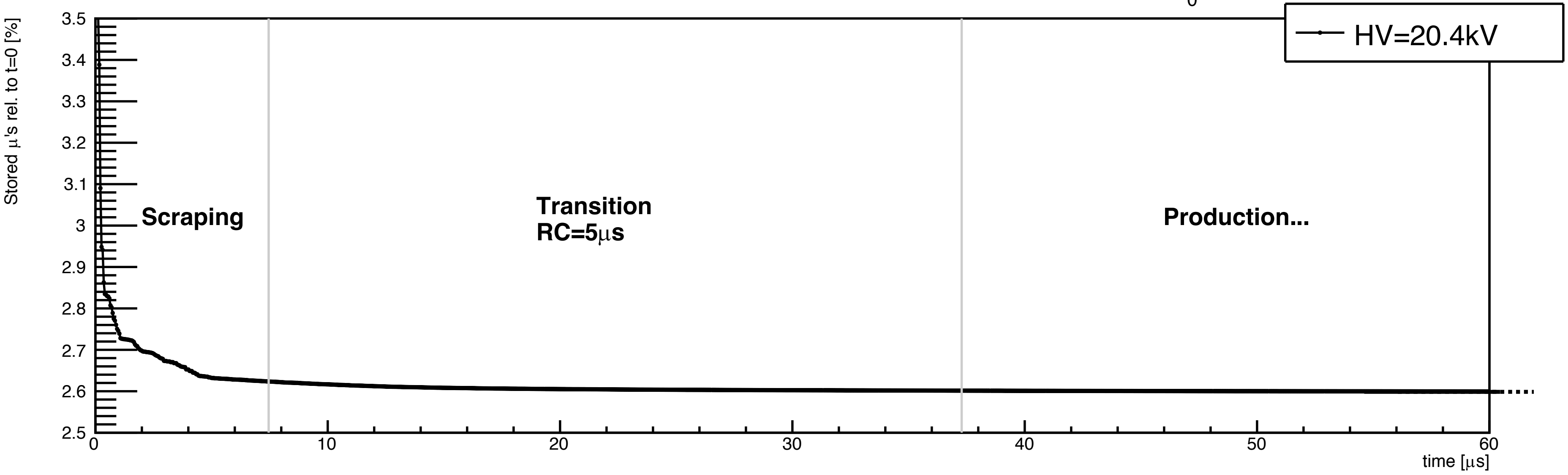
Muon g-2 Storage Ring: Simulation-> Stages

- We follow a sequence of stages similar to those done at BNL and that has been implemented for E989.

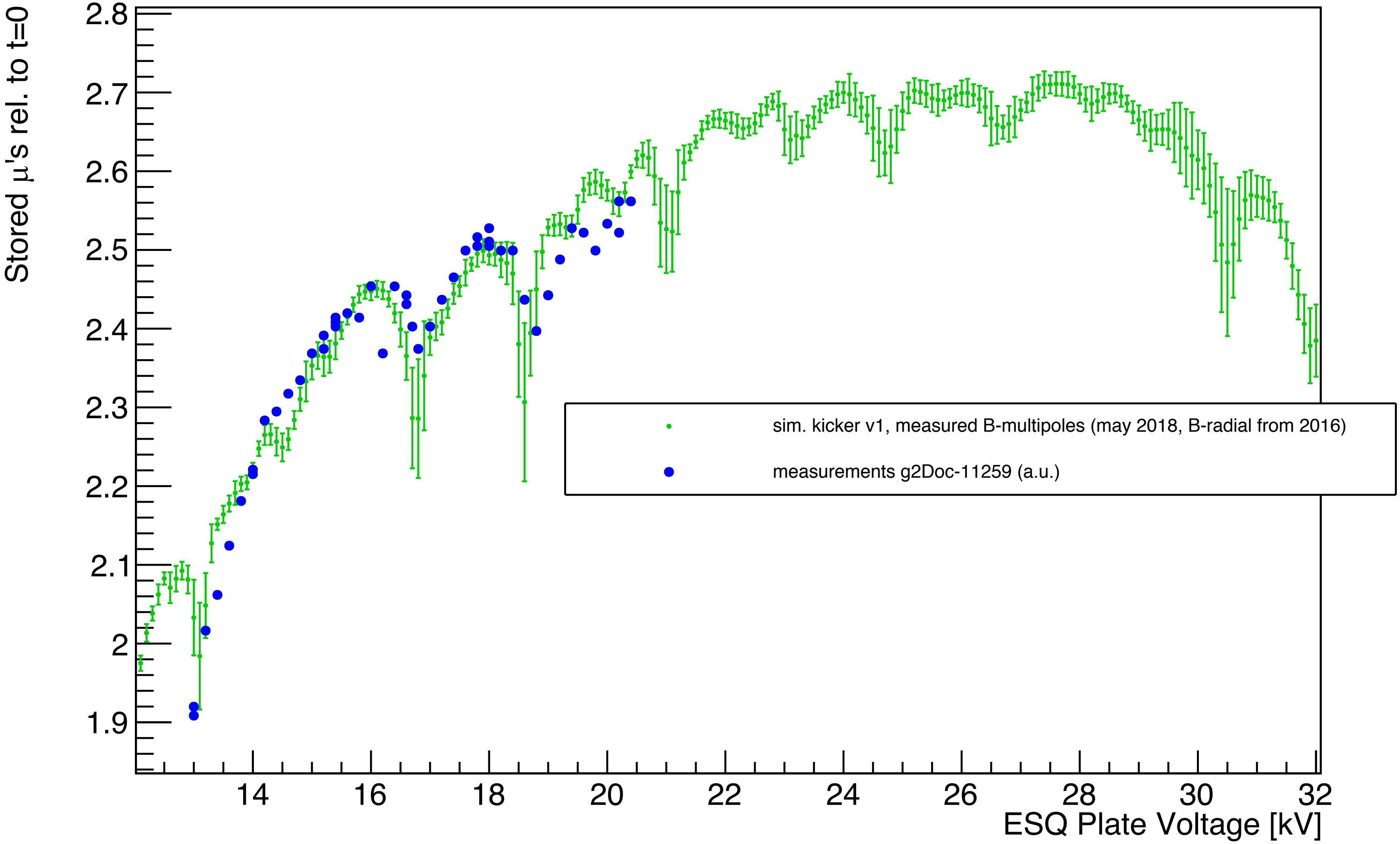

"Traditional" Scraping phase

Transition phase

Production phase


Stored μ's (sim. with kicker) vs. time (1st, $V_0*17/24$, 2nd step=V_0)

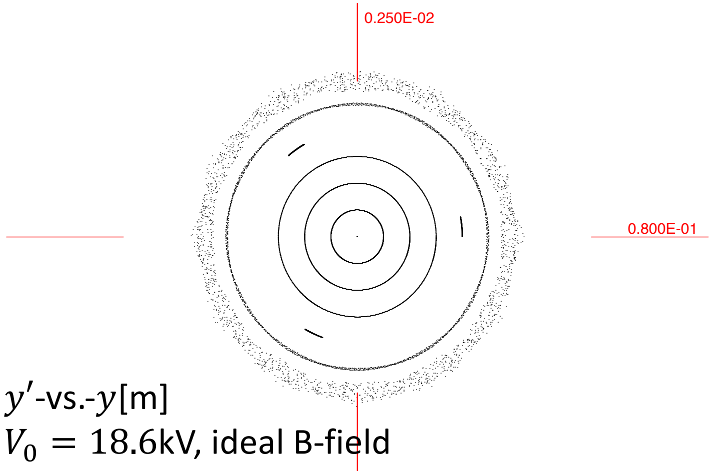
Stored muons vs. HV[kV] (t=186μs)



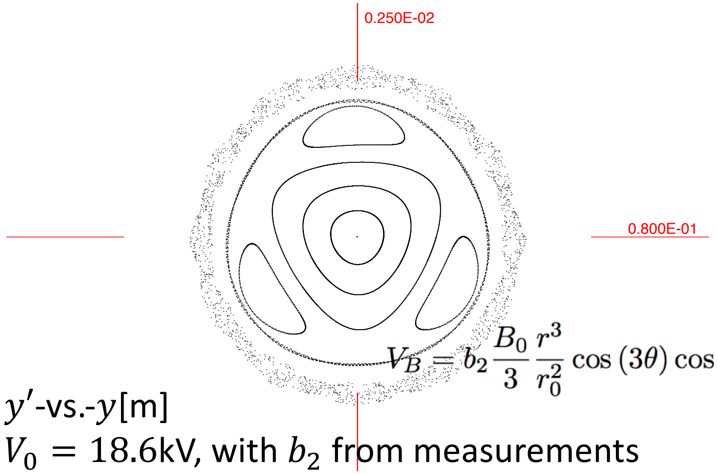
Muon g-2 Storage Ring: Studies

Betatron resonances

- Muon losses contribute to the systematics of a_μ^{E989} by introducing a slowly changing modification to the normal exponential decay and by possibly changing $g-2$ phase



y'-vs.-y[m]
$V_0 = 18.6$kV, ideal B-field



$$V_B = b_2 \frac{B_0}{3} \frac{r^3}{r_0^2} \cos(3\theta) \cos\varphi$$

y'-vs.-y[m]
$V_0 = 18.6$kV, with b_2 from measurements

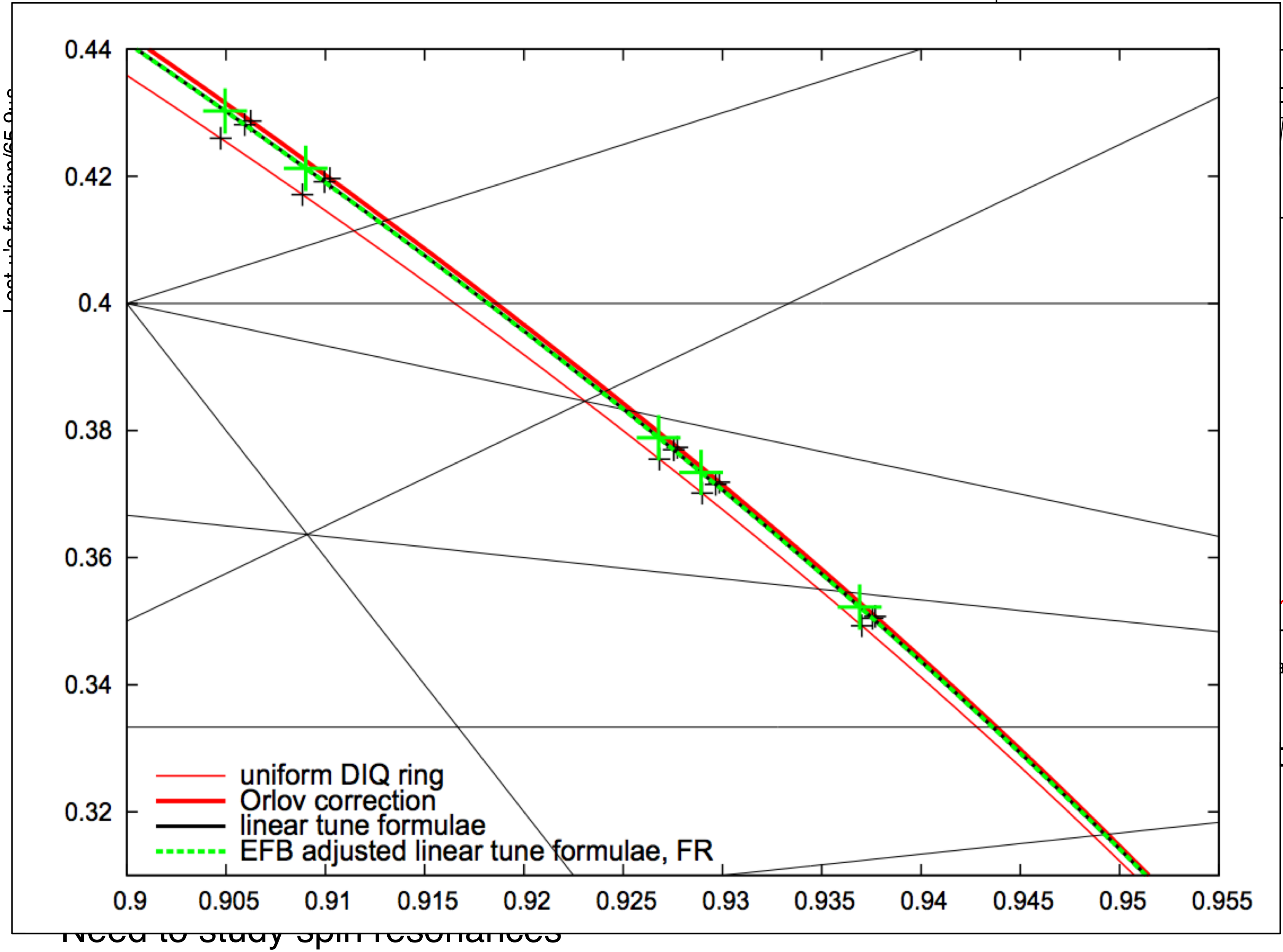
- Specific configurations of the storage ring (i.e. ESQ HV's and/or magnetic field multipoles) drive resonances that drastically increase muon losses during production.

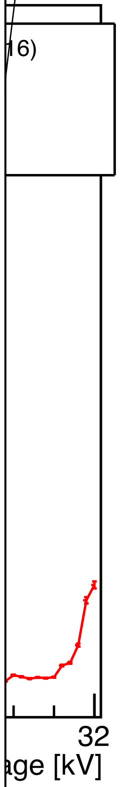
- The understanding of observed resonances is tantamount to a deeper characterization of the storage ring EQS plates misalignment and nonlinear effects.

Betatron resonances: HV scan

$2\nu_x - 2\nu_y = 1$, B-8th pole

…





ted

Need to study spin resonances

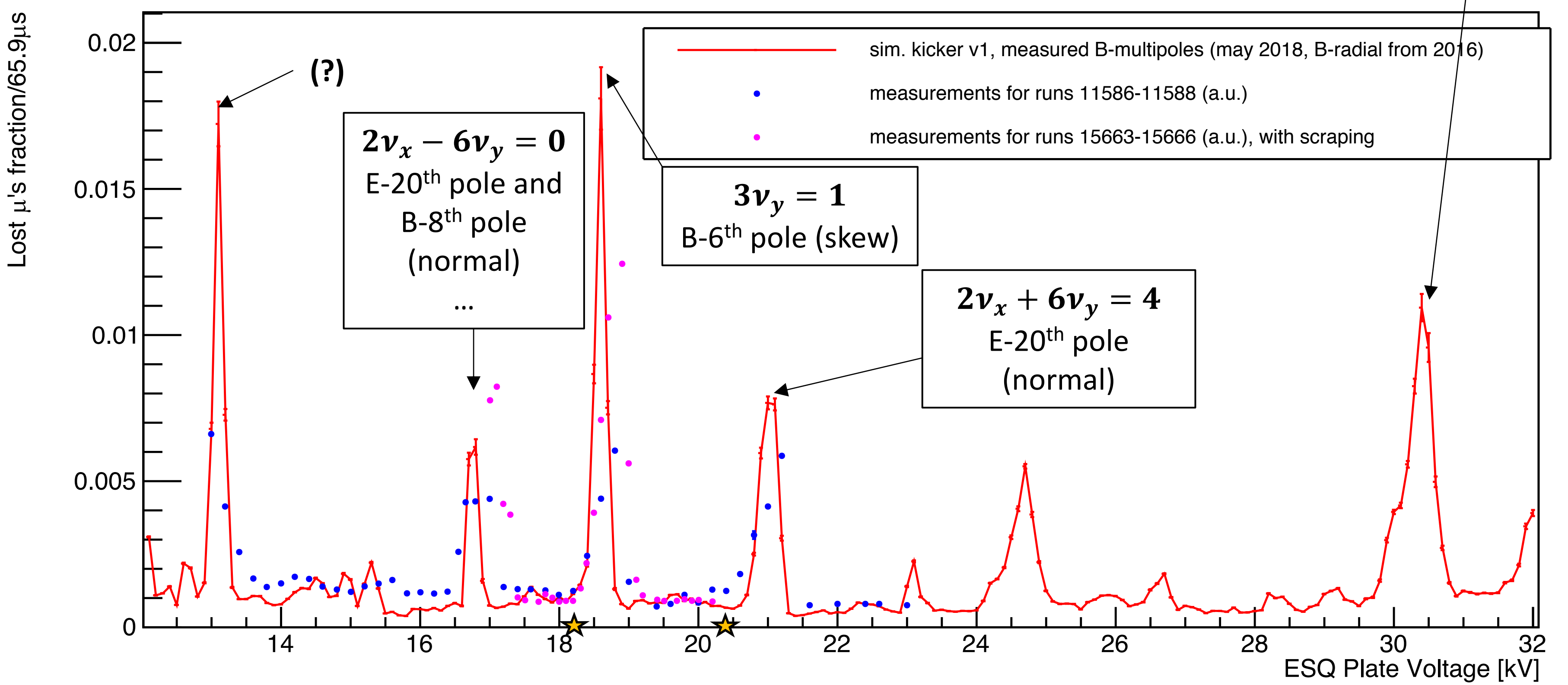
Betatron resonances: **HV scan**

$2\nu_x - 2\nu_y = 1$, B-8th pole

…

Revise…

lost muons (sim) vs. HV[kV] (t=121-186µs)



$2\nu_x - 6\nu_y = 0$

E-20th pole and

B-8th pole

(normal)

…

$3\nu_y = 1$

B-6th pole (skew)

$2\nu_x + 6\nu_y = 4$

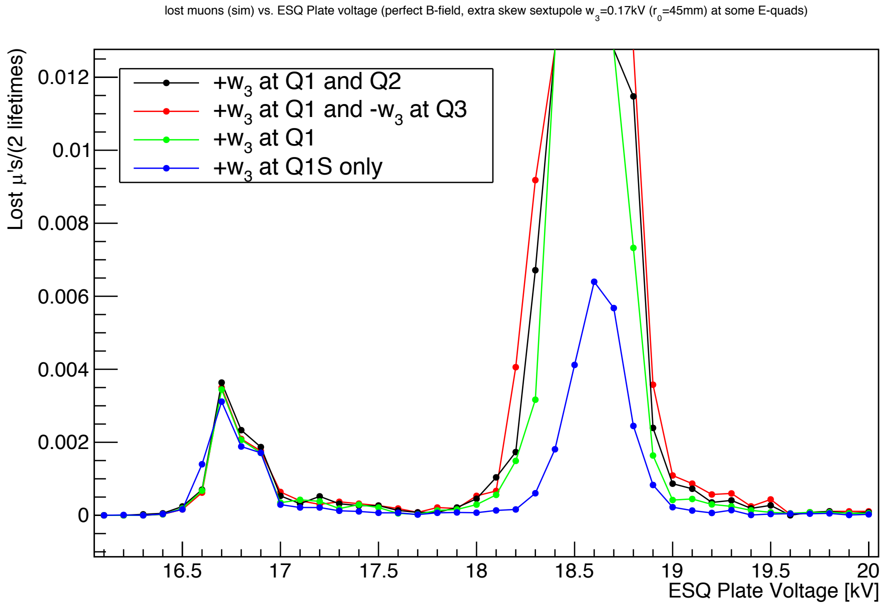
E-20th pole

(normal)

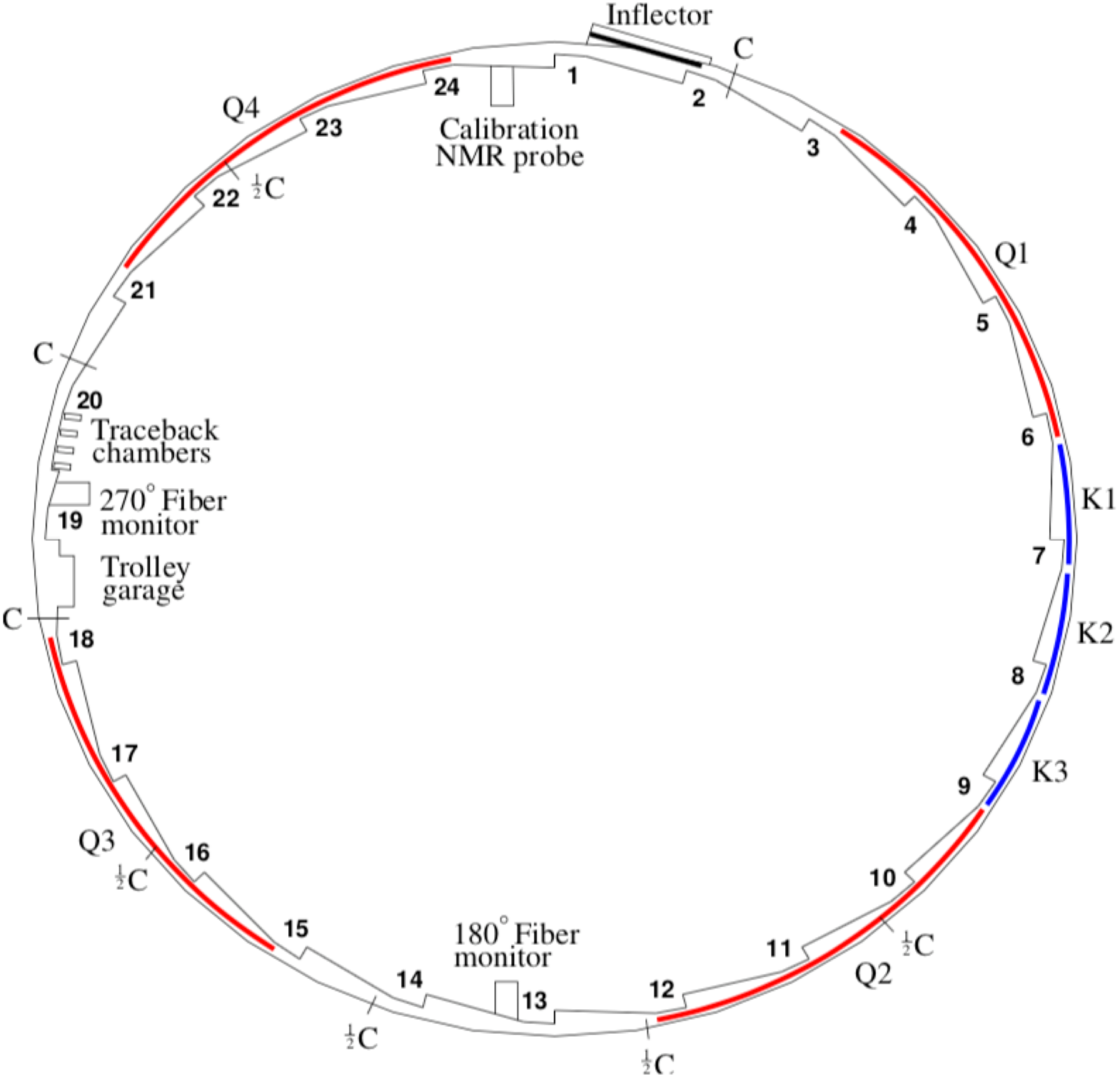
- Aim to maintain lost muon fraction <10^{-4} per muon lifetime
- Measurements: Calorimeters read MIP (Minimum Ionizing Particles) deposited energy (~170MeV) and apply time cuts to detect lost muons.
- Cuts and background-subtraction measurement technique by S. Ganguly
- Optimal HV at ~28kV
- Need to study spin resonances

Betatron resonances: ~18.6kV resonance (from E-fields)

- Precise measurements of EQS plates alignment are done during shutdown. Hard to do during runs
- ~18.6kV resonance was unexpected while quad scan measurements were performed
- ~18.6kV resonance condition is $3\nu_y = 1$ produced by skew sextupole term fields
  - Non-perfect B-field contributes to skew sextupole term
  - Misaligned EQS plates may introduce electric skew sextupole field



- From the electric side, w_3 =0.17kV ($r_0 = 4.5$cm) at Q1S is enough to recreate observed peak at $\sim$18.6kV.


Inflector
C
1
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4
5
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7
8
9
10
11
12
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15
16
17
18
19
20
21
22
23
24
Calibration NMR probe
Q1
Q2
Q3
Q4
K1
K2
K3
½C
C
Traceback chambers
270° Fiber monitor
Trolley garage
180° Fiber monitor

Betatron resonances: **~18.6kV resonance (from E-fields)**

- Considering all the other multipoles associated with w_3 =0.17kV ($r_0 = 4.5$cm) at Q1S from Table 6, NIM-A quad paper (Y. Semertzidis et al., 503(3):476, 2003):

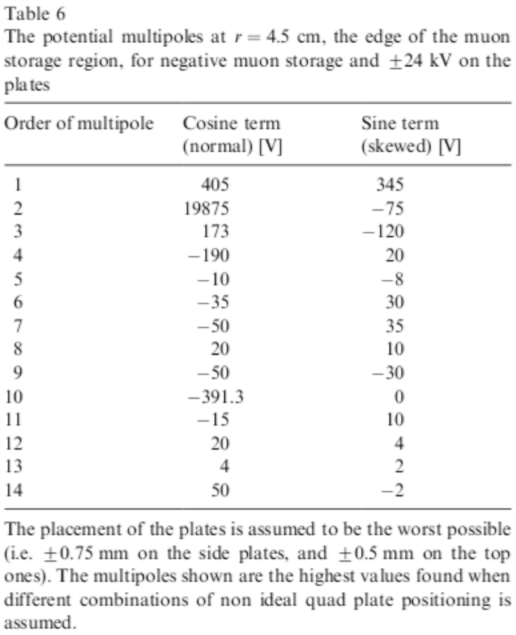
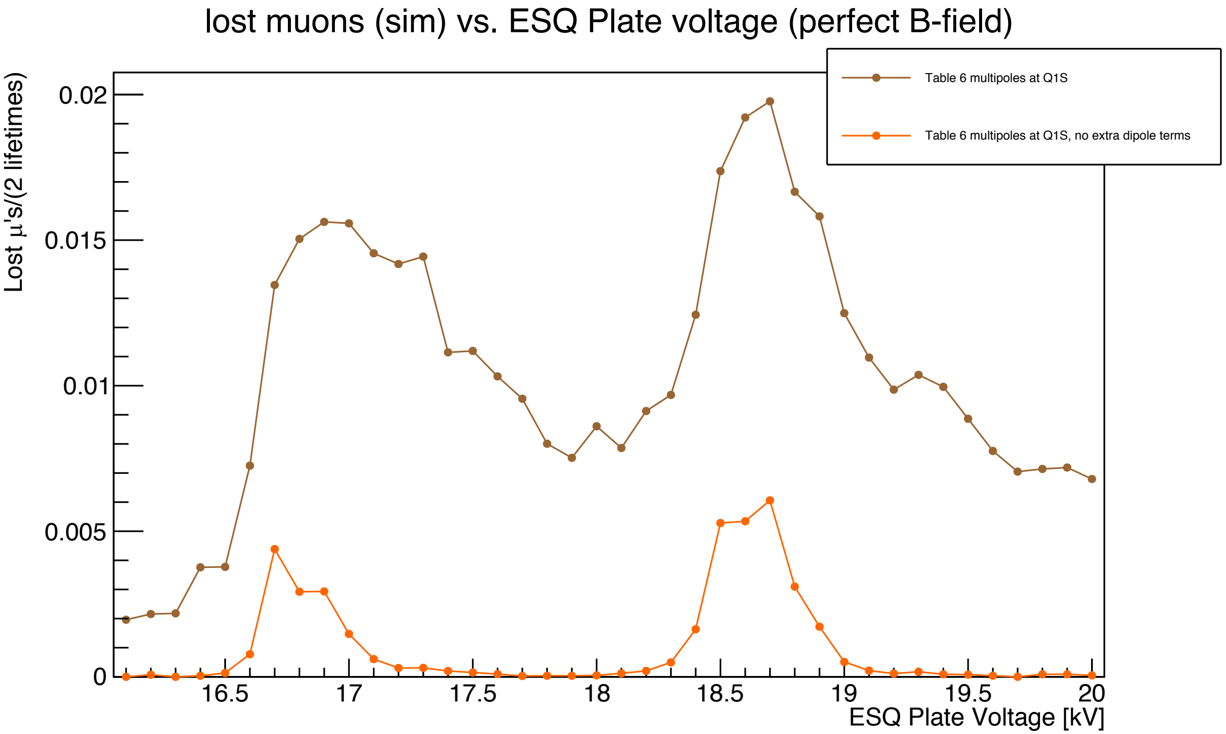


Table 6
The potential multipoles at $r = 4.5$ cm, the edge of the muon storage region, for negative muon storage and ±24 kV on the plates

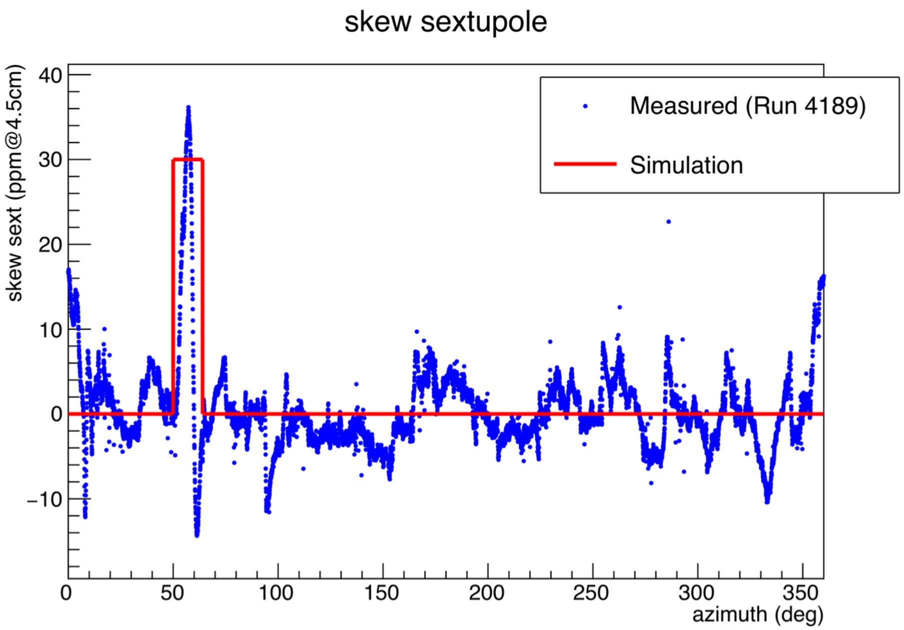
Order of multipole	Cosine term (normal) [V]	Sine term (skewed) [V]
1	405	345
2	19875	−75
3	173	−120
4	−190	20
5	−10	−8
6	−35	30
7	−50	35
8	20	10
9	−50	−30
10	−391.3	0
11	−15	10
12	20	4
13	4	2
14	50	−2

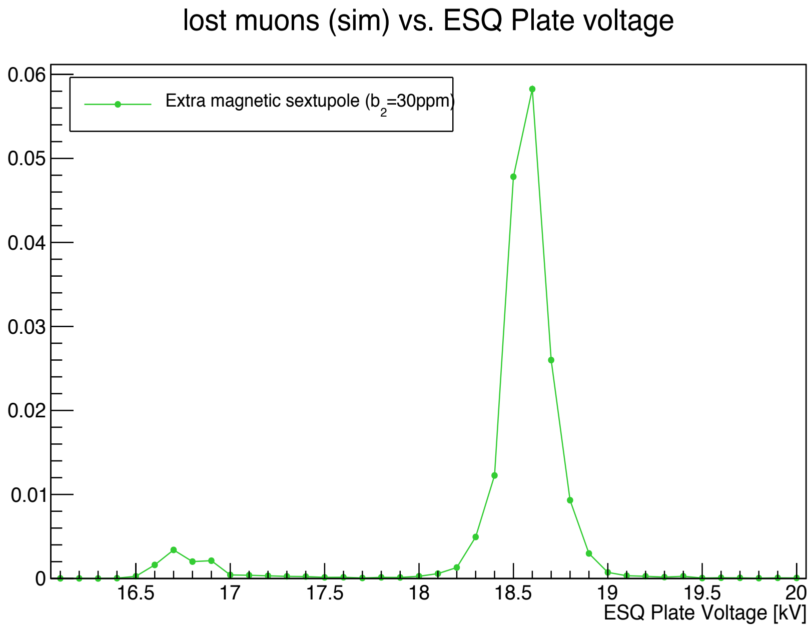
The placement of the plates is assumed to be the worst possible (i.e. ±0.75 mm on the side plates, and ±0.5 mm on the top ones). The multipoles shown are the highest values found when different combinations of non ideal quad plate positioning is assumed.



- The extra dipole terms from Table 6 introduce muon losses all around, which makes the expected peaks at ~ 17kV and ~ 18.7kV from measurements to lose their observed sharpness.

Betatron resonances: ~18.6kV resonance (from B-fields)


skew sextupole

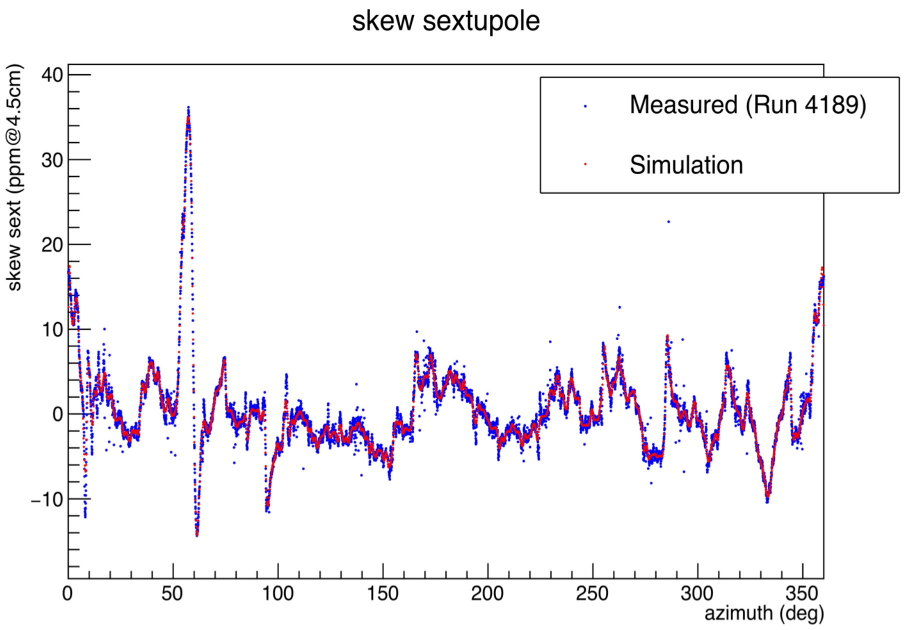

lost muons (sim) vs. ESQ Plate voltage

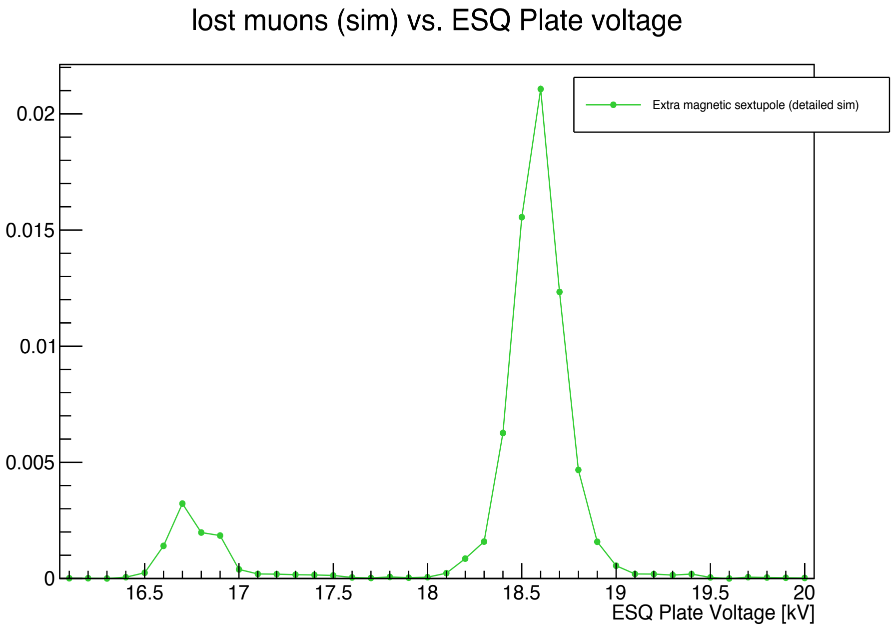
- Based on recent measurements of the magnetic skew sextupole* (blue dots), we add to the ideal g – 2 ring simulation model in COSY a skew sextupole $b_2 = 30$ppm as shown by the red line above to mimic the tallest peak at ~56°

- Simulation results show the high sensitivity of the $3\nu_y = 1$ resonance to magnetic sextupole fields.

* J. Grange et al. *Field results trolley run 5/5 evening*. Elog-1161, May 2018.

Betatron resonances: ~18.6kV resonance (from B-fields)


skew sextupole

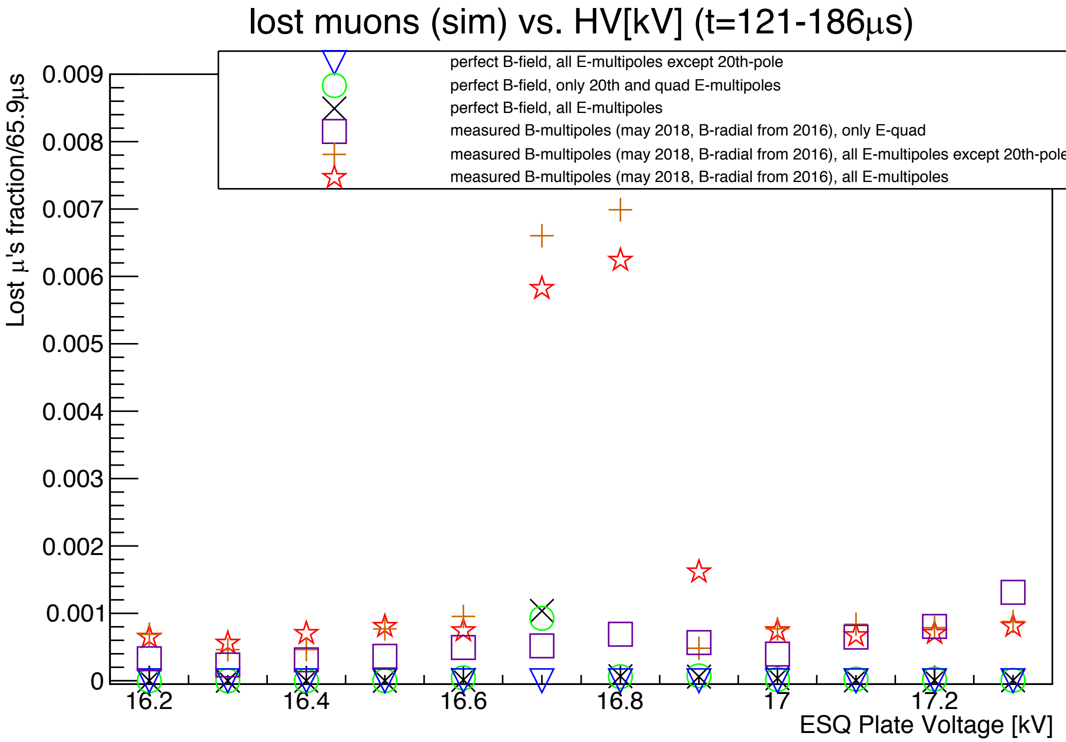

lost muons (sim) vs. ESQ Plate voltage

- To better evaluate measurements from the simulation side, a more detailed representation of the measured term was implemented (see red dots above)
- Simulation results for this case recreate observations
- Other factors such as damping due to imperfect vacuum and different time scales between simulations (128μs) and measurements (700μs) may be the main sources of discrepancy.

Betatron resonances: **~16.8kV resonance**

- ~16.8kV resonance was not expected when quad scan measurements became available. EQS misalignments or B-field errors thought to be the culprits

- Effect of interplay between B-field errors and EQS multipoles on muon losses not explored for previous $g-2$ experiments

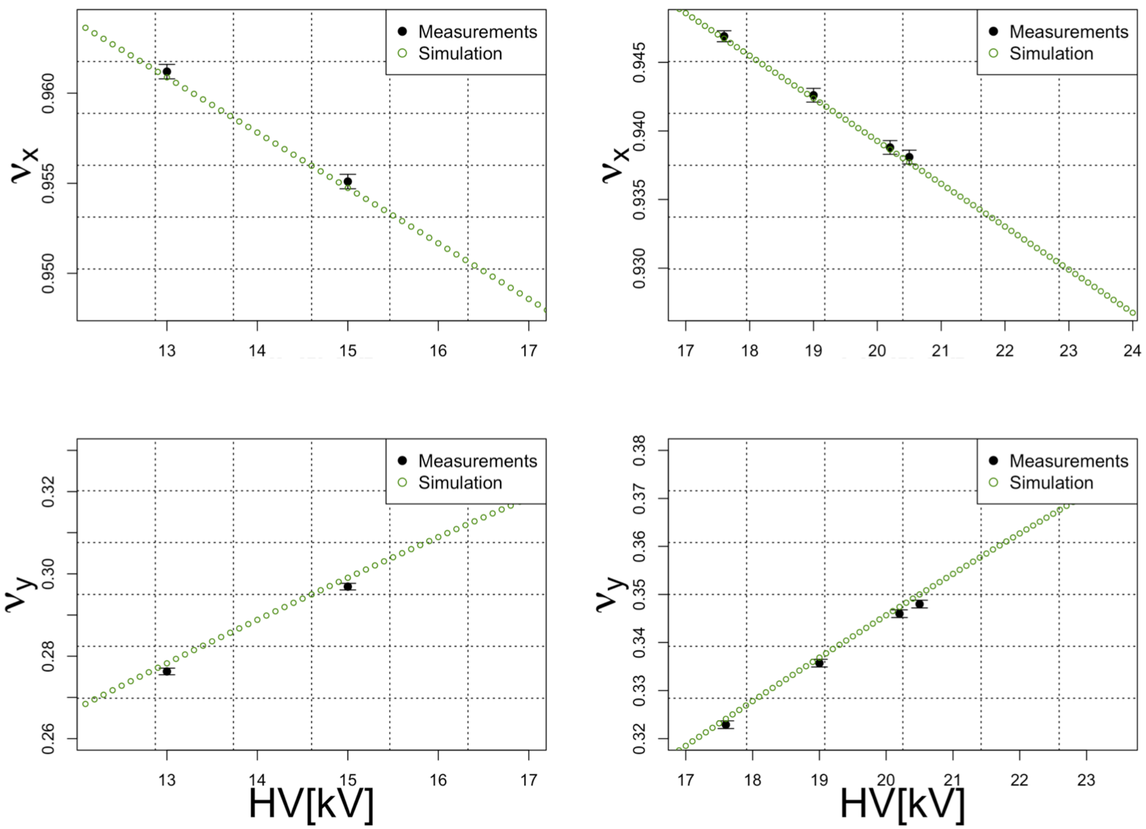


$lv_x + mv_y = N$			
HV (KV)	l	m	N
16.75	1	-3	0

from EQS 20th-pole and magnetic normal octupole

- Lower order magnetic nonlinearities can boost higher order electric resonances. Same mechanism may happen for spin resonances.

Tunes and f_{CBO}: Tunes measurements vs. simulation



- Tunes relate to High-Voltage on ESQ plates

$$\nu_x \approx \sqrt{1 - \frac{13}{30}\frac{R}{vB_0}\frac{\boldsymbol{\partial E_y}}{\boldsymbol{\partial y}}}$$

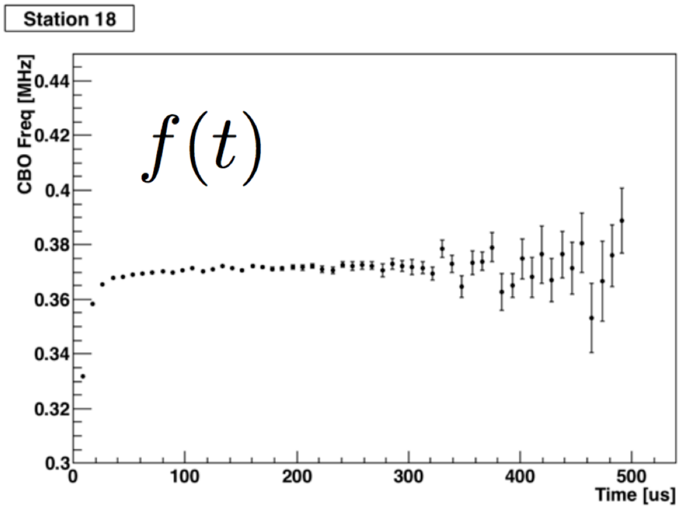
$$\nu_y \approx \sqrt{1 - \nu_x^2}$$

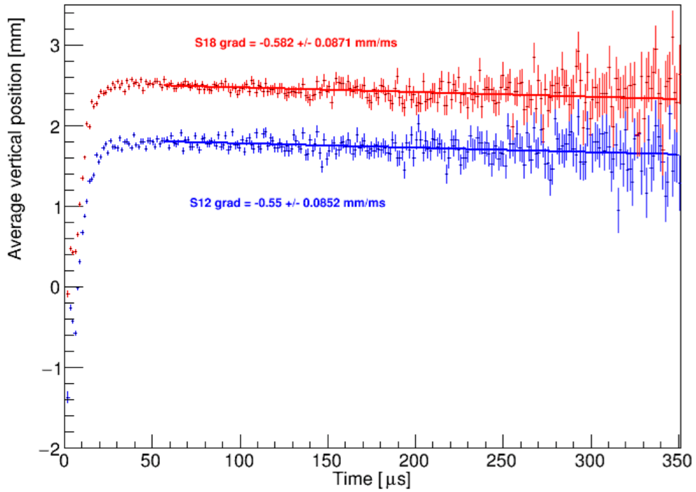
- f_{CBO} results from muon bunch cyclotron frequency and tune

$$fCBO_{0x} = f_{c0}(1 - \nu_x),\ fCBO_{0y} = f_{c0}\nu_y$$

- Comparison of data with detailed simulations of muon beam properties allow to identify systematic error sources (EQS plates misalignments, HV's, and more)
- Horizontal tunes in agreement with measurements. Vertical tunes discrepancies may come from amplitude/momentum tune shifts or EQS damaged resistors
- Measurements done by A. Chapelain.

Tunes and f_{CBO}: Time-evolving f_{CBO}

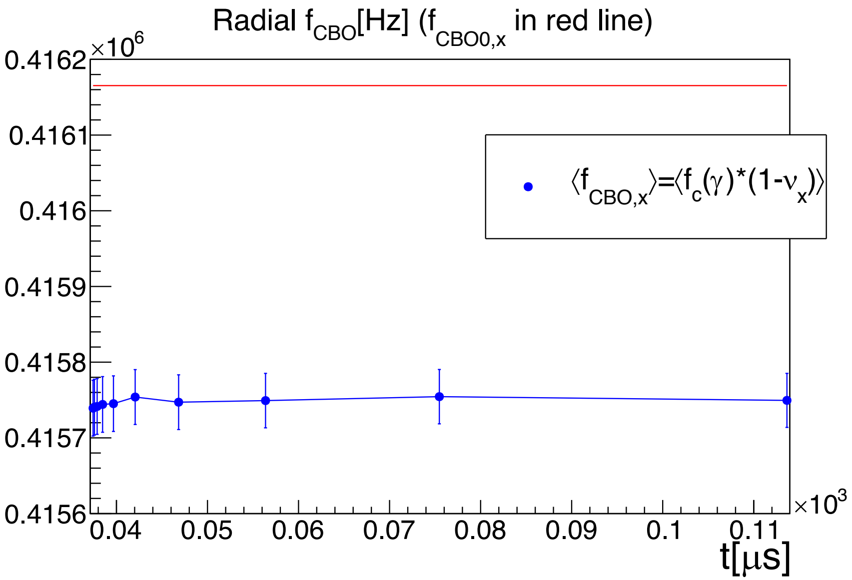




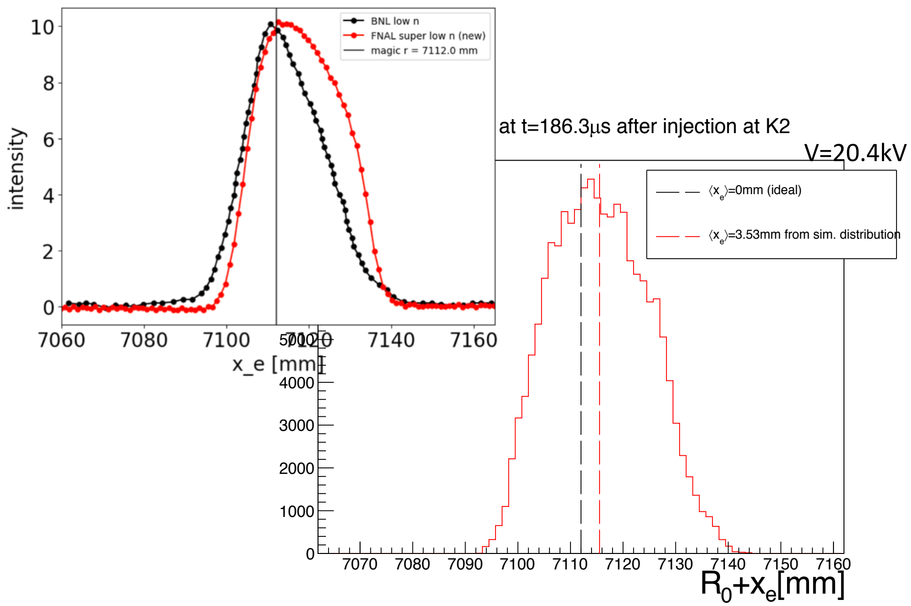
- From data fitting, horizontal f_{CBO} increases by about 1.6% when data set < 50µs was removed
- Average vertical position observed to fall ~0.5mm/ms
- Tracking simulations with nonlinear detuning considered did not show varying f_{CBO} nor average vertical position at late times
- Model of damaged EQS resistors recreates observations best
- Measurements done by Tracker team

Source: All Displayed
Trace Title: resistor17-top...
Data Format
Format: ASCII
SubFormat: Time & Amplitude
Delimiter: Comma
Auto Save (save on each new trigger)
Save Files In Directory: C:\Documents and Settings\g2muon
Next File Will Be Saved To: XXresistor17-top-plate-10kv00004.txt
Disk Space
Size : 20.8 GB
Free : 9.09 GB

Tunes and f_{CBO}: Time-evolving f_{CBO} (nonlinear detuning)



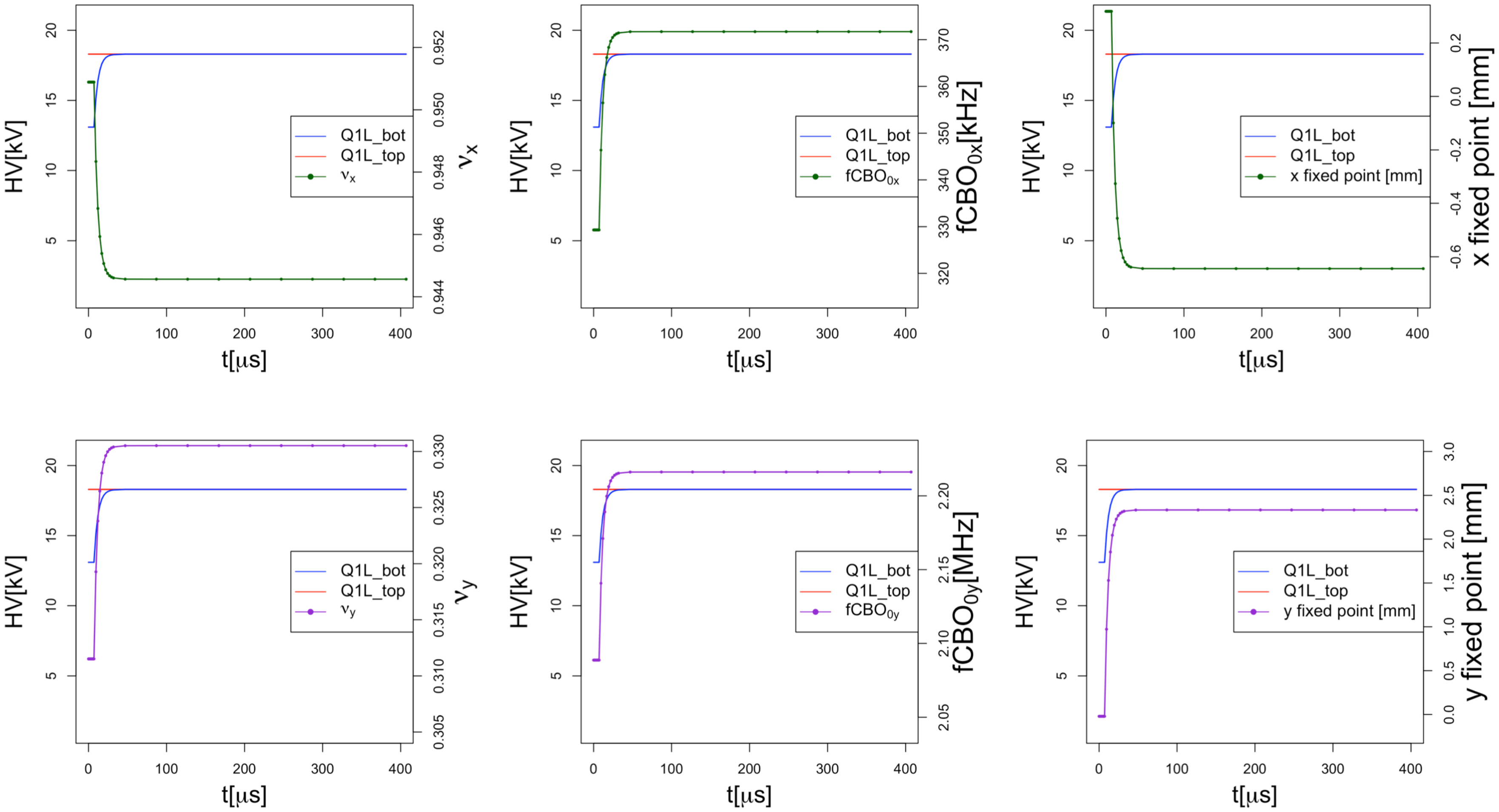
x_e distribution



- Amplitude and time dependent tune shifts from tune maps

- This approach does not yield vertical beam average position observed either

Tunes and f_{CBO}: Time-evolving f_{CBO} (nominal)

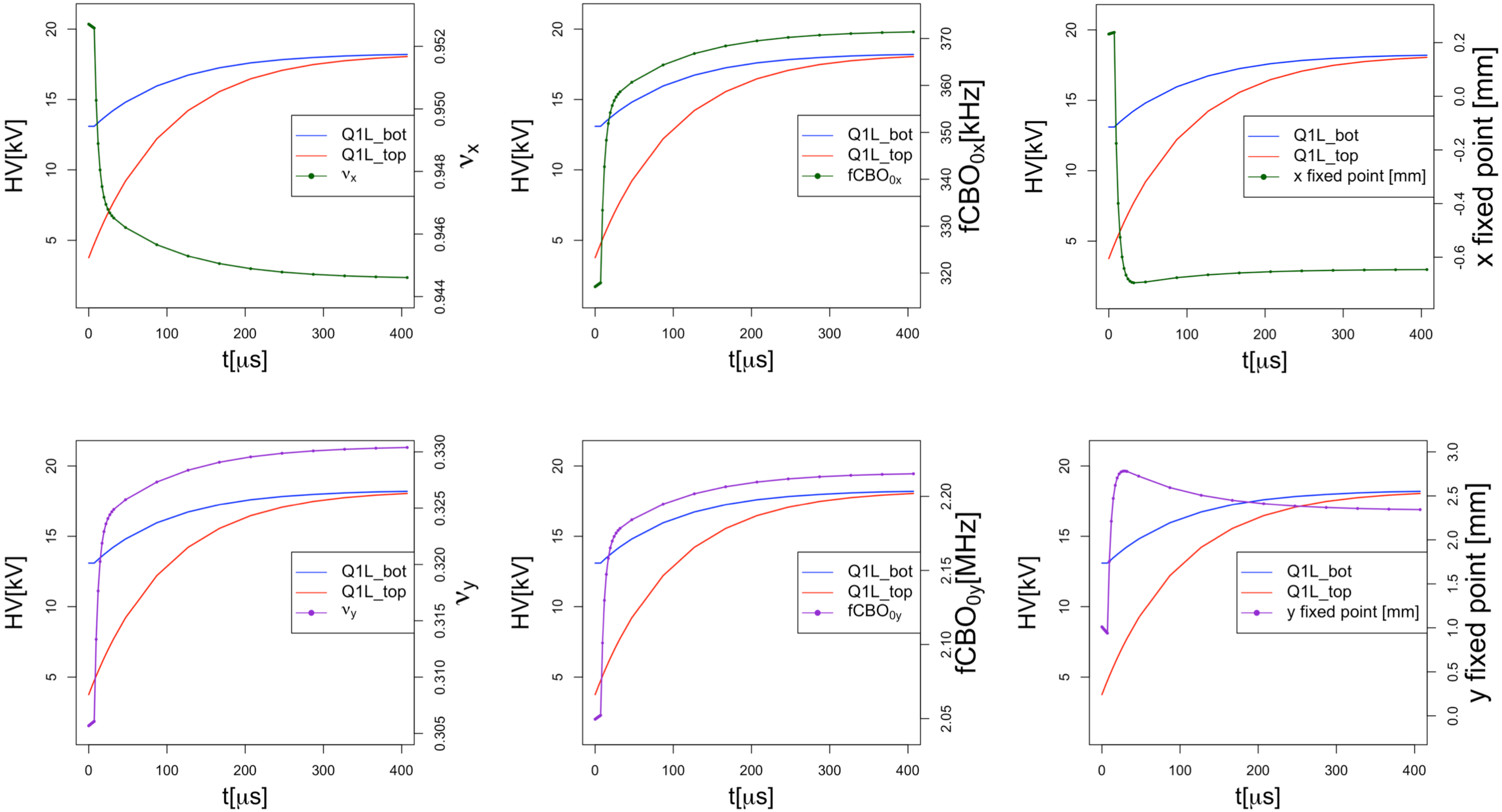


$t = 0$ when beam is injected

$fCBO_{0x} = f_{c0}(1-\nu_x)$, $fCBO_{0y} = f_{c0}\nu_y$ where $f_{c0} = \frac{1}{2\pi}\frac{eB_0}{\gamma_0 m}$

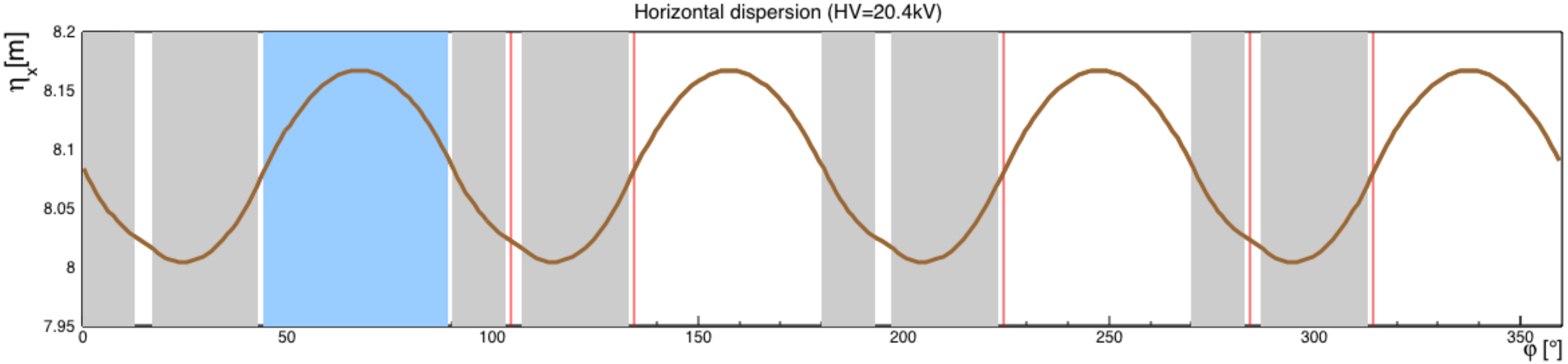
"fixed point" below corresponds to the x, y coordinates of the closed orbit at K2 center

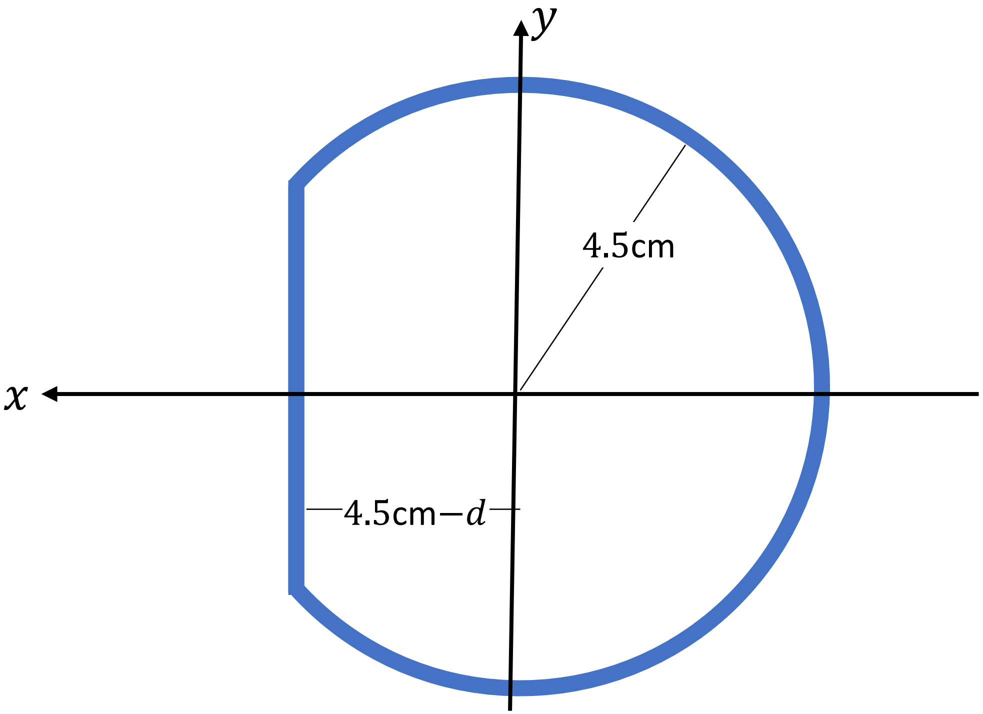
Tunes and f_{CBO}: Time-evolving f_{CBO} (damaged resistors)



- Tunes calculated from transport maps around closed orbits for different HV settings
- Fixed points from transport maps around ideal orbit
- HCBO and VCBO increase ~1.26% and ~0.61%, respectively, from 120μs to 400μs
- Vertical fixed point does fall after 50μs, but nonlinearly

Momentum distribution and collimation



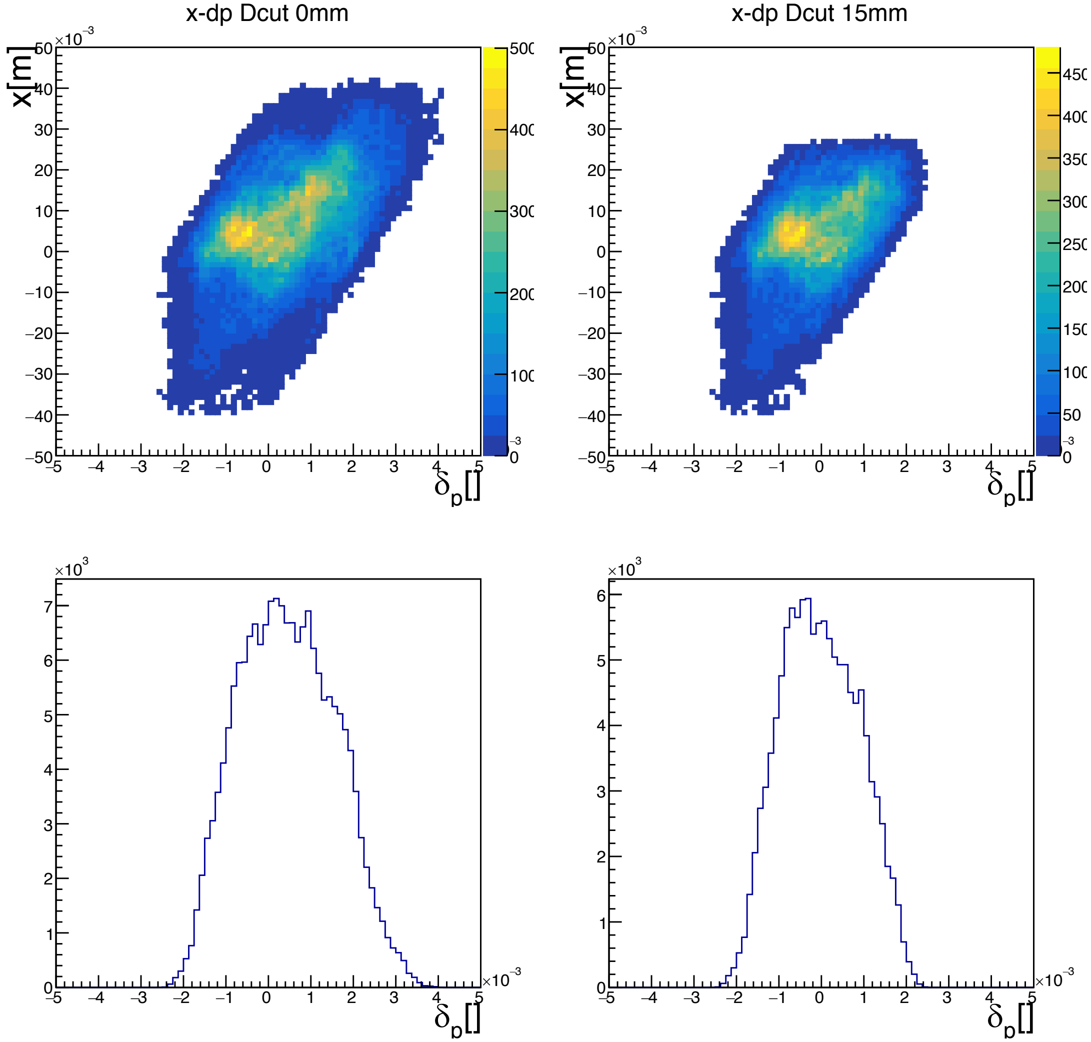


- Increment of **E-field correction** from beam momentum offset $\left\langle\frac{\Delta p}{p_0}\right\rangle \neq 0$ due to inj. kicker imperfect pulse. Special beam collimation could mitigate this issue

$$C_E = -2n(1-n)\beta^2\frac{\langle x_e^2\rangle}{\rho_0^2}$$

- Studies of D-shape collimators for several d's

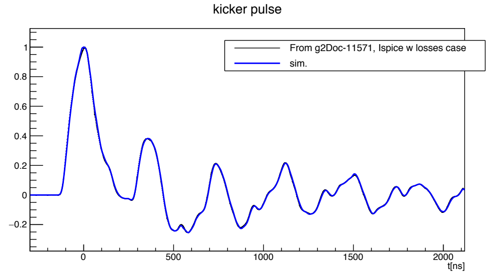
Momentum distribution and collimation



- Due to dispersion, D-shape collimators remove higher-momentum muons

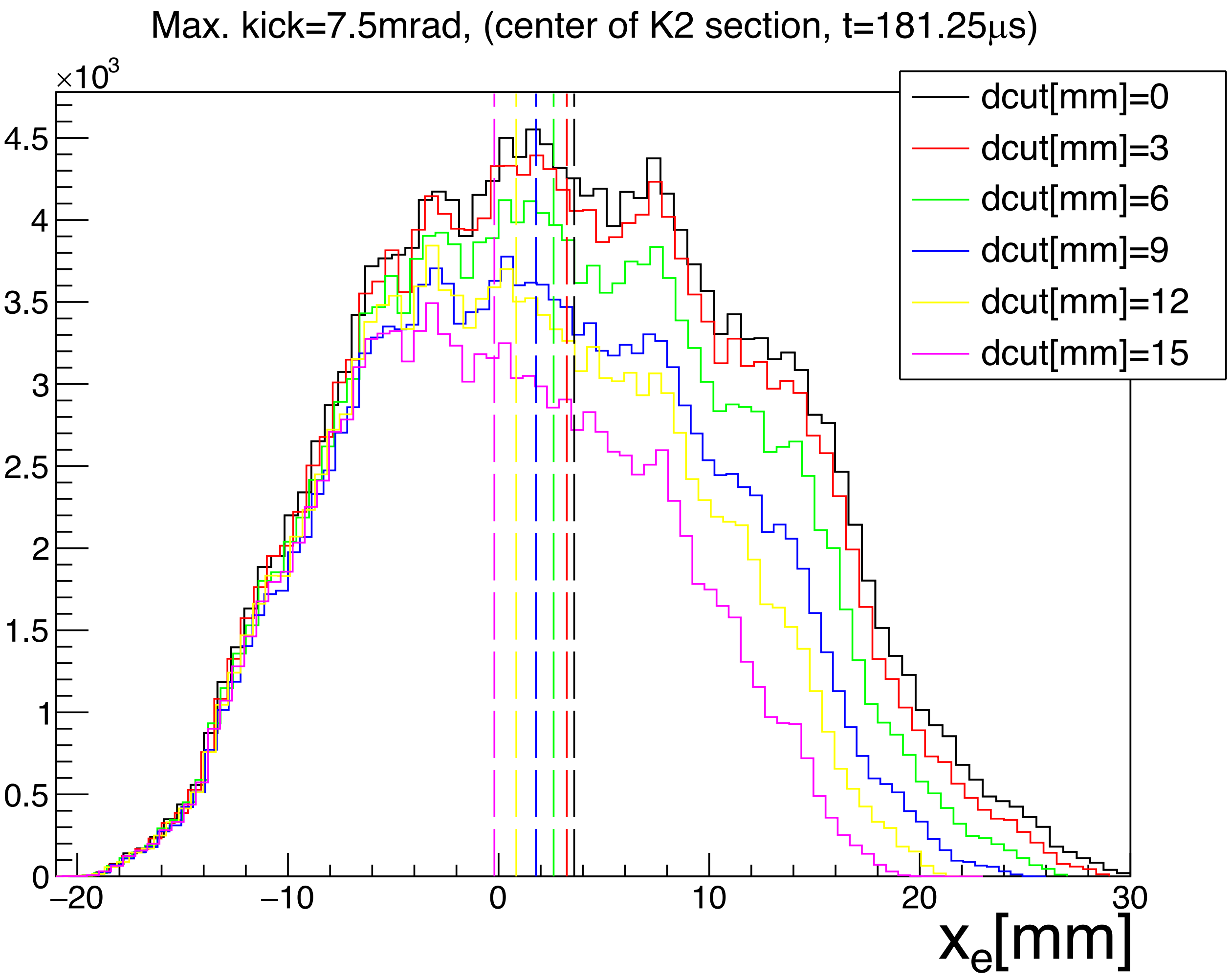
- More muons removed as d increases.

- Low-momentum muons barely affected by D-shape



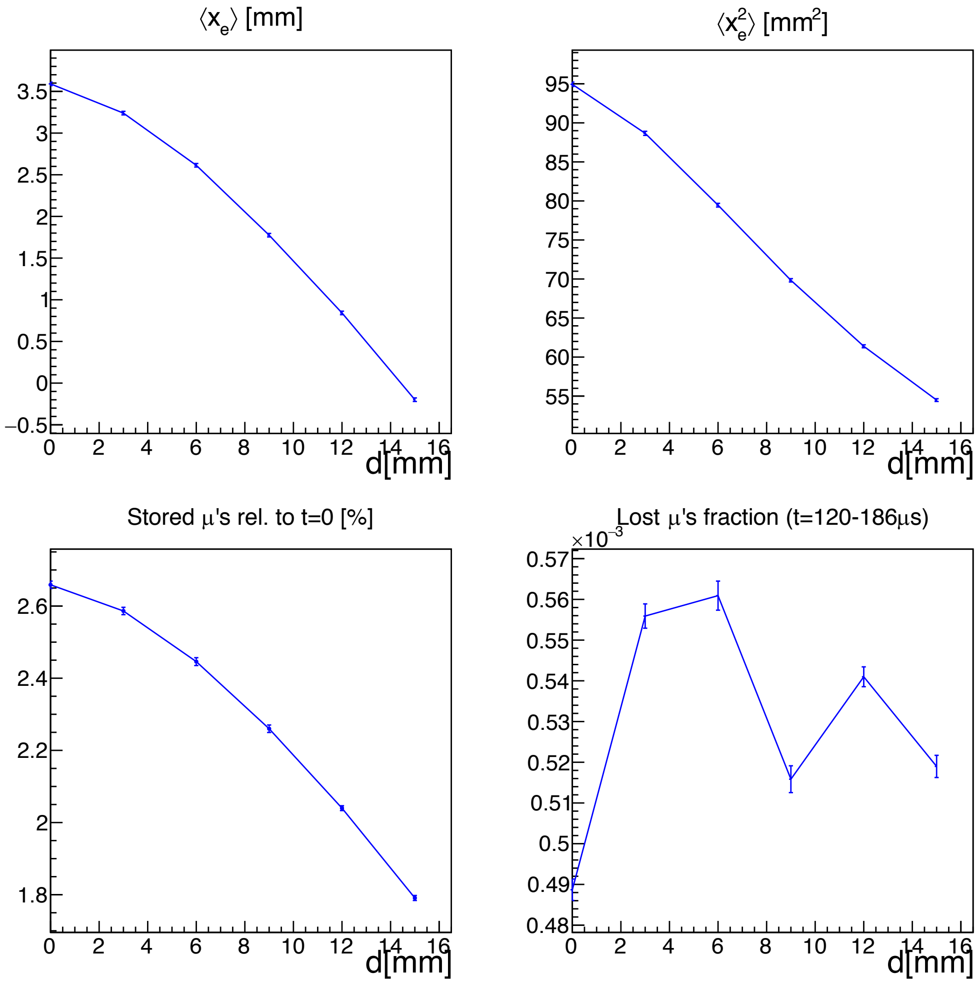
*Kick pulse in simulations, where $\theta_{max} = 7.5\text{mrad}$

Momentum distribution and collimation



- $\langle x_e^2 \rangle$ is reduced with D-shaped collimators at the expense of storing less muons

Momentum distribution and collimation



- E.g. for d=12mm, stored muons fraction reduced from 2.6% to 2%, but $\langle x_e^2 \rangle$ reduced from 95 to 60mm^2

- Lost muons fractions not much affected by d-cut in collimators

- For higher HV setting, η_x increases ---> D-shape collimation more effective.

Conclusions

- Wide range of HV simulations show configurations to suppress betatron resonance effects and maximize stored muons fraction
  - Betatron resonance simulations and measurements reveal interplay of low order magnetic nonlinearities with electric resonances
  - Such mechanism could drive spin resonances. Will prepare spin simulations, normal form theory frame and do measurements
  - Driving resonance mechanisms possible to understand with COSY simulation

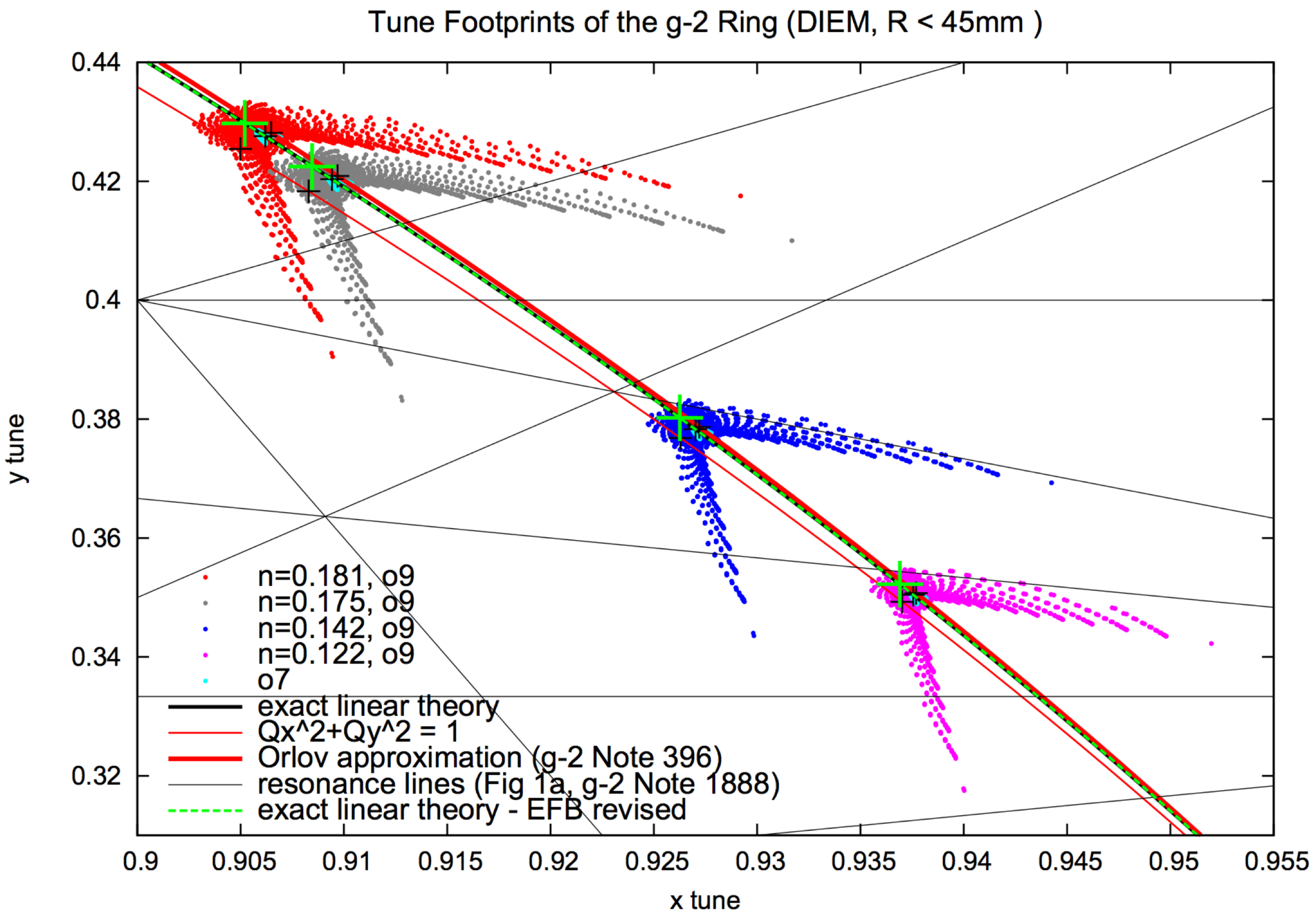
- Simulations with damaged EQS resistors describe observed behavior of f_{CBO} and beam average positions
  - Add more details of damaged resistors recently measured
  - Do tracking and study spin, lost and stored muons rates to assess impact on commissioning runs

- Special collimation to reduce systematics of E-field correction
  - Reduction of stored muons limits method

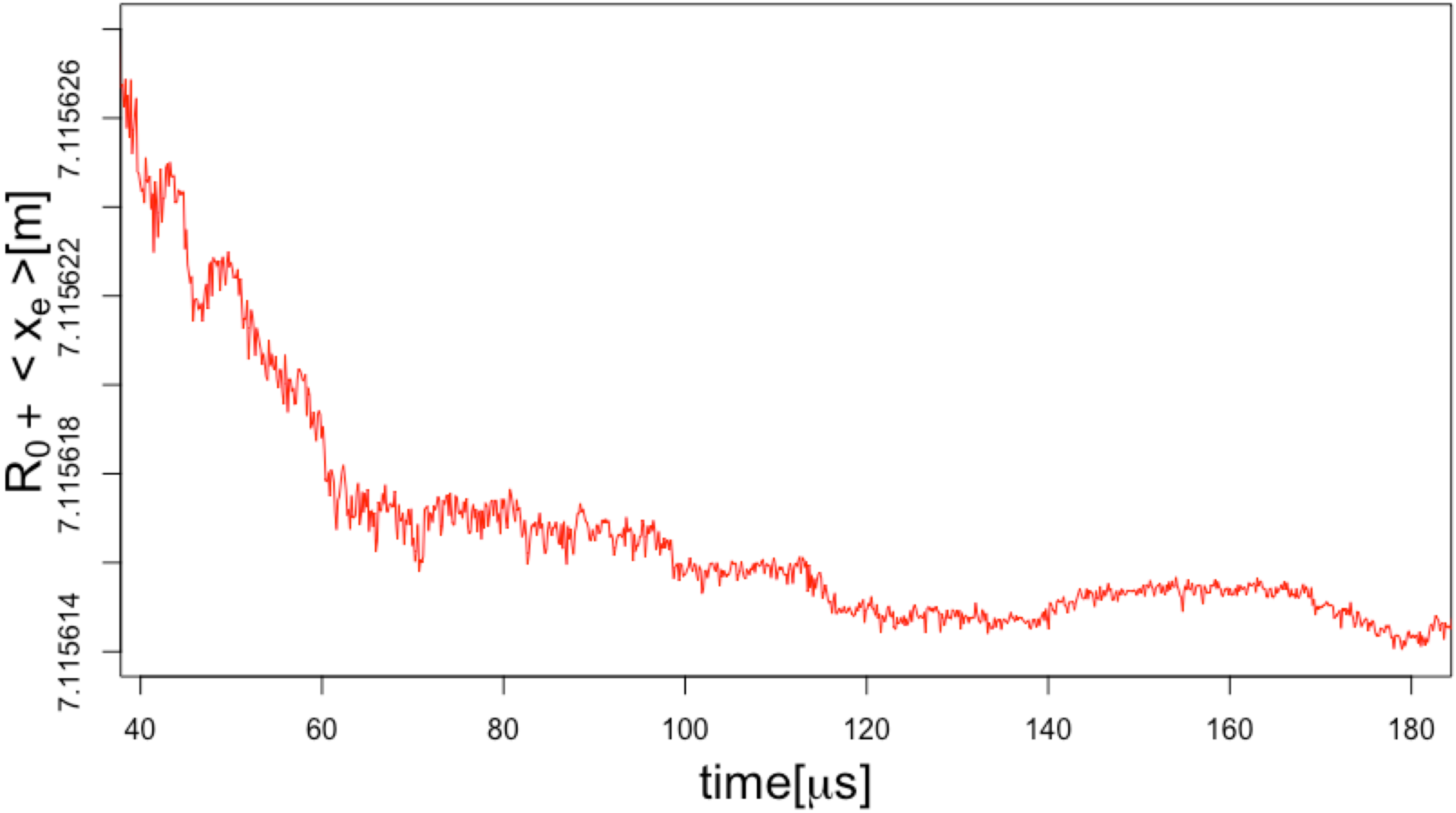
- Next: add inflector to connect beam delivery and storage ring simulations, dispersion measurements along end-to-end beamlines, RF scraping, $g-2$ phase tracking.

BACKUP

Amplitude-dependent tune shifts



V=20.4kV


$R_0 + < x_e > [m]$
7.115614
7.115618
7.115622
7.115626
40
60
80
100
120
140
160
180
time[μs]

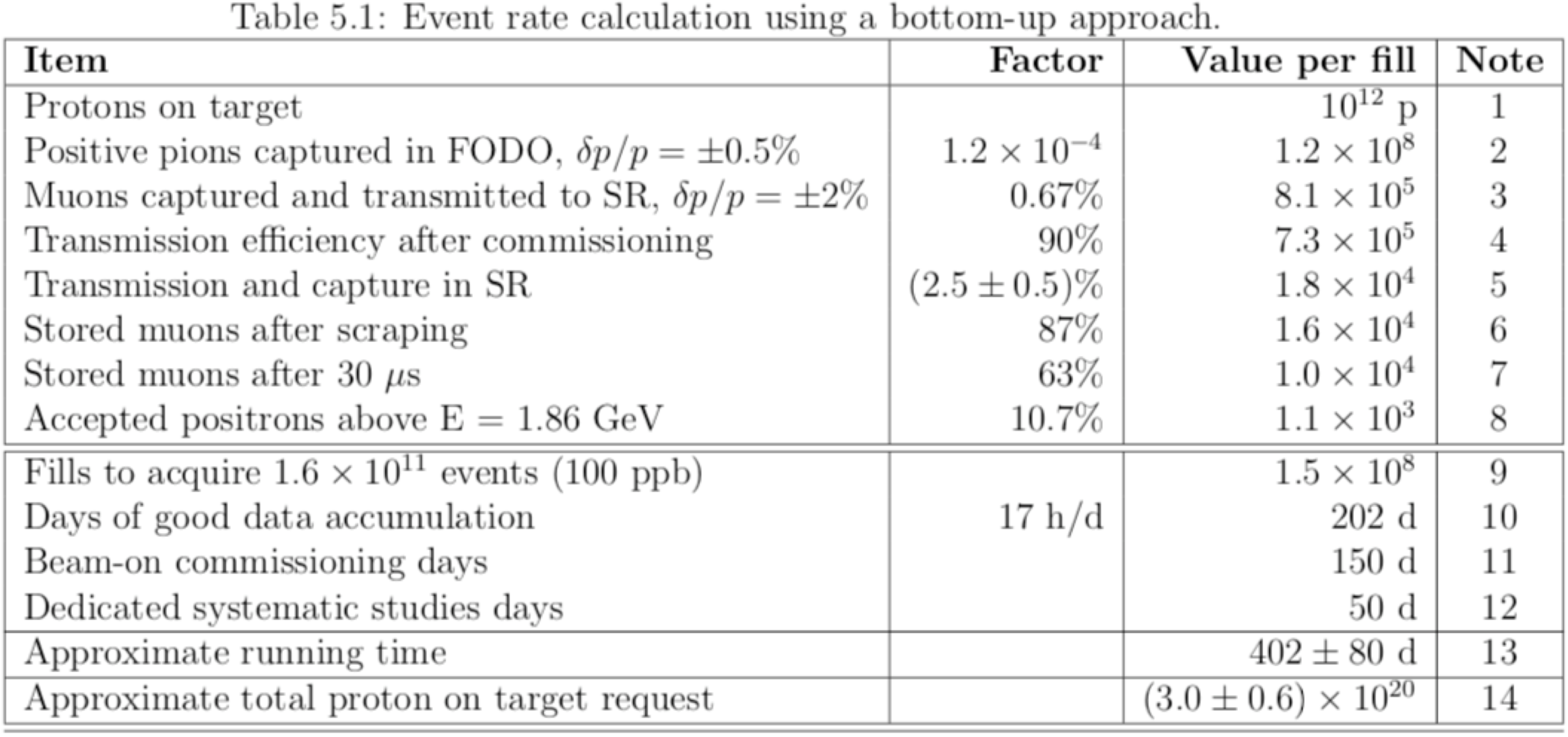


Table 5.1: Event rate calculation using a bottom-up approach.

Item	**Factor**	**Value per fill**	**Note**
Protons on target		10^{12} p	1
Positive pions captured in FODO, $\delta p/p = \pm 0.5\%$	1.2×10^{-4}	1.2×10^{8}	2
Muons captured and transmitted to SR, $\delta p/p = \pm 2\%$	0.67%	8.1×10^{5}	3
Transmission efficiency after commissioning	90%	7.3×10^{5}	4
Transmission and capture in SR	$(2.5 \pm 0.5)\%$	1.8×10^{4}	5
Stored muons after scraping	87%	1.6×10^{4}	6
Stored muons after 30 μs	63%	1.0×10^{4}	7
Accepted positrons above E = 1.86 GeV	10.7%	1.1×10^{3}	8
Fills to acquire 1.6×10^{11} events (100 ppb)		1.5×10^{8}	9
Days of good data accumulation	17 h/d	202 d	10
Beam-on commissioning days		150 d	11
Dedicated systematic studies days		50 d	12
Approximate running time		402 ± 80 d	13
Approximate total proton on target request		$(3.0 \pm 0.6) \times 10^{20}$	14

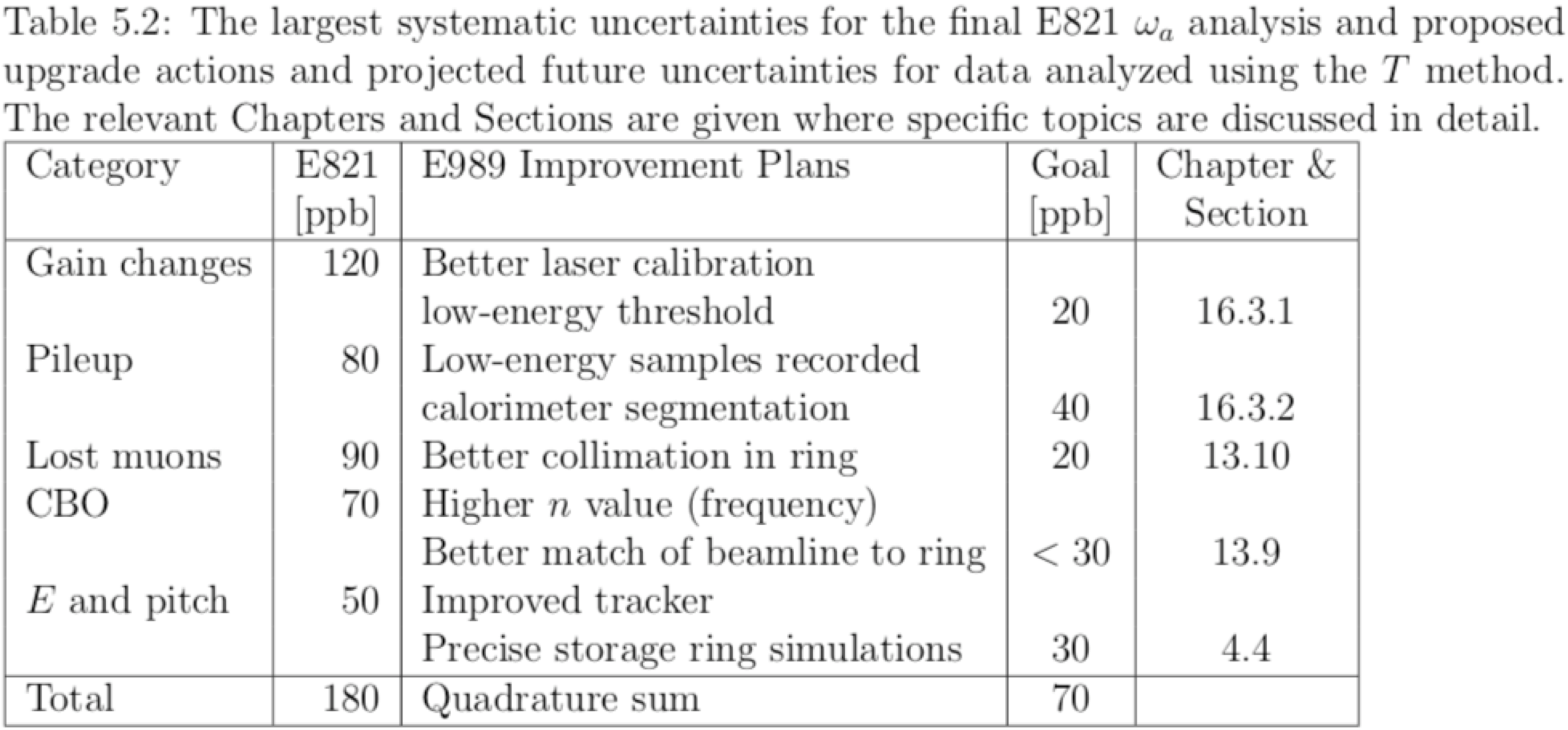


Table 5.2: The largest systematic uncertainties for the final E821 ω_a analysis and proposed upgrade actions and projected future uncertainties for data analyzed using the T method. The relevant Chapters and Sections are given where specific topics are discussed in detail.

Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]	Chapter & Section
Gain changes	120	Better laser calibration low-energy threshold	20	16.3.1
Pileup	80	Low-energy samples recorded calorimeter segmentation	40	16.3.2
Lost muons	90	Better collimation in ring	20	13.10
CBO	70	Higher n value (frequency) Better match of beamline to ring	< 30	13.9
E and pitch	50	Improved tracker Precise storage ring simulations	30	4.4
Total	180	Quadrature sum	70	